Black Hole Throats and Large Quantum Fluctuations

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Based on work with J. de Boer, I. Messamah, and D. Van den Bleeken

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Motivation

Puzzles from Black Hole Physics

- Field Theory reasoning fails to explain qualitative features of QG:
  1. Information Loss
  2. Holography
  3. Black hole entropy

- How does standard effective field theory (EFT) break-down in QG?

Potential New Features in Quantum Gravity?

- Failure of locality (at horizon scales)?
- Large scale quantum effects?
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Approach

Deep Throats in String Theory

- Study families of black hole *like* solutions in string theory.
- Soln’s support throats of arbitrary depth mimicking horizons of SUSY BHs.
- Throats end in smooth cap and have no large curvature ⇒ EFT implies quantum corrections negligible.

Quantization

- Geometries are backreaction of system of D-branes.
- D-branes at weak coupling described by SUSY QM ⇒ tractable!
- Phase space at strong and weak coupling related by SUSY.
- After quantization throat destroyed by *macroscopic* quantum fluctuations!

Based On

- *A bound on the entropy of supergravity?* [arXiv:0906.0011]
- *Quantizing $\mathcal{N} = 2$ Multicenter Solutions.* [arXiv:0807.4556]
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D-branes

Setup

- Wrap branes on cycles of 6-d compactification manifold (Calabi-Yau).
- Branes sit at points $\vec{x}_a \in \mathbb{R}^3$.
- “Integrate out” internal degrees of freedom.

D-brane Theory (at weak-coupling)

- $\mathcal{N} = 4, d = 1$ theory (SUSY QM).
- Coords become world-line fields, $\vec{x}_a(\tau)$, encoding brane dynamics.
- Coupling, $g_s$, is free parameter from spacetime point of view.
- SUSY ground states: zeros of potential from brane interactions.

Figure: Positions $\vec{x}_a$ minimize potential.
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![Diagram of D-branes and compactification manifold](image)

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D-branes
Quantum Mechanics

- Branes have electron-monopole like interactions.
- First order part of Lagrangian fixed by SUSY
  \[ L^{(1)} = \sum_a (-U_a(x)D_a + \vec{A}_a(x) \cdot \dot{\vec{x}}_a) + \text{fermions} \]
- \((x^i_a(\tau), D_a(\tau))\) are bosonic world-line fields.
- \(U(x)_a\) and \(A^i_a(x)\) functionals fixed by SUSY \(\Rightarrow g_s\) independent.
- Protected terms fix SUSY phase space and symplectic form.

Commutators (from symplectic form)
\[ [x^i_{ab}, x^j_{ab}] \sim \epsilon^{ijk} x^k_{ab} \]

Note: \(\bar{x}_{ab} := \bar{x}_a - \bar{x}_b\) self-conjugate (consistent with electron-monopole interaction).
Branes have electron-monopole like interactions.

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As \( g_s \) tuned up D-branes couple to gravity and backreact.

At strong coupling supergravity is a better effective description than SQM.

At \( g_s \sim 0 \) brane lives on \( \mathbb{R}^{1,3} \).

For \( g_s \gg 1 \) branes warp spacetime \( \Rightarrow \) generate geometry.
From D-branes to Supergravity

Solutions

- Branes backreact giving SUSY solutions to 4d, $\mathcal{N} = 2$ sugra.
- Also lift to soln of 5d, $\mathcal{N} = 1$ sugra (but will not discuss).

4-d fields

$$ds^2 = -\frac{1}{\Sigma(x)} (dt + \omega(x))^2 + \Sigma(x) \, dx^i dx^i,$$

$$t^A = B^A + i J^A, \quad A^A = \ldots$$

- Original brane coords $\vec{x}_a$ parameterize soln’s via dependence of $\Sigma(x)$ and $\omega(x)$ on $H(x)$.

Solutions specified in terms of:

$$H(x) = \sum_{a=1}^{N} \frac{\Gamma_a}{|x - x_a|} + h$$
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- Original brane coords $\tilde{x}_a$ parameterize soln’s via dependence of $\Sigma(x)$ and $\omega(x)$ on $H(x)$.

Solutions specified in terms of:

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\]
Angular Momentum of Solutions

- \( \omega(x) \) in metric implies solutions are **stationary** but not **static**.
- Angular momentum carried between *each pair* of centers \( \vec{J}_{ab} \).

### Intrinsic Angular Momentum

\[
\vec{J} = \sum_{a < b} \vec{J}_{ab} = \frac{1}{2} \sum_{a < b} \frac{\langle \Gamma_a, \Gamma_b \rangle \vec{x}_{ab}}{r_{ab}}.
\]

- Asymptotic value of \( \omega(x) \).
- \( \langle \Gamma_a, \Gamma_b \rangle \) electric-magnetic pairing \( \Rightarrow \) crossed EM fields.

- Brane commutator \([x^i_{ab}, x^j_{ab}] \sim \epsilon^{ijk} x^k_{ab}\) corresponds to quantizing \( \vec{J}_{ab} \).
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SUSY Phase Space

BPS Constraint Equations

\[ r_{ab} = |\vec{x}_a - \vec{x}_b| \text{ must satisfy:} \]

\[ \sum_{a, a \neq b} \frac{\langle \Gamma_a, \Gamma_b \rangle}{r_{ab}} = \langle h, \Gamma_a \rangle \]

- Constraint eqns minimize potential from gravity and scalars.
- For \( N \) centers solution space to above \( 2N - 2 \) dim.
- Dimension even \( \Rightarrow \) good because sol space is phase space!
- This is because \( \{\vec{x}_a\} \) parameterizing soln’s are self-conjugate.

Weak-Strong Equivalence

Constraint eqns exactly match min of brane \( (g_s \sim 0) \) potential!
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Scaling Solutions

Special Family of Solutions

Consider a family of solutions parameterized by $\lambda$ such that $x_{ab} \sim \lambda$.

Figure: D-brane QM Regime ($g_s \sim 0$)
- Brane wavefn’s have little overlap.
- Approximately semi-classical.

Figure: Supergravity Regime ($g_s \gg 1$)
- Smooth multicentered sugra solution.
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Scaling Solutions

\( \lambda \ll 1 \)

As \( \lambda \to 0 \) centers meld to long **but smooth** throat ending in a cap.

- Centers very close together.
- Less phase space with \( |\vec{x}_{ab}| \sim \lambda \).
- Non-commutative nature of coords becomes relevant.
- Throat depth scales inversely to \( \lambda \).
- Solutions smooth for all \( x_{ab} > 0 \).
Scaling Solutions

\[ \lambda \sim 0 \]

In this regime throat depth very \( \lambda \)-sensitive.

- Outside semi-classical regime.
- \( \sim 1 \) unit of phase space in this region.
- Quantum fluctuations large.
  \[ \langle x_{ab} \rangle \sim \mathcal{O}(\hbar), \quad |\delta \vec{x}_{ab}| \sim |\vec{x}_{ab}| \]

- Metric scale \( g_{ij}(x) \sim \lambda^{-2} \) as \( x_{ab} \sim \lambda \).
- Geodesic distance between centers remains finite and large.
Large Quantum Fluctuations

\( \lambda \sim 0 \) at weak-coupling \((g_s \sim 0)\)

- Brane system very quantum when \( \lambda \sim 0 \) because

\[
[x^i, x^j] \sim \epsilon^{ijk} x^k
\]

- Define \( \lambda_{\text{crit}} \) such that \( |x_{ab}| < \lambda_{\text{crit}} \) occupies less than one unit of phase space.

- States localized near \( x_{ab} \sim \lambda_{\text{crit}} \) cannot be semi-classical:

\[
\sigma_x \sim \sqrt{\langle x_{ab}^2 \rangle - \langle x_{ab} \rangle^2} \sim \langle x_{ab}^i \rangle
\]

\( \lambda \sim 0 \) at strong coupling \((g_s \gg 1)\)

- Throat depth very sensitive to \( x_{ab} \).

- As \( x_{ab} \rightarrow 0 \) throat deeper but geometry stays smooth.

- Geometries corresponding to \( x_{ab} < \lambda_{\text{crit}} \) necessarily quantum!!
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Phase Space

Phase Space Density

Solutions corresponding to $\lambda \sim 0$ occupy very little phase space volume.

Figure: Phase space as a function of $\lambda$ (schematically).
Main Result:

Plank size cells in phase space contain soln’s that differ on macroscopic scales!

What have we learned?

- Supersymmetric non-renormalization implies phase space volume fixed even if spacetime volume increases.
- Black hole like throats can look quantum near horizon (where spacetime is smooth).
- Supersymmetry gives us control and intuition but result is not totally unexpected and should be more general.

Not Unexpected:

- Consistent with Holography: phase space in QG scales with area not volume.
- This is the kind of effect that might help resolve information loss.
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