Top-Antitop Production at Hadron Colliders

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Plan of the Talk

- General Introduction
  - Top Quark at the Tevatron and LHC Perspectives

- Status of the Theoretical calculations
  - Total Cross Section at NLO
  - Analytic Two-Loop QCD Corrections

- Conclusions
Top Quark

With a mass of $m_t = 173.1 \pm 1.3$ GeV, the TOP quark (the up-type quark of the third generation) is the heaviest elementary particle produced so far at colliders.

Because of its mass, top quark is going to play a unique role in understanding the EW symmetry breaking $\Rightarrow$ Heavy-Quark physics crucial at the LHC.

Two production mechanisms

- $pp(\bar{p}) \rightarrow t\bar{t}$

- $pp(\bar{p}) \rightarrow t\bar{b}, tq'(\bar{q}')$, $tW^-$

Top quark does not hadronize, since it decays in about $5 \cdot 10^{-25}$ s (one order of magnitude smaller than the hadronization time) $\Rightarrow$ opportunity to study the quark as single particle

- Spin properties
- Interaction vertices
- Top quark mass

Decay products: almost exclusively $t \rightarrow W^+b \ (|V_{tb}| \gg |V_{td}|, |V_{ts}|)$
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Top Quark @ Tevatron

Events measured at Tevatron

\[ \sigma_{t\bar{t}} \sim 7 \text{pb} \]

\[ p\bar{p} \rightarrow t\bar{t} \rightarrow W^+ b W^- \bar{b} \rightarrow l\nu l\nu b\bar{b} \]

Dilepton \( \sim 10\% \)

Lep+jets \( \sim 44\% \)

All jets \( \sim 46\% \)

2 high-\( p_T \) lept, \( \geq 2 \) jets and ME

NO lept, \( \geq 6 \) jets and low ME

1 isol high-\( p_T \) lept, \( \geq 4 \) jets and ME
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Background Processes

\[ W^+ \text{jets} \]

\[ \text{QCD} \]

\[ \text{Drell-Yan} \]

\[ \text{Di-boson} \]

Top Quark @ Tevatron

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- \( p\bar{p} \rightarrow t\bar{t} \rightarrow W^+bW^-b \rightarrow l\nu l\nu b\bar{b} \)  
  - Dilepton \( \sim 10\% \)

- \( p\bar{p} \rightarrow t\bar{t} \rightarrow W^+bW^-b \rightarrow l\nu q\bar{q}'b\bar{b} \)  
  - Lep+jets \( \sim 44\% \)

- \( p\bar{p} \rightarrow t\bar{t} \rightarrow W^+bW^-b \rightarrow q\bar{q}'q\bar{q}'b\bar{b} \)  
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Background Processes

Reduction of the background: b-tagging crucial
Top Quark @ Tevatron

- **Total Cross Section**
  \[
  \sigma_{t\bar{t}} = \frac{N_{\text{data}} - N_{\text{bkgr}}}{\epsilon L} = 7.0 \pm 0.6 \text{ pb} \quad (m_t = 175 \text{ GeV})
  \]

- **Top-quark Mass**
  \[
  m_t = 173.1 \pm 1.3 \text{ GeV} \quad (0.75\%)
  \]

- **$W$ helicity fractions** $F_i = B(t \rightarrow bW^+(\lambda_W = i = -1, 0, 1)) \quad (F_0 + F_+ + F_- = 1)$
  \[
  \frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta^*} = \frac{3}{4} F_0 \sin^2 \theta^* + \frac{3}{8} F_- (1 - \cos \theta^*)^2 + \frac{3}{8} F_+ (1 + \cos \theta^*)^2
  \]
  \[
  F_0 = 0.66 \pm 0.16 \pm 0.05 \quad F_+ = -0.03 \pm 0.06 \pm 0.03
  \]

- **Spin correlations measured fitting the double distribution**
  \[
  \frac{1}{N} \frac{d^2 N}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4} (1 + \kappa \cos \theta_1 \cos \theta_2)
  \]
  \[
  -0.455 < \kappa < 0.865 \quad (68\% \text{ CL})
  \]

- **Forward-Backward Asymmetry**
  \[
  A_{FB} = (19.3 \pm 6.5(\text{sta}) \pm 2.4(\text{sys}))\%
  \]
LHC Perspectives

Cross Section

- With $100 \text{ pb}^{-1}$ of accumulated data an error of $\Delta \sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 15\%$ is expected (dominated by statistics!)
- After 5 years of data taking an error of $\Delta \sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 5\%$ is expected

Top Mass

- With $1 \text{ fb}^{-1}$ Mass accuracy: $\Delta m_t \sim 1 - 3 \text{ GeV}$

Top Properties

- $W$ helicity fractions and spin correlations with $10 \text{ fb}^{-1} \implies 1-5\%$
- Top-quark charge. With $1 \text{ fb}^{-1}$ we could be able to determine $Q_t = 2/3$ with an accuracy of $\sim 15\%$

Sensitivity to new physics

- all the above mentioned points
- Narrow resonances: with $1 \text{ fb}^{-1}$ possible discovery of a $Z'$ of $M_{Z'} \sim 700 \text{ GeV}$ with $\sigma_{pp \to Z' \to t\bar{t}} \sim 11 \text{ pb}$
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Top-Anti Top Pair Production

According to the factorization theorem, the process \( h_1 + h_2 \rightarrow t\bar{t} + X \) can be sketched as in the figure:

\[
\sigma_{h_1,h_2}^{t\bar{t}} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1, \mu_F) f_{h_2,j}(x_2, \mu_F) \hat{\sigma}_{ij}(\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R)
\]

\[
s = (p_{h_1} + p_{h_2})^2, \quad \hat{s} = x_1 x_2 s
\]
The Partonic Cross Section: Tree-Level

\[ q(p_1) + \bar{q}(p_2) \rightarrow t(p_3) + \bar{t}(p_4) \]

\[ g(p_1) + g(p_2) \rightarrow t(p_3) + \bar{t}(p_4) \]

Dominant at Tevatron
\[ \sim 85\% \]

Dominant at LHC
\[ \sim 90\% \]

\( \sigma_{tt\bar{t}}^{LO}(LHC, m_t = 171 \text{ GeV}) = 583 \text{ pb} \pm 30\% \)

\( \sigma_{tt\bar{t}}^{LO}(Tev, m_t = 171 \text{ GeV}) = 5.92 \text{ pb} \pm 44\% \)
The Partonic Cross Section: NLO

Fixed Order

- The NLO QCD corrections are quite sizable: +25% at Tevatron and +50% at LHC.
  - Scales variation ±15%.
  - Nason, Dawson, Ellis '88-'90; Beenakker, Kuijf, van Neerven, Smith '89-'91;
    Mangano, Nason, Ridolfi '92; Frixione et al. '95; Czakon and Mitov '08.

- Mixed NLO QCD-EW corrections are small: -1% at Tevatron and -0.5% at LHC.
  - Beenakker et al. '94 Bernreuther, Fuecker, and Si '05-'08
  - Kühn, Scharf, and Uwer '05-'06; Moretti, Nolten, and Ross '06.

All-order Soft-Gluon Resummation

- Leading-Logs (LL)
  - Laenen et al. '92-'95; Berger and Contopanagos '95-'96; Catani et al. '96.

- Next-to-Leading-Logs (NLL)
  - Kidonakis and Sterman '97; R. B., Catani, Mangano, and Nason '98.

- Next-to-Next-to-Leading-Logs (NNLL) under study.
  - Moch and Uwer '08; Beneke et al. '09; Czakon et al. '09; Kidonakis '09

\[
\sigma_{NNLO+NLL}^{tt}(\text{Tev}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 7.61 \pm 0.53(6.9\%) \text{ (scales)} \pm 0.36(4.8\%) \text{ (PDFs)} \text{ pb}
\]

\[
\sigma_{NNLO+NLL}^{tt}(\text{LHC}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 908 \pm 85(9.3\%) \text{ (scales)} \pm 29(3.2\%) \text{ (PDFs)} \text{ pb}
\]


Experimental requirements for $\sigma_{tt\bar{t}}$:

- **Tevatron** $\Delta \sigma/\sigma \sim 10\% \implies \sim \text{ok with the theory!}$
- **LHC** (14 TeV, high luminosity) $\Delta \sigma/\sigma \sim 5\% \ll \text{NLO theoretical prediction!!}$

Kidonakis-Vogt and Moch-Uwer, Langenfeld-Moch-Uwer, presented recently approximated NNLO results for $\sigma_{tt\bar{t}}$ including:

- scale dependence at NNLO
- NNLL soft-gluon contributions
- Coulomb corrections

This drastically reduces the uncertainty (factorization/renormalization scale dependence) to the level predicted for LHC: $\sim 4-6\%$, and indicate that a COMPLETE NNLO computation is indeed needed in order to match the experimental precision of LHC.
The NNLO calculation of the top-quark pair hadro-production requires several ingredients:

- **Virtual Corrections**
  - two-loop matrix elements for $q\bar{q} \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$
  - interference of one-loop diagrams

  Körner et al. ’05–’08; Anastasiou and Aybat ’08

- **Real Corrections**
  - one-loop matrix elements for the hadronic production of $t\bar{t} + 1$ parton
  - tree-level matrix elements for the hadronic production of $t\bar{t} + 2$ partons

  Dittmaier, Uwer and Weinzierl ’07–’08

- **Subtraction Terms**
  - Both matrix elements known for $t\bar{t} + j$ calculation, BUT subtraction up to 1 unresolved parton, while in a complete NNLO computation of $\sigma_{tt}$ we need subtraction terms with up to 2 unresolved partons.

  $\Rightarrow$ Need an extension of the subtraction methods at the NNLO.

  Gehrmann-De Ridder, Ritzmann ’09, Daleo et al. ’09, Boughezal et al. ’10, Glover, Pires ’10

  Very recently: for double real in $\sigma_{tt}$, method proposed by Czakon, arXiv:1005.0274
Next-to-Next-to-Leading Order

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Two-Loop Corrections to $q\bar{q} \rightarrow t\bar{t}$

$$|\mathcal{M}|^2 (s, t, m, \varepsilon) = \frac{4\pi^2 \alpha_s^2}{N_c} \left[ A_0 + \left( \frac{\alpha_s}{\pi} \right) A_1 + \left( \frac{\alpha_s}{\pi} \right)^2 A_2 + \mathcal{O}(\alpha_s^3) \right]$$

$$A_2 = A_2^{(2\times0)} + A_2^{(1\times1)}$$

$$A_2^{(2\times0)} = N_c C_F \left[ N_c^2 A + B + \frac{C}{N_c^2} + N_l \left( N_c D_l + \frac{E_l}{N_c} \right) ight. \left. + N_h \left( N_c D_h + \frac{E_h}{N_c} \right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h \right]$$

218 two-loop diagrams contribute to the 10 different color coefficients

- The whole $A_2^{(2\times0)}$ is known numerically
  - Czakon '08.

- The coefficients $D_i$, $E_i$, $F_i$, and $A$ are known analytically (agreement with num res)
  - R. B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

- The poles of $A_2^{(2\times0)}$ (and therefore of $B$ and $C$) are known analytically
  - Ferroglia, Neubert, Pecjak, and Li Yang '09
Two-Loop Corrections to $q\bar{q} \rightarrow t\bar{t}$

- $D_i, E_i, F_i$ come from the corrections involving a closed (light or heavy) fermionic loop:

- $A$ the leading-color coefficient, comes from the planar diagrams:

- The calculation is carried out analytically using:
  - Laporta Algorithm for the reduction of the dimensionally-regularized scalar integrals (in terms of which we express the $|\mathcal{M}|^2$) to the Master Integrals (MIs)
  - Differential Equations Method for the analytic solution of the MIs
Master Integrals for $N_l$ and $N_h$

18 irreducible two-loop topologies (26 MIs)

Master Integrals for the Leading Color Coeff

For the leading color coefficient there are 9 additional irreducible topologies (19 MIs)

Example: Box for the Leading Color Coeff

\[
\begin{bmatrix}
A_{-4} = \frac{x^2}{24(1 - x)^4(1 + y)}, \\
A_{-3} = \frac{x^2}{96(1 - x)^4(1 + y)} \left[ -10G(-1; y) + 3G(0; x) - 6G(1; x) \right], \\
A_{-2} = \frac{x^2}{48(1 - x)^4(1 + y)} \left[ -5\zeta(2) - 6G(-1; y)G(0; x) + 12G(-1; y)G(1; x) + 8G(-1, -1; y) \right], \\
A_{-1} = \frac{x^2}{48(1 - x)^4(1 + y)} \left[ -13\zeta(3) + 38\zeta(2)G(-1; y) + 9\zeta(2)G(0; x) + 6\zeta(2)G(1; x) - 24\zeta(2)G(-1/y; x) \\
+ 24G(0; x)G(-1, -1; y) - 24G(1; x)G(-1, -1, -1; y) - 12G(-1/y; x)G(-1, -1, y) \\
-12G(-y; x)G(-1, -1, -1; y) - 6G(0; x)G(0, -1; y) + 6G(-1/y; x)G(0, -1; y) + 6G(-y; x)G(0, -1; y) \\
+ 12G(-1; y)G(1, 0; x) - 24G(-1; y)G(1, 1; x) - 6G(-1; y)G(-1/y, 0; x) + 12G(-1; y)G(-1/y, 1; x) \\
- 6G(-1; y)G(-y, 0; x) + 12G(-1; y)G(-y, 1; x) + 16G(-1, -1, -1; y) - 12G(-1, 0, -1; y) \\
- 12G(0, -1, -1; y) + 6G(0, 0, -1; y) + 6G(1, 0, 0; x) - 12G(1, 0, 1; x) - 12G(1, 1, 0; x) + 24G(1, 1, 1; x) \\
- 6G(-1/y, 0, 0; x) + 12G(-1/y, 0, 1; x) + 6G(-1/y, 1, 0; x) - 12G(-1/y, 1, 1; x) + 6G(-y, 1, 0; x) \\
- 12G(-y, 1, 1; x) \right]
\end{bmatrix}
\]

Example: Box for the Leading Color Coeff

\[ \frac{1}{m^6} \sum_{i=-4}^{-1} A_i \epsilon^i + O(\epsilon^0) \]

\[ A_{-4} = \frac{x^2}{24(1-x)^4(1+y)} , \]

\[ A_{-3} = \frac{x^2}{96(1-x)^4(1+y)} \left[ -10G(-1; y) + 3G(0; x) - 60G(1; x) \right] \]

\[ A_{-2} = \frac{x^2}{48(1-x)^4(1+y)} \left[ -5\zeta(2) - 6G(-1; y)G(0; x) + \frac{x}{1+y} \right] \]

\[ A_{-1} = \frac{x^2}{48(1-x)^4(1+y)} \left[ -13\zeta(3) + 38\zeta(2)G(-1; y) + 9\zeta(2)G(0; x) + \frac{6\zeta(2)}{1+y} \right] \]

1- and 2-dim GHPLs
GHPLs

One- and two-dimensional Generalized Harmonic Polylogarithms (GHPLs) are defined as repeated integrations over set of basic functions. In the case at hand

\[ f_w(x) = \frac{1}{x - w}, \quad \text{with} \quad w \in \left\{ 0, 1, -1, -y, -\frac{1}{y}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2} \right\} \]

\[ f_w(y) = \frac{1}{y - w}, \quad \text{with} \quad w \in \left\{ 0, 1, -1, -x, -\frac{1}{x}, 1 - \frac{1}{x} - x \right\} \]

The weight-one GHPLs are defined as

\[ G(0; x) = \ln x, \quad G(w; x) = \int_0^x dt f_w(t) \]

Higher weight GHPLs are defined by iterated integrations

\[ G\underbrace{(0, 0, \cdots, 0)}_{n}; x) = \frac{1}{n!} \ln^n x, \quad G(w, \cdots; x) = \int_0^x dt f_w(t) G(\cdots; t) \]

Shuffle algebra. Integration by parts identities

Remiddi and Vermaseren ’99, Gehrmann and Remiddi ’01-’02, A. Aglietti and R. B. ’03, Vollinga and Weinzierl ’04, R. B., A. Ferroglia, T. Gehrmann, and C. Studerus ’09
Coefficient $A$

Finite part of $A$

Threshold expansion versus exact result

$\eta = \frac{s}{4m^2} - 1$, \hspace{1cm} \phi = -\frac{t - m^2}{s}$

$\beta = \sqrt{1 - \frac{4m^2}{s}}$

partonic c.m. scattering angle $= \frac{\pi}{2}$

Numerical evaluation of the GHPLs with GiNaC C++ routines.

Vollinga and Weinzierl '04
Two-Loop Corrections to $gg \rightarrow t\bar{t}$

$$|\mathcal{M}|^2 (s, t, m, \varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c} \left[ A_0 + \left( \frac{\alpha_s}{\pi} \right) A_1 + \left( \frac{\alpha_s}{\pi} \right)^2 A_2 + \mathcal{O} (\alpha_s^3) \right]$$

$$A_2 = A_2^{(2\times0)} + A_2^{(1\times1)}$$

$$A_2^{(2\times0)} = (N_c^2 - 1) \left( N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h ight.$$  

$$+ N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh} \right)$$

789 two-loop diagrams contribute to 16 different color coefficients

- No numeric result for $A_2^{(2\times0)}$ yet
- The poles of $A_2^{(2\times0)}$ are known analytically
  
  Ferroglia, Neubert, Pecjak, and Li Yang '09

- The coefficients $A, E_l - I_l$ can be evaluated analytically as for the $q\bar{q}$ channel
  
  R. B., Ferroglia, Gehrmann, von Manteuffel and Studerus, in prep.

Conclusions

In the last 15 years, Tevatron explored top-quark properties reaching a remarkable experimental accuracy. The top mass could be measured with $\Delta m_t/m_t = 0.75\%$ and the production cross section with $\Delta \sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 9\%$. Other observables could be measured only with bigger errors.

At the LHC the situation will further improve. The production cross section of $t\bar{t}$ pairs is expected to reach the accuracy of $\Delta \sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 5\%$!!

This experimental precision demands for more accurate theoretical predictions. Quantum corrections have to be unavoidably taken into account.

For the production cross section, $\sigma_{t\bar{t}}$, a complete NNLO analysis is mandatory in order to reach the experimental accuracy expected in 3-4 years from now.

In spite of a big activity of different groups, many ingredients are still missing.

In this talk I briefly reviewed the analytic evaluation of the two-loop matrix elements, afforded using the Laporta algorithm for the reduction to the MIs and the Differential Equations method for their analytic evaluation. To date, the corrections involving a fermionic loop (light or heavy) in the $q\bar{q}$ channel are completed, together with the leading color coefficient. Analogous corrections in the $gg$ channel can be calculated with the same technique and are at the moment under study.