Multi-jet merging with NLO matrix elements

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1 In collaboration with Thomas Gehrmann, Stefan Höche, Frank Krauss & Marek Schönherr
Multi-jet merging at tree level

 POWHEG

 Multi-jet merging with full NLO

 Conclusions and Outlook

 JHEP 0111 (2001) 063, JHEP 0205 (2002) 046,

Review: Multi-jet merging at tree level

Two approaches to real higher-order corrections

Matrix Element

\[(+)\text{ Exact to fixed order}\]
- Perturbative series breaks down due to large logarithms

Parton Shower

\[(+)\text{ Resums logarithmically enhanced contributions to all orders}\]
- Only approximation for real ME

Goal: Combine advantages $\implies$ ME$\otimes$PS

- Describe particular final state by ME (hard radiation)
- Don’t spoil the inclusive picture provided by the PS (intra-jet evolution)

Problem: PS and higher-order ME describe the same final state!
Solution: Phase space slicing by parton separation criterion $Q_{\text{cut}}$
Multi-jet merging at tree level

Features and shortcomings

Example

Diphoton production at Tevatron

- Recently published by DØ \cite{d0}
- Isolated hard photons with:
  - $E_\perp^1 > 21$ GeV
  - $E_\perp^2 > 20$ GeV
  - $|\eta_\gamma| < 0.9$
  - Isolation: $E_\perp (R = 0.4) - E_\perp^\gamma < 2.5$ GeV
- Here: Azimuthal angle between the diphoton pair

ME⊗PS simulation using \textsc{Sherpa} 1.2.2 with QCD+QED interleaved shower and merging as in \cite{sherpa}

Conclusions

Shapes described very well even for this non-trivial process/observable for both:
- Hard region, e.g. $\Delta \Phi_{\gamma\gamma} \to 0$
- Soft region, e.g. $\Delta \Phi_{\gamma\gamma} \to \pi$

Total cross section too low \Rightarrow Virtual matrix elements needed
The POWHEG master formula

**Why POWHEG?**
- Matches full NLO matrix element and parton shower for the hardest emission
- Generates positive weights (almost always)
- Independent of shower algorithm
- Will use it as basic ingredient in ME⊗PS@NLO

**Master formula**  
\[
d\sigma_{NLO} = \bar{B}(\Phi_B)d\Phi_B \left[ \bar{\Delta}(k_{\perp},0) + \sum \int_{k_{\perp},0} d\Phi_R |_B \frac{R(\Phi_R)}{B(\Phi_B)} \bar{\Delta}(k_{\perp}) \right]
\]

Two problems to be solved:
1. **Differential NLO cross section**  
   \( \bar{B} = B + V + I + d\Phi_R |_B [R - S] \)
   - Born and Real MEs from automated tree-level generators
   - Virtual ME e.g. via Binoth Les Houches Accord [Comput. Phys. Commun. 181:1612-1622, 2010]
   - Special integrator for extra emission on top of Born phase space

2. **POWHEG Sudakov**  
   \( \bar{\Delta}(k_{\perp}) = \exp \left( - \sum \int_{k_{\perp}} d\Phi_R |_B \frac{R(\Phi_R)}{B(\Phi_B)} \right) \)
Generating the POWHEG Sudakov

Exponentiating $\Gamma(k_{\perp}) = \sum \int_{k_{\perp}} d\Phi R|B \frac{R}{B}$

Standard MC technique to dice $k_{\perp}$ from Sudakov: \cite{comput.phys.commun.82:74-90,1994}

- Find invertible overestimate $\tilde{\Gamma}$
- Dice random number $\#_1$ and calculate $k_{\perp} = \tilde{\Gamma}^{-1}(-\log \#_1)$
- Accept $k_{\perp}$ with weight $w = \frac{\Gamma(k_{\perp})}{\tilde{\Gamma}(k_{\perp})}$

Tricky bit: Find suitable $\tilde{\Gamma}$

$\tilde{\Gamma}$ should be sufficiently similar to $\Gamma$

- Otherwise the efficiency in the last step will be very low
- But still has to overestimate in full phase space

Good candidate: Splitting functions in parton showers

- In \textsc{Sherpa}: Dipole-like parton shower based on CS subtraction \cite{jhep.0803:038,2008}
- Multiplied with automatically determined enhancement factors such that always overestimating
POWHEG results from Sherpa for $p\bar{p} \rightarrow Z+\text{jets}$ at 1960 GeV

- LO predictions scaled with K-factor 1.2
- POWHEG and ME⊗PS with up to 1-jet agree within a few percent
- Both show improvements over ordinary parton shower predictions in the region of hard emissions
Multi-jet merging with full NLO matrix elements in **Sherpa**

**ME⊗PS “master formula” for first emission**

\[
\frac{d\sigma_{NLO}}{d\Phi} = B(\Phi_B) d\Phi_B \left[ \Delta(k_{\perp}, 0) + \sum \int_{k_{\perp}, 0} d\Phi_{R\mid B} \left( \Theta(Q_{cut} - Q) K_{ab} \Delta(k_{\perp}) + \Theta(Q - Q_{cut}) \frac{R(\Phi_R)}{B(\Phi_B)} \Delta(k_{\perp}) \right) \right]
\]

Already very similar to **POWHEG**, but:

- \(\Delta(k_{\perp})\) doesn’t contain \(R/B\) ⇒ \([\ldots]\) is only approximately unitary

- \(B\) instead of \(\bar{B}\) ⇒ LO accuracy only
How can this be improved to (almost) full NLO accuracy? \textit{JHEP 1006:039,2010}

- Replace unresolved and PS part by POWHEG
- Apply K-factor for $R$ such that $\bar{B}/B$ is reproduced

recently implemented by Hamilton & Nason \textit{JHEP 1006:039,2010} in a simplified version:
- \textbf{global} K-factor to generate $\bar{B}/B$ $R$
- we use a \textbf{local} K-factor $\bar{B}(\Phi_B)/B(\Phi_B)$ as above
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Conclusions and Outlook

Results for $e^+e^- \rightarrow \text{jets}$

- POWHEG and MENLOPS with up to one additional jet agree well
- No “kink” around merging cut

$log(y_{\text{cut}}) = -2.25$

- Higher order tree-level MEs improve description in some regions of phase space
MENLOPS results for $p\bar{p} \rightarrow Z+jets$ at 1960 GeV

- Generated with $Q_{\text{cut}} = 20$ GeV
- $k_T$ jets with $p_{\text{min}}^\perp = 20$ GeV
- POWHEG and MENLOPS with up to 1-jet agree fairly well
- For observables sensitive to additional hard emissions the merging of higher order MEs is necessary
Stability of the merging

- Generated with $Q_{\text{cut}} = 15, 20$ and $40 \text{ GeV}$ for up to 3 jets
- Only very small variations in results

⇒ Stable and consistent MENLOPS merging
**Conclusions**

- Tree-level ME+PS merging works well for shapes, but needs K-factor for cross section
- **POWHEG** reproduces full NLO cross section and shape of first emission but fails for additional hard radiation
- Combination of full NLO and higher order tree-level MEs with shower achieves both of the above
- Recently much progress and already first implementations
- Automation within **SHERPA** framework in reach
- Full NLO only in core process, not in higher order corrections . . .

**Outlook**

- . . . yet
- Application to more processes
- Public availability in a **SHERPA** release, as simple to use as tree-level merging
Main idea

Phase space slicing for extra QCD radiation:
- Hard emissions from matrix element
- Soft/collinear emissions from parton shower

Two main ingredients

**Shower on top of higher order MEs**

**Problem:** ME only gives final state, no history as shower input

**Solution:** Backward-clustering (running the shower reversed)

⇒ Branching history corresponding to the ME final state

**Avoid double-counting of emissions**

- Populate full real-emission phase space with *either* ME or PS
- Make MEs exclusive by rejection of events with hard shower emissions