The b-quark mass and the heavy-light decay constant from lattice HQET

Nicolas Garron

University of Edinburgh

ALPHA Collaboration

Motivations

To constraint the standard model and see a signal of new physics the theoretical uncertainties should be decreased .

Two examples:

- Theoretical uncertainty on the inclusive determination on $|V_{ub}|$ dominated by the one of the b-quark mass $\delta V_{ub}/V_{ub} \sim 4\,\delta m_{\rm b}/m_{\rm b}$

  Now $\delta m_{\rm b} = 40\ \mathrm{MeV} \Rightarrow \delta V_{ub}/V_{ub} = 3.5\%$ [Hitlin et al. 09]

- $\mathcal{B}_R(B_s \to \mu^+\mu^-) = F_{B_s}^2(C_{\rm SM} + \tan^6\beta_{\rm MSSM})$

$\Rightarrow$ In the B sector, high precision results are needed

$\Rightarrow$ Lattice HQET is a natural candidate to study heavy-light mesons
It is theoretically sound and can give precise results

Effective theories for heavy quark

Momentum of a heavy quark (inside a hadron) $p = m_Q v + k$
Interaction with light dof $k \sim \Lambda_{\rm QCD} \ll m_Q$
Separate the higher and lower components of the heavy quark, and find an effective Lagrangian (see eg [Grozin '02])

$$\mathcal{L}_{\rm eff}^{\rm heavy} = \bar{\psi}_{\rm h}(x)\left[iv.D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g\sigma.G}{4m_Q} + ...\right]\psi_{\rm h}(x)$$

Different choices of lattice implementation

- Expansion in $\Lambda_{\rm QCD}/m_Q$: HQET $\rightarrow$ This talk
- Expansion in v and $1/am_Q$: NRQCD
- Fermilab Method [El-Khadra et al '96]
- Relativistic heavy quarks [Aoki et al '01, Christ et al, Lin et al '06]

(Note a recent proposal by ETMC for B physics [ETMC '10])

See talks by E. Gamiz and M Della Morte in the lattice session

Should you like (lattice) HQET ?

- pros
  - Theoretically well defined, (continuum limit, renormalization)
  - Can be implemented non-perturbatively
  - The static propagator is numerically cheap
  - In many cases the $1/m$ terms are doable
  - Convergence expected to be fast

- cons
  - Effective theory, not QCD
  - Linear divergence in the static energy [Eichten & Hill '90]

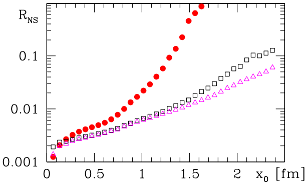
$$E^{\mathrm{stat}} \simeq \frac{19.95}{12\pi^2} \times \frac{g_0^2}{a} + \dots$$

  - Ratio $\mathrm{Noise/signal} \longrightarrow exp\,(E^{\mathrm{stat}} x_o)$
    $\Rightarrow$ Can one get a signal ?

"Recent" improvements in HQET

- Conceptual improvement:
  Non Perturbative matching with HQET [Heitger & Sommer 03]
  ⇒ Subtractions of the divergences

- Technical improvement:
  1. Reduction of the Ratio Noise/Signal
  [Della Morte, Dürr, Heitger, Molke, Rolf, Shindler, Sommer '03]



  2. Application on variational techniques and all to all propagators
  [Blossier, Della Morte, von Hippel, Mendes, Sommer '09]

HQET at zero velocity on the lattice

The static part is given by the Eichten-Hill action [Eichten & Hill 90]

$$S_{\mathrm{stat}} = a^4 \sum_x \overline{\psi}_{\mathrm{h}}(x) D_0 \psi_{\mathrm{h}}(x)$$

with $P_+\psi_{\mathrm{h}} = \psi_{\mathrm{h}}\,,\quad \overline{\psi}_{\mathrm{h}} P_+ = \overline{\psi}_{\mathrm{h}}\,,\quad P_+ = \frac{1}{2}(1+\gamma_0)$

The static energy contains a linear divergence ($\propto 1/a$) which is absorbed by m_{bare}

$$m_{\mathrm{B}} = E^{\mathrm{stat}} + m_{\mathrm{bare}}$$

The $1/m$ corrections are the kinetic and chromomagnetic terms

$$\mathcal{O}_{\mathrm{kin}} = -\overline{\psi}_{\mathrm{h}}(\mathbf{D}^2)\psi_{\mathrm{h}} \qquad \mathcal{O}_{\mathrm{spin}} = -\overline{\psi}_{\mathrm{h}}(\sigma\cdot\mathbf{B})\psi_{\mathrm{h}}$$

with coefficient $\omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}$ $\Rightarrow$ Classically $\omega_{\mathrm{kin}} = \omega_{\mathrm{spin}} = 1/(2m)$

HQET coefficients $m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}$ are determined non-perturbatively
$\Rightarrow$ renormalizability

HQET computation on the lattice

We want to compute hadronic quantities at the $1/m$ order of hqet, for example

$$
\begin{aligned}
m_{\mathrm{B}} &= m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} E^{\mathrm{spin}} + \omega_{\mathrm{spin}} E^{\mathrm{spin}} \\
\langle 0 | A_0^{\mathrm{HQET}} | \mathrm{B} \rangle &= Z_{\mathrm{A}}^{\mathrm{HQET}} \Big(\langle 0 | A_0^{\mathrm{stat}} | \mathrm{B} \rangle + \omega_{\mathrm{kin}} \langle 0 | \mathrm{A}_0^{\mathrm{kin}} | \mathrm{B} \rangle + \omega_{\mathrm{spin}} \langle 0 | \mathrm{A}_0^{\mathrm{spin}} | \mathrm{B} \rangle \Big)
\end{aligned}
$$

$\Rightarrow$ To achieve such a computation, one needs:

- large volume matrix element and energies $E^{\mathrm{stat}}, E^{\mathrm{kin}}, \langle 0 | A_0^{\mathrm{stat}} | \mathrm{B} \rangle, \ldots$
  $\rightarrow$ use variational techniques on top of all-to-all propagators
- HQET parameters $m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, Z_{\mathrm{A}}^{\mathrm{HQET}}, \ldots$
  $\rightarrow$ non perturbative matching

Strategy [Heitger & Sommer 03]

$$
\begin{tikzcd}
 & \boxed{\text{QCD } (am_b \ll 1)} & \boxed{\text{HQET}} \\
L_1 \sim 0.5\ \mathrm{fm} & \Phi^{\mathrm{QCD}}(L_1, am_{\mathrm{b}}) \arrow[r, leftrightarrow, "\text{Matching}"] & \Phi^{\mathrm{HQET}}(L_1) \arrow[d, leftrightarrow] \\
L_2 = 2L_1 & & \left[\Phi^{\mathrm{HQET}}(L_2) - \Phi^{\mathrm{HQET}}(L_1)\right] \arrow[d, leftrightarrow] \\
L_{\mathrm{inf}} & \Phi(L_{\mathrm{inf}}, am_{\mathrm{b}}) \arrow[r, leftrightarrow] & \left[\Phi^{\mathrm{HQET}}(L_{\mathrm{inf}}) - \Phi^{\mathrm{HQET}}(L_2)\right] \\
 & \boxed{\text{Experiment}} &
\end{tikzcd}
$$

Strategy [Heitger & Sommer 03]

- Simulate QCD in small volume $L_1 \sim 0.5\ \mathrm{fm}$ with $am_{\mathrm{b}} \ll 1$.
  Compute a set of observables and take the continuum limit $\Phi(L_1, m_{\mathrm{q}})$

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- Compute the corresponding quantities at a given order of the effective theory for various lattice spacing a.
  Impose the matching $\Rightarrow$ HQET parameters for these values of the lattice spacings .

  e.g. static meson mass $\quad \Gamma^{\mathrm{QCD}}(L_1, m_\mathrm{q}) = m_\mathrm{bare}(m_\mathrm{q}, a) + \Gamma^{\mathrm{stat}}(L_1, a)$

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- Perform another simulation of HQET, with the same a's but in a larger volume, for example $L_2 = 2L_1$.

  Use the HQET parameters computed in the previous step, to obtain the observables in the volume L_2, and take their continuum limit $\Phi(L_2, m_{\mathrm{q}})$ (cancelation of the divergences).

  Static meson mass:

$$\begin{aligned} \Gamma(L_2, m_{\mathrm{q}}) &= \lim_{a\to 0} \left(\Gamma^{\mathrm{stat}}(L_2, a) - \Gamma^{\mathrm{stat}}(L_1, a)\right) + \Gamma^{\mathrm{QCD}}(L_1, m_{\mathrm{q}}) \\ &= L_1\, \sigma^{\mathrm{m}}(\bar{g}^2(L_1)) + \Gamma^{\mathrm{QCD}}(L_1, m_{\mathrm{q}}) \end{aligned}$$

Strategy [Heitger & Sommer 03]

- Simulate QCD in small volume $L_1 \sim 0.5\ \mathrm{fm}$ with $am_\mathrm{b} \ll 1$.
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&= L_1\, \sigma^\mathrm{m}(\bar{g}^2(L_1)) + \Gamma^\mathrm{QCD}(L_1, m_\mathrm{q})
\end{aligned}
$$

- Restart from step 1, with $\Phi^\mathrm{QCD}(L_1, m_\mathrm{q}) \to \Phi(L_2, m_\mathrm{q})$ until the volume is large enough to compute hadronic quantities

Strategy

For the meson mass at the static order, we obtain for various quark masses $m_{\rm q}$

$$\begin{aligned}\Gamma(L_\infty, m_{\rm q}) \quad &= \quad \lim_{a\to 0}\left(\Gamma^{\rm stat}(L_\infty, a) - \Gamma^{\rm stat}(L_2, a)\right) + \lim_{a\to 0}\left(\Gamma^{\rm stat}(L_2, a) - \Gamma^{\rm stat}(L_1, a)\right) \\ &\quad + \Gamma^{\rm QCD}(L_1, m_{\rm q})\end{aligned}$$

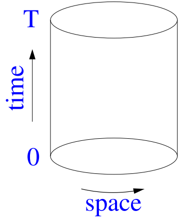
And we interpolate at the B(s)-meson mass to obtain the b-quark mass

In the case of the heavy-light decay constant we use the parameters interpolated at the obtained value of the b-quark mass, and obtain $F_{\rm B_{(s)}}$

Implementation : Schrödinger functional [Lüscher '92, Sint '94]

Implementation: Schrödinger functional of size $T \times L^3$

- Dirichlet boundary conditions in time (at $x_0 = 0$ and $x_0 = T$)
- Periodic boundary conditions in space, up to a phase $\Psi(x + \hat{k}L) = e^{i\theta}\Psi(x)$.



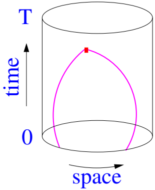
Transition amplitude for $C(x_0 = 0) \to C'(x_o = T)$

$$\begin{aligned} \mathcal{Z}[C', C] &= \langle C' | \mathrm{e}^{-\mathbb{H}\mathrm{T}} \mathbb{P} | \mathrm{C} \rangle \\ &= \sum_{n=0}^{\infty} \mathrm{e}^{-E_n T} \psi_n[C'] \psi_n[C]^* \end{aligned}$$

Implementation: 2-point functions in QCD

Boundary to current correlators

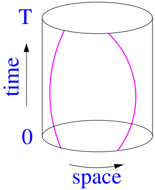
$$f_{\mathrm{A}}(x_0) \quad = \quad -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle (A_{\mathrm{I}})_0(x)\,(\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z}))\right\rangle$$



and boundary to boundary correlator

$$f_1 \quad = \quad -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar{\zeta}'_{\mathrm{b}}(\mathbf{y}')\gamma_5\zeta'_{\mathrm{l}}(\mathbf{z}'))\,(\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z}))\right\rangle$$

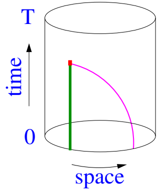
$$k_1 \quad = \quad -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar{\zeta}'_{\mathrm{b}}(\mathbf{y}')\gamma_{\mathrm{k}}\zeta'_{\mathrm{l}}(\mathbf{z}'))\,(\bar{\zeta}_{\mathrm{b}}(\mathbf{y})\gamma_{\mathrm{k}}\zeta_{\mathrm{l}}(\mathbf{z}))\right\rangle$$



Implementation: 2-point functions in the static theory

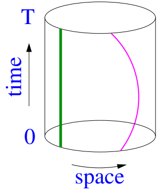
Boundary to current correlators

$$f_{\rm A}^{\rm stat}(x_0) \quad = \quad -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle (A_{\rm I}^{\rm stat})_0(x)\,(\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



and boundary to boundary correlator

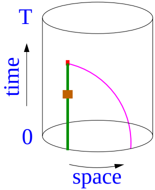
$$f_1^{\rm stat} = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar\zeta'_{\rm h}(\mathbf{y}')\gamma_5\zeta'_{\rm l}(\mathbf{z}'))\,(\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



Implementation: 2-point functions at the $1/m$ order

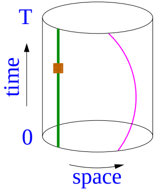
Boundary to current correlators

$$f_{\rm A}^{\rm kin}(x_0) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z},u}\left\langle \,{\rm A}_0^{\rm stat}(x)\, O^{\rm kin}(u)\,(\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



Boundary to boundary correlator

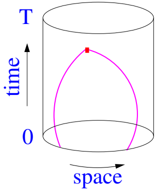
$$f_1^{\rm kin} = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}',u}\left\langle(\bar\zeta'_{\rm h}(\mathbf{y}')\gamma_5\zeta'_{\rm l}(\mathbf{z}'))\, O^{\rm kin}(u)\,(\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$

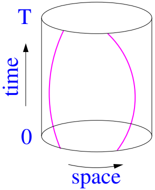


And the same for $f_{\rm A}^{\rm spin}$, $f_1^{\rm spin}$

F_B, including $1/m$ corrections

From the current-to-boundary f_A and the boundary-to-boundary f_1 correlators





Build an observables related to the decay constant :

$$\Phi_2^{\mathrm{QCD}} = \ln\left(\frac{-f_{\mathrm{A}}(x_0)}{\sqrt{f_1}}\right) \quad \xrightarrow{L\gg 1} \quad \ln\left(\frac{1}{2}F_{\mathrm{B}}\sqrt{m_{\mathrm{B}}L^3}\right)$$

At the $1/m$ oder of HQET

$$\begin{aligned}\Phi_2^{\mathrm{HQET}} \quad &= \quad \ln Z_{\mathrm{A}}^{\mathrm{HQET}} + \ln\left(\frac{-f_{\mathrm{A}}^{\mathrm{stat}}}{\sqrt{f_1^{\mathrm{stat}}}}\right) \\ &+ \quad \underbrace{c_{\mathrm{A}}^{\mathrm{HQET}}\frac{f_{\delta\mathrm{A}}^{\mathrm{stat}}}{f_{\mathrm{A}}^{\mathrm{stat}}} + \omega_{\mathrm{kin}}\left(\frac{f_{\mathrm{A}}^{\mathrm{kin}}}{f_{\mathrm{A}}^{\mathrm{stat}}} - \frac{1}{2}\frac{f_1^{\mathrm{kin}}}{f_1^{\mathrm{stat}}}\right) + \omega_{\mathrm{spin}}\left(\frac{f_{\mathrm{A}}^{\mathrm{spin}}}{f_{\mathrm{A}}^{\mathrm{stat}}} - \frac{1}{2}\frac{f_1^{\mathrm{spin}}}{f_1^{\mathrm{stat}}}\right)}_{1/m}\end{aligned}$$

The observables (II)

$\Phi_1 =$	$L\Gamma^{\rm P} =$	$L\Gamma^{\rm stat}$	$+Lm_{\rm bare}^{\rm HQET}$	$+c_{\rm A}^{\rm HQET} L\Gamma_{\delta{\rm A}}^{\rm stat}$	$+\omega_{\rm kin} LE^{\rm kin}$	$+\omega_{\rm spin} LE^{\rm spin}$
$\Phi_2 =$	$\ln\left(\frac{-f_{\rm A}}{\sqrt{f_1}}\right) =$	$\zeta_{\rm A}$	$+\ln Z_{\rm A}^{\rm HQET}$	$+c_{\rm A}^{\rm HQET}\rho_{\delta A}$	$+\omega_{\rm kin}\Psi^{\rm kin}$	$+\omega_{\rm spin}\Psi^{\rm spin}$
$\Phi_3 =$	$R_{\rm A} =$	$R_{\rm A}^{\rm stat}$		$+c_{\rm A}^{\rm HQET} R_{\delta{\rm A}}$	$+\omega_{\rm kin} R_{\rm A}^{\rm kin}$	$+\omega_{\rm spin} R_{\rm A}^{\rm spin}$
$\Phi_4 =$	$\frac{1}{4}(R_1^{\rm P} + 3R_1^{\rm V}) =$	$R_1^{\rm stat}$			$+\omega_{\rm kin} R_1^{\rm kin}$	
$\Phi_5 =$	$\frac{3}{4}\ln\left(\frac{f_1}{k_1}\right) =$					$\omega_{\rm spin}\frac{f_1^{\rm spin}}{f_1^{\rm stat}}$

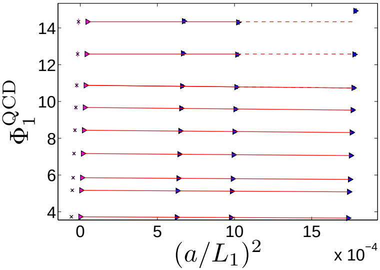
We define the 5 dimensional vectors Φ, η, ω and a 5 by 5 matrix ϕ

$$\Phi(L, m_{\rm q}) = \lim_{a\to 0}\left[\phi(L,a)\,\omega(m_{\rm q},a) + \eta(L,a)\right]$$

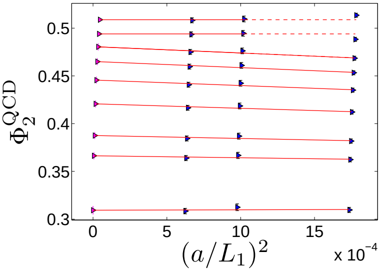
Results

Continuum extrapolation of the QCD observables

The meson mass



The heavy-light decay constant



The RGI quark masses M are such that $z = L_1 M \in (4, 6, 7, 9, 11, 13, 15, 18, 21)$

$L_1/r_0 = 0.9 \Rightarrow L_1 \sim 0.45\ \mathrm{fm}$

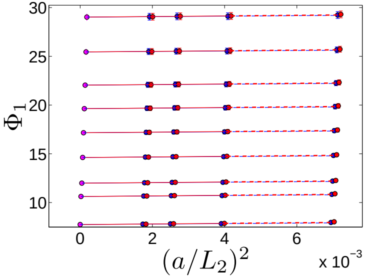
$L_1/a = 40, 32, 24(20)$

$\beta = 6.638, 6.4574, 6.2483$

Results

Continuum extrapolation of the static observables in L_2

The meson mass



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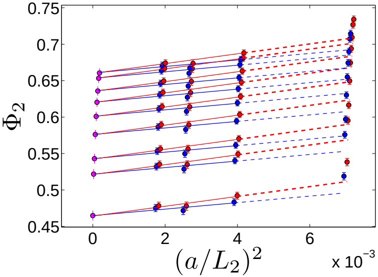

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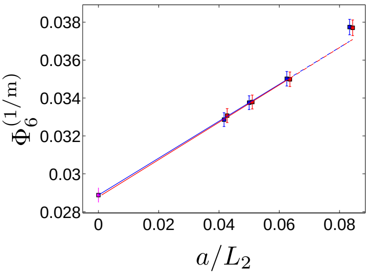
$L_1/a = 12, 10, 8$ $\quad$ $L_1/a = 24, 20, 16$

$\beta = 5.758, 5.619, 5.4689$ $\quad$ Point with $L_2/a = 32, L_1/a = 16$ will be added in the near future

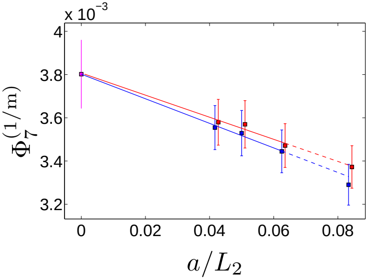
Results

Continuum extrapolation of the $1/m$ observables in L_2

kinetic observable



magnetic observable



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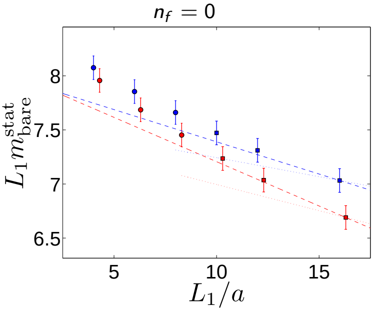
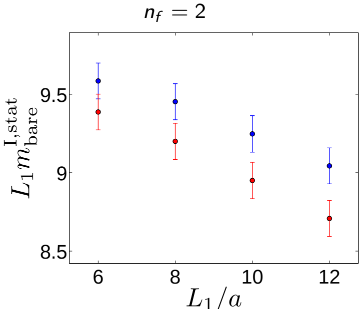
Results

Example of static parameter: $m_{\mathrm{bare}}^{\mathrm{stat}}$





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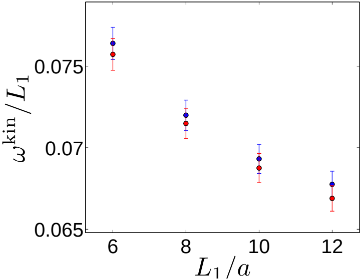
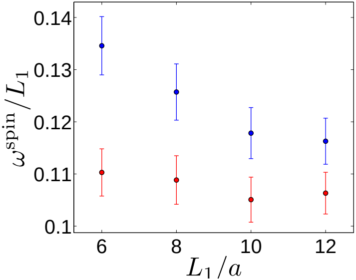
Results

Example of $1/m$ parameters





The RGI quark masses M are such that $z = L_1 M \in (4, 6, 7, 9, 11, 13, 15, 18, 21)$

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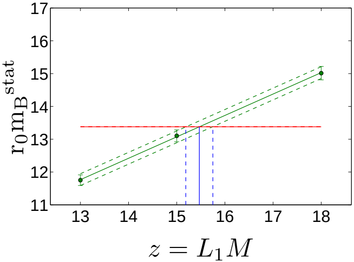
Results

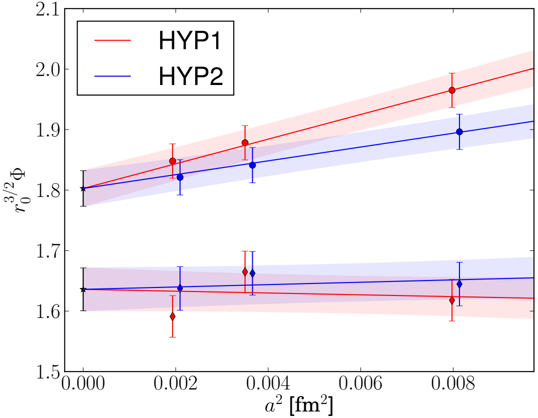
Interpolation at the physical mass, in the static approximation for $n_f = 2$



The RGI quark masses M are such that $z = L_1 M \in (13, 15, 18)$

Results

Continuum limit of $F_{\mathrm{B_s}}$ for $n_f = 0$



Only static (upper curve) and static + 1/m

Conclusion - Status of the project

- $n_f = 0$
  - ▷ b-quark mass [Alpha '06]

$$m_{\rm b}(m_{\rm b}) = \underbrace{4.350(64)}_{\text{static}}\ \text{GeV}\ \underbrace{-0.049(29)}_{\mathrm{O}(\Lambda^2/m_{\rm b})}\ \text{GeV} + \mathrm{O}(\Lambda^3/m_{\rm b}^2)$$

  - ▷ I. HQET parameters [Alpha '10]
  - ▷ II. Spectoscopy [Alpha '10]
  - ▷ III. Decay constant (submitted in June)

$$F_{\rm B_s}^{\rm stat} = 229 \pm 6\ \text{MeV} \qquad F_{\rm B_s}^{\rm stat+1/m} = 219 \pm 8\ \text{MeV}$$

- $n_f = 2$
  - ▷ HQET parameter : almost finished
  - ▷ Large volume part: preliminary results (1 lattice spacing)
    See Talk by Della Morte

VERY PRELIMINARY $m_{\rm b}(m_{\rm b})^{\rm stat} = 4.255(25)(50)(??)$ $\qquad m_{\rm b}(m_{\rm b})^{\rm HQET} = 4.276(25)(50)(??)$

Akdowledgments

Thanks to

- the members of the Alpha collaboration, and in particular to Benoît Blossier, Michele Della Morte, Patrick Fritzsch, Jochen Heitger, Georg von Hippel, Bjorn Leder, Tereza Mendes, Hubert Simma, Rainer Sommer, Nazario Tantalo.
- the organizers of ICHEP 2010
- you for your attention