Progress in Lattice QCD

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Before My Talk

- 30 min is too short for comprehensive review
  $\Rightarrow$ selected topics
- Avoid just flashing latest results
  $\Rightarrow$ explain difficulties and perspectives
- Avoid comparison between different lattice actions or computational techniques

Research Areas of Lattice QCD

Investigate nonperturbative effects of the strong interaction through numerical simulations

- Hadron spectrum and other fundamental quantities
  Schaefer[323], Morningstar[274], Kaneko[366], Scholz[432], Rodriguez-Quintero[1228]
- Dynamics of strong interaction
  hadron-hadron scattering, resonances, bound states
  Aoki[338]
- Phenomenological applications
  Renner[1226], Alexandrou[346], Della Morte[284], Gamiz[560], Ramos[261], Matthias Von Hippel[1008]
- QCD at finite temperature and finite density
  Philiipsen[301]
- Others (QCD-like gauge theories etc.)
  Fleming[1234], Bogolubsky[223]

Plan of Talk

§1. Brief Introduction to Lattice QCD

§2. Hadron Spectrum (1981～)

§3. $\pi\pi \rightarrow \rho$ Resonance (2007～)

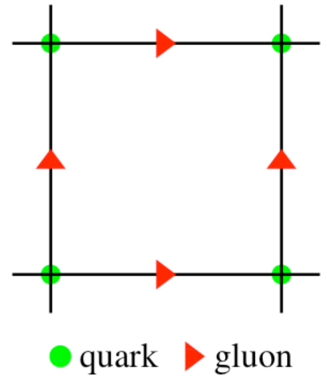
§4. Nuclei from Lattice QCD (2007～)

§5. Summary

§1. Brief Introduction to Lattice QCD

Green functions in path integral formulation on 4-dim. space-time lattice

$$\langle \mathcal{O}[A_\mu, q, \bar{q}] \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}q \mathcal{D}\bar{q}\ \mathcal{O}[A_\mu, q, \bar{q}]\ \exp\left\{ -\int d^4x \mathcal{L}[A_\mu, q, \bar{q}] \right\}$$



Numerical integration with Monte Caro method

$$\langle \mathcal{O}[A_\mu, q, \bar{q}] \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}[A_\mu^{(i)}, q^{(i)}, \bar{q}^{(i)}]$$

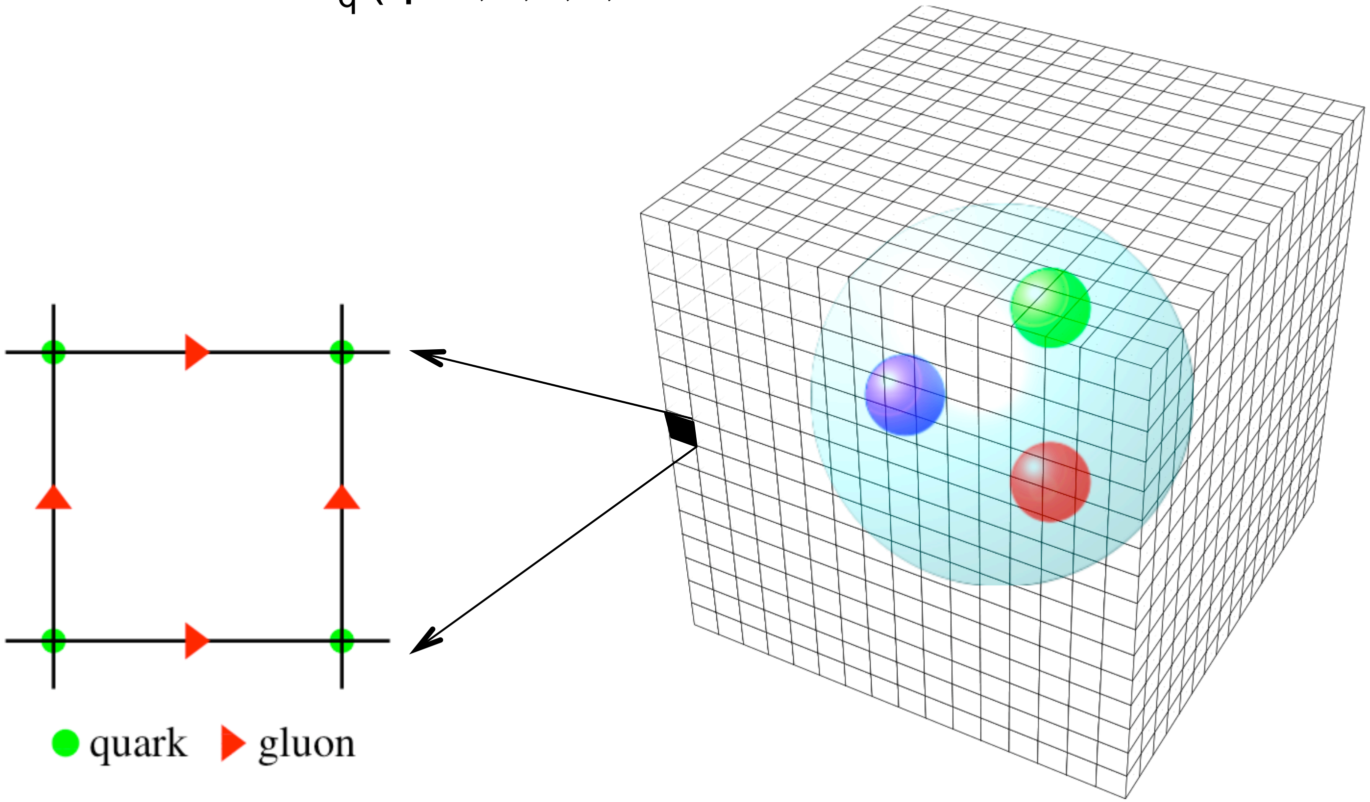
$A_\mu^{(i)}, q^{(i)}, \bar{q}^{(i)}$: i-th configuration

statistical error ∝ $1/\sqrt{N}$

Simulation Parameters

Few parameters

- 4-dim. volume : V=NX･NY･NZ･NT
- lattice spacing : a (as function of bare coupling g)
- quark masses : m_q (q=u,d,s,c,b,t)



Major Systematic Errors

- Finite volume effects

  ⇒ larger V=NX･NY･NZ･NT

- Finite lattice spacing effects

  $\Lambda_{QCD} \ll 1/a$, currently $m_b > 1/a$

  ⇒ smaller a

- Quench approximation (neglect quark vacuum polarization)

  ⇒ 2+1 ($m_u=m_d \neq m_s$) flavor simulation

- Chiral extrapolation

  ⇒ simulations at physical quark masses (physical point)

Need heavier computational cost to diminish the systematic errors

cost ∝ (physical vol.)$^{1.25}$･(lattice spacing)$^{-6\sim-7}$･(quark mass)$^{-2\sim-3}$

§2. Hadron Spectrum

Fundamental quantities both in physical and technical sense

<u>physical side</u>

physical input ⇒ $m_u, m_d, m_s, \ldots$ ⇒ reproduce hadron spectrum?

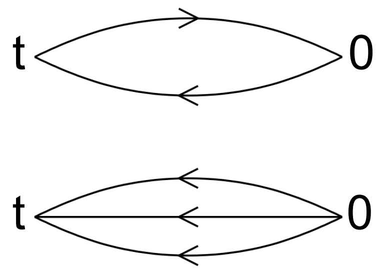
(ex. m_π, m_K, m_Ω) validity of QCD / determination of m_q

<u>technical side</u>

hadron correlators in terms of quark fields

$$\langle \mathcal{O}_h(t)\mathcal{O}_h^\dagger(0)\rangle \overset{t\gg 0}{\approx} C\exp\left(-m_h t\right)$$ ⇒ extract m_h by fit

quark diagrams from Wick contractions



meson

baryon

History of Hadron Spectrum Calculation

1981 first calculation of hadron masses in quenched approx.

Hamber-Parisi

demonstrate the possibility of first principle calculations

1996～2000 precision measurement in quenched approx.

CP-PACS

clear deviation from the experiment

2000～ initiate 2+1 flavor QCD simulations

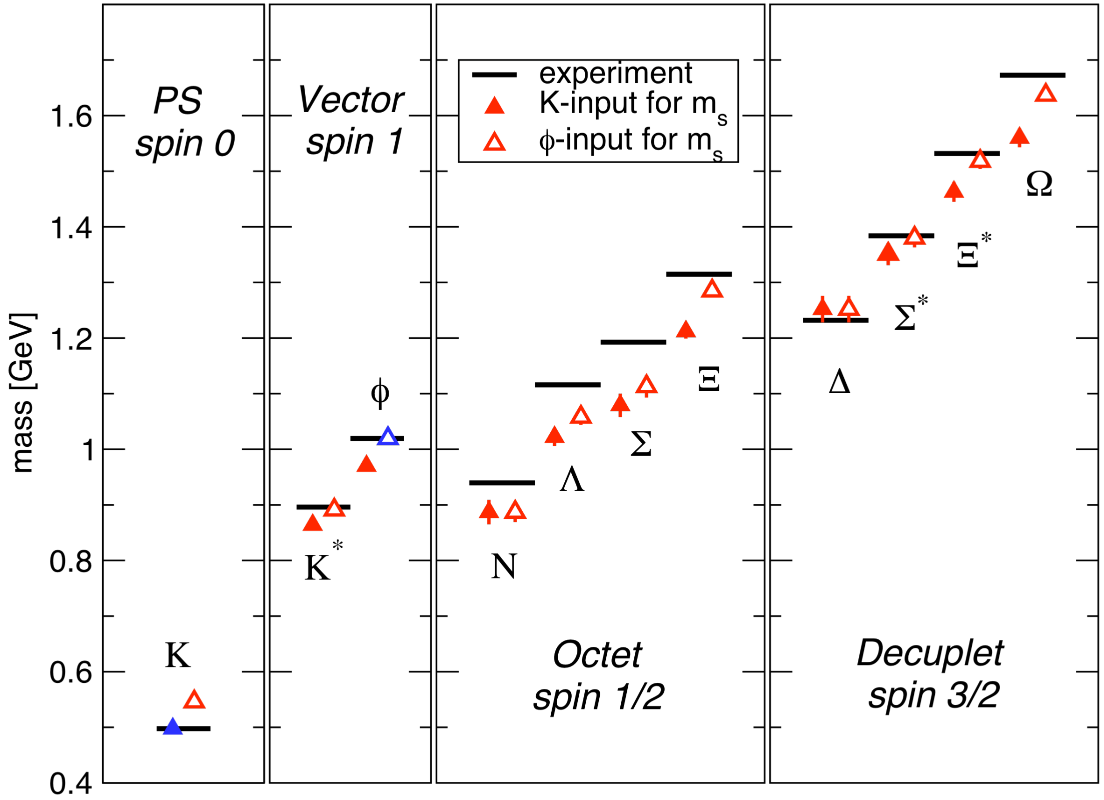
CP-PACS/JLQCD, MILC, …

incorporate u,d,s vacuum polarization effects

Hadron Spectrum in Quenched QCD

physical input m_π, m_K or m_φ, m_ρ $\Rightarrow$ m_u=m_d, m_s, a

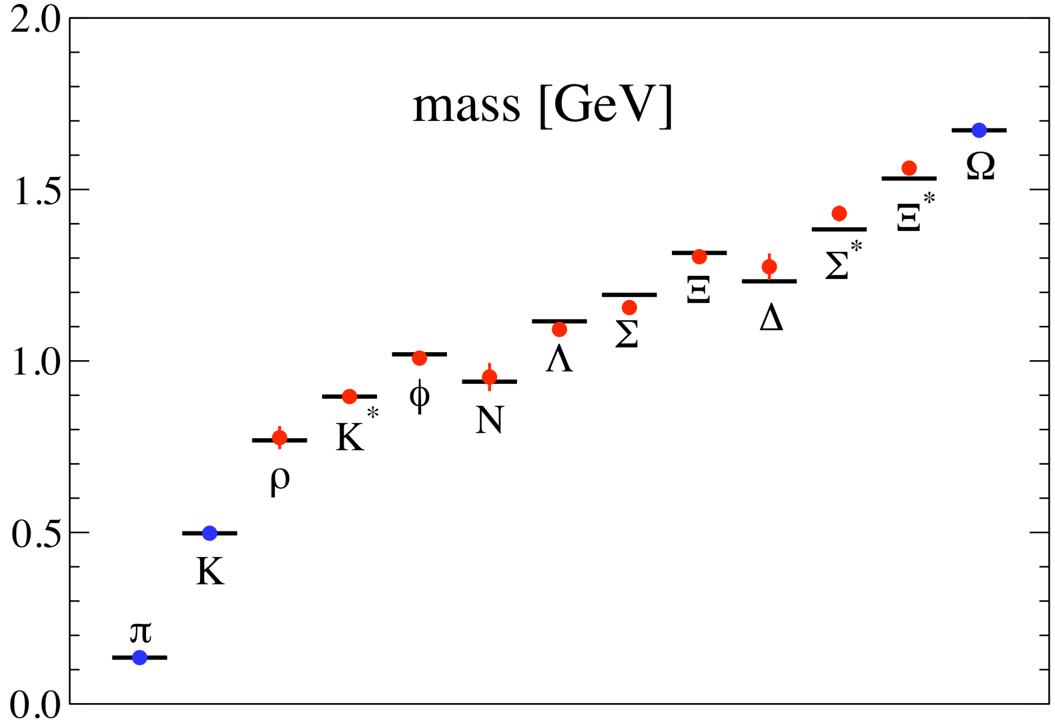
CP-PACS 00



～10% deviation from experimental values

Hadron Spectrum in 2+1 Flavor QCD

physical input m_π, m_K, m_Ω $\Rightarrow$ m_u=m_d, m_s, a



PACS-CS 09

consistent within 2～3% error bars

similar results are obtained by other groups
MILC, RBC/UKQCD, BMW, ...

What Next ?

- 2+1 ($m_u = m_d \neq m_s$) flavor simulation at the physical point

  PACS-CS 10

  avoid chiral extrap. with 200～300MeV < m_π < 500～600MeV

- 1+1+1 ($m_u \neq m_d \neq m_s$) flavor simulation at the physical point
  - electromagnetic interactions

    quenched study: Eichten et al. 96, Blum et al. 07,10

    MILC@lat10, BMW@lat10
  - u-d quark mass difference

$K^0(d\bar{s})$ 497.6MeV

1%

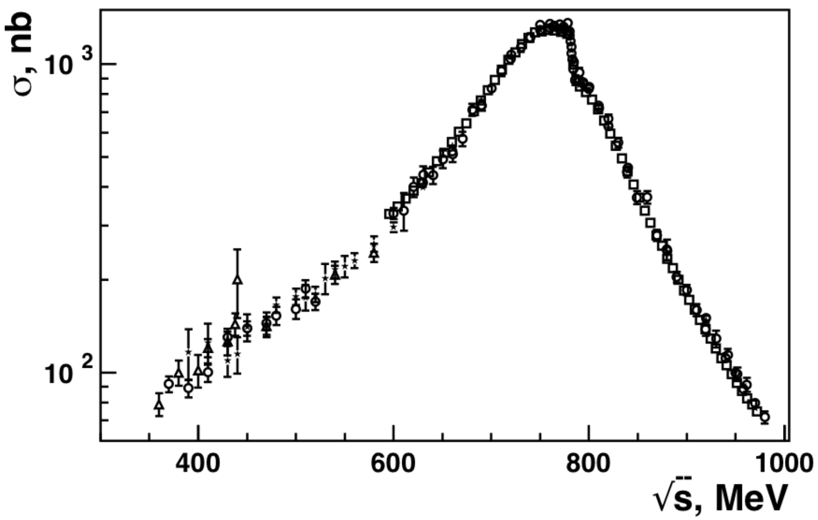
$K^+(u\bar{s})$ 493.7MeV

- Direct treatment of resonances in lattice QCD

  decay width of resonance states

§3. ππ→ρ resonance

ρ resonance in $e^+e^-\to\pi^+\pi^-$ cross section



Achasov 06

Difficulties in lattie QCD:

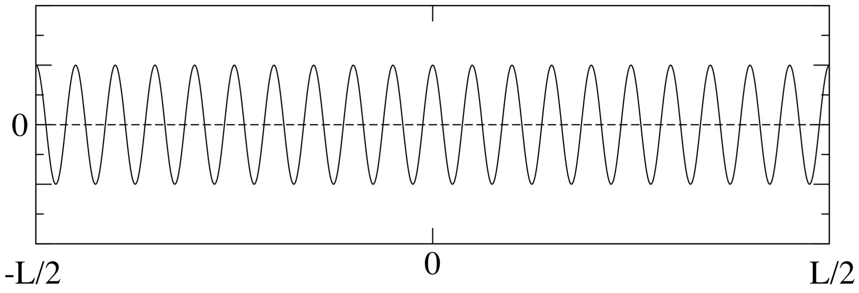
- 2 or 2+1 flavor QCD simulations at $2m_\pi<m_\rho$
- how to treat resonances or scattering in a finite box?

Basic Idea

Finite size formula: Lüscher 91,86

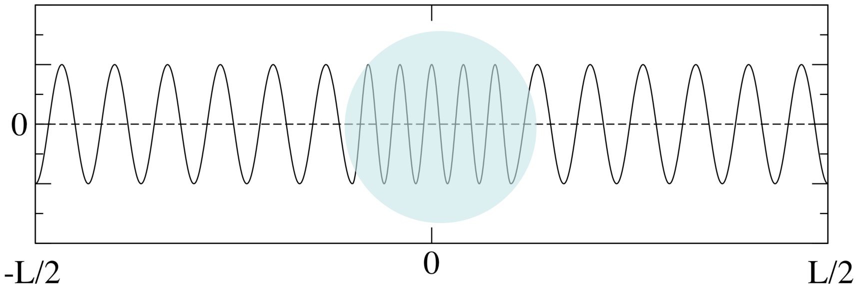
energy shift due to interaction in a finite box ⇒ phase shift

ex. 1-dim. Schrödinger wave function ψ(x,y)=f(x-y) with periodic BC



free

$$p_n = \frac{2\pi}{L} n \quad n \in Z$$



interacting

$$\exp\left(2i\delta(k)\right)\exp\left(ikL\right) = 1$$

$$k(\neq p_n) \Leftrightarrow \delta(k)$$

Strategy

1). Choose ππ kinematics such that $\sqrt{s} \sim m_\rho$

2). Calculate correlation matrix → extract energy eigen values

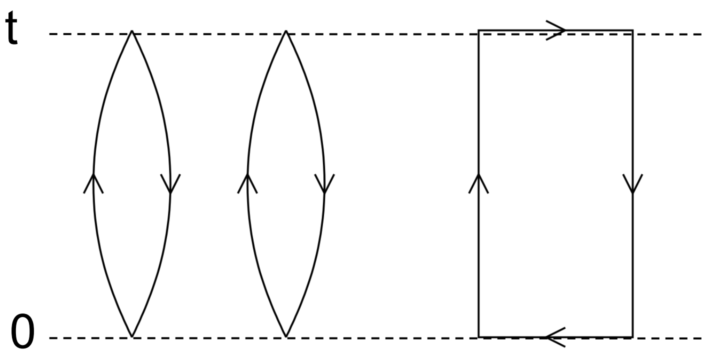
$$\begin{pmatrix} \langle \mathcal{O}_{\pi\pi}(t)\mathcal{O}^\dagger_{\pi\pi}(0)\rangle & \langle \mathcal{O}_{\pi\pi}(t)\mathcal{O}^\dagger_{\rho}(0)\rangle \\ \langle \mathcal{O}_{\rho}(t)\mathcal{O}^\dagger_{\pi\pi}(0)\rangle & \langle \mathcal{O}_{\rho}(t)\mathcal{O}^\dagger_{\rho}(0)\rangle \end{pmatrix}$$

$$\xrightarrow{\text{diagonalize}} \begin{pmatrix} C_1 \exp(-W_1(k_1)t) & 0 \\ 0 & C_2 \exp(-W_2(k_2)t) \end{pmatrix}$$

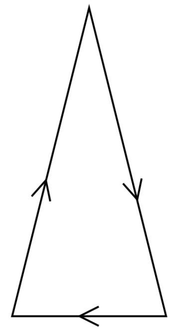
3). Momentum k_i → phase shift $\delta(k_i)$ by Lüscher's formula

Correlation Matrix with Quark Diagram

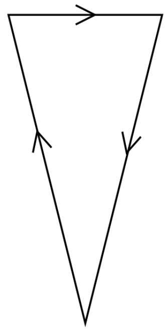
$$\langle \mathcal{O}_{\pi\pi}(t)\mathcal{O}^\dagger_{\pi\pi}(0)\rangle$$



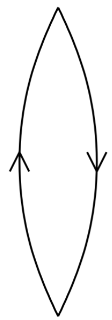
$$\langle \mathcal{O}_{\rho}(t)\mathcal{O}^\dagger_{\pi\pi}(0)\rangle$$



$$\langle \mathcal{O}_{\pi\pi}(t)\mathcal{O}^\dagger_{\rho}(0)\rangle$$



$$\langle \mathcal{O}_{\rho}(t)\mathcal{O}^\dagger_{\rho}(0)\rangle$$

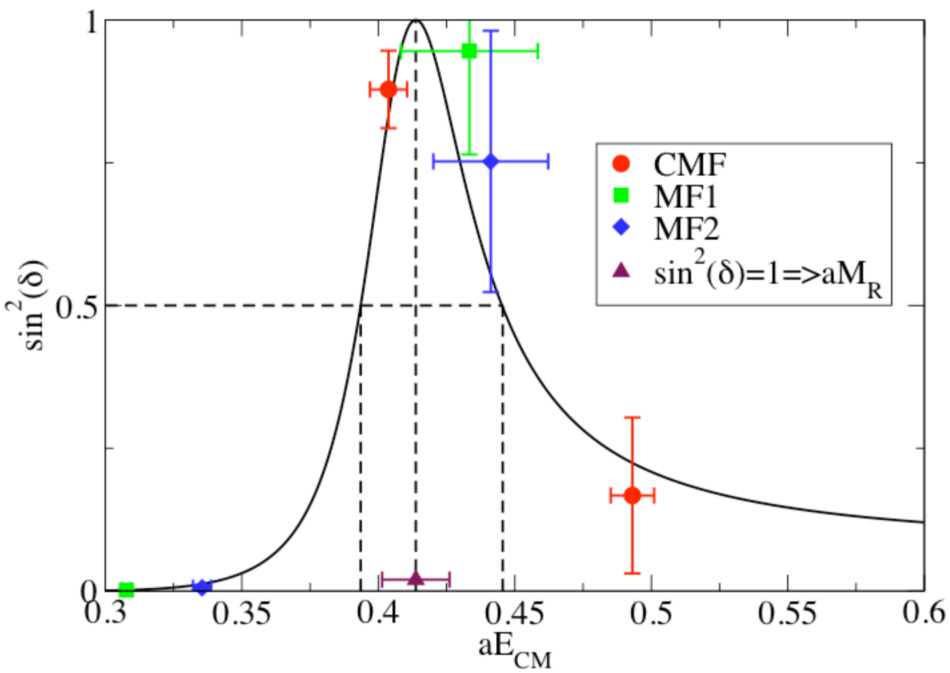


more complicated, more difficult

Decay Width from δ(k)

hard to accumulate data at different kinematics

⇒ some assumption for fit formula



ETMC 10,@lat10

2 flavor, m_{π}=309MeV

Effective ρ→ππ Coupling Constant $g_{\rho\pi\pi}$

Simple phenomenological description of ρ→ππ decay by

$$\mathcal{L}_{\rm int} = g_{\rho\pi\pi}\epsilon_{abc}\rho^a_\mu \pi^b \partial^\mu \pi^c$$

Phase shift δ(k) in terms of $g_{\rho\pi\pi}$

$$\tan\delta(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{\sqrt{s}(m_\rho^2 - s)} \qquad (2k)^2 = s - (2m_\pi)^2$$

$g_{\rho\pi\pi}$, m_ρ are fit parameters ⇒ fig in previous slide

"physical value" of $g_{\rho\pi\pi}$

$$\Gamma^{\rm ph} = \frac{(g^{\rm ph}_{\rho\pi\pi})^2}{6\pi}\frac{(k^{\rm ph})^3}{(m^{\rm ph}_\rho)^2} = 149.1(8) \ \ \text{MeV} \ \Rightarrow \ g^{\rm ph}_{\rho\pi\pi} = 5.98(2)$$

Current Status

Only recently 2 or 2+1 flavor QCD simulations at $2m_\pi < m_\rho$ make the investigation possible

$$\boxed{g_{\rho\pi\pi}^{\rm ph} = 5.98(2)}$$

group	year	#flavor	m_π [MeV]	$g_{\rho\pi\pi}$
CP-PACS	07	2	330	6.25(67)
QCDSF	lat08	2	240～810	5.3(+2.1)(-1.5)
ETMC	10,lat10	2	290～480	6～7
PACS-CS	lat10	2+1	410	5.24(51)
BMW	lat10	2+1	200,340	5.5(2.9),6.6(3.4)

§4. Nuclei from lattice QCD

Previous studies of multi-nucleon system

- $\Lambda\Lambda$ system ⇒ H dibaryon: unbound
  Mackenzie-Thacker 85, Iwasaki et al. 88, Pochinski et al. 99
  Wetzorke et al. 00, Wetzorke-Karsch 02

- NN system ⇒ Deuteron: unbound
  Fukugita et al. 95, NPLQCD 06, Ishii et al. 08, NPLQCD 09

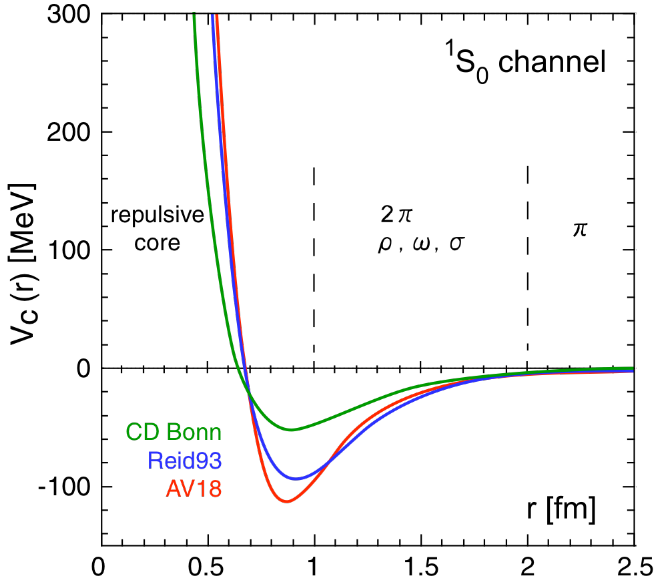
- NNN system ⇒ Triton: unbound
  NPLQCD 09

What about nuclear force?

attractive potential with repulsive core is reproduced?

Nuclear Force

phenomenological model

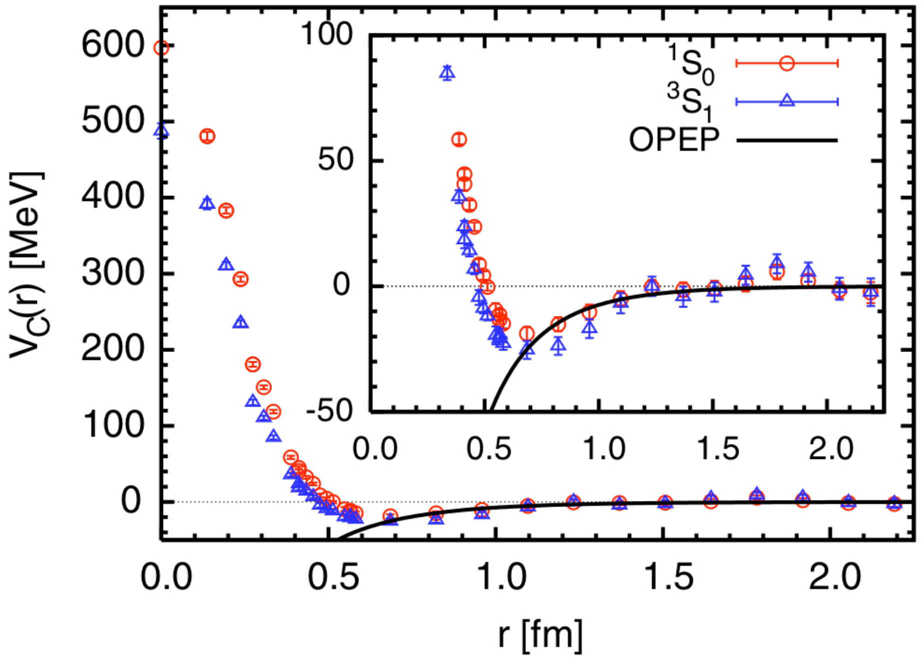


Ishii-Aoki-Hatsuda 07

based on equal-time BS amplitude

$$\varphi_E(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|6q,E\rangle$$

quench, m_N=1.34GeV



Qualitative features are reproduced from first principles

Recent Developments

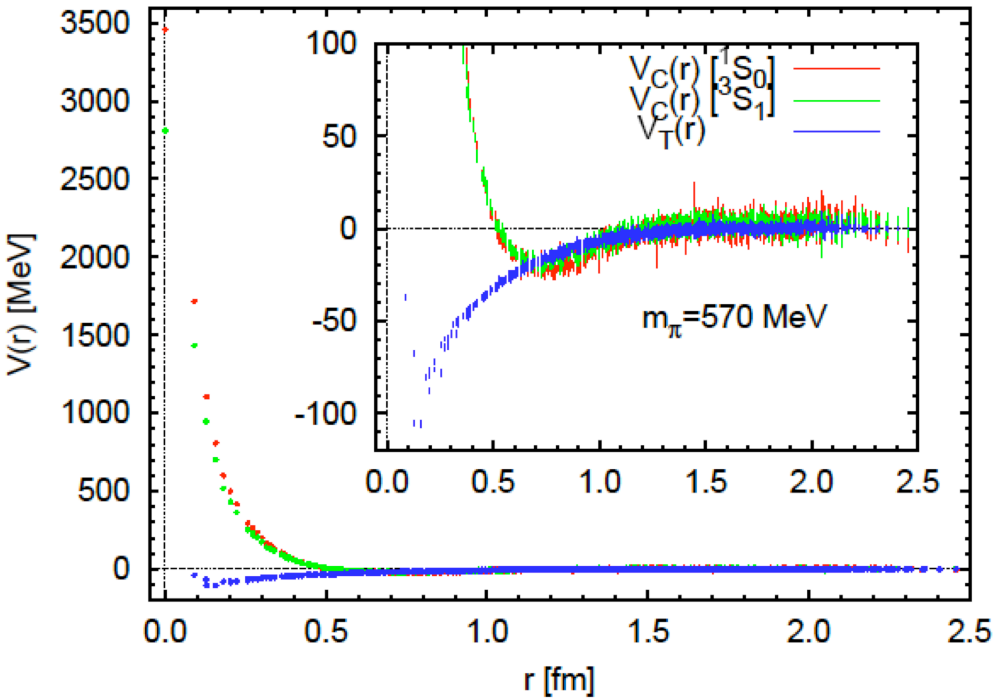
Aoki[338]

Measurements in 2+1 flavor QCD

m_N=1.43GeV

$V_C(r)$: center potential
$V_T(r)$: tensor potential

$$V_C(r) + V_T(r)\left[\frac{3}{r^2}(\sigma_1\cdot\vec{r})(\sigma_2\cdot\vec{r}) - (\sigma_1\cdot\sigma_2)\right]$$



no repulsive core in $V_T(r)$

⇒ lighter quark masses, YN and YY interactions

Direct Construction of Nuclei on the Lattice

Exploratory study for ^{4}He and ^{3}He nuclei

Yamazaki-YK-Ukawa 10

large binding energy ΔE_{4He}=28.3 MeV

^{4}He has double magic numbers (Z=2,N=2)

Difficulties in multi-nucleon systems

(1) No. of Wick contractions

(2) how to distinguish bound state from scattering state?

Wick Contractions

He nucleus correlator in terms of quark fields

$$\langle \mathcal{O}_{^4\mathrm{He}}(t)\mathcal{O}^\dagger_{^4\mathrm{He}}(0)\rangle \overset{t\gg 0}{\approx} C\exp\left(-m_{^4\mathrm{He}}t\right) \qquad \Delta E_{^4\mathrm{He}} = E_{^4\mathrm{He}} - 4E_N$$

^{4}He operator consists of two protons (udu) and two neutrons (dud)

⇒ No. of Wick contraction: $N_u! \times N_d! = (2N_p+N_n)! \times (2N_n+N_p)!$

^{4}He: $6! \times 6! = 518400$

^{3}He: $5! \times 4! = 2880$

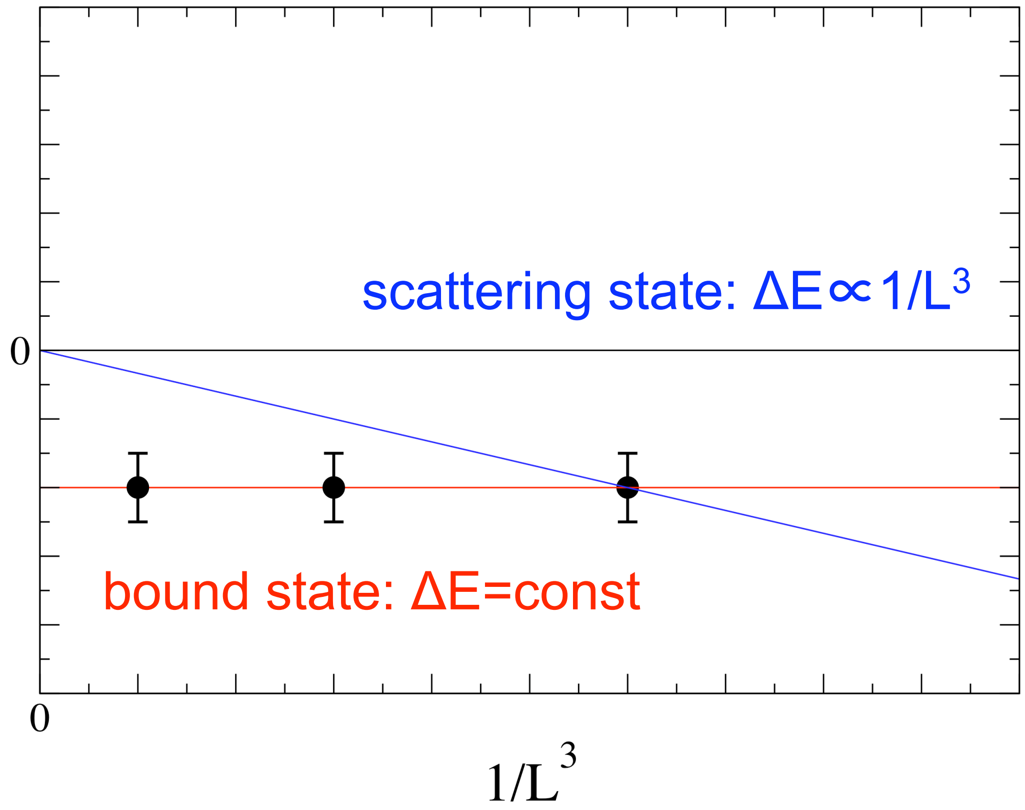
cf. N-N: $3! \times 3! = 36$

^{12}C: $18! \times 18! \sim 4 \times 10^{31}$

independent quark diagrams are reduced by imposing $m_u = m_d$

Identification of Bound State in a Finite Box

ΔE<0 both for bound state and attractive scattering state

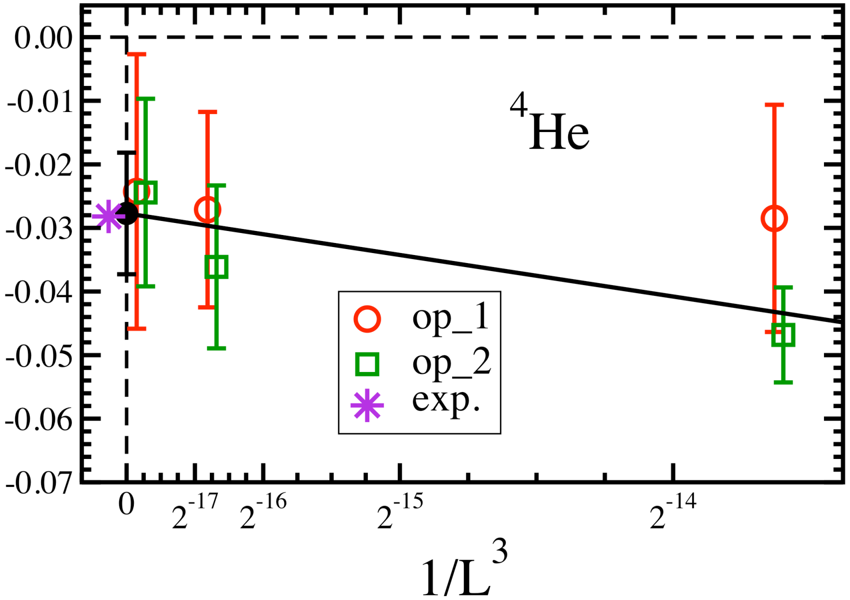


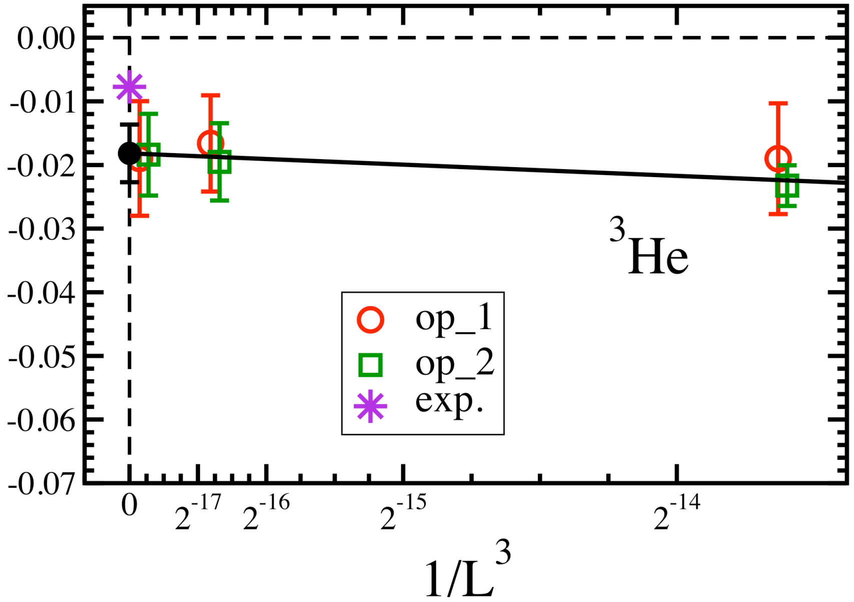
mandatory to check volume dependence of ΔE

Volume dependence of ΔE_{4He}

Yamazaki-YK-Ukawa 10

Exploratory study with m_N=1.6 GeV in quenched QCD





same order to experimental values

§5. Summary

- 2+1 ($m_u = m_d \neq m_s$) flavor simulation near the physical point
  ⇒ 1+1+1 ($m_u \neq m_d \neq m_s$) flavor simulation at the physical point

- Realistic study of resonances
  ⇒ tool to investigate new quark composite states (tetraquark etc.)

- Direct construction of helium nuclei in lattice QCD
  ⇒ physics of nuclei based on first principles