Update to the Bodek-Yang Unified Model for Electron- and Neutrino- Nucleon Scattering Cross sections

Arie Bodek
University of Rochester
Un-ki Yang
University of Manchester

Saturday July 24 15:00-15:30
Neutrino Session
Salle Maillot, ICHEP 2010 Paris, France
Describe DIS, resonance, even photo-production ($Q^2=0$) in terms of quark-parton model. With PDFS, it is straightforward to convert charged-lepton scattering cross sections into neutrino cross section.

**Challenge:**
- Understanding of high $x$ PDFs at very low $Q^2$?
- Understanding of resonance scattering in terms of quark-parton model?
- What happens near $Q^2=0$?

Bodek-Yang LO approach (pseudo NNLO)
Use effective LO PDFs with a new scaling variable, $\xi_w$ to absorb target mass, higher twist, missing QCD higher orders

+ add low $Q^2$ K factors

$$
\xi_w = \frac{Q^2 + m_f^2 + O(m_f^2 - m_i^2) + A}{m_N (1 + (1 + Q^2 / y^2)^{1/2} + B)} \rightarrow Xbj = \frac{Q^2}{2 M_N}
$$

Bodek-Yang – 2010
1. Start with GRV98 LO ($Q^2_{min} = 0.80 \text{ GeV}^2$)
   - dashed line- describe $F_2$ data at high $Q^2$
2. Replace the $X_{bj}$ with a new scaling, $\xi_w$
3. Multiply all PDFs by $K$ factors for photo prod.
   limit and higher twist
   
   \[ \frac{\sigma(\gamma)}{4\pi\alpha} = \frac{F_2(x, Q^2)}{Q^2 + C_{sea}} \]
   
   \[ K_{sea} = \frac{Q^2}{Q^2 + C_{sea}} \]
   
   \[ K_{val} = \frac{(1 - G_{D}^2(Q^2))}{Q^2 + C_{2V}} \]
   
   \[ \frac{Q^2 + C_{1V}}{Q^2 + C_{1V}} \]
   
   motivated by Adler Sum rule
   where $G_{D}^2(Q^2) = \frac{1}{1 + \frac{Q^2}{0.71}}$
4. Freeze the evolution at $Q^2 = Q^2_{min}$
   - $F_2(x, Q^2 < 0.8) = K(Q^2) \times F_2(X_w, Q^2 = 0.8)$

- Fit to all DIS $F_2$ P/D (with low $x$ HERA data)
  $A = 0.418, B = 0.222$

\[ C_{sea} = 0.381, C_{1V} = 0.604, C_{2V} = 0.485 \]
\[ \chi^2/DOF = 1268 / 1200 \quad \text{Solid Line} \]

**A**: initial state binding/TM+ higher order QCD

**B**: final state mass $m_f^2$, $\Delta m^2$.

**K Factor**: Photo-prod limit ($Q^2 = 0$), Adler sum-rule

**For 2xF1 use R1998**

Fit only precise charged lepton scattering data.
No neutrino data and No Resonance data included in the fit.

**2004 update:**
Separate $K$ factors for uv, dv,us,ds

**For XF3**: we assumed $V=A$ in 2003-2004

\[ X_{bj} = \frac{Q^2}{2 M_{\nu}} \]

\[ \xi_w = \frac{Q^2 + m_f^2 + \alpha(m_f^2 - m_i^2) + A}{M_{\nu} (1 + (1 + Q^2/\nu^2)^{1/2}) + B} \]
Fit results GRV98 + B-Y 2004 (SLAC, BCDMS, NMC) H + D

Proton experiment data fit

Deuteron experiment data fit

F2 proton  Bodek-Yang – 2010  F2 deuterium
Resonance F2 proton

Resonance F2 deuterium

Fit works on resonance region - Resonance data are not included in the fit!!!
How model uses only H and D data. For lepton/muon cross sections on nuclear targets - need to correct for Nuclear Effects measured in e/muon expt. Use also for neutrino expt. (Note nuclear effects can be different for neutrinos)

Comparison of Fe/D F2 data
In resonance region (JLAB)
Versus DIS SLAC/NMC data
In $\xi_T$ (C. Keppel 2002).

Figure 5. The ratio of $F_2$ data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments (SLAC, NMC, E665). The band shows the uncertainty of the parametrized curve from the statistical and systematic errors in the experimental data [16].
The BY model is used for the inelastic part of the cross section.

People use the 2004 Bodek-Yang model above $W=1.8$ and other other models for quasielastic, the delta, and the 1520 resonance region and match to Bodek-Yang in the $W=1.8$ region.

- Find that predicted total neutrino and antineutrino cross sections are lower than high energy measurements (5%).
- The antineutrino to neutrino ratio is also a little low.

2009 updates:

1. Correct for the difference in scaling violations for $F2$ and $XF3$
2. Correct for Vector is not equal to Axial at low $Q2$.
3. Make model work down to $W=1.1$ to provide smoother matching

For higher precision:

Need to apply radiative corrections to $F2$

Need to include $c$-$c\overline{b}ar$ sea high energy (no $c$-$c\overline{b}ar$ sea in GRV98).

Need to account for the differences in nuclear effects between electrons and neutrinos.
2009 Update 1: $H(x) = \text{NLO Correction to } xF_3$

- Scaling variable, $\xi w$ absorbs higher order QCD and higher twist in $F_2$, but $xF_3$ may be different.
  
  ($F_2$ data was used in the fitting our corrections to leading order PDF)

- 1st Update: Use double ratio correction $H(x)$ from QCD

\[
\frac{xF_3(\text{NLO})}{xF_3(\text{LO})} \bigg/ \frac{F_2(\text{NLO})}{F_2(\text{LO})}
\]

$\Rightarrow$ not 1 but indep. of $Q^2$

NLO ratio: using VFS
• Parameterized $x F_3$ correction
  as a function of $x = H(x)$
• Neutrino cross section down by 1%
• Anti-neutrino cross section up by 3%
In our neutrino previous cross section model we assumed $K_{\text{axial}}(Q^2) = K_{\text{vector}}(Q^2)$. This is only true for free quarks (which is a correct assumption for $Q^2 > 0.5 \text{ GeV}^2$).

However: We expect that axial-vector is not suppressed at $Q^2=0$.

- **2009 Update 2**: $K_{\text{axial}} = 1$ as a first try.
Axial-contribution

CCFR diff. cross at $E_\nu = 55$ GeV

$K_{axial} = Q^2/(Q^2 + C)$

Black line GRV98; red line with B-Y 2004 and $K_{axial} = K_{vector}$, blue $K_{axial} = 1$

better

Bodek-Yang - 2010
$\sigma(\gamma\text{-proton}) = 4\pi\alpha/Q^2 \times F_2(\xi_w, Q^2)$

where $F_2(\xi_w, Q^2) = Q^2 / (Q^2 + C) \times F_2(\xi_w)$

Update 3: In resonance region, duality works down to $Q^2=0.5 \text{ GeV}^2$, but breaks down at $Q^2=0$.
Not important for the Vector part, since $Q^2=0$ contributes zero to the vector part of the neutrino cross section.

Proton low $Q$ electroproduction

Update 3: Improve the model so that it is also valid in the resonance region at $Q^2=0$.
We will fix it by applying a low Ehad K factor.
Important for axial part

Proton Q2=0 photoproduction

Deuteron Q2=0 photo

Photo-production (Deuteron)
\[ \sigma(\gamma\text{-deuteron}) = 4\pi\alpha/Q^2 \times F_2(\xi_w, Q^2) \]

where \( F_2(\xi_w, Q^2) = Q^2/(Q^2 + C) \times F_2(\xi_w) \)

Update (3): apply a low \( \nu \) (Ehad) K factor

\[ K(\nu) = (\nu^2 + C_{2\nu})/\nu^2 \]

Where \( C_{2\nu} = 0.20 \)

It makes duality work for resonance all the way to \( Q^2=0 \).

So vector part is now modeled everywhere including resonance region down to very low \( \nu \) and very low \( Q^2 \).

For a heavy nucleus, Fermi motion will smear all of the resonances.
Low Ehad K(ν) factor extends the validity of the model for electron scattering in the resonance region down to Q2=0

- Proton data
- (note that for nuclear targets the resonances will be smeared by Fermi Motion)

Photo-production Q2=0

Black line includes Low Ehad K factor
Red line no Low ν K factor

Electroproduction - proton
Summary and Discussions

- We updated our Effective LO model with $\xi_w$ and $K(Q^2)$ factors.
- (1) Updated to account for the difference in the higher order QCD corrections between $F_2$ and $X_F$. This is accounted for with a $H(x)$ factor. Therefore, the axial part is also well described for $Q^2 > 1$ GeV$^2$, where axial and vector are expected to be the same.
- (2) Updated to use $K_{\text{axial}}(Q^2)=1$ for both the resonance and inelastic continuum region. This is expected since we know that neutrino quasielastic and resonance production form factor are not zero at $Q^2=0$.
- The lowest $Q^2$ bins in the neutrino and antineutrino measured differential cross sections favor $K_{\text{axial}}(Q^2)=1$. The total cross section as measured in high energy neutrino scattering favors $K_{\text{axial}}(Q^2)=1$. --> To be investigated in MINERvA.
- (3) Updated to include a low Ehad factor $K(\nu)$ to describe all charged lepton inelastic continuum as well as all resonance data including photo-production data down to $Q^2=0$. The vector part of the neutrino cross section is now modeled very well. Note: By Gauge Invariance, the vector structure functions must go to zero at $Q^2=0$ for both resonances and inelastic continuum.
Backup-slides
Separate K factors for uv, dv, us, ds provided additional parameters. They provide separate tuning for H and D data, but are not important for Heavy nuclei.
Fit results GRV98 + B-Y 2004 muon scattering

NMC [Proton target]

NMC [Deuteron target]
All DIS $e/\mu F_2$ data are well described.

Photo-production data ($Q^2=0$) also work: thus included in the latest fit.

$2xF_1$ data (Jlab/SLAC) also work:
using $F_2(\xi_w)+R1998$

line GRV98 + B–Y 2004
Apply nuclear corrections using $e/m$ scattering data.

- Calculate $F_2$ and $xF_3$ from the modified PDFs with $\xi_w$.
- Use $R=\text{Rworld}$ fit to get $2xF_1$ from $F_2$.
- Implement charm mass effect through $\xi_w$ slow rescaling algorithm, for $F_2$, $2xF_1$, and $XF_3$.

Our model describes CCFR diff. cross sect. ($E_\nu=30-300$ GeV) well. Note that no neutrino data was included in fit. (However, let's look in more detail.)
Comparison with updated model (assume $V=A$)

CCFR Fe data/ (GRV98 + B-Y 2004)

Model underestimates neutrino data at lowest $x$ bin. At high energy, some may be from missing radiative corrections and $c\bar{c}$ contribution.
Model underestimates neutrino data at lowest x bin. At high energy, some may be from missing radiative corrections and c-cbar contribution.
Model underestimates antineutrino data at lowest x bin — also lowest Q2. At high energy, some may be from missing radiative corrections and c-cbar contribution.
How is the model be used

- Duality is not expected to work for quasielastic or the delta. This is because these cross section have definite isospin final states. Therefore PDFs will not give the correct ratio of neutrino vs antineutrino and proton versus neutron scattering for quasielastic and delta production.
- Duality should work in the region of higher resonances since these regions include several resonances with different isospins.
- People use the 2004 Bodek-Yang model above $W=1.8$
- They used other models for quasielastic, the delta, and the 1520 resonance region and matched them to Bodek-Yang in the $W=1.8$ region.
- 2009 -> make it work in the delta region
Total cross sections
Bodek-Yang 2004 used above $W=1.8$ AND
matched to resonance and quaselastic models.

Find that predicted total neutrino and antineutrino cross sections
are lower than high energy measurements (5%). The antineutrino
to neutrino ratio is also low.

Some may come from the need to apply radiative corrections and include the c-cbar sea at
very high energy (no c-cbar sea in GRV98). Some may be differences in nuclear effects
between electrons and neutrinos—But is this all?

Total Cross Sections

$$r = \frac{\sigma^\nu}{\sigma^{\bar{\nu}}}$$
Note: GRV98 + B-Y 2004 is for free nucleons (H+D). Electron and muon data are corrected for radiative corrections. In addition, GRV98 has no charm sea.

Published neutrino differential cross sections:

1. Have no radiative corrections
2. Are on nuclear targets
3. Have contributions from XF3 and include both axial and vector contributions.
4. Some are at very high energy which include a contribution from the charm sea.

In order to compare to neutrino data:

1. We need to account for difference in the scaling violations in XF3 and F2 (2009 update 1)
2. We need to make duality work in the resonance region at very low Q2 if we want to match to the resonance region, (2009 update 2)
3. We need to account for difference in axial and vector structure functions at low Q2 (2009 update 3)
4. We need to apply an X dependent nuclear correction to convert from H-D to nuclei.
   1. However, nuclear effects may be different for muons and neutrinos, different for axial versus vector, different for F2, XF3 (will be studied in MINERvA)
   2. We should add radiation to GRV98 + B-Y 2004 (or radiatively correct the neutrino data) - not done
   3. We should add charm sea contribution at very large energy (not done)
CCFR diff. cross at $E\nu = 35 \text{ GeV}$

$K_{\text{axial}} = K_{\text{vector}} = \frac{Q^2}{Q^2 + C}$

$K_{\text{axial}} = 1$

Black line GRV98; red line with B-Y 2004

And $K_{\text{axial}} = K_{\text{vector}}$, blue $K_{\text{axial}} = 1$
• Use Kaxial (Q2)=1 for now, but it will be tuned further in the future.
• We can tune the axial vector K factor by including low Q2 neutrino and antineutrino differential cross sections in the fit

However, the electron data has been radiatively corrected. A proper comparison to neutrino differential cross sections needs to include both radiative corrections and the c-cbar contribution at high energies (which are not included in the GRV98 PDFs). And what about the nuclear effects?

We plan tune Kaxial (Q2) to get better agreement with the neutrino and antineutrino measured total cross sections (Here we need to separately add the quasielastic, delta and c-cbar contributions, (but no need to include the radiative corrections since these integrate away in the total cross section). We will have this comparison soon

• In the future more detailed information on the axial form factor would come from MINERvA: by combining JUPITER Jlab (e-N vector) with the MINERvA (neutrino-N vector+axial) data.
• There could be different nuclear effects (e vs ν), F2 vs xF3, and for axial F2 versus vector F2. This will also be studied in MINERvA