Hadronic contribution to g-2 from lattice QCD

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Muon g-2

- muon $g-2$: high precision theory, exp. and BSM implications

$$a_\mu = \frac{g_\mu - 2}{2}$$

- radiative corrections cause g to differ from 2

$$a_\mu = \frac{\alpha}{2\pi} + \alpha^2 A_{\text{pt}} + \alpha^2 A_{\text{had}} + \mathcal{O}(\alpha^3)$$

- leading-order hadronic contribution dominates theory uncertainty

$$a_\mu^{\text{had}} \equiv \alpha^2 A_{\text{had}}$$

- first lattice QCD calculation to control most major sources of error

Theory and Experimental Precision

- experimental measurement at BNL [Muon G-2, PRD73:072003, 2006]

$$a_\mu^{\text{ex}} = 11659208.0(6.3) \times 10^{-10} \text{ [0.54 ppm]}$$

- standard model "prediction" [Jegerlehner, Nyffeler Phys.Rept.477, 2009]

$$a_\mu^{\text{th}} = 11659179.0(6.5) \times 10^{-10} \text{ [0.58 ppm]}$$

- discrepancy between theory and experiment

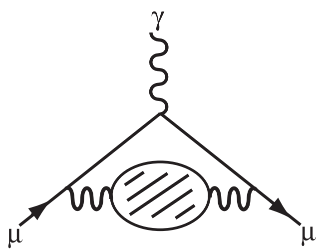
$$a_\mu^{\text{ex}} - a_\mu^{\text{th}} = 29.0(9.1) \times 10^{-10} \text{ [}3.2\,\sigma\text{]}$$

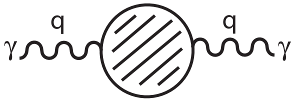
- leading-order hadronic contribution dominates theory error

$$a_\mu^{\text{had}} = 690.3(5.3) \times 10^{-10} \text{ [60\% of theory error]}$$

Lattice Calculation

- vacuum polarization by quarks or equivalently hadrons





- vacuum polarization function

$$\pi_{\mu\nu}(q^2) = \int d^4x\, e^{iq\cdot(x-y)} \langle J_\mu(x) J_\nu(y) \rangle = (q_\mu q_\nu - q^2 \delta_{\mu\nu}) \pi(q^2)$$

- leading-order hadronic contribution [Blum, PRL95:052001, 2003]

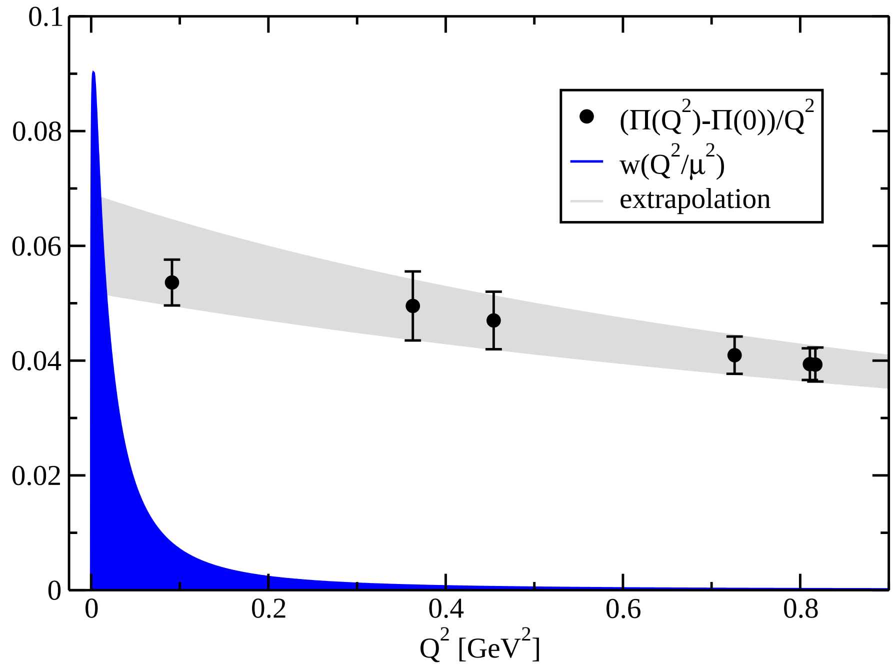
$$a_\mu^{\text{had}} = \frac{\alpha^2}{\pi^2} \int_0^\infty dQ^2 w(Q^2/m_\mu^2) \frac{(\pi(Q^2) - \pi(0))}{Q^2}$$

- w is known function of Q^2/m_μ^2 only

Dominant Challenge

- the leading-order hadronic contribution

$$a_\mu^{\text{had}} = \frac{\alpha^2}{\pi^2}\int_0^\infty dQ^2 w(Q^2/m_\mu^2)\frac{(\pi(Q^2)-\pi(0))}{Q^2}$$



- $w(Q^2/m_\mu^2)$ is maximum at $Q^2 = (\sqrt{5}-2)m_\mu^2 \approx 0.003$ GeV^2
- lowest momentum on lattice is $Q^2_{\text{min}} = (2\pi/T)^2 \approx 0.06$ GeV^2

- $N_F = 2$ maximally-twisted mass fermions from ETMC
- physical observables are accurate to $\mathcal{O}(a^2)$ at maximal twist

a [fm]	$L^3 \times T/a^4$	L [fm]	m_π [MeV]					
0.079	$20^3 \times 40$	1.6		350				
0.079	$24^3 \times 48$	1.9		340*	420*	480	520	650
0.079	$32^3 \times 64$	2.5	290	330*				
0.063	$24^3 \times 48$	1.5				450		
0.063	$32^3 \times 64$	2.0		330*		450*	520	

- disconnected diagrams included for 5 ensembles (denoted by $*$)
- we calculate with a quenched s, degenerate with u and d
- see [PoS LATTICE2008:129, (2008)] for more details

Q^2 Extrapolation

- must match to a smooth function in Q^2 to integrate

$$a_\mu^{\text{had}} = \frac{\alpha^2}{\pi^2}\int_0^\infty dQ^2 w(Q^2/m_\mu^2)\frac{(\pi(Q^2)-\pi(0))}{Q^2}$$

- analyticity suggests polynomials

$$\pi(Q^2) = \sum_n a_n (Q^2)^n$$

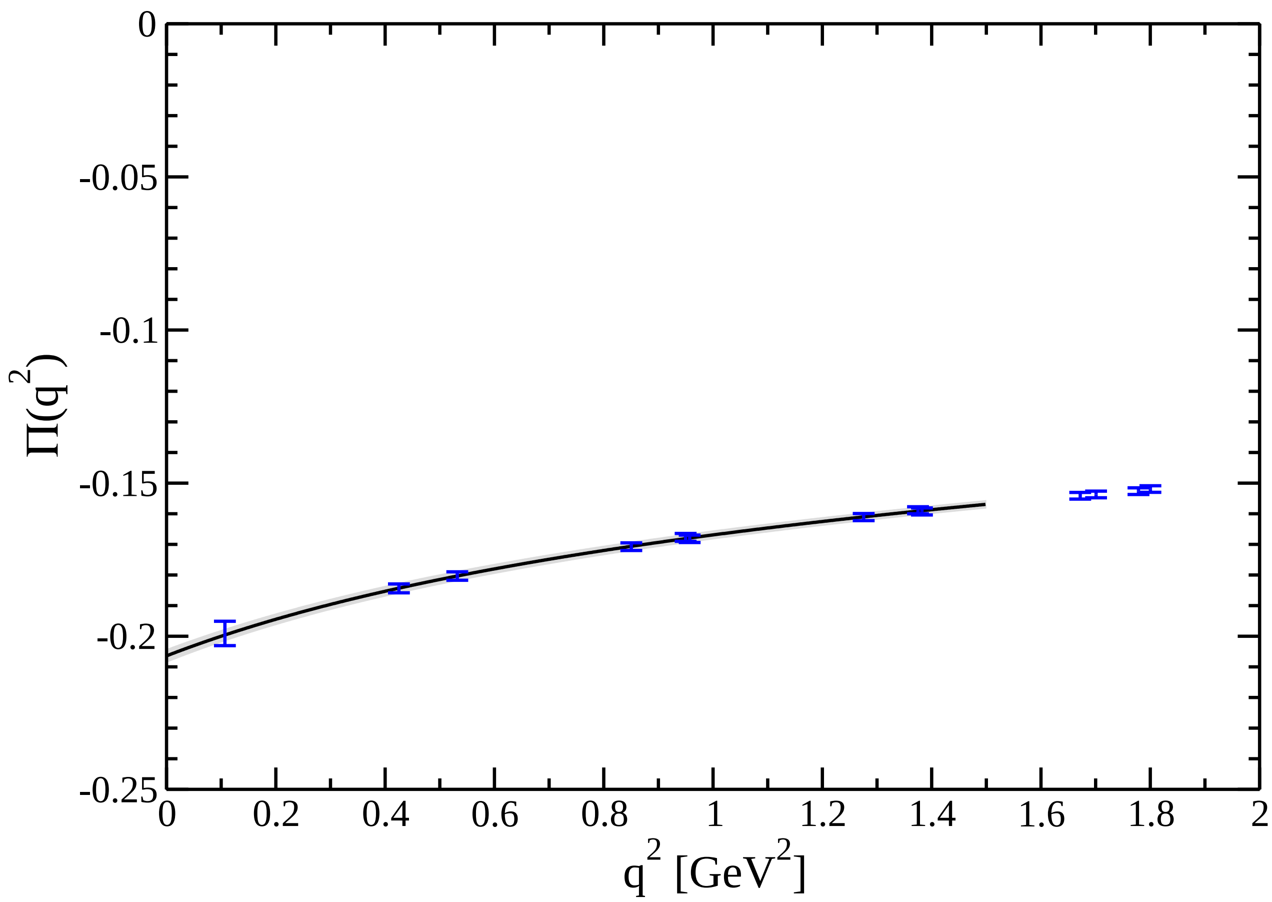
- physical models/chiral pert. theory suggestions ρ contribution

$$\pi(Q^2) = -\frac{f_v^2}{3}\left[\frac{3}{Q^2+m_\rho^2}+\frac{1}{Q^2+m_\omega^2}\right]$$

- systematics from functional choice are part of finite-size effect

Example Extrapolation

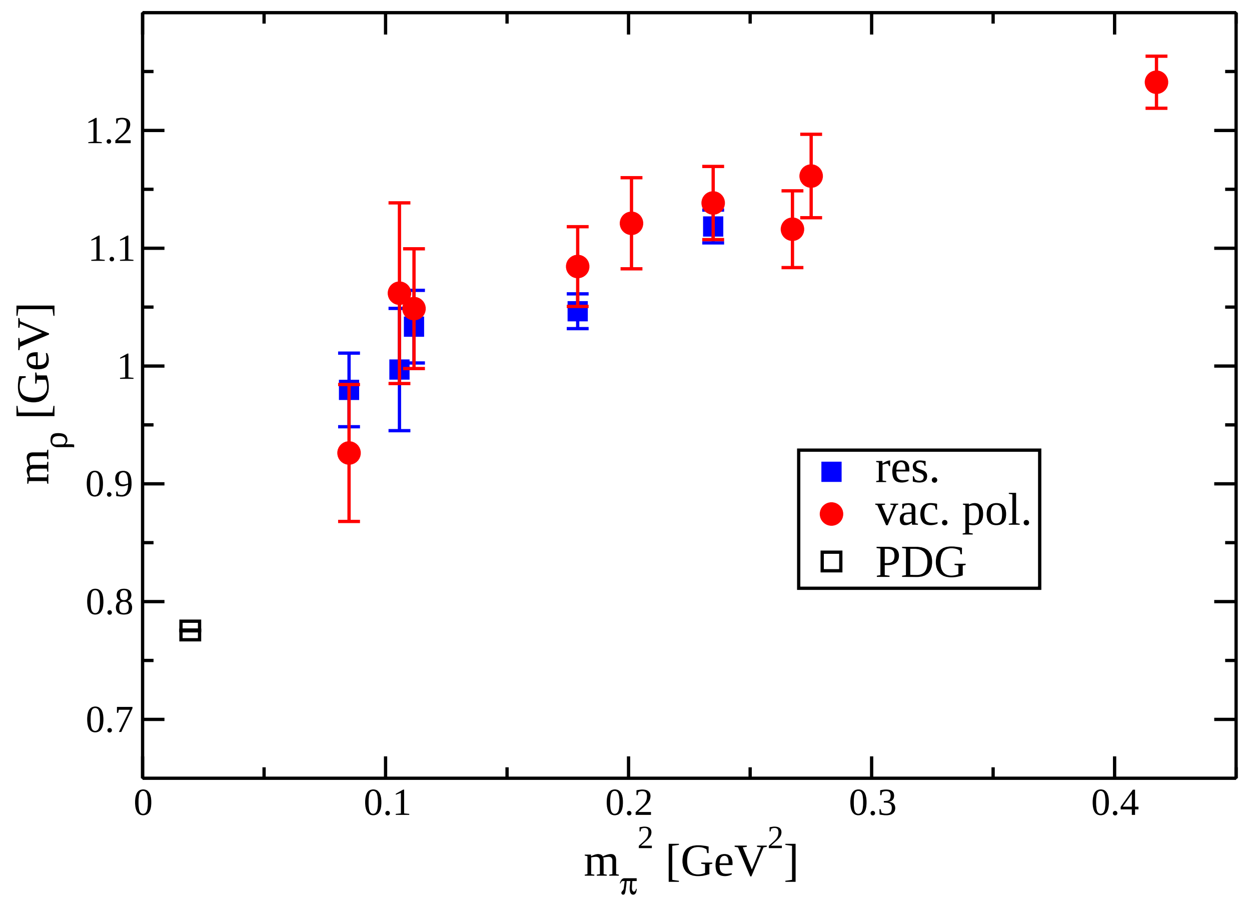
- example extrapolation of $\pi(Q^2)$ used to calculate a_μ^{had}



- one example of 12: $a = 0.079$ fm, $m_\pi = 420$ MeV, $L/a = 24$

Rho Decay

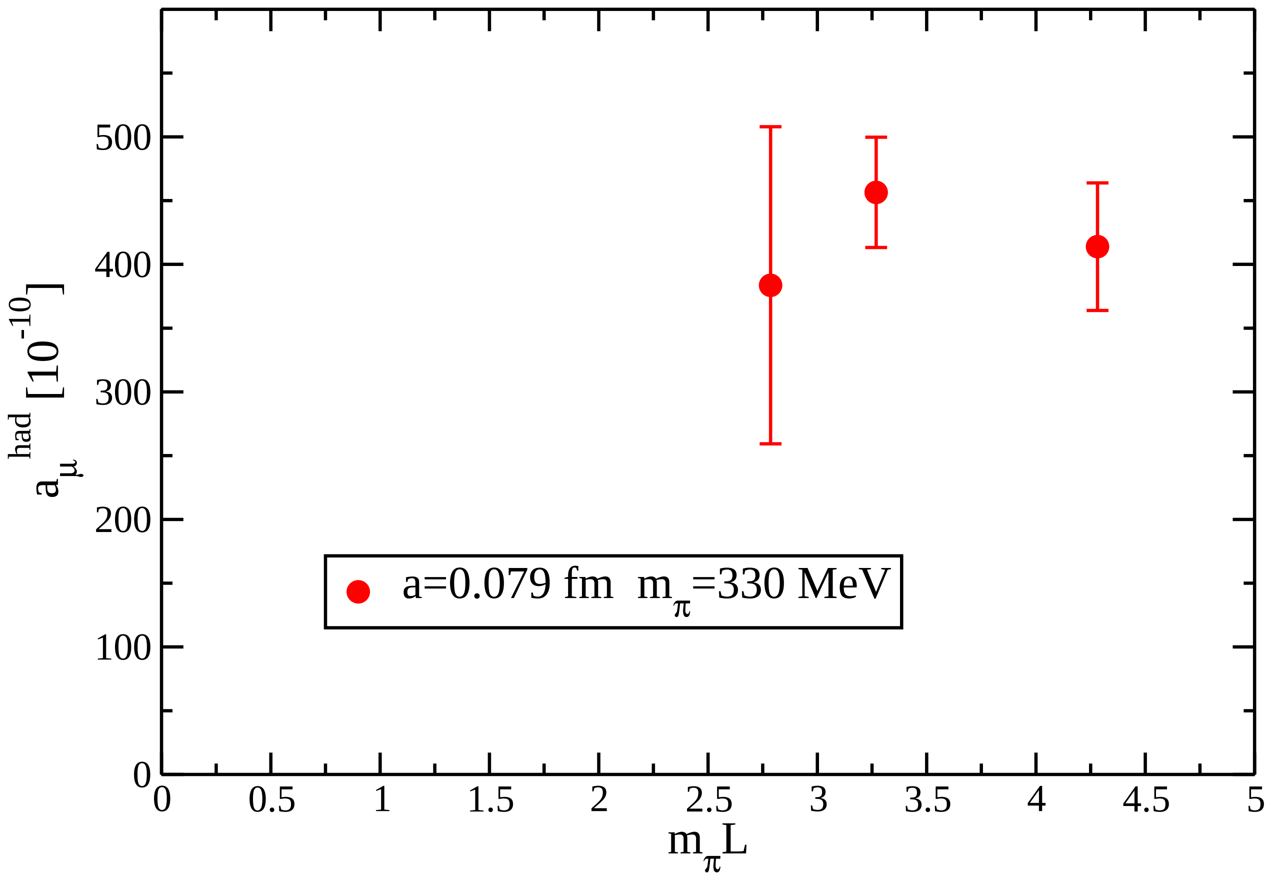
- m_ρ from $\pi(Q^2)$ agrees with resonance masses from X. Feng [LAT2010]



- similar agreement between m_ρ, f_ρ and standard ETMC calc.

Finite-Size Effects

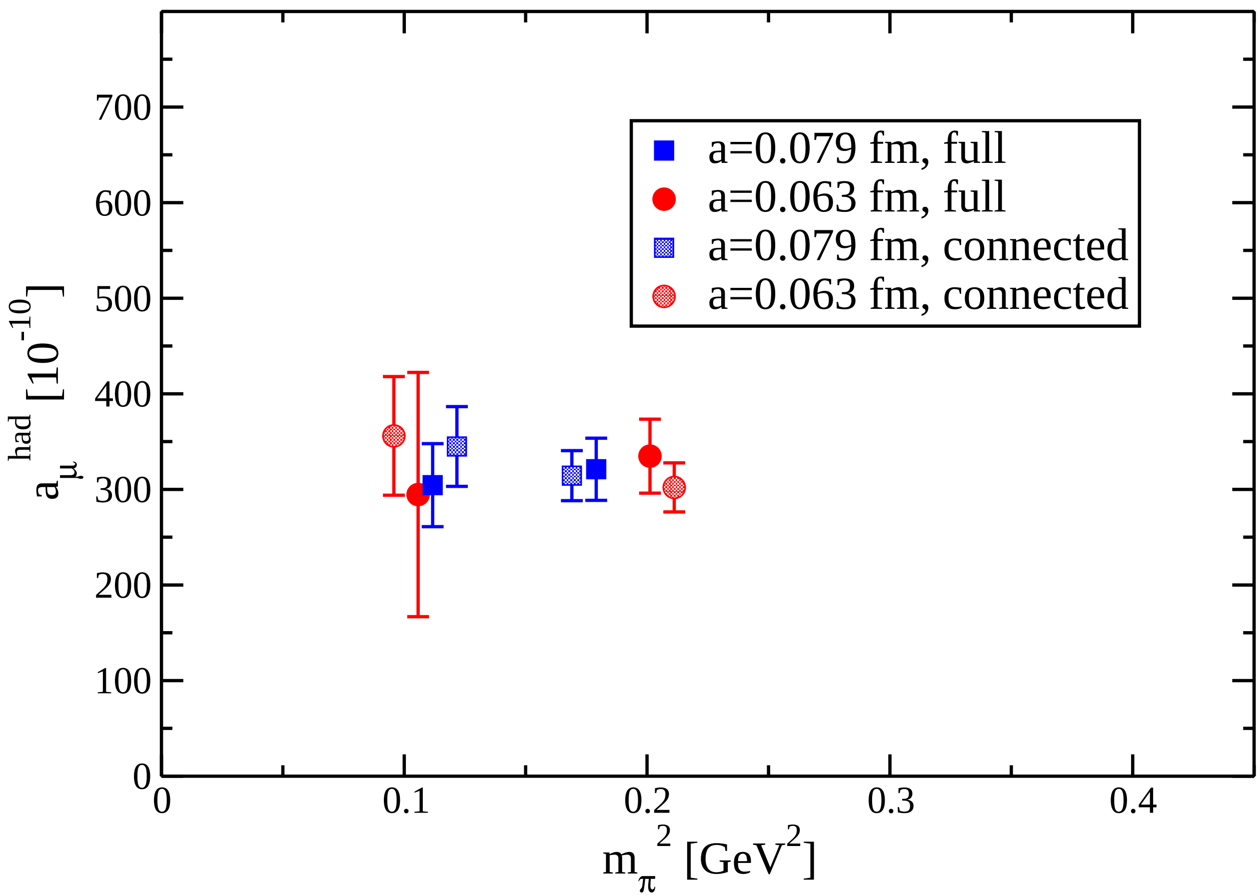
- no statistically significant finite-size effects seen



- same conclusion from vol. study with $m_\pi = 450$ Mev, $a = 0.063$ fm

Disconnected Contributions for $\mathrm{N_f = 2}$

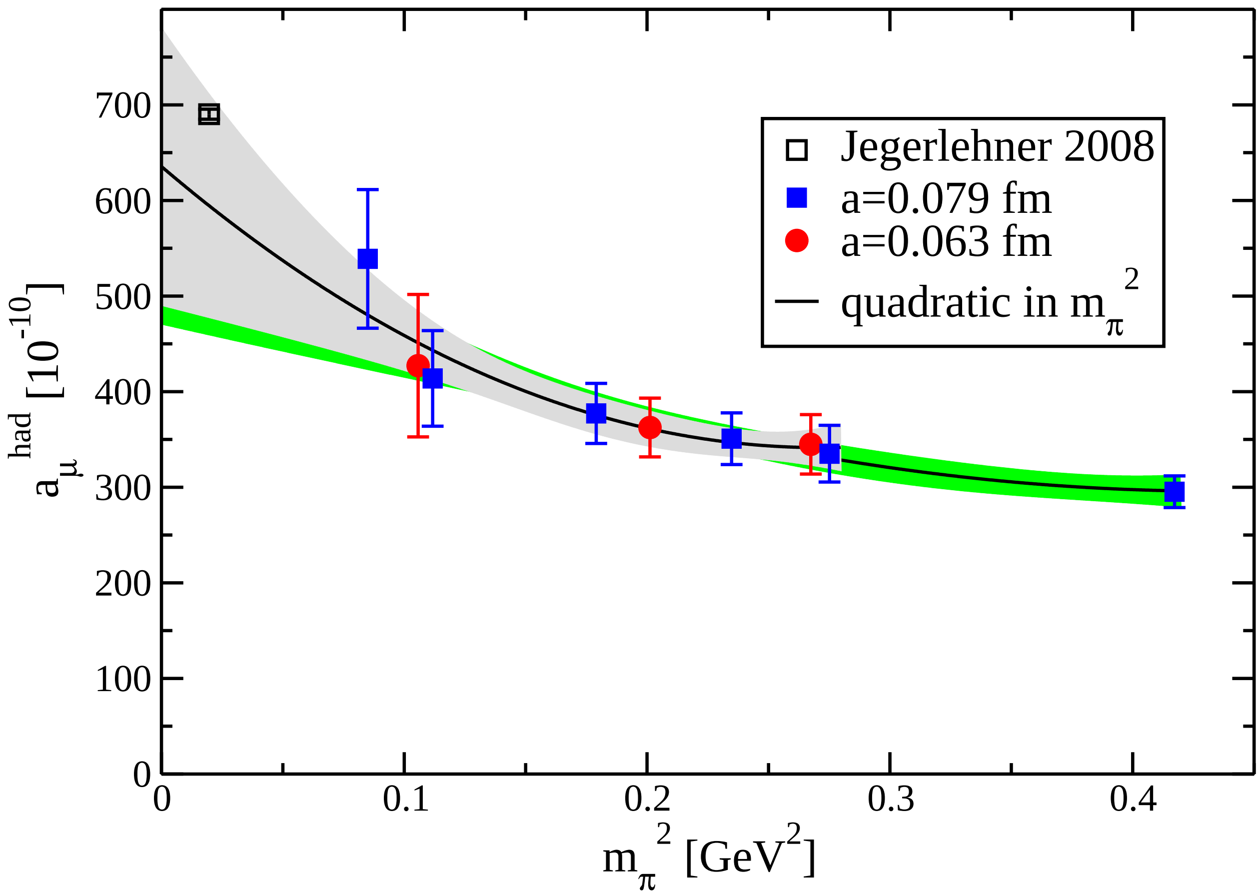
- shift is within 1σ of connected contribution



- disc. contribution increases noise but not insurmountable

Hadronic Contribution to a_μ

- finite-size effects and disc. contr. already shown to be within errors



- lattice artifacts are also apparently small compared to the errors

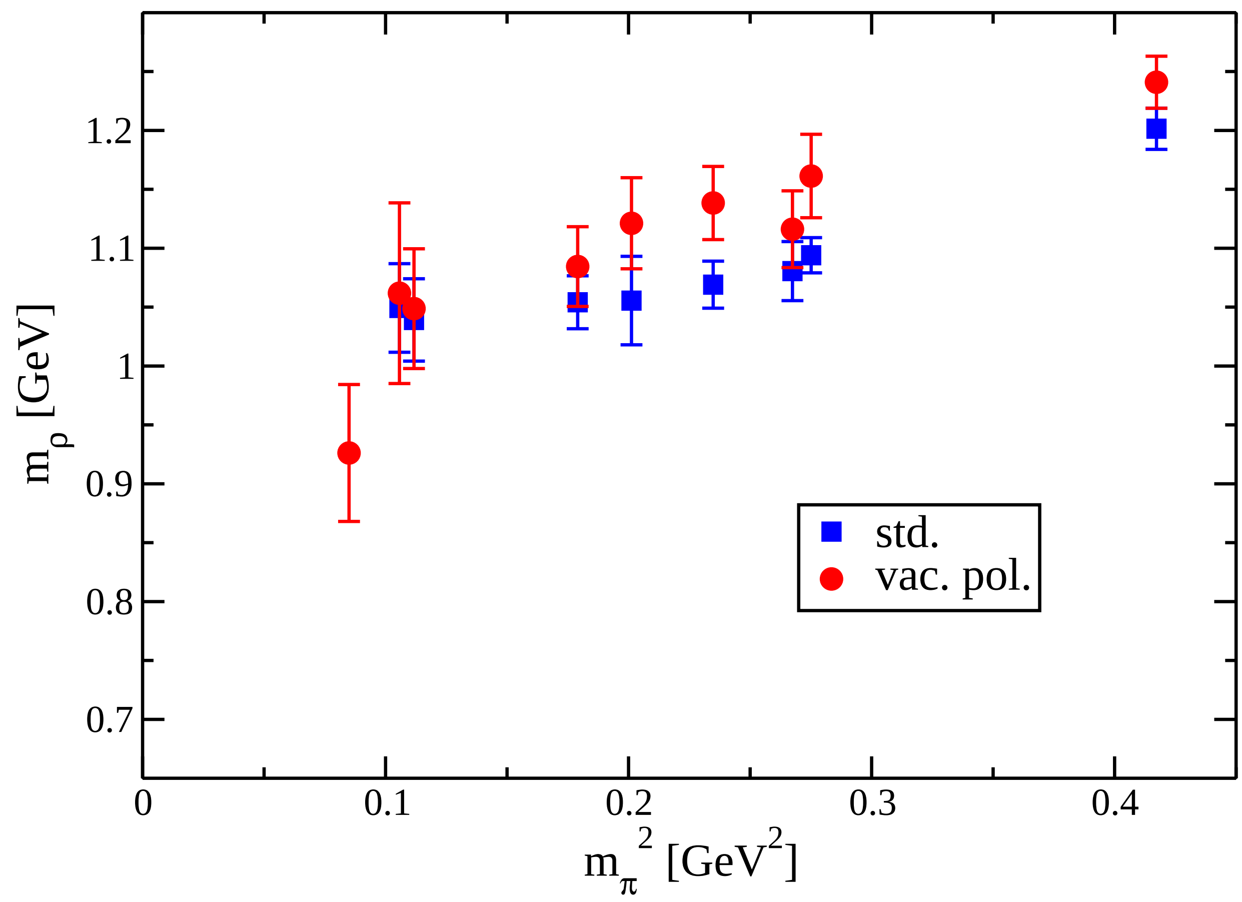
Conclusions

- calculated the leading-order hadronic contribution to muon $g-2$
- studied quark mass dependence from $m_\pi = 290$ to 650 MeV
- examined lattice artifacts, finite-size effects and discon. diagrams
- prelim. result $a_\mu^{\text{had}} = 595 \pm 120$ agrees with SM result 690 ± 5

Extra Slides

Rho Mass

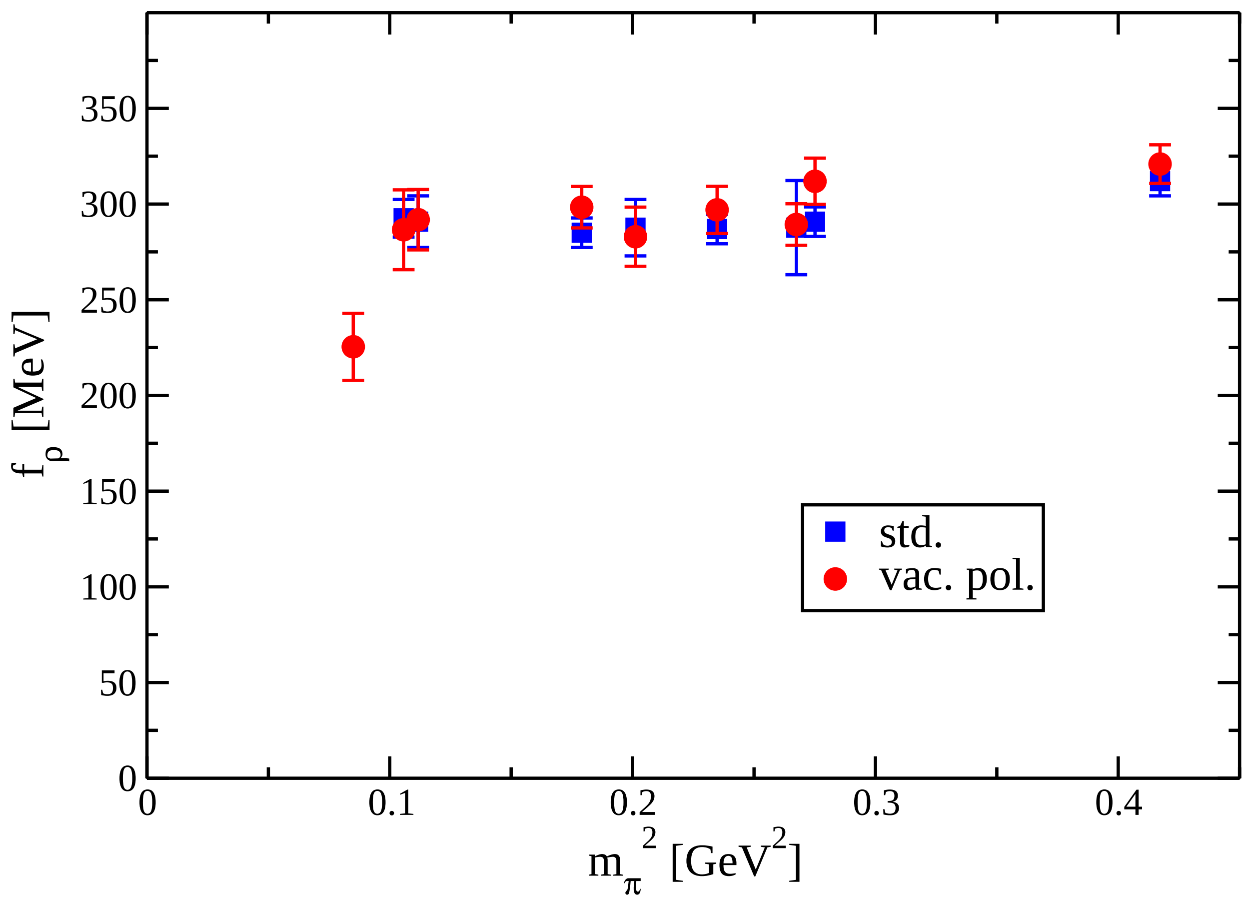
- calculate m_ρ from $\sum_i \langle J_i(\vec{q}=0,t) J_i(\vec{q}=0,0)\rangle$



- compare to standard calc. from ETMC [PRD80:054510, 2009]

Vector Decay Constant

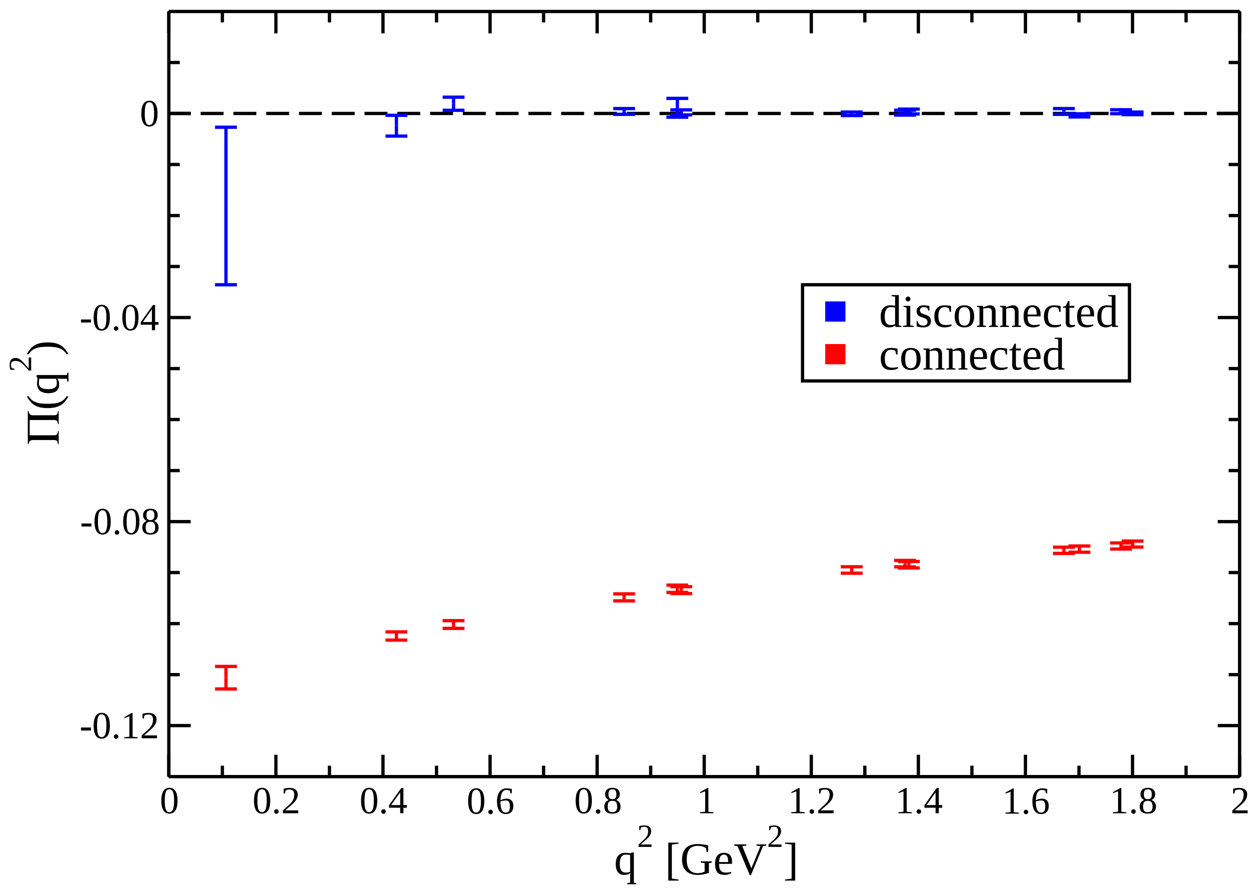
- calculate f_ρ from $\sum_i \langle J_i(\vec{q}=0,t) J_i(\vec{q}=0,0) \rangle$



- compare to standard calc. from ETMC [PRD80:054510, 2009]

Disconnected Example

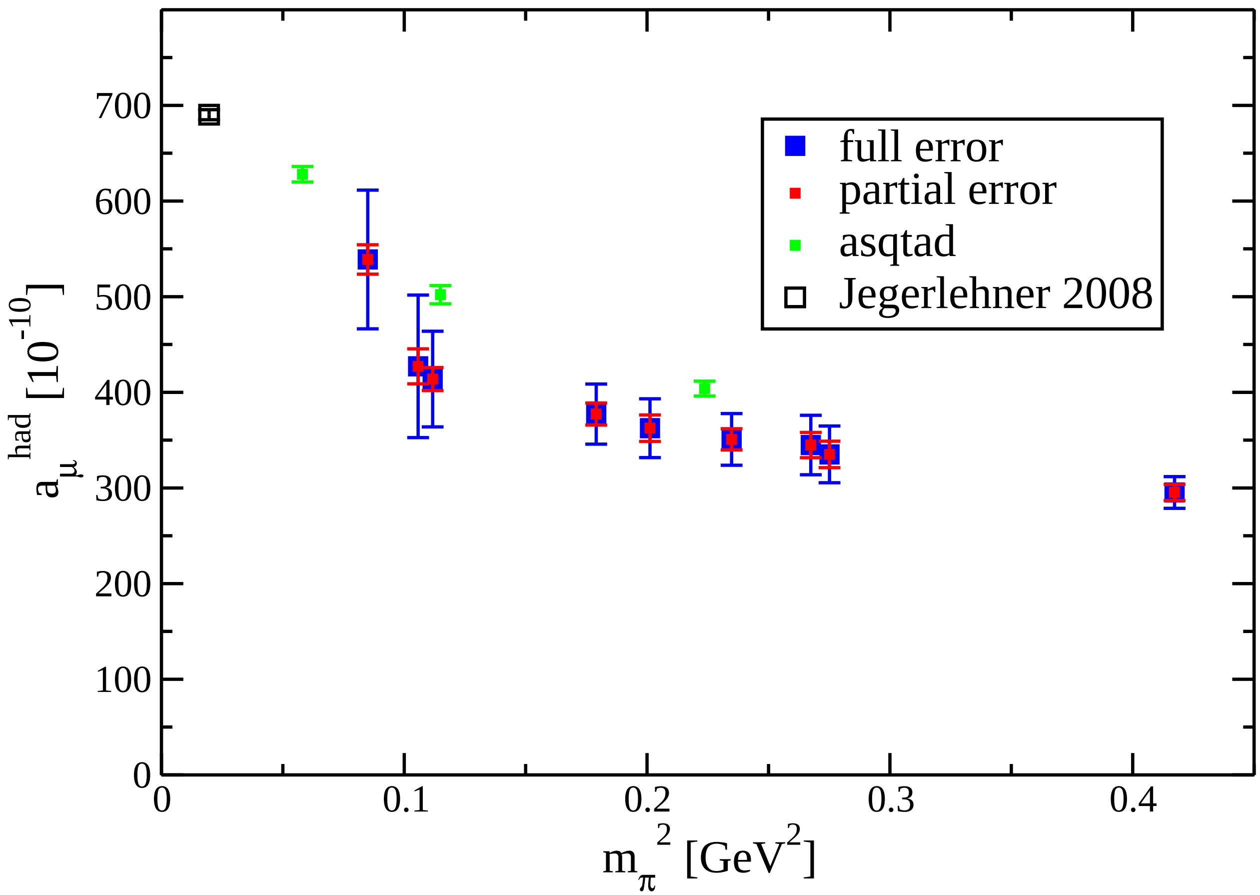
- disconnected contribution from M. Petschlies



- one example of five: $a = 0.079$ fm, $m_\pi = 420$ MeV, $L/a = 24$

Error Analysis

- full error includes error propagation of m_ρ into a_μ^{had}



- our partial error analysis gives similar results as for asqtad