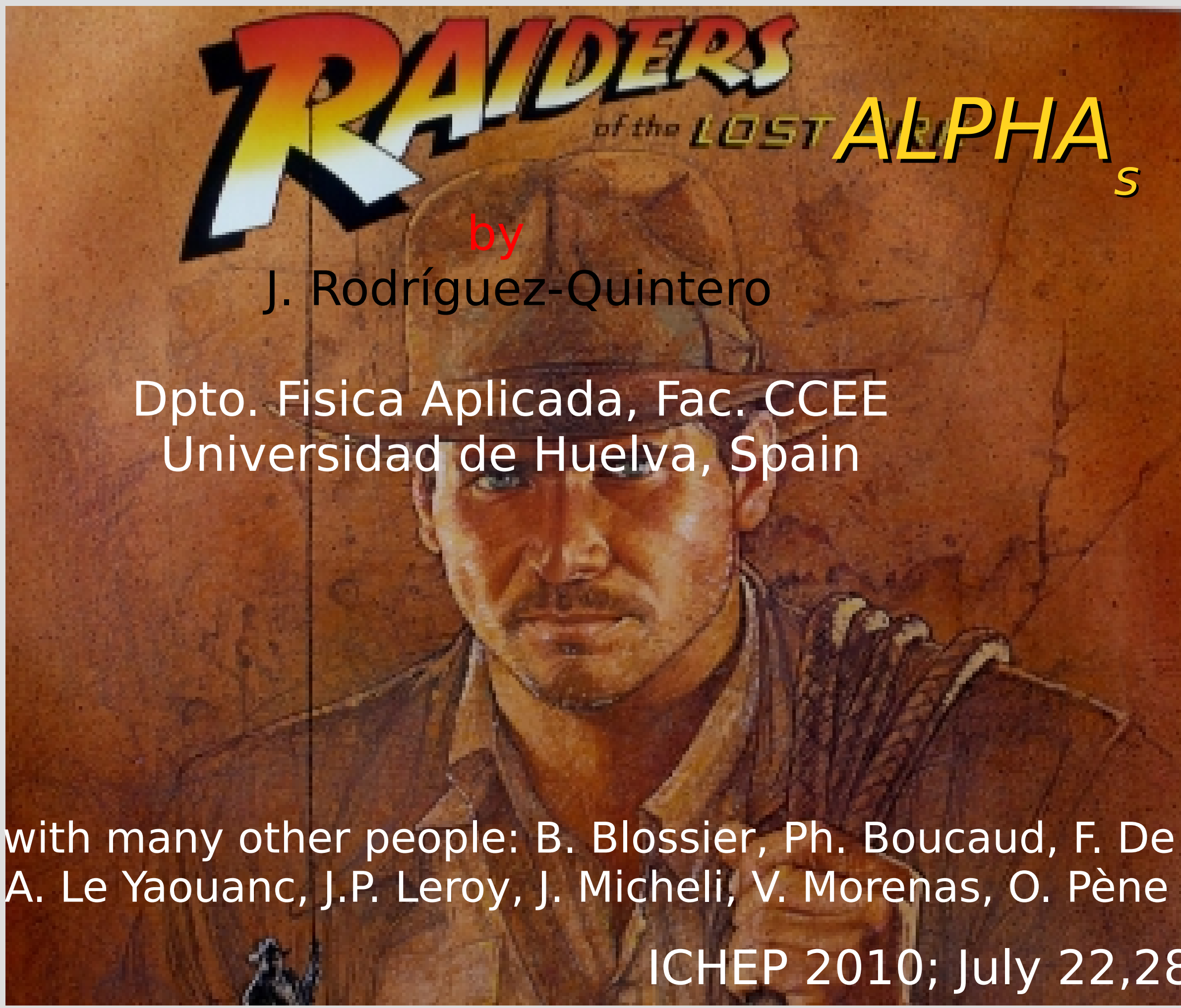
ALPHA$_s$ from Lattice QCD

by
J. Rodríguez-Quintero

Dpto. Fisica Aplicada, Fac. CCEE
Universidad de Huelva, Spain

In collaboration with many other people: B. Blossier, Ph. Boucaud, F. De Soto, M. Gravina, A. Le Yaouanc, J.P. Leroy, J. Micheli, V. Morenas, O. Pène

Lattice QCD and



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ICHEP 2010; July 22,28; Paris 20010.

QCD: The running of ALPHA$_s$

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = \sum_q \left(i\bar{q}\not{D}\, q - m_q \bar{q} q \right) - \frac{1}{4} (D \times A)^2}$$

$$\boxed{(D^\mu \times A^\nu)_a \equiv \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g \sum f_{abc}\, A_b^\mu A_c^\nu}$$

$$\boxed{\left[\lambda^a, \lambda^b\right] = i f_{abc}\, \lambda^c}$$

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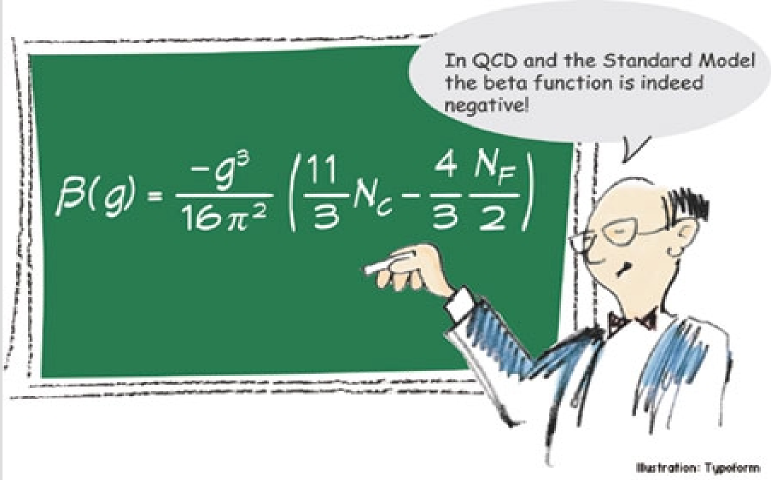

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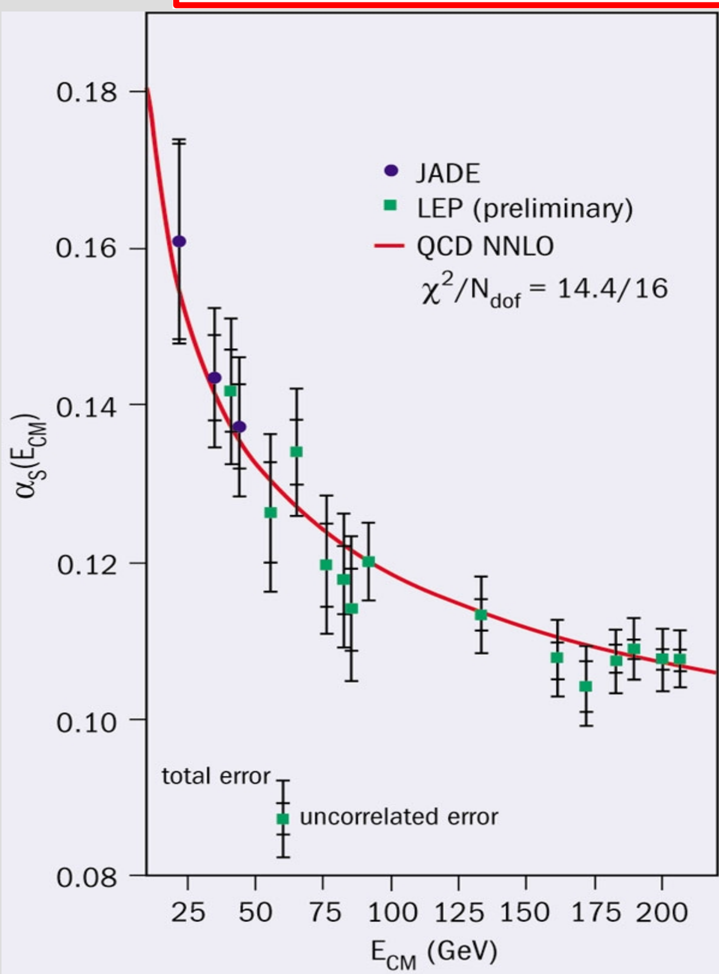
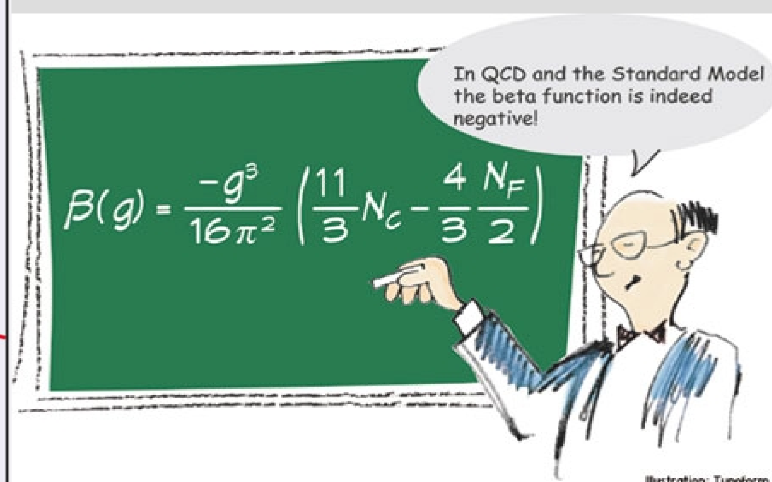
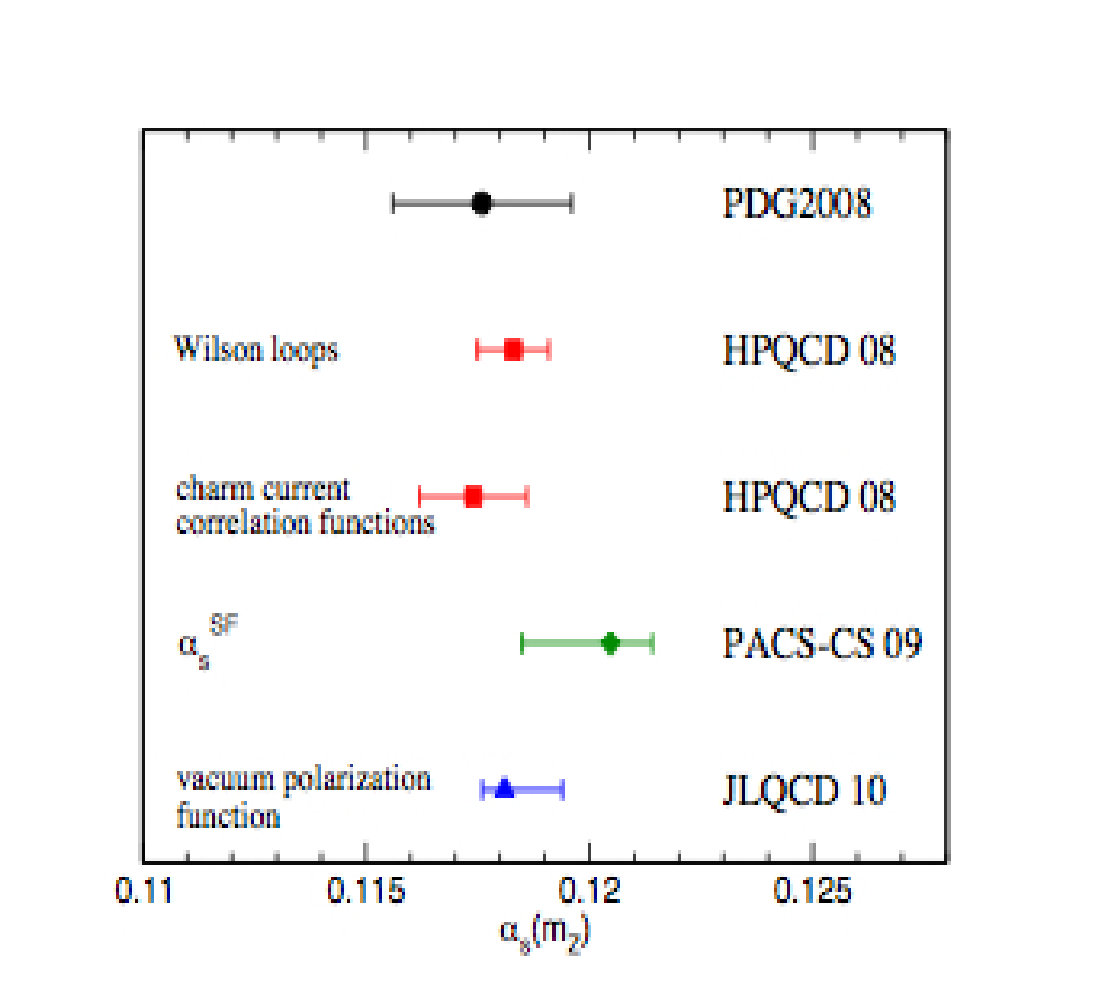




ALPHA$_s$ from Lattice QCD:

the many raiders...

Very recent $N_f=2+1$ & $N_f=4$ computations:



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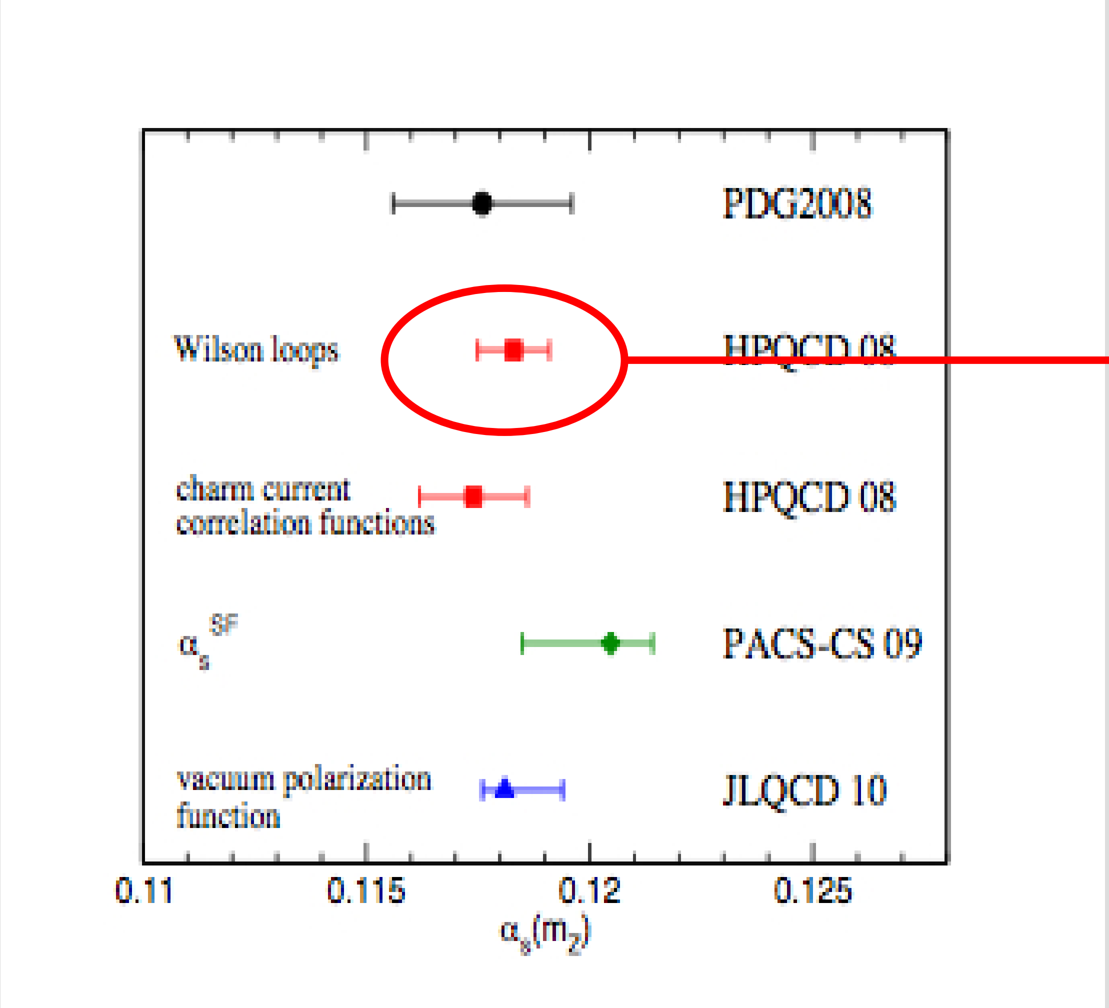

C.T.H. Davies et al.;
Phys. Rev. D78(2008)114507

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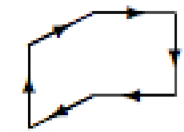
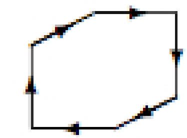
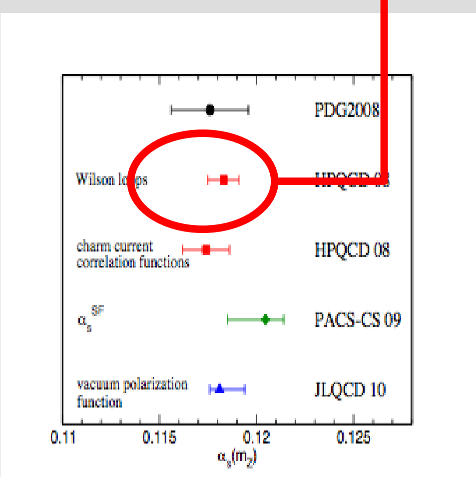
BR =  CC = 

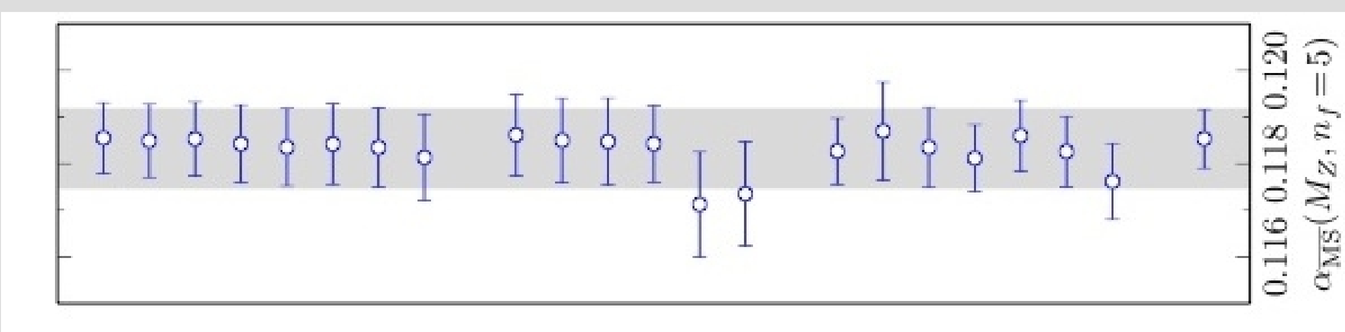
$$W_{mn} \equiv \frac{1}{3}\langle 0|\,\mathrm{Re}\,\mathrm{Tr}\,\mathrm{P}\,e^{-ig\oint_{nm} A\cdot dx}\,|0\rangle,$$

$$q^2\frac{d\alpha_V(q)}{dq^2} = -\beta_0\alpha_V^2 - \beta_1\alpha_V^3 - \beta_2\alpha_V^4 - \beta_3\alpha_V^5$$

$$Y = \sum_{n=1}^{\infty} c_n \alpha_V^n(d/a)$$

$$\alpha_0 \equiv \alpha_V(7.5\,\mathrm{GeV}, n_f=3) \rightarrow \alpha_{\overline{\mathrm{MS}}}(M_Z, n_f=5)$$





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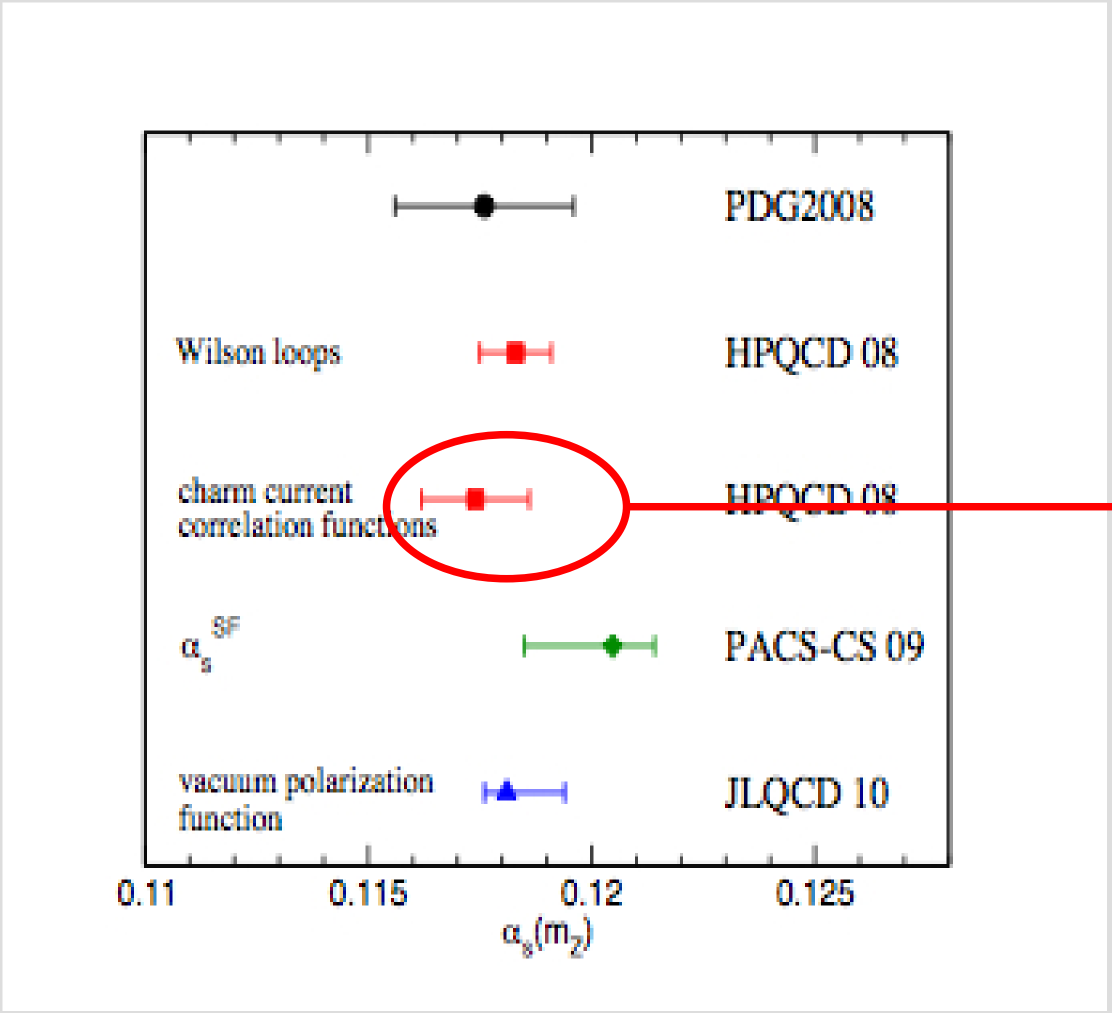

I. Allison et al.;
Phys. Rev. D78(2008)054513

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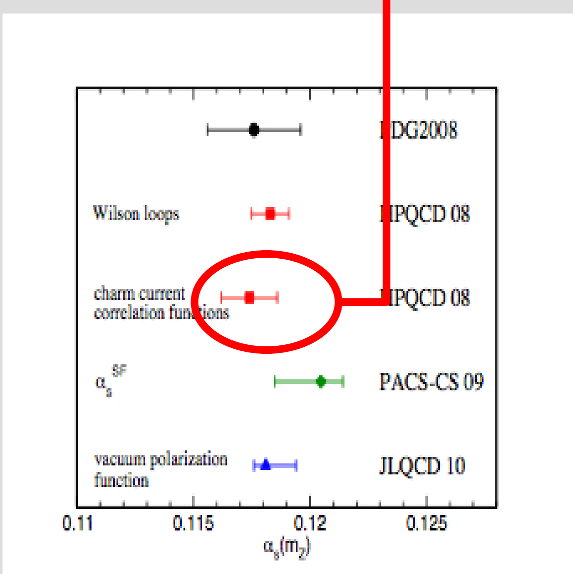
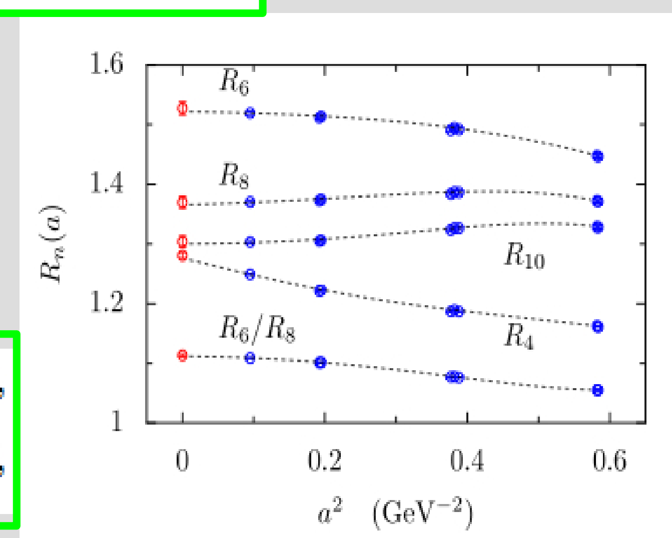
$$G(t) \equiv a^6 \sum_{\mathbf{x}} (am_{0c})^2 \langle 0 | j_5(\mathbf{x},t) j_5(0,0) | 0 \rangle$$

$$G_n \equiv \sum_t (t/a)^n G(t),$$

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4, \\ \dfrac{am_{\eta_c}}{2am_{\text{pole},c}^{(0)}} \left(G_n/G_n^{(0)}\right)^{1/(n-4)} & \text{for } n \geq 6, \end{cases}$$

$$R_n \equiv \begin{cases} r_4(\alpha_{\overline{\text{MS}}}, \mu/m_c) & \text{for } n = 4, \\ \dfrac{r_n(\alpha_{\overline{\text{MS}}}, \mu/m_c)}{2m_c(\mu)/m_{\eta_c}} & \text{for } n \geq 6, \end{cases}$$





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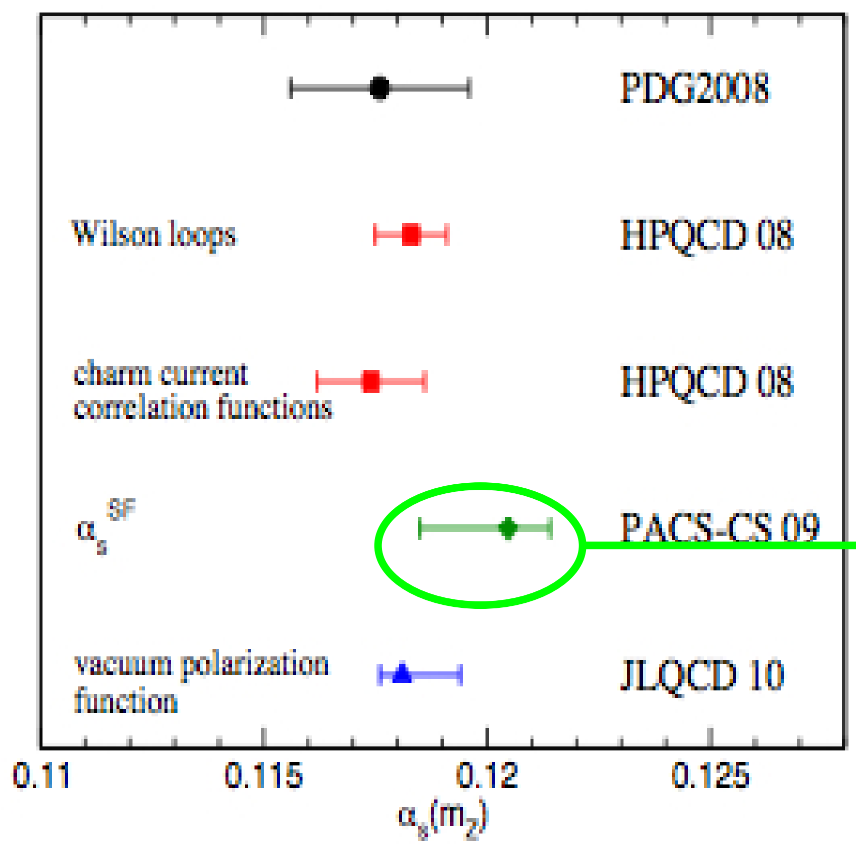

S. Aoki et al.; JHEP10(2009)053

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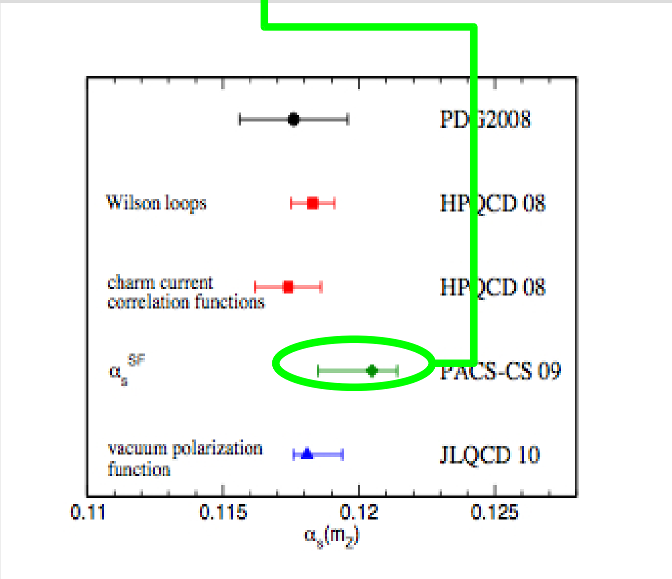
S. Aoki et al.; JHEP10(2009)053

$$S_g = \frac{\beta}{N}\sum_{C\in S_0} W_0(C,g_0^2)\mathrm{Re\,tr}\,(1-P(C)) + \frac{\beta}{N}\sum_{C\in S_1} W_1(C,g_0^2)\mathrm{Re\,tr}\,(1-R(C)),$$

$$S_f[U,\psi,\overline{\psi}] = a^4\sum_x \overline{\psi}\,(D_W+m_0)\,\psi,$$

$$D_W = \frac{1}{2}\left(\gamma_\mu\left(\nabla_\mu+\nabla_\mu^*\right) - a\nabla_\mu^*\nabla_\mu\right) - c_{\mathrm{SW}}\frac{1}{4}\sigma_{\mu\nu}P_{\mu\nu}.$$



- Iwasaki action for the gauge fields
- Wilson Clover for the fermions

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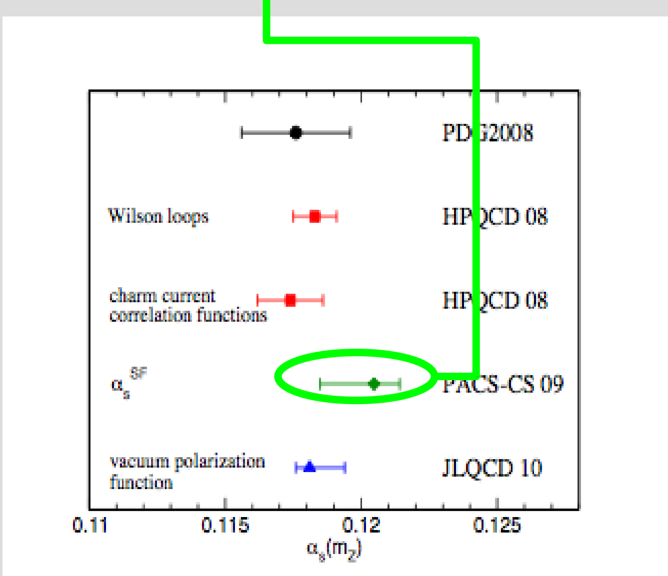
$$\Lambda_{\rm SF} = \frac{1}{L}\left(b_0\bar{g}(L)\right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\bar{g}(L)}\right) \exp\left(-\int_0^{\bar{g}(L)} dg\left(\frac{1}{\beta(g)}+\frac{1}{b_0 g^3}-\frac{b_1}{b_0^2 g}\right)\right),$$



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$$\frac{1}{\bar{g}^2(L)} = \frac{1}{k}\left.\frac{\partial\Gamma[V_\mu]}{\partial\eta}\right|_{\eta=0},$$

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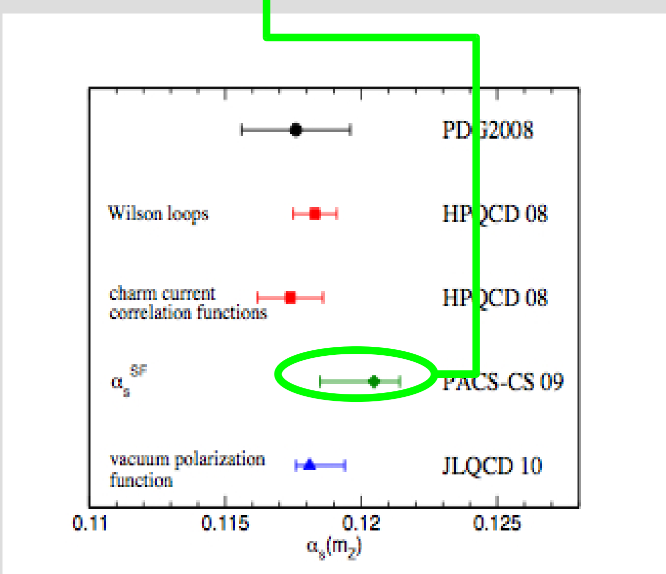

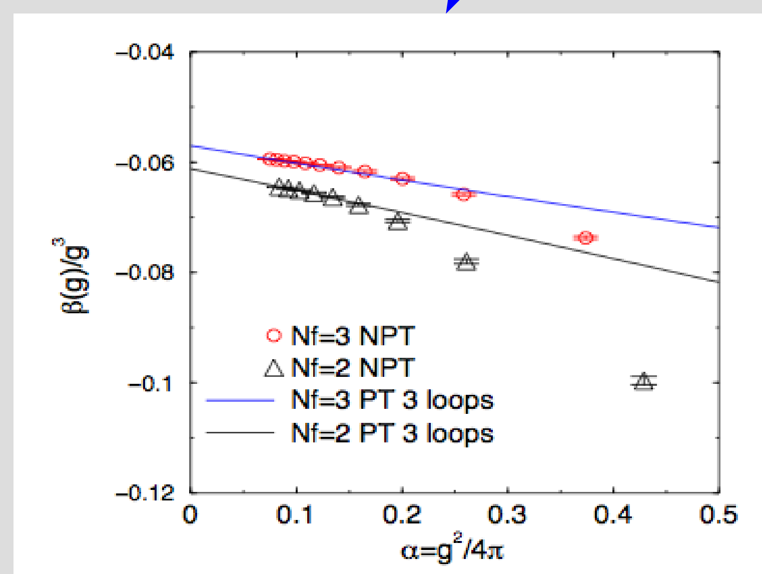

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$$\Sigma\left(u,\frac{a}{L}\right) = \bar{g}^2(2L)\Big|_{u=\bar{g}^2(L)}.$$

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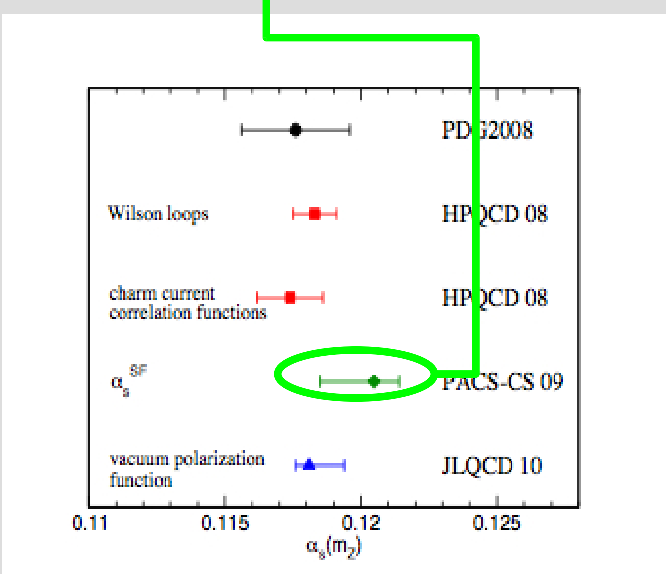

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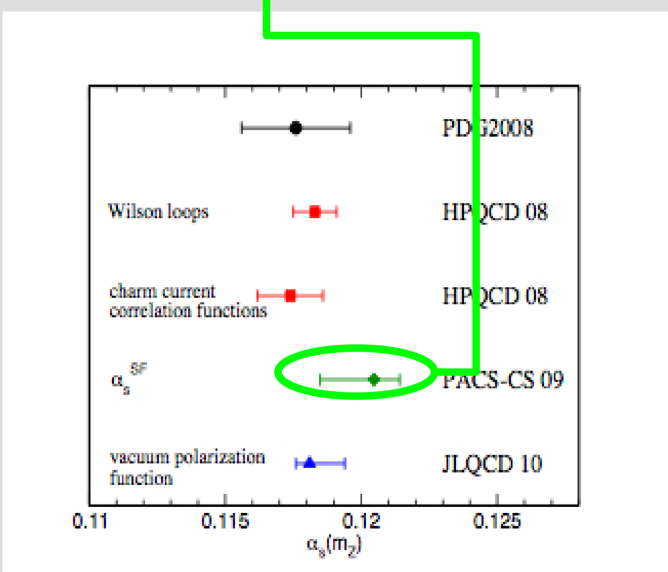

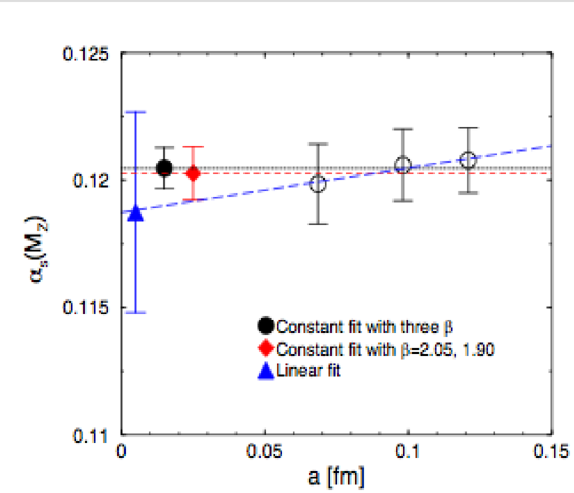

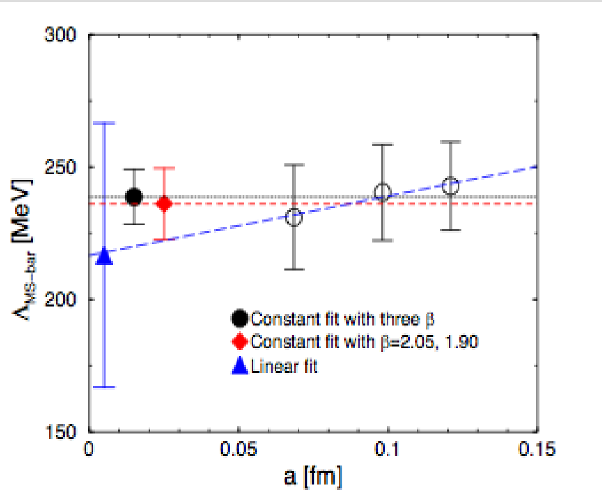

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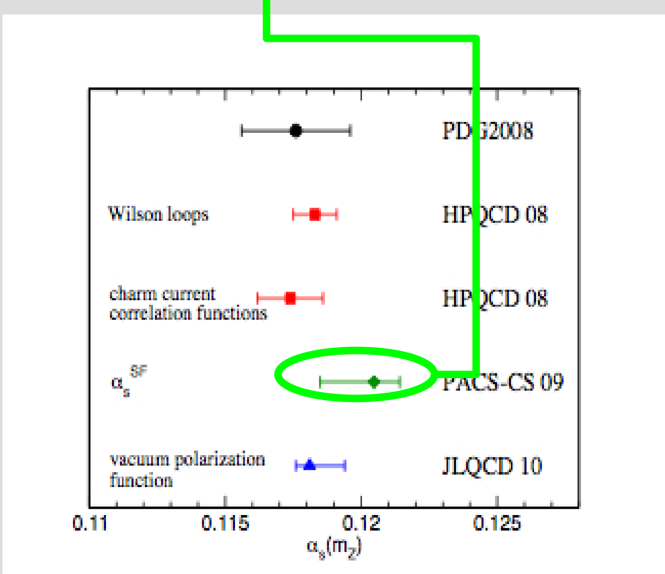

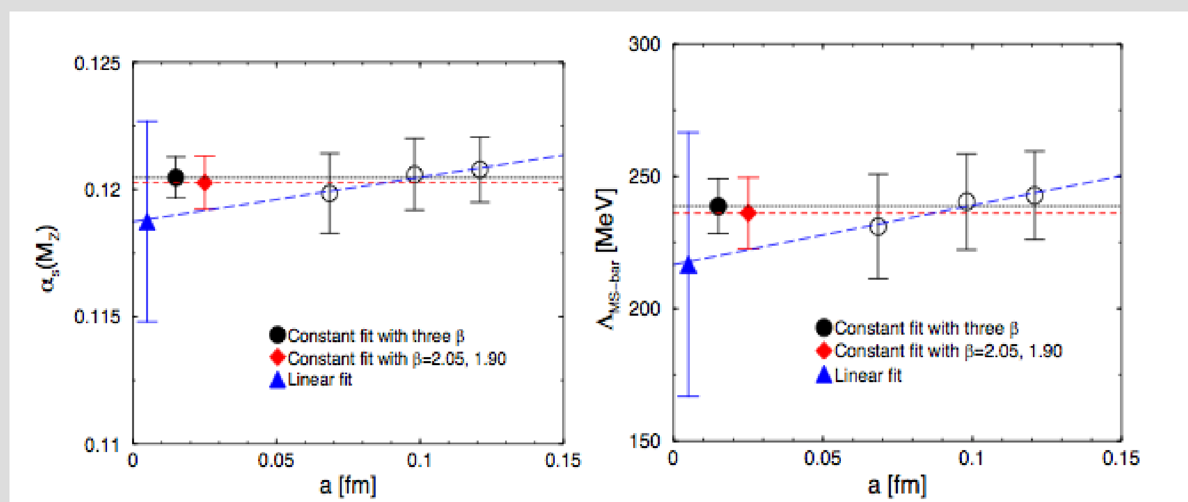

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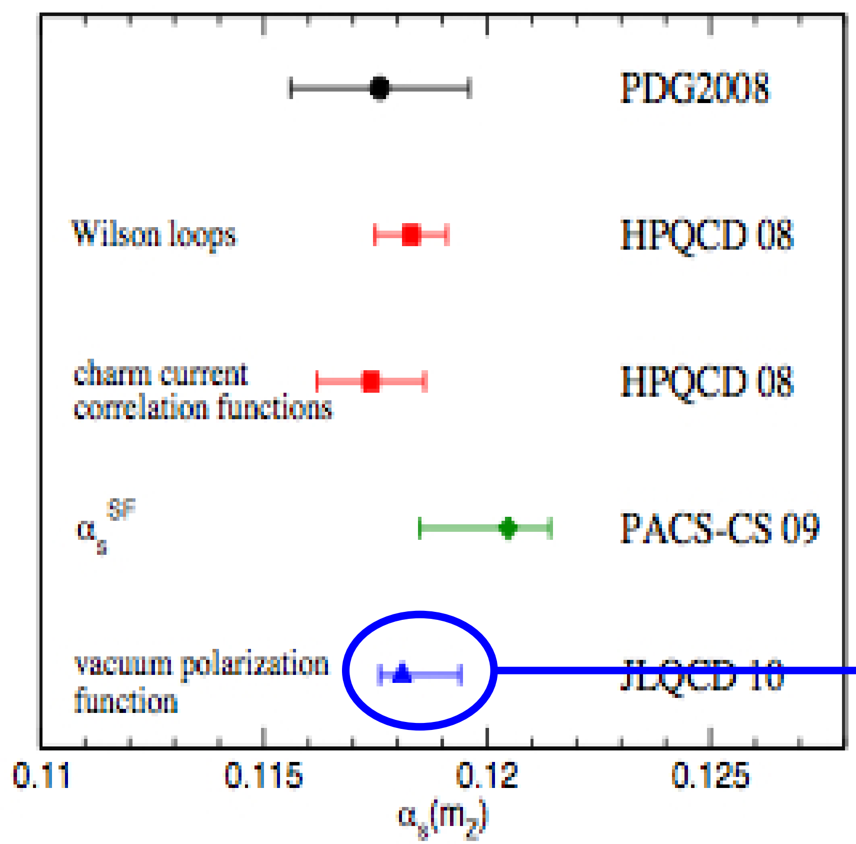

E. Shintani et al.;
arXiv:1002.0371

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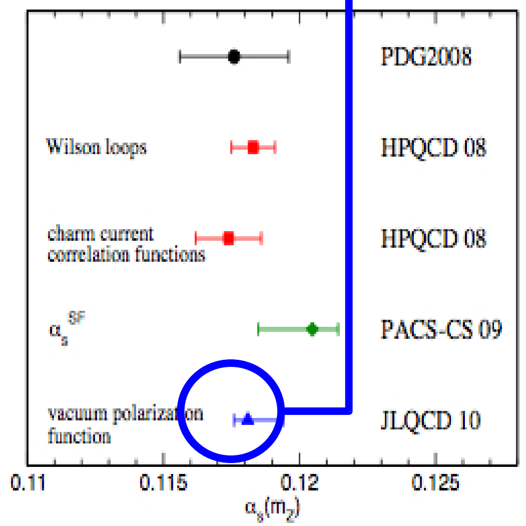
- Overlap fermions

$$\langle J^a_\mu J^b_\nu \rangle(Q) = \delta^{ab}\left[(\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu)\Pi^{(1)}_J(Q) - Q_\mu Q_\nu \Pi^{(0)}_J(Q)\right].$$

$$\begin{aligned}\Pi_{V+A}|_{\mathrm{OPE}}(Q^2,\alpha_s) &= c + C_0(Q^2,\mu^2,\alpha_s)\\ &+ C_m^{V+A}(Q^2,\mu^2,\alpha_s)\frac{\bar m^2(Q)}{Q^2}\\ &+ \sum_{q=u,d,s} C_{\bar qq}^{V+A}(Q^2,\alpha_s)\frac{\langle m\bar qq\rangle}{Q^4}\\ &+ C_{GG}(Q^2,\alpha_s)\frac{\langle(\alpha_s/\pi)GG\rangle}{Q^4} + \mathcal{O}(Q^{-6}).\end{aligned}$$



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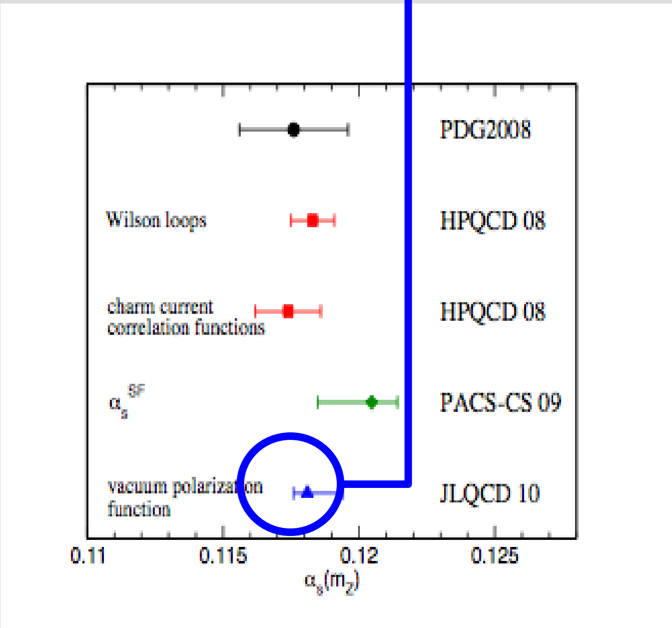
Unknown parameters to be fitted!!



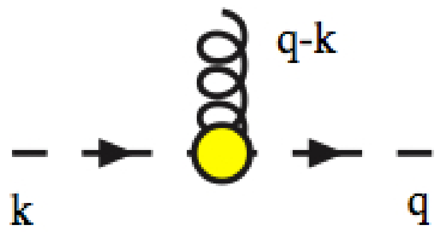
$$\Lambda^{(3)}_{\bar{MS}} = 0.247(5)$$

ALPHA$_s$ from Lattice QCD:

The ghost-gluon coupling

The ghost-gluon vertex:

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$$



$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

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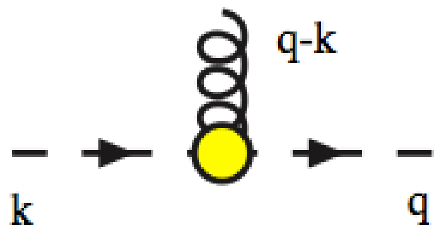

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The strong coupling:

$$g_R(\mu^2) = \lim_{\Lambda\to\infty} Z_g^{-1}(\mu^2,\Lambda^2) g_0(\Lambda^2)$$

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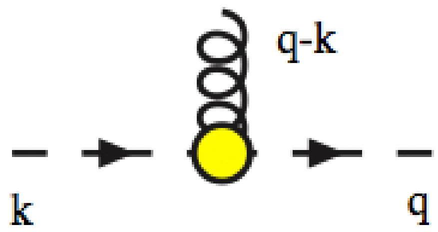

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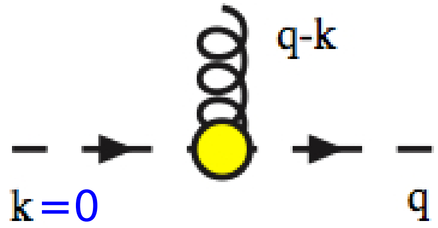

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In Taylor scheme

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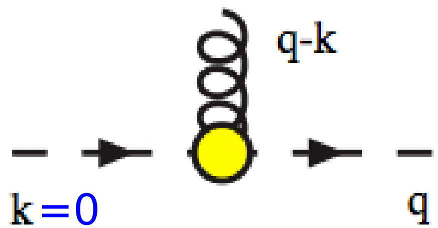

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$\to 1$

In Taylor scheme & Landau gauge

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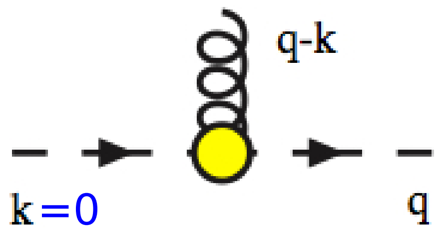
The ghost-gluon vertex:

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$$



$$= g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

The strong coupling:

$$g_R(\mu^2) = \lim_{\Lambda\to\infty} Z_g^{-1}(\mu^2,\Lambda^2)\, g_0(\Lambda^2) = \lim_{\Lambda\to\infty} \frac{Z_3^{1/2}(\mu^2,\Lambda^2)\widetilde{Z}_3(\mu^2,\Lambda^2)}{\widetilde{Z}_1(\mu^2,\Lambda^2)}\, g_0(\Lambda^2)$$

1

In Taylor scheme & Landau gauge

$$\boxed{\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2)\widetilde{Z}_3^2(\mu^2,\Lambda^2)}$$

ALPHA$_s$ from Lattice QCD:

The ghost-gluon coupling

Lattice: $\frac{1}{L^2} \ll p^2 \ll \frac{1}{a^2}$

Propagators in Landau gauge

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U] \; \mathcal{D}[\Phi] O e^{-S[U,\Phi]}$$

$$A_\mu(x + \frac{\hat{\mu}}{2}) = \frac{1}{2iag_0} \left(U_\mu(x) - U_\mu^\dagger(x) \right)$$

- In Landau gauge: $\left[\text{minimizing} \qquad F_U[g] = \mathrm{Re} \sum_x \sum_\mu \left(1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x + \hat{\mu}) \right) \right]$

$$D_{\mu\nu}^{ab}(k) = \int d^4x \, d^4y e^{ik\cdot(x-y)} \langle A_\mu^a(x) A_\nu^b(y) \rangle_U = \frac{G(k^2)}{k^2} \delta^{ab} \, \delta_{\mu\nu}^T(k)$$

- In MOM renormalization scheme:

ALPHA$_s$ from Lattice QCD:

The ghost-gluon coupling

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$$G^{ab}(k) = \int d^4x\, d^4y e^{ik\cdot(x-y)} \left(M^{-1}\right)^{ab}_{xy} = -\frac{F(k^2)}{k^2}\delta^{ab}$$

- In MOM renormalization scheme:

ALPHA$_s$ from Lattice QCD:

The ghost-gluon coupling

Lattice: $\frac{1}{L^2} \ll p^2 \ll \frac{1}{a^2}$

Propagators in Landau gauge

$$\left(G^{(2)}\right)^{ab}_{\mu\nu}(p^2,\Lambda) = \frac{G(p^2,\Lambda)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$\left(F^{(2)}\right)^{a,b}(p^2,\Lambda) = -\delta_{ab}\,\frac{F(p^2,\Lambda)}{p^2}$$

Renormalized in non-perturbative MOM-scheme:

$$G_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda)$$

$$F_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} \widetilde{Z}_3^{-1}(\mu^2,\Lambda)\,F(p^2,\Lambda)$$

$$\boxed{G_R(\mu^2,\mu^2) = F_R(\mu^2,\mu^2) = 1}$$

ALPHA$_s$ from Lattice QCD:

Matching Lattice and PTh

4-loops perturbation theory[3]: $p \gg \Lambda_{QCD}$

$$
\begin{aligned}
\alpha_T(\mu^2) \quad &= \quad \frac{4\pi}{\beta_0 t}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t}+\frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t)-\frac{1}{2}\right)^2+\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}-\frac{5}{4}\right)\right) \\
&+ \quad \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0}+\frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2\log^3(t)+5\log^2(t)+\left(4-6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}\right)\log(t)-1\right)\right) \; , t = \ln\frac{\mu^2}{\Lambda_T^2}
\end{aligned}
$$

$$
\beta_T(\alpha_T) = \frac{d\alpha_T}{d\ln\mu^2} = -4\pi\sum_{i=0}\widetilde{\beta}_i\left(\frac{\alpha_T}{4\pi}\right)^{i+2}, \quad \frac{\Lambda_{\overline{\mathrm{MS}}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}} = 0.541449
$$

ALPHA$_s$ from Lattice QCD:

Matching Lattice and PTh

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$$\begin{aligned}\alpha_T(\mu^2) &= \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t) - \frac{1}{2}\right)^2 + \frac{\widetilde{\beta}_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right) \\ &+ \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}\right)\log(t) - 1\right)\right), \; t = \ln\frac{\mu^2}{\Lambda_T^2}\end{aligned}$$

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K.G. Chetyrkin, Nucl. Phys. B710 (2005) 499

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$$+ \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2 \right.\right.$$

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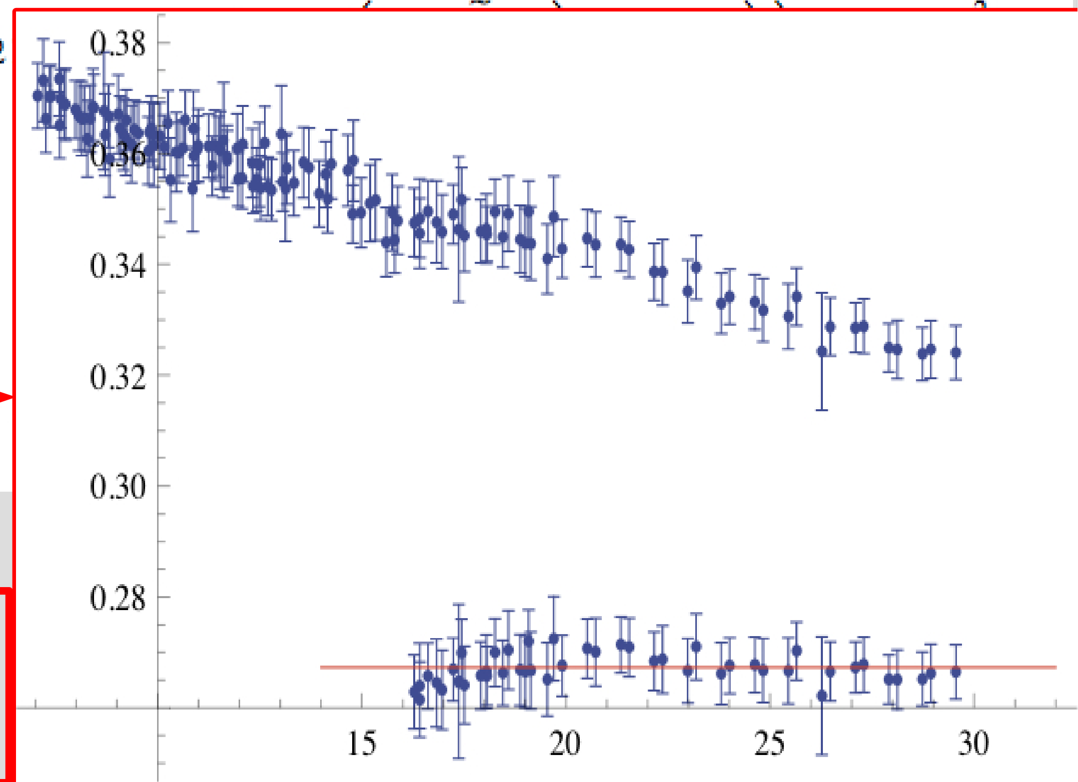

Ph. Boucaud et al, PRD79(2009)014508

Quenched QCD: $N_f=0$

ALPHA$_s$ from Lattice QCD:

Matching Lattice and PTh

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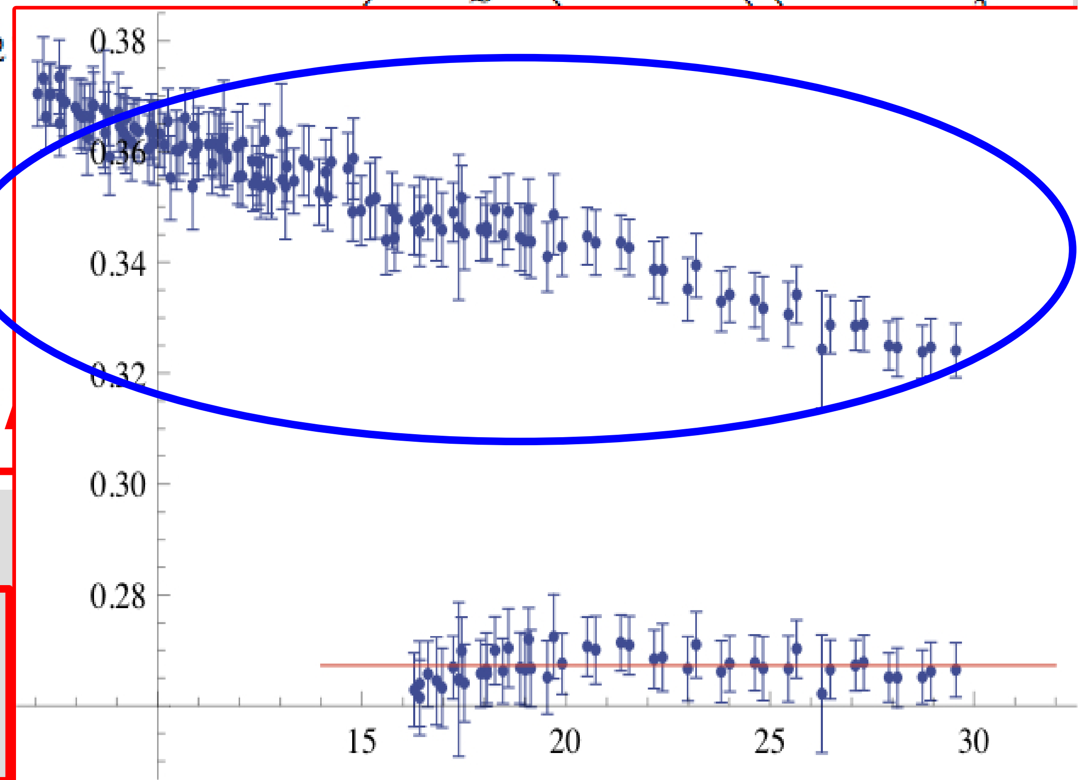


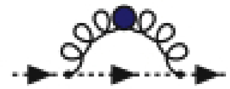
Ph. Boucaud et al, PRD79(2009)014508
Quenched QCD: $N_f=0$

ALPHA$_s$ from Lattice QCD:

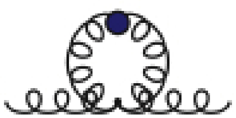
Matching Lattice and PTh

OPE power corrections

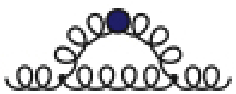
$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\text{pert}})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab} = 2\times$$



$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\text{pert}})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab}_{\mu\nu} =$$



$$+\,2\times$$

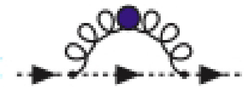


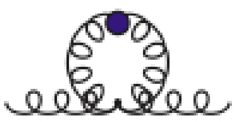
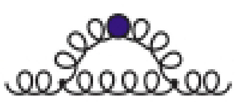
$$F_R(q^2,\mu^2) = F_{R,\text{pert}}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)\,, \quad G_R(q^2,\mu^2) = G_{R,\text{pert}}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)$$

ALPHA$_s$ from Lattice QCD:

Matching Lattice and PTh

OPE power corrections

$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\rm pert})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab} = 2\times$$ 

$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\rm pert})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab}_{\mu\nu} =$$  $+\,2\times$ 

$$F_R(q^2,\mu^2) = F_{R,\rm pert}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)\,,\quad G_R(q^2,\mu^2) = G_{R,\rm pert}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)$$

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$$1-\gamma_0^{A^2}/\beta_0 = 1-\frac{105-8N_f}{132-8N_f} = \frac{9}{44-\frac{8}{3}N_f}$$

ALPHA$_s$ from Lattice QCD:

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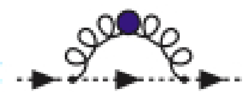

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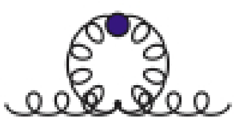

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Ph. Boucaud et al, PRD79(2009)014508

ALPHA$_s$ from Lattice QCD:

Matching Lattice and PTh

OPE power corrections

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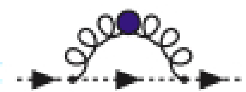

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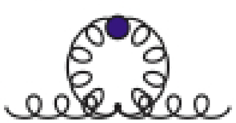
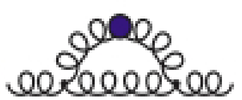


$+\ 2\times$



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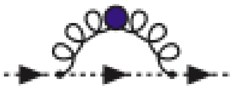
Chetyrking & Maier, arXiv:0911.0594
At the three-loop level !!!

ALPHA$_s$ from Lattice QCD:

Matching Lattice and PTh

OPE power corrections

$$(F^{(2)})^{ab}(q^2) \;=\; (F^{(2)}_{\text{pert}})^{ab}(q^2) + w^{ab}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab} = 2\times$$ 

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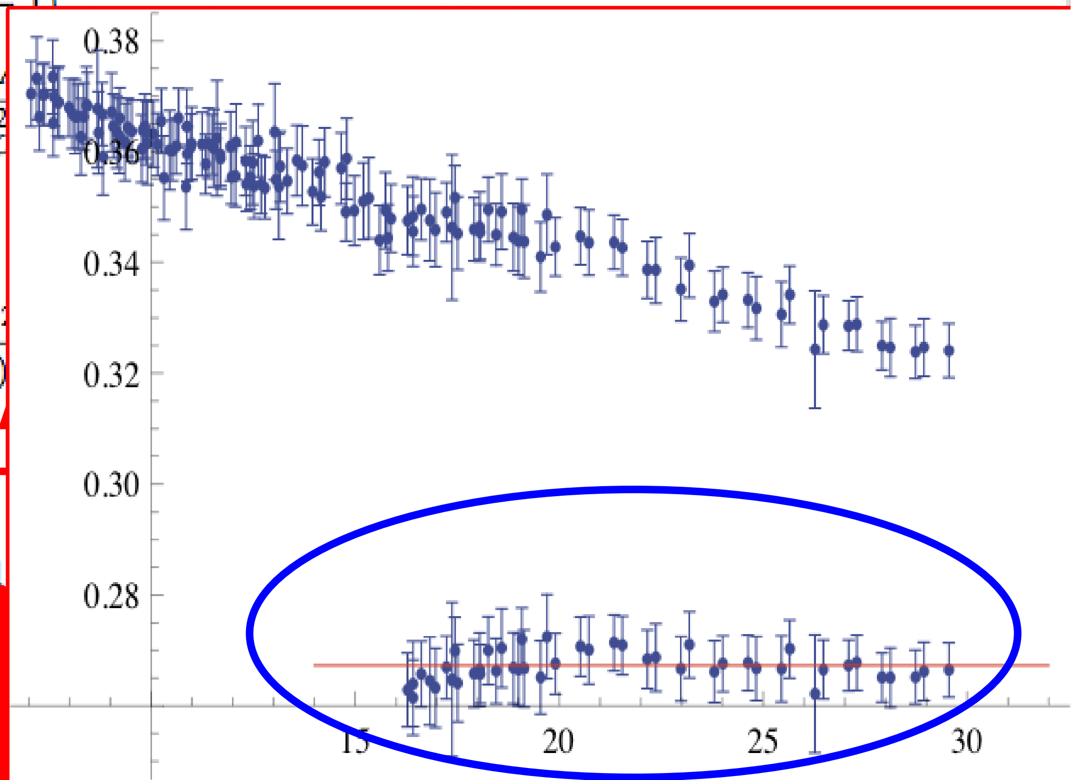

Ph. Boucaud et al, PRD79(2009)014508
Quenched QCD: $N_f=0$

ALPHA$_s$ from Lattice QCD:

N_f=2 twisted-mass QCD

European Twisted Mass Collaboration

Fermions: twisted-mass action

$$S^{\mathrm{F}}_{\mathrm{tm}} = a^4 \sum_x \left\{ \bar{\chi}_x \left[D_{\mathrm{W}} + m_0 + i\gamma_5 \tau_3 \mu_q \right] \chi_x \right\}$$

Gauge fields: tlSym action

$$S_g = \frac{\beta}{3} \sum_x \left(b_0 \sum_{\substack{\mu,\nu=1 \\ 1 \le \mu < \nu}}^{4} \{1 - \mathbb{R}\mathrm{eTr}(U^{1\times1}_{x,\mu,\nu})\} + b_1 \sum_{\substack{\mu,\nu=1 \\ \mu \neq \nu}}^{4} \{1 - \mathbb{R}\mathrm{eTr}(U^{1\times2}_{x,\mu,\nu})\} \right) \quad , \quad \beta \equiv 6/g_0^2$$

$(b_0 = 1 - 8b_1 \, , b_1 = -1/12)$ $+$ **Maximal twist :** $\underline{\mathcal{O}(a^2)}$

$$\begin{array}{llcll} V = 24^3 \times 48 & \beta & = & 3.9 & \mu = 0.004,\ 0.0064,\ 0.010 \\ V = 32^3 \times 64 & \beta & = & 4.05 & \mu = 0.003,\ 0.006,\ 0.008,\ 0.012 \\ & \beta & = & 4.2 & \mu = 0.0065 \end{array}$$

Artefacts: $\underline{\mathcal{O}(a^2 \Lambda^2_{QCD})}$, $\underline{\mathcal{O}(a^2 p^2)}$, $\underline{\mathcal{O}(a^2 \mu^2)}$

ALPHA$_s$ from Lattice QCD:

N_f=2 twisted-mass QCD

Ghost and gluon on the lattice

Landau gauge

$$F_U[g] = \mathrm{Re}\left[\sum_x \sum_\mu \mathrm{Tr}\left(1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x+\mu)\right)\right]$$

Gluon:

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\left(\frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0}\right)$$

$$\left(G^{(2)}\right)^{a_1 a_2}_{\mu_1 \mu_2}(p) = \langle A^{a_1}_{\mu_1}(p) A^{a_2}_{\mu_2}(-p)\rangle$$

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Ghost:

$$\left(F^{(2)}\right)^{ab}(x-y) \equiv \langle \left(M^{-1}\right)^{ab}_{xy} \rangle , \quad M(U) = -\frac{1}{N} \nabla \cdot \widetilde{D}(U)$$

$$\widetilde{D}(U)\eta(x) = \frac{1}{2}\left(U_\mu(x)\eta(x+\mu) - \eta(x)U_\mu(x) + \eta(x+\mu)U^\dagger_\mu - U^\dagger_\mu(x)\eta(x)\right)$$

ALPHA$_s$ from Lattice QCD:

Lattice artefacts

$O(4)$ breaking: $H(4)$ discretization artefacts[8]

Orbit labeled by $H(4)$-invariants: $p^{[2n]} = \sum_{\mu=1}^{4} p_\mu^{2n}, n = 1, 2, 3$

Momentum on the lattice: $\tilde{p}_\mu = \frac{1}{a} \sin a p_\mu \ , \ \ p_\mu = \frac{2\pi n}{Na} \qquad n = 0, 1, \cdots, N$

$$a^2 \tilde{p}^2 \equiv \sum_{\mu=1}^{4} a^2 \tilde{p}_\mu^2 = a^2 p^2 + c_1 a^4 p^{[4]} + \cdots = a^2 p^2 \left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots \right)$$

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$$a^2\tilde{p}^2 \equiv \sum_{\mu=1}^{4} a^2\tilde{p}_\mu^2 = a^2p^2 + c_1 a^4 p^{[4]} + \cdots = a^2p^2 \left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots\right)$$

If $\epsilon = a^2 p^{[4]}/p^2 << 1$ ...

$$Q(a^2\tilde{p}_\mu^2, a^2\Lambda^2) \equiv Q\left(a^2p^2\left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots\right), a^2\Lambda^2\right)$$

$$= Q(a^2p^2, a^2\Lambda^2) + \left.\frac{dQ}{d\epsilon}\right|_{\epsilon=0} a^2 \frac{p^{[4]}}{p^2} + \cdots$$

ALPHA$_s$ from Lattice QCD:

Lattice artefacts

$O(4)$ breaking: $H(4)$ discretization artefacts[8]

Orbit labeled by $H(4)$-invariants: $p^{[2n]} = \sum_{\mu=1}^{4} p_\mu^{2n}, n = 1, 2, 3$

Momentum on the lattice: $\tilde{p}_\mu = \frac{1}{a} \sin a p_\mu \ , \ \ p_\mu = \frac{2\pi n}{Na} \qquad n = 0, 1, \cdots, N$

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$$\boxed{R = R_0 + R_1 a^2 p^2}$$

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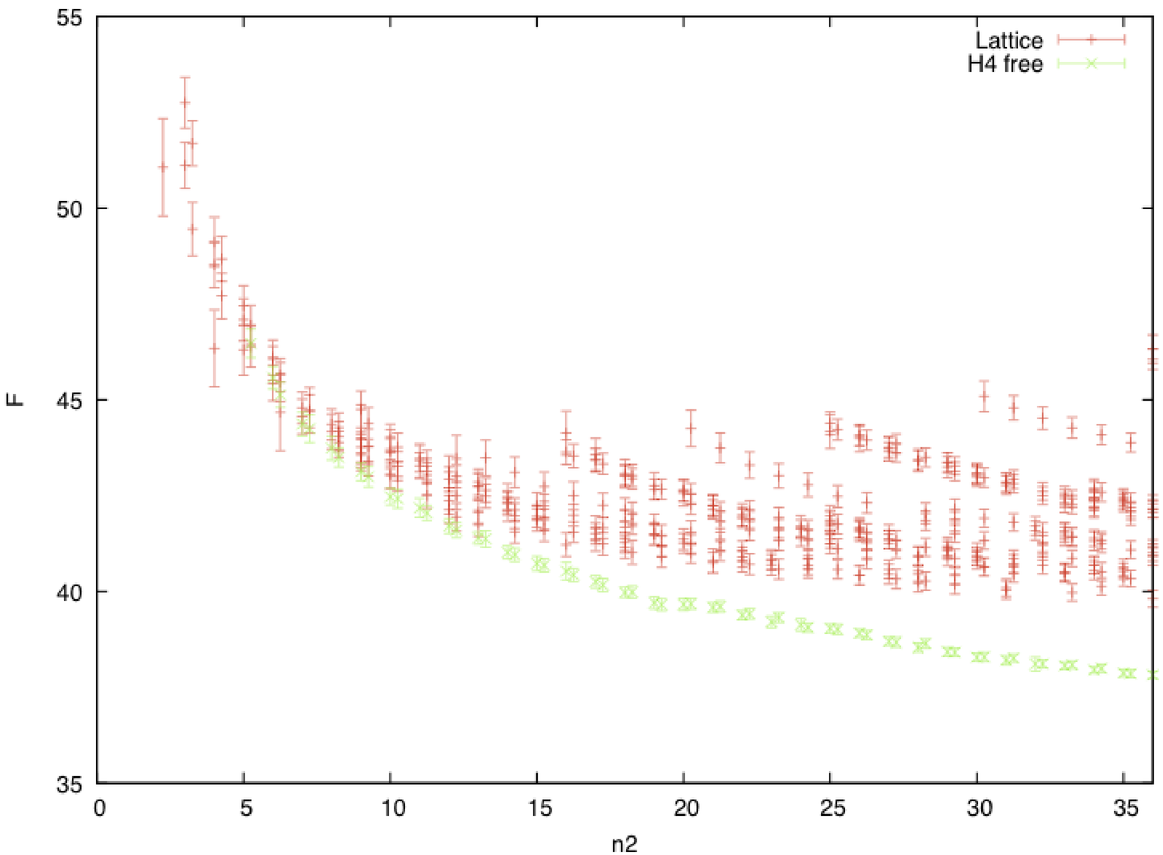

$n = 0, 1, \cdots, N$

$1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots \Big)$

$p^2 << 1 \ldots$

$) \equiv Q\left(a^2p^2\left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots\right), a^2\Lambda^2\right)$

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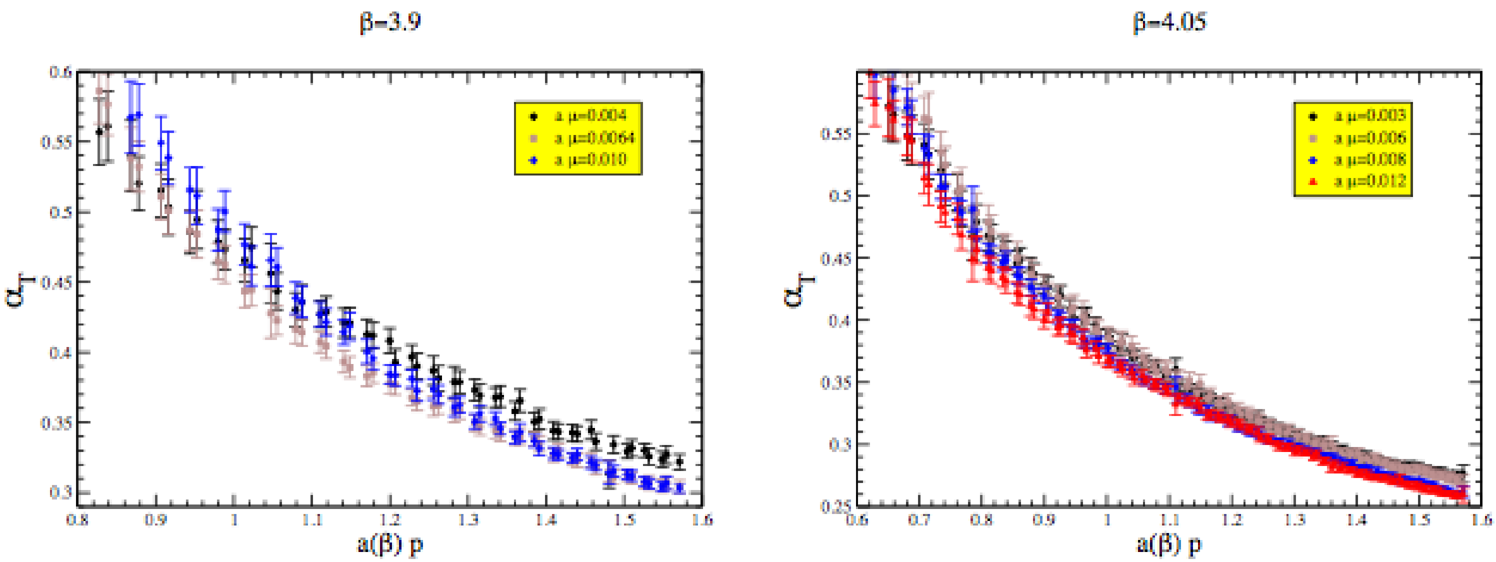
ALPHA$_s$ from Lattice QCD:

Lattice artefacts

Quark mass artefacts



ALPHA$_s$ from Lattice QCD:

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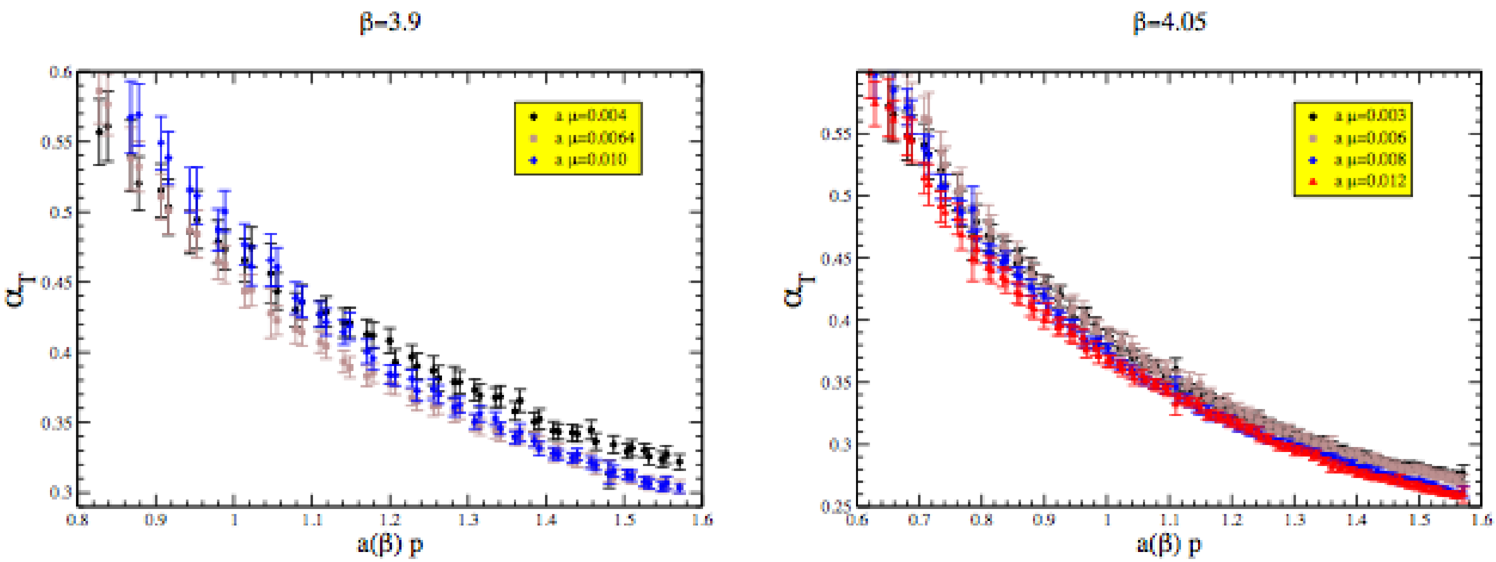
Quark mass artefacts



$\mathcal{O}(a^2\mu_q^2)$ dependence

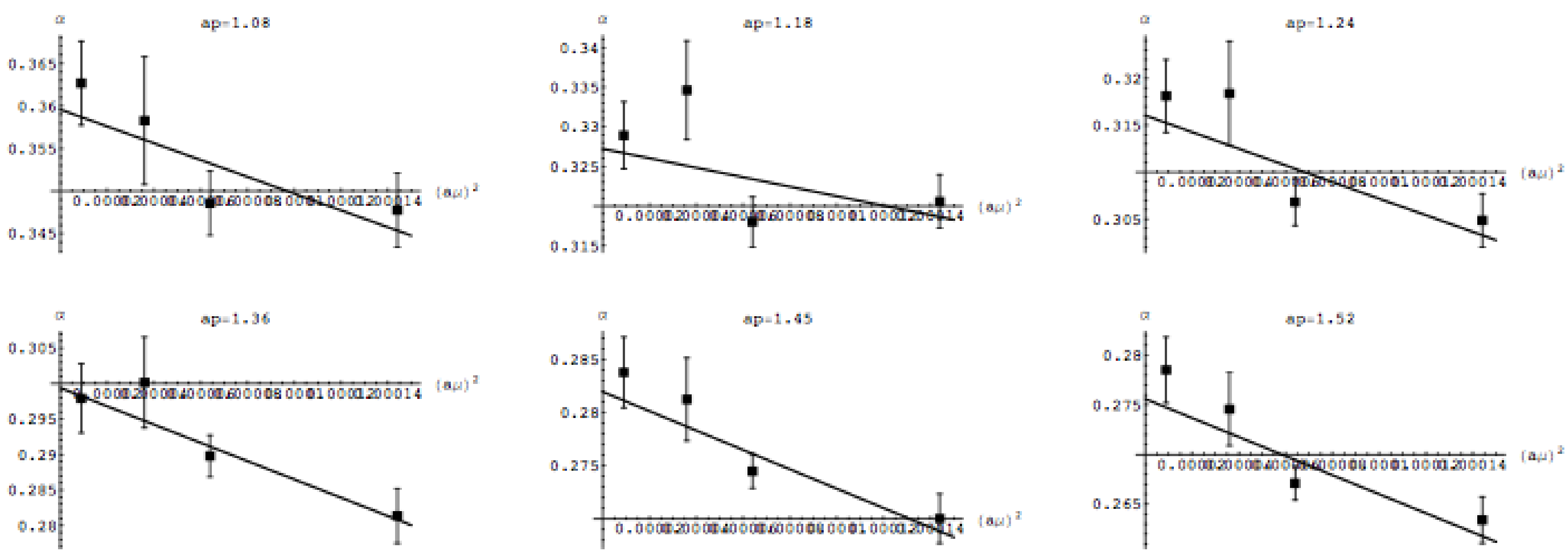
$$\begin{aligned}
\widehat{\alpha}_T(a^2p^2, a^2\mu_q^2) &= \frac{g_0^2(a^2)}{4\pi}\widehat{G}(a^2p^2, a^2\mu_q^2)\widehat{F}^2(a^2p^2, a^2\mu_q^2) \\
&= \widehat{\alpha}_T(a^2p^2, 0) + \frac{\partial\widehat{\alpha}_T}{\partial(a^2\mu_q^2)}\left(a^2p^2\right)\, a^2\mu_q^2 + \cdots
\end{aligned}$$

ALPHA$_s$ from Lattice QCD:

Lattice artefacts

Quark mass artefacts

Example: $\beta = 4.05$



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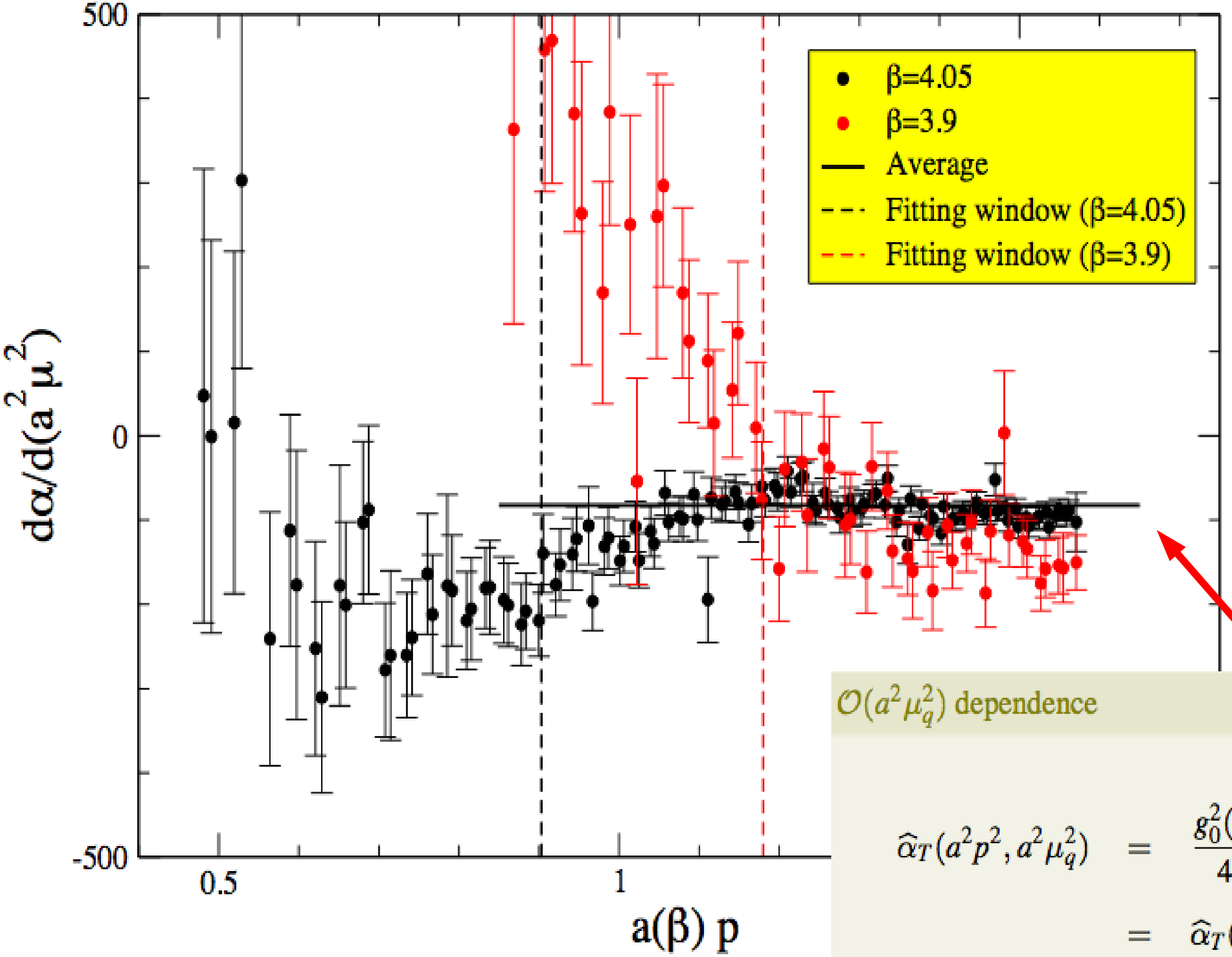
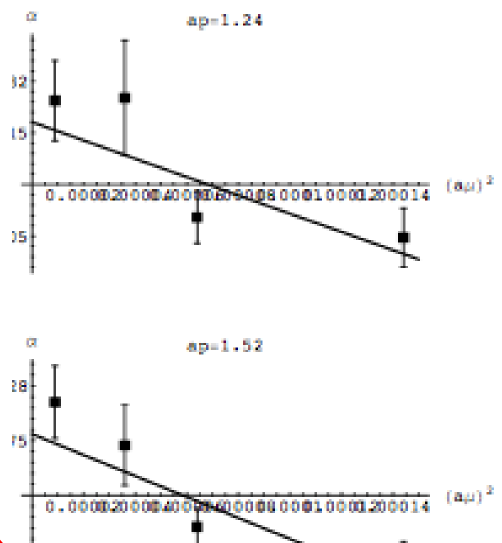
ALPHA$_s$ from Lattice QCD:

Lattice artefacts

Quark mass artefacts





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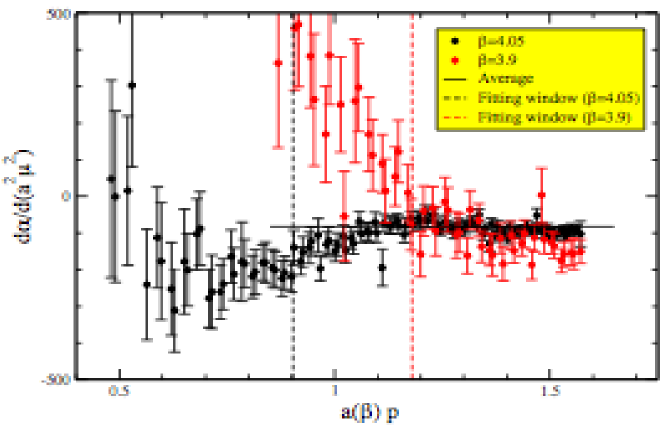
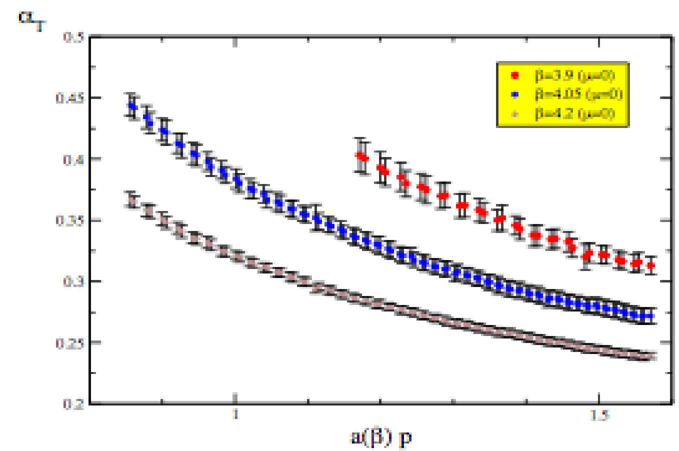
ALPHA$_s$ from Lattice QCD:

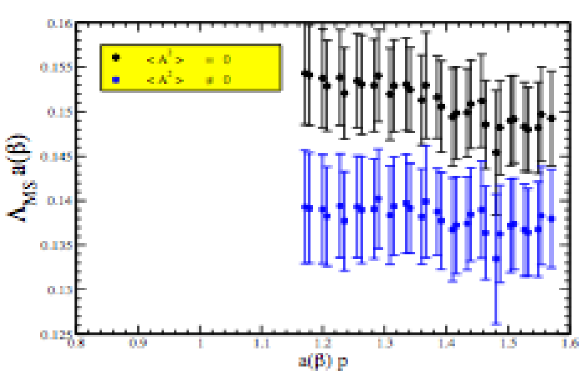
After curing lattice artefacts



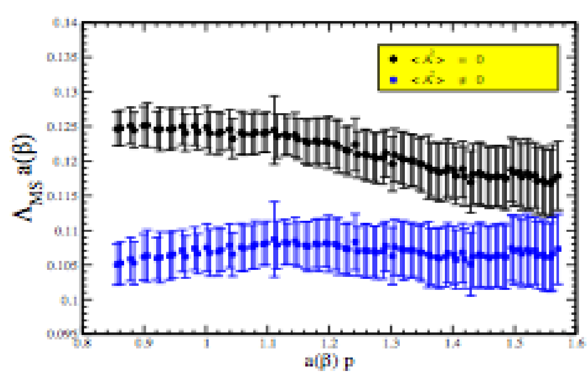


$R_0 = -92(11), \quad p \geq p_{\min} \simeq 2.8$ GeV (a(3.9)=0.0801(14) fm^9)

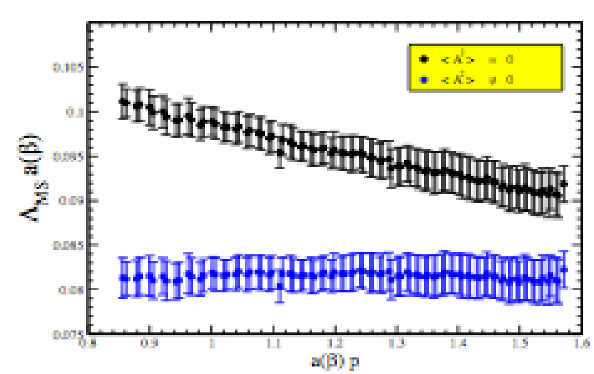
Requiring a plateau for $\Lambda_{\overline{MS}}$ in the window $ap \geq p_{\min}$



$\beta = 3.9$



$\beta = 4.05$



$\beta = 4.2$

ALPHA$_s$ from Lattice QCD:

After curing lattice artefacts

Global fit and calibration of lattice spacing

$$\chi^2\left(a(\beta_0)\Lambda_{\overline{\mathrm{MS}}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)}\right) = \sum_{j=0}^{2}\sum_{i} \frac{\left(\Lambda_i(\beta_j) - \frac{a(\beta_j)}{a(\beta_0)} a(\beta_0)\Lambda_{\overline{\mathrm{MS}}}\right)^2}{\delta^2(\Lambda_i)}$$

Variables: $a(\beta_0)\Lambda_{\overline{\mathrm{MS}}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)}$

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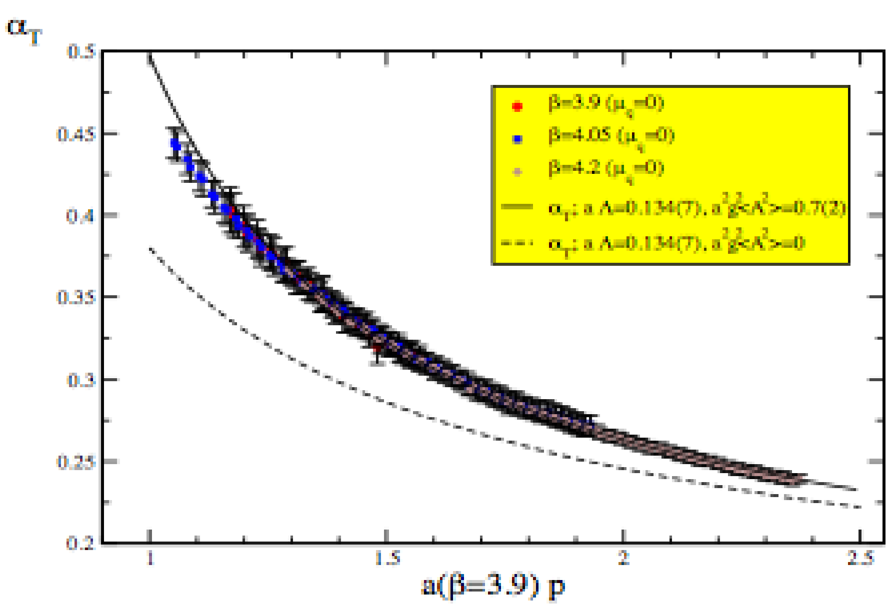
	This paper	String tension[10]
$a(3.9)/a(4.05)$	1.224(23)	1.255(42)
$a(3.9)/a(4.2)$	1.510(32)	1.558(52)
$a(4.05)/a(4.2)$	1.233(25)	1.241(39)
$\Lambda_{\overline{\mathrm{MS}}}a(3.9)$	0.134(7)	
$g^2\langle A^2\rangle a^2(3.9)$	0.70(23)	



ALPHA$_s$ from Lattice QCD:

After curing lattice artefacts

Global fit and calibration of lattice spacing

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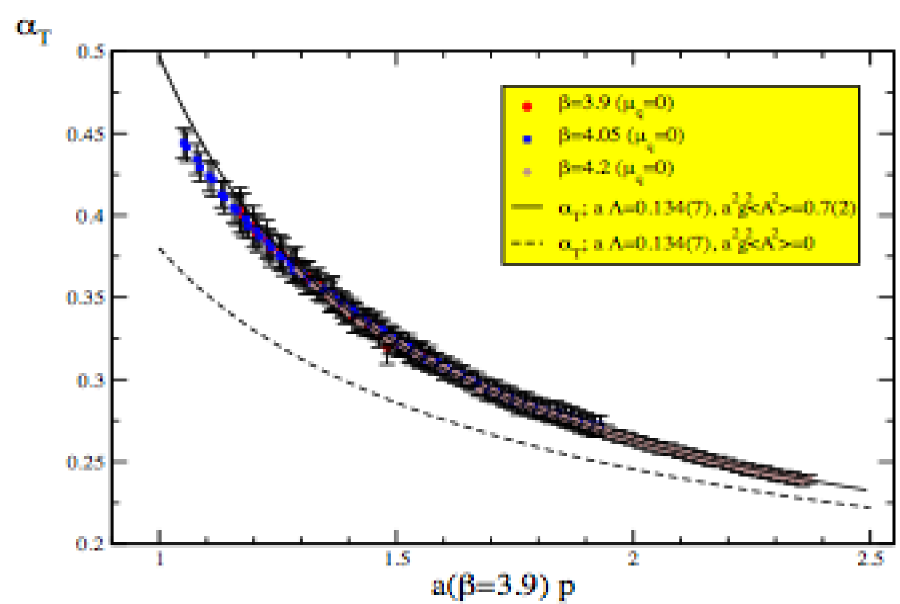


ALPHA$_s$ from Lattice QCD:

Systematic deviations

Higher orders for Wilson coefficient...

	One loop	Two loops	Three loops	Four loops
$\Lambda_{\overline{MS}} a(3.9)$	0.134(7)	0.136(7)	0.137(7)	0.138(7)
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	0.52(18)	0.44(14)	0.39(14)

Three-loop versus four-loop perturbative coupling constant...

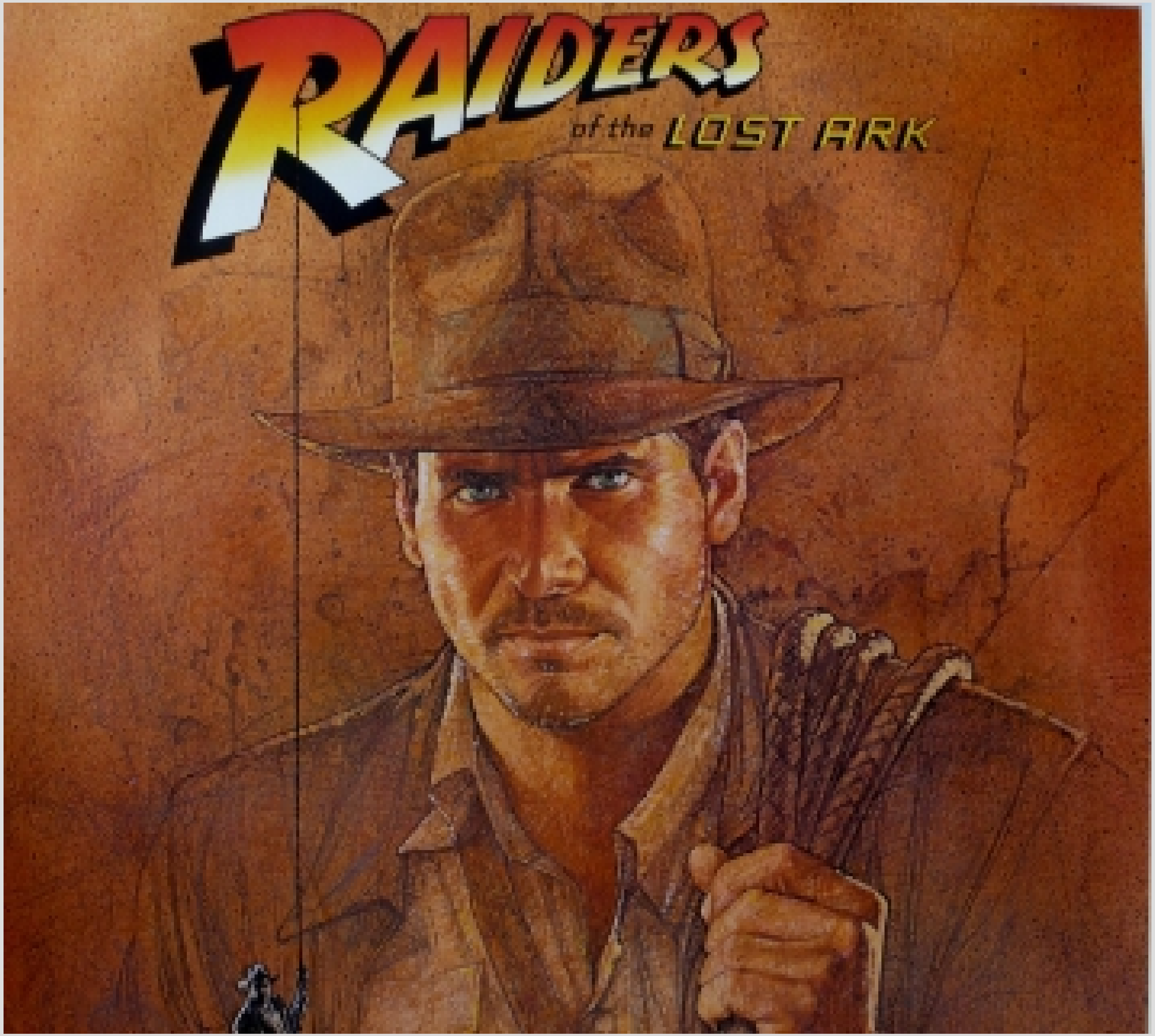
	Four loops	Three loops
$a(3.9)/a(4.05)$	1.224(23)	1.229(23)
$a(3.9)/a(4.2)$	1.510(32)	1.510(29)
$a(4.05)/a(4.2)$	1.233(26)	1.234(25)
$\Lambda_{\overline{MS}} a(3.9)$	0.134(7)	0.125(6)
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	0.80(20)

Higer orders in OPE...unstable!

$$\alpha_{T,P4}(\mu^2) = \alpha_T(\mu^2) + \frac{c_4}{\mu^4}$$

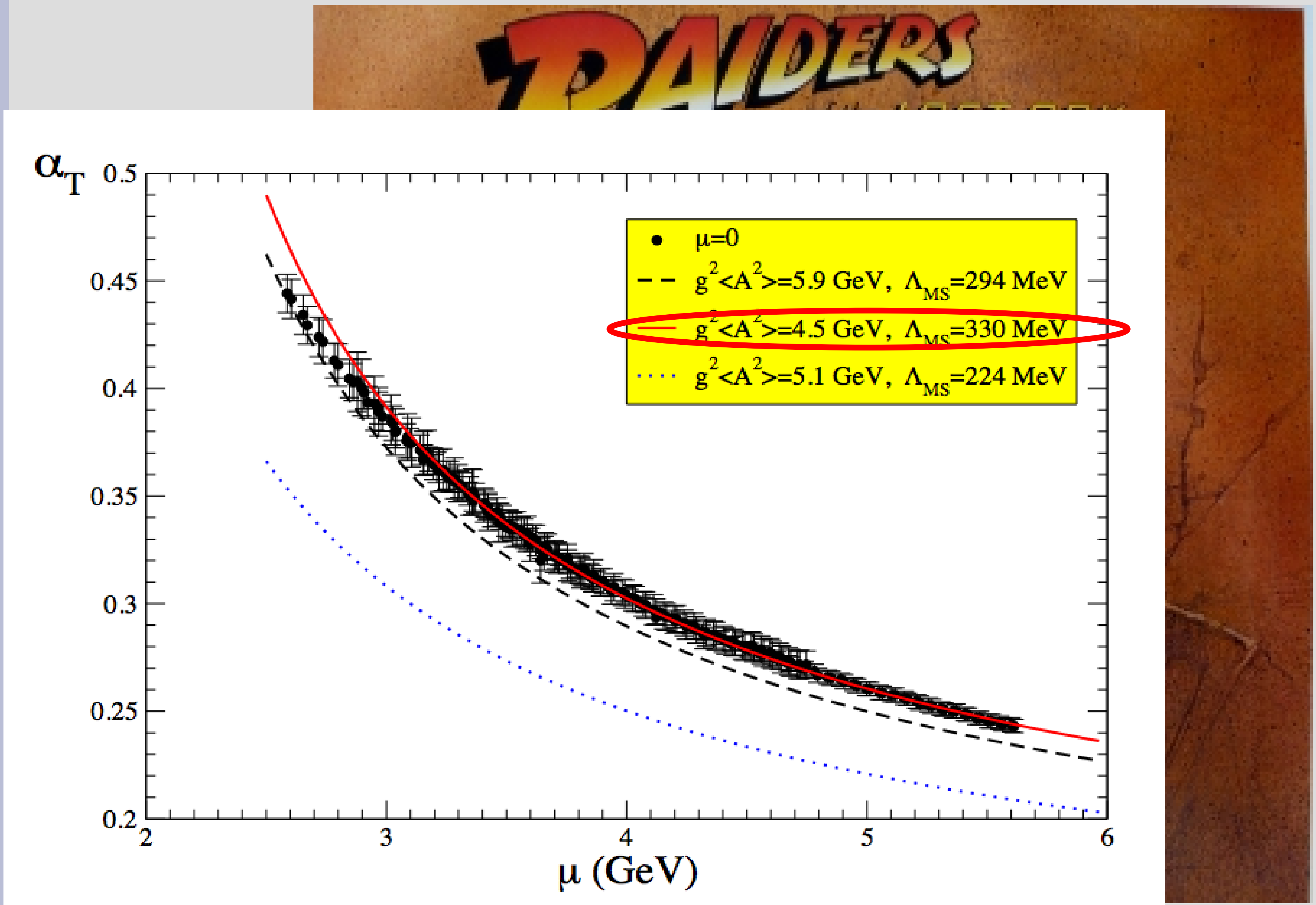
ALPHA$_s$ from Lattice QCD:

Conclusions



ALPHA$_S$ from Lattice QCD:

Conclusions



ALPHA$_s$ from Lattice QCD:

Conclusions

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$N_f = 2$:

$$\Lambda_{\overline{MS}} = (330 \pm 23 \pm 22_{-33})\,\text{MeV}$$
$$g^2(q_0^2)\langle A^2\rangle_{q_0} = \left(4.2 \pm 1.5 \pm 0.7^{+?}\right)\text{GeV}^2\,, \quad q_0 \sim 10\,\text{GeV}$$

$N_f = 0$:

$$\Lambda_{\overline{MS}} = 224^{+8}_{-5}\text{Mev}$$
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Outlooks

ETMC new configurations:

$$N_f = 4$$
$$N_f = 2+1+1$$

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Thank you!!!

Outlooks

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