Probing the theoretical description of central exclusive production

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Overview.

Central exclusive production and the Durham model

Next-to-leading order corrections

Phenomenological impact
Central exclusive production

- Central exclusive production is the process

\[ h_1(p_1) + h_2(p_2) \rightarrow h_1(p'_1) \oplus X \oplus h_2(p'_2) \]

- Can provide, potentially unique, information on the central system:
  - Quantum number filter (non \( J^{PC} = 0^{++} \) suppressed).
  - Invariant mass, with resolution \( \sim 2-3 \text{ GeV (per event)} \), via a missing mass method (Albrow & Rostovtsev - arXiv:hep-ph/0009336).

- Di-jet, \( \chi_c \) and \( \gamma\gamma \) production observed by CDF at the Tevatron (\textit{Phys. Rev. D77, Phys. Rev. Lett. 102, 99}).

- Feasibility at the LHC studied by the FP420 R&D collaboration (arXiv:0806.0302).

- For a recent review see Albrow, TC & Forshaw arXiv:1006.1289 (to be published in Progress in Particle and Nuclear Physics).
Theoretical predictions - the Durham model

- Central exclusive production calculated in perturbative QCD by Khoze, Martin & Ryskin.

- Schematically:
The Durham model - cross-section

- The cross-section is assumed to factorise as (Khoze, Martin & Ryskin, *Eur. Phys. J. C*23)

\[
\frac{\partial \sigma}{\partial \hat{s} \partial y \partial p_{1\perp}^2 \partial p_{2\perp}^2} = S^2 e^{-b(p_{1\perp}^2 + p_{2\perp}^2)} \frac{\partial \mathcal{L}}{\partial \hat{s} \partial y} \, d\hat{\sigma}(gg \rightarrow X) .
\]

- Sub-process cross-section:

\[
d\hat{\sigma}(gg \rightarrow X) = \frac{1}{2\hat{s}} |\tilde{\mathcal{M}}(gg \rightarrow X)|^2 \, dPS_X
\]

where,

\[
\tilde{\mathcal{M}}(gg \rightarrow X) = \frac{1}{2} \frac{1}{N^2 - 1} \sum_{a_1 a_2} \sum_{\lambda_1 \lambda_2} \delta_{a_1 a_2} \delta_{\lambda_1 \lambda_2} M_{\lambda_1 \lambda_2}^{a_1 a_2} (gg \rightarrow X) .
\]

The sum over equal helicities here gives the $J_z = 0$ selection rule.
The Durham model - effective luminosity

- Effective luminosity, \( \frac{\partial L}{\partial \hat{s} \partial y} \), given by

\[
\frac{\partial L}{\partial \hat{s} \partial y} = \frac{1}{\hat{s}} \left( \frac{\pi}{N^2 - 1} \int \frac{dQ_\perp^2}{Q_\perp^4} f_g(x_1, x_1', Q_\perp^2, \mu^2) f_g(x_2, x_2', Q_\perp^2, \mu^2) \right)^2.
\]

- The kinematics are such that \( x_i' \ll x_i \). In this limit:

\[
f_g(x, x', Q_\perp^2, \mu^2) \approx R_g \frac{\partial}{\partial \ln Q_\perp^2} \left( \sqrt{T(Q_\perp, \mu)} xg(x, Q_\perp^2) \right).
\]

- \( T(Q_\perp, \mu) \) is a Sudakov factor and \( R_g \) accounts for the off-forward kinematics \( (x_i' \neq x_i) \).
The focus of this talk will be the Sudakov factor, $T(Q_{\perp}, \mu)$.

Sudakov factor previously found to be given by (Kaidalov, Khoze, Martin & Ryskin Eur. Phys. J. C33)

$$T(Q_{\perp}, \mu) = \exp \left( - \int_{Q_{\perp}^2}^{\hat{s}/4} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{\alpha_s(k_{\perp}^2)}{2\pi} \int_0^{1-\Delta} dz \left[ zP_{gg}(z) + \sum_q P_{qg}(z) \right] \right)$$

where

$$\Delta = \frac{k_{\perp}}{k_{\perp} + \mu}, \quad \mu = 0.62\sqrt{\hat{s}}.$$

To collect all terms of order $\alpha_s^n \ln^m(\hat{s}/Q_{\perp}^2)$, with $m = 2n, 2n - 1$, require precise upper $z$ and lower $k_{\perp}^2$ cutoffs.
Form of the Durham result - Sudakov factor (2)

- To understand the lower limit, consider the $k_\perp \sim Q_\perp$ region in the BFKL formalism. This leads to the replacement:

$$\int_{k_0} \frac{d^2 k_\perp}{k_\perp^2} \rightarrow \int_{k_0} \frac{d^2 k_\perp}{k_\perp^2} \left( 1 - \frac{Q^2_\perp}{k_\perp^2 + (Q_\perp - k_\perp)^2} \right) \approx \int_{Q_\perp^2} \frac{d^2 k_\perp}{k_\perp^2}$$

i.e. the region with $k_\perp^2 < Q_\perp^2$ is cancelled.

- Upper $z$ limit corresponds to soft gluons. Fix it by exploiting unitarity (Bloch-Nordsieck theorem).
Form of the Durham result - Sudakov factor (3)

- Calculate $\sigma(gg \rightarrow Hg)$. By unitarity, soft logarithms in this process will be equal and opposite to those in the $gg \rightarrow H$ process.

- KMR obtain

$$\sigma(gg \rightarrow gH) \propto \int \frac{d k_{\perp}^2}{k_{\perp}^2} \frac{C_A \alpha_s}{\pi} \left( \ln(0.62) + \ln \left( \frac{m_H}{k_{\perp}} \right) - \frac{11}{12} \right)$$

$$= \int \frac{d k_{\perp}^2}{k_{\perp}^2} \frac{\alpha_s}{2\pi} \int_0^{1-\Delta} dz \ z P_{gg}(z)$$

- We find that this result is not correct. Specifically, we find one should replace $0.62 \rightarrow 1$ (TC, J. Forshaw - JHEP 1001).
Next-to-leading order corrections - our calculation

- Take the process $qq \rightarrow qHq$.

- Imaginary part of the amplitude dominates, $A \approx i\Im(A)$, so use the Cutkosky rules.

- Compute the one-loop corrections to each side of the cut

- Use these to extract the Sudakov factor.
Next-to-leading order diagrams

- Full set of diagrams with the Higgs to the right of the cut (not including those related by $x_1 \leftrightarrow x_2$)
Method of calculation (1)

- Use \( m_{\text{top}} \to \infty \) effective theory (Shifman et al, Voloshin, Ellis et al).

- Interaction described by an effective Lagrangian:

\[ \mathcal{L}_{\text{eff}} = -\frac{H}{4} C_1^R(\mu) \, G^a_{\mu\nu} G^{a\mu\nu} + \cdots \]

where

\[ C_1^R(\mu) = -\frac{1}{3v} \frac{\alpha_s(\mu)}{\pi} \left(1 + \frac{11}{4} \frac{\alpha_s(\mu)}{\pi}\right) + \mathcal{O}(\alpha_s^3) \]
Need to calculate tensor integrals:

\[ I^{\mu_1 \cdots \mu_m}(d; \{\nu_k\}_{k=1}^N) = \int \frac{d^d k}{i \pi^{d/2}} \frac{k^{\mu_1} \cdots k^{\mu_m}}{(k + q_1)^{2\nu_1} \cdots (k + q_N)^{2\nu_N}} \]

Two steps:

1. **Tensor reduction to scalar integrals** (Davydychev):

   \[ I^{\mu_1 \cdots \mu_m}(d; \{\nu_k\}_{k=1}^N) = \sum c^{\mu_1 \cdots \mu_m} I(d'; \{\nu'_k\}_{k=1}^N) \]

   where \(d + m \leq d' \leq d + 2m\) and \(\nu'_k \geq \nu_k\).

2. **Integral recursion**: Reduce scalar integrals to a known basis set of “Master Integrals”.
\[ A_{\text{NLO}} \approx A_0 \int \frac{dQ^2_\perp}{Q^4_\perp} \left( \frac{2 \alpha_s(Q^2_\perp)}{\pi} N \int_0^{Q^2_\perp} \frac{dk^2_\perp}{(k^2_\perp)^{1+\epsilon}} \int_0^{1-k_\perp/|Q_\perp|} P_{qq}(z)dz \right. \\
+ 2\epsilon_G(Q^2) \ln \left( \frac{s}{Q^2_\perp} \right) \\
- \int_{Q^2_\perp}^{m^2_H/4} \frac{dk^2_\perp}{k^2_\perp} \frac{\alpha_s(k^2_\perp)}{2\pi} \int_0^{1-k_\perp/m_H} dz \left[ zP_{gg}(z) + \sum_q P_{qg}(z) \right] \right) \\
\]

- Which should be compared with what we would expect expanding out the Durham Sudakov:

\[ A_{\text{NLO}} \approx A_0 \int \frac{dQ^2_\perp}{Q^4_\perp} \left( \int_{Q^2_\perp}^{m^2_H/4} \frac{dk^2_\perp}{k^2_\perp} \frac{\alpha_s(k^2_\perp)}{2\pi} \int_0^{1-\Delta} dz \left[ zP_{gg}(z) + \sum_q P_{qg}(z) \right] \right) \\
\]

with

\[ \Delta = \frac{k_\perp}{k_\perp + \mu} , \quad \mu = 0.62m_H . \]
Implications

- New scale suppresses the amplitude relative to the original Durham predictions.

- The suppression increases with central system mass.

- To understand the size of the effect, consider the full (i.e. no cuts) central exclusive Higgs cross-section at the LHC (14 TeV).

- Approximately a factor two difference.
Comments on predictions at the Tevatron

- Would be interesting to see the effect on predictions for observed processes at the Tevatron ($\gamma\gamma$, di-jets, $\chi_c$).

- However, typical theoretical uncertainties (unintegrated pdfs, soft-survival factor, etc.) of a similar size, so unlikely to find disagreement.

- Di-jet production is especially interesting. The fit is worst at high mass - where the change in Sudakov factor has the largest effect. Could lead to a better shape.
Summary and outlook

- Have computed the subset of next-to-leading order corrections sensitive to the central exclusive production Sudakov factor.

- We find that the Durham result must be modified, by the replacement $\mu = 0.62 \sqrt{\hat{s}} \rightarrow \sqrt{\hat{s}}$.

- Decreases the cross-section by a factor $\sim 2$ for $\sqrt{\hat{s}}$ in the range 80-560 GeV.

- May improve the shape of the di-jet invariant mass distribution at the Tevatron.

- Corrections computed so far form part of the full next-to-leading order corrections. Also required are:
  - Other partonic channels in addition to $qq$.
  - Emissions across the cut (so far only computed in the logarithmic approximation).
Back up slides
Form of the Durham result - pdf evolution

- $p_\perp$ ordered ladders evolve pdfs to scale $Q_\perp$

- Corrections to the Higgs vertex, after final $s$-channel emission, generate Sudakov factor, $T$. 
Form of the Durham result - pdf and Sudakov derivatives

- Final rung gives

\[ g(x, Q^2_\parallel) \propto \sqrt{T} \frac{\alpha_s}{2\pi} \sum_{a=q, g} \int \frac{dx}{x} \tilde{P}_{ga} \left( \frac{x_1}{x} \right) a(x, Q^2_\perp) \]

\[ \approx \sqrt{T} \frac{\partial g(x_1, Q^2_\perp)}{\partial \ln Q^2_\perp} + g(x_1, Q^2_\perp) \frac{\partial \sqrt{T}}{\partial \ln Q^2_\perp} \]

- First term generated by DGLAP equation.

- Second term due to lack of plus-prescription for final emission:

\[ P_{gg}(z) \propto \left( \frac{1}{1-z} \right)_+ = \frac{1}{1-z} - \delta(1-z) \int_0^1 \frac{dz'}{1-z'} \]

\[ \tilde{P}_{gg}(z) \propto \frac{1}{1 - z + \frac{Q^2_\perp}{(1-z)m^2_H}} \]