Non-universal, Non-anomalous U(1)´ in Anomaly Mediated SUSY Breaking

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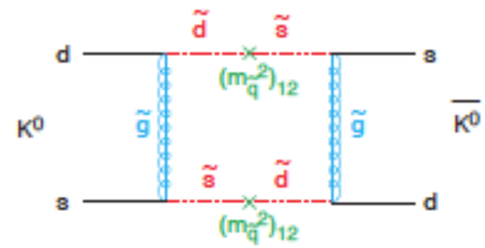
Introduction

- SUSY: solution to gauge hierarchy problem
- SUSY must be broken: sparticles at the LHC
- soft SUSY Lagrangian:

$$\mathcal{L}_{\rm soft} = -(m^2)^j{}_i \phi^i \phi_j - \left(\frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}M_\alpha\lambda_\alpha\lambda_\alpha + \text{h.c.}\right)$$



- generic models: total number of parameter = 124
- SUSY flavor and SUSY CP problem: soft parameters give additional sources of flavor violation and CP violation
- SUSY breaking mechanisms
    - gravity mediation: mSUGRA B.C.s to avoid large flavor violation
    - gauge mediation: controlling flavor violation, UV sensitive
        - low gravitino mass: severe cosmological constraints

Anomaly Mediated SUSY Breaking

- soft masses generated by conformal anomaly
- RG invariance ⇒ UV insensitive

$$
\begin{aligned}
M_i &= m_{3/2}\frac{\beta_{g_i}}{g_i} \\
A_{ijk} &= -m_{3/2}\beta_Y^{ijk} \\
(m_0^2)^i_j &= \frac{1}{2}m_{3/2}^2\mu\frac{d}{d\mu}\gamma^i_j \\
b^{ij} &= \kappa m_{3/2}\mu^{ij} - m_{3/2}\beta_\mu^{ij}
\end{aligned}
$$

sparticle masses depend on low energy dynamics only

- highly predictive: reduction of number of parameters
  - all soft SUSY breaking masses determined by $m_{3/2}$
- natural solution to SUSY flavor and SUSY CP problems
  - RG evolution: contributions to flavor violation magically cancelled
  - no additional CPV phases other than those in the Yukawa matrices and those associated with B term and mu term
- heavier gravitino mass $m_{3/2} \simeq (4\pi)^2 m_{\tilde{q}} \simeq 100 \text{ TeV}$ ⇒ saving thermal leptogenesis

Slepton Mass Problem

- anomalous dimensions for 3rd generation

$$
\begin{aligned}
16\pi^2\gamma_{H_1} &= 3\lambda_b^2 + \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{H_2} &= 3\lambda_t^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{L} &= \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{Q} &= \lambda_b^2 + \lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{3}{2}g_2^2 - \tfrac{1}{30}g_1^2, \\
16\pi^2\gamma_{t^c} &= 2\lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{8}{15}g_1^2, \\
16\pi^2\gamma_{b^c} &= 2\lambda_b^2 - \tfrac{8}{3}g_3^2 - \tfrac{2}{15}g_1^2, \\
16\pi^2\gamma_{\tau^c} &= 2\lambda_\tau^2 - \tfrac{6}{5}g_1^2,
\end{aligned}
$$

- for 1st & 2nd generations:
  - gauge contributions dominate
  - negative slepton masses: breaking EM
- AMSB in the minimal form is ruled out

Slepton Mass Problem

- Existing solutions:
  - adding common mass^2 term (mAMSB):

$$(\overline{m}^2)^i_j = \frac{1}{2} m_{3/2}^2 \mu \frac{d}{d\mu} \gamma^i_j + m_0^2 \delta_{ij}$$

not RG invariant, loss UV sensitivity

  - adding D-term contribution: I. Jack and D.R.T. Jones (1999)

$$(\overline{m}^2)^i_j = \frac{1}{2} m_{3/2}^2 \mu \frac{d}{d\mu} \gamma^i_j + \zeta q_i \delta_{ij}$$

ζ : D-term
q_i : charge of chiral superfield

    - if no mixed anomalies with respect to the SM:

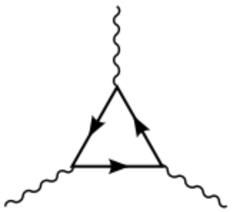
    $(\overline{m}^2)^i_j$: RG invariance ⇒ UV insensitive solution

    - U(1)´ forbids proton decay arising from
      - dim-4 R-parity operators
      - Planck induced higher dimensional operators $\frac{QQQL}{M_{pl}}$

Slepton Mass Problem

- D-term contribution:
- generation independent U(1)´ :
  - in the presence of 3 RH neutrinos: $U(1)_Y$, $U(1)_{B\text{-}L}$
  - no SUSY flavor problem
- generation-dependent U(1)´ :
  - can explain fermion mass hierarchy and mixing a la Froggatt-Nielsen
  - flavor violation can be induced:
  - high U(1)´ scale:
    - flavor violation under control
    - interesting predictions
  - existing solution: anomalous U(1)´
    - with mixed anomaly cancelled by Green-Schwarz mechanism

Anomalous vs Non-anomalous U(1)´



- anomaly cancellations: relating charges of different fermions
  - $[U(1)´]^3$ condition generally difficult to solve
- most models utilized anomalous U(1)´:
  - mixed anomaly: cancelled by Green-Schwarz mechanism
  - $[U(1)´]^3$ anomaly: cancelled by exotic fields besides RH neutrinos
  - U(1)´ broken at fundamental string scale
  - earlier claim that U(1)´ has to be anomalous to be compatible with SU(5) while giving rise to realistic fermion mass and mixing patterns Ibanez, Ross, 1994

constraints not as stringent

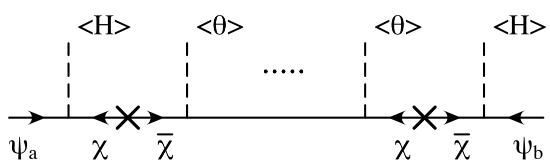
- non-anomalous U(1)´ can be compatible with SUSY SU(5) while giving rise to realistic fermion mass and mixing patterns M.-C.C, D.R.T. Jones, A. Rajaraman, H.B. Yu, 2008
  - no exotics other than 3 RH neutrinos
  - U(1)´ also forbids Higgs-mediated proton decay

Solution with Non-universal U(1)´

- Superpotential

$$W = Y_u H_u Q u^c + Y_d H_d Q d^c + Y_e H_d L e^c + Y_\nu H_u L \nu^c + Y_{\nu\nu} \Xi \nu^c \nu^c + \mu H_u H_d$$

- Yukawa sector: determined by U(1)’ charges, flavor structure from FN



$$Y_{ij} \sim \left(y_{ij} \frac{\Phi}{\Lambda} \right)^{3|q_i + q_j + q_H|}$$

- suppressed mu term

$$\mu \sim \left(\mu_{ud} \frac{\Phi}{\Lambda} \right)^{3|q_{H_u} + q_{H_d} - 1/3|} \Phi$$

Solution with Non-universal U(1)´

- sparticle masses: pure AMSB contributions + D-term contributions

$$\begin{aligned} \bar{m}_Q^2 &= m_Q^2 + \zeta q_{Q_i} \delta_j^i, \\ \bar{m}_{u^c}^2 &= m_{u^c}^2 + \zeta q_{u_i} \delta_j^i, \\ \bar{m}_{d^c}^2 &= m_{d^c}^2 + \zeta q_{d_i} \delta_j^i, \end{aligned} \qquad \begin{aligned} \bar{m}_L^2 &= m_L^2 + \zeta q_{L_i} \delta_j^i, \\ \bar{m}_e^2 &= m_e^2 + \zeta q_{e_i} \delta_j^i, \end{aligned} \qquad \begin{aligned} \bar{m}_{H_u}^2 &= m_{H_u}^2 + \zeta q_{H_u}, \\ \bar{m}_{H_d}^2 &= m_{H_d}^2 + \zeta q_{H_d}, \end{aligned}$$

- search for solutions that satisfy:
    - all 6 anomaly cancellation conditions
    - realistic quark masses (6), charged lepton masses (3), neutrino masses and mixing angles (6)
    - electroweak symmetry breaking
    - all squark and slepton masses positive

Anomaly Cancellation Conditions

- 6 anomaly cancellation conditions:

$$
\begin{aligned}
[SU(3)]^2 U(1)'_F \; &: \; \sum_i \left[2q_{Q_i} - (-q_{u_i}) - (-q_{d_i})\right] = 0 \, , \\
[SU(2)_L]^2 U(1)'_F \; &: \; \sum_i \left[q_{L_i} + 3q_{Q_i}\right] = 0 \, , \\
[U(1)_Y]^2 \, U(1)'_F \; &: \; \sum_i \Bigg[2 \times 3 \times \left(\frac{1}{6}\right)^2 q_{Q_i} - 3 \times \left(\frac{2}{3}\right)^2 (-q_{u_i}) - 3 \times \left(-\frac{1}{3}\right)^2 (-q_{d_i}) \\
&\qquad + 2 \times \left(-\frac{1}{2}\right)^2 q_{L_i} - (-1)^2 (-q_{e_i})\Bigg] = 0 \, , \\
\left[U(1)'_F\right]^2 U(1)_Y \; &: \; \sum_i \Bigg[2 \times 3 \times \left(\frac{1}{6}\right) q_{Q_i}^2 - 3 \times \left(\frac{2}{3}\right) \times (-q_{u_i})^2 - 3 \times \left(-\frac{1}{3}\right)(-q_{d_i})^2 \\
&\qquad + 2 \times \left(-\frac{1}{2}\right)(q_{L_i})^2 - (-1)(-q_{e_i})^2\Bigg] = 0 \, , \\
U(1)'_F - \mathrm{gravity} \; &: \; \sum_i \left[6q_{Q_i} + 3q_{u_i} + 3q_{d_i} + 2q_{L_i} + q_{e_i} + q_{N_i}\right] = 0 \, , \\
[U(1)'_F]^3 \; &: \sum_i \left[3\big(2(q_{Q_i})^3 - (-q_{u_i})^3 - (-q_{d_i})^3\big) + 2(q_{L_i})^3 - (-q_{e_i})^3 - (-q_{N_i})^3\right] = 0
\end{aligned}
$$

Finding the Solutions

- convenient parametrization

$$
\begin{aligned}
q_{Q_1} &= -\frac{1}{3}q_{L_1} - 2a, \\
q_{Q_2} &= -\frac{1}{3}q_{L_2} + a + a', \\
q_{Q_3} &= -\frac{1}{3}q_{L_3} + a - a', \\
q_{u_1} &= -\frac{2}{3}q_{L_1} - q_{e_1} - 2b, \\
q_{u_2} &= -\frac{2}{3}q_{L_2} - q_{e_2} + b + b', \\
q_{u_3} &= -\frac{2}{3}q_{L_3} - q_{e_3} + b - b',
\end{aligned}
$$

$$
\begin{aligned}
q_{d_1} &= \frac{4}{3}q_{L_1} + q_{e_1} - 2c, \\
q_{d_2} &= \frac{4}{3}q_{L_2} + q_{e_2} + c + c', \\
q_{d_3} &= \frac{4}{3}q_{L_3} + q_{e_3} + c - c', \\
q_{N_1} &= -2q_{L_1} - q_{e_1} - 2d, \\
q_{N_2} &= -2q_{L_2} - q_{e_2} + d + d', \\
q_{N_3} &= -2q_{L_3} - q_{e_3} + d - d'.
\end{aligned}
$$

Finding the Solutions

- in total: 12 charges for chiral superfields
- further conditions:
  - heavy 3rd generation Yukawa couplings not suppressed + hierarchy

$$q_{Q_3} + q_{u_3} + q_{H_u} = 0 \qquad q_{Q_3} + q_{d_3} + q_{H_d} = 1 \qquad q_{L_3} + q_{e_3} + q_{H_d} = 1$$

$$c' = -a' \qquad b' = -1/2 - a'$$

  - mild suppression for 3rd generation neutrino Dirac mass + charged lepton mass hierarchy + maximal atm mixing

$$q_{L_3} + q_{N_3} + q_{H_u} = 2 \qquad q_{L_2} = q_{L_3} \qquad q_{e_2} = q_{e_3} + 2 \qquad q_{e_1} = q_{e_3} + 3$$

  - anomaly cancellation conditions further reduce the number of parameters
  - in total, two parameters parametrizing class of solutions

Solution (I)

- charges that satisfy all requirements are NOT pretty
- nevertheless, it is remarkable that solutions exist at all

Field	$U(1)'$ charge	Field	$U(1)'$ charge
L_1	$q_{L_1} = 3/2$	Q_1	$q_{Q_1} = 1003/450$
L_2	$q_{L_2} = 1/2$	Q_2	$q_{Q_2} = 1447/225$
L_3	$q_{L_3} = 1/2$	Q_3	$q_{Q_3} = 983/225$
e_1^c	$q_{e_1} = 31228381/1586700$	u_1^c	$q_{u_1} = -21278009/1586700$
e_2^c	$q_{e_2} = 29641681/1586700$	u_2^c	$q_{u_2} = -28164287/1586700$
e_3^c	$q_{e_3} = 26468281/1586700$	u_2^c	$q_{u_3} = -40540547/1586700$
N_1	$q_{N_1} = -31757281/1586700$	d_1^c	$q_{d_1} = 10200251/528900$
N_2	$q_{N_2} = -31757281/1586700$	d_2^c	$q_{d_2} = 548909/21156$
N_3	$q_{N_3} = -31757281/1586700$	d_3^c	$q_{d_3} = 1390561/105780$
H_u	$q_{H_u} = 35724031/1586700$	Φ	$q_\Phi = -1/3$
H_d	$q_{H_d} = -27261631/1586700$	Ξ	$q_\Xi = 28583881/793350$

Resulting Yukawa Sector

$$Y_U \sim \begin{pmatrix} (\lambda)^{|2q_{Q_1}+q_{u_1}+q_{H_u}|} & (\lambda)^{|q_{Q_1}+q_{u_2}+q_{H_u}|} & (\lambda)^{|q_{Q_1}+q_{u_3}+q_{H_u}|} \\ (\lambda)^{|q_{Q_2}+q_{u_1}+q_{H_u}|} & (\lambda)^{|q_{Q_2}+q_{u_2}q_{H_u}|} & (\lambda)^{|q_{Q_2}+q_{u_3}+q_{H_u}|} \\ (\lambda)^{|q_{Q_3}+q_{u_1}+q_{H_u}|} & (\lambda)^{|q_{Q_3}+q_{u_2}+q_{H_u}|} & (\lambda)^{|q_{Q_3}+q_{u_3}+q_{H_u}|} \end{pmatrix}$$

$$= \begin{pmatrix} (\lambda)^{10} & (\lambda)^{\frac{283}{50}} & (\lambda)^{|-\frac{107}{50}|} \\ (\lambda)^{\frac{67}{50}} & (\lambda)^{|-3|} & (\lambda)^{|-\frac{54}{5}|} \\ (\lambda)^{|-\frac{607}{50}|} & (\lambda)^{\frac{39}{5}} & (\lambda)^{0} \end{pmatrix}$$

$$Y_D \sim \begin{pmatrix} (\lambda)^{|q_{Q_1}+q_{d_1}+q_{H_d}|} & (\lambda)^{|q_{Q_1}+q_{d_2}+q_{H_d}|} & (\lambda)^{|q_{Q_1}+q_{d_3}+q_{H_d}|} \\ (\lambda)^{|q_{Q_2}+q_{d_1}+q_{H_d}|} & (\lambda)^{|q_{Q_2}+q_{d_2}+q_{H_d}|} & (\lambda)^{|q_{Q_2}+q_{d_3}+q_{H_d}|} \\ (\lambda)^{|q_{Q_3}+q_{d_1}+q_{H_d}|} & (\lambda)^{|q_{Q_3}+q_{d_2}+q_{H_d}|} & (\lambda)^{|q_{Q_3}+q_{d_3}+q_{H_d}|} \end{pmatrix}$$

$$= \begin{pmatrix} (\lambda)^{5} & (\lambda)^{\frac{583}{50}} & (\lambda)^{|-\frac{57}{50}|} \\ (\lambda)^{|-\frac{183}{50}|} & (\lambda)^{3} & (\lambda)^{|-\frac{49}{5}|} \\ (\lambda)^{\frac{357}{50}} & (\lambda)^{\frac{69}{5}} & (\lambda)^{1} \end{pmatrix}$$

$$Y_E \sim \begin{pmatrix} (\lambda)^{|q_{L_1}+q_{e_1}+q_{H_d}|} & (\lambda)^{|q_{L_1}+q_{e_2}+q_{H_d}|} & (\lambda)^{|q_{L_1}+q_{e_3}+q_{H_d}|} \\ (\lambda)^{|q_{L_2}+q_{e_1}+q_{H_d}|} & (\lambda)^{|q_{L_2}+q_{e_2}+q_{H_d}|} & (\lambda)^{|q_{L_2}+q_{e_3}+q_{H_d}|} \\ (\lambda)^{|q_{L_3}+q_{e_1}+q_{H_d}|} & (\lambda)^{|q_{L_3}+q_{e_2}+q_{H_d}|} & (\lambda)^{|q_{L_3}+q_{e_3}+q_{H_d}|} \end{pmatrix}$$

$$= \begin{pmatrix} (\lambda)^{5} & (\lambda)^{4} & (\lambda)^{2} \\ (\lambda)^{4} & (\lambda)^{3} & (\lambda)^{1} \\ (\lambda)^{4} & (\lambda)^{3} & (\lambda)^{1} \end{pmatrix}$$

$$Y_N \sim \begin{pmatrix} (\lambda)^{|q_{L_1}+q_{N_1}+q_{H_u}|} & (\lambda)^{|q_{L_1}+q_{N_2}+q_{H_u}|} & (\lambda)^{|q_{L_1}+q_{N_3}+q_{H_u}|} \\ (\lambda)^{|q_{L_2}+q_{N_1}+q_{H_u}|} & (\lambda)^{|q_{L_2}+q_{N_2}+q_{H_u}|} & (\lambda)^{|q_{L_2}+q_{N_3}+q_{H_u}|} \\ (\lambda)^{|q_{L_3}+q_{N_1}+q_{H_u}|} & (\lambda)^{|q_{L_3}+q_{N_2}+q_{H_u}|} & (\lambda)^{|q_{L_3}+q_{N_3}+q_{H_u}|} \end{pmatrix}$$

$$= \begin{pmatrix} (\lambda)^{3} & (\lambda)^{3} & (\lambda)^{3} \\ (\lambda)^{2} & (\lambda)^{2} & (\lambda)^{2} \\ (\lambda)^{2} & (\lambda)^{2} & (\lambda)^{2} \end{pmatrix}$$

$$\lambda = \left(\frac{\langle\phi\rangle}{\Lambda}\right)^3 = \left(\frac{\langle\phi'\rangle}{\Lambda}\right)^3 = 0.22$$

$$Y_{NN} \sim \begin{pmatrix} (\lambda)^{|2q_{N_1}|} & (\lambda)^{|q_{N_1}+q_{N_2}|} & (\lambda)^{|q_{N_1}+q_{N_3}|} \\ (\lambda)^{|q_{N_2}+q_{N_1}|} & (\lambda)^{|2q_{N_2}|} & (\lambda)^{|q_{N_2}+q_{N_3}|} \\ (\lambda)^{|q_{N_3}+q_{N_1}|} & (\lambda)^{|q_{N_3}+q_{N_2}|} & (\lambda)^{|2q_{N_3}|} \end{pmatrix}$$

$$= \begin{pmatrix} (\lambda)^{|-\frac{31757281}{793350}|} & (\lambda)^{|-\frac{31757281}{793350}|} & (\lambda)^{|-\frac{31757281}{793350}|} \\ (\lambda)^{|-\frac{31757281}{793350}|} & (\lambda)^{|-\frac{31757281}{793350}|} & (\lambda)^{|-\frac{31757281}{793350}|} \\ (\lambda)^{|-\frac{31757281}{793350}|} & (\lambda)^{|-\frac{31757281}{793350}|} & (\lambda)^{|-\frac{31757281}{793350}|} \end{pmatrix}$$

Predicted Sparticle Spectrum (I)

- SOFTSUSY 3.0

$$\zeta = 1.5 \times (100GeV)^2$$

$$\tan\beta = 10$$

$$sign(\mu) = -1$$

Field	Mass (GeV)	Field	Mass (GeV)
$h0$	115	$\tilde{c}_R$	754
$H0$	212	$\tilde{s}_L$	747
$A0$	212	$\tilde{s}_R$	1014
$H+$	227	$\tilde{t}_1$	364
$\tilde{g}$	880	$\tilde{t}_2$	780
χ_1	134	$\tilde{b}_1$	744
χ_2	362	$\tilde{b}_2$	905
χ_3	509	$\tilde{e}_L$	324
χ_4	517	$\tilde{e}_R$	247
$\chi_1^{\pm}$	134	$\tilde{\mu}_L$	300
$\chi_2^{\pm}$	516	$\tilde{\mu}_R$	214
$\tilde{u}_L$	825	$\tilde{\tau}_1$	112
$\tilde{u}_R$	796	$\tilde{\tau}_2$	300
$\tilde{d}_L$	829	$\tilde{\nu}_{eL}$	314
$\tilde{d}_R$	963	$\tilde{\nu}_{\mu L}$	289
$\tilde{c}_L$	743	$\tilde{\nu}_{\tau L}$	287

Solution (II)

- $U(1)_Y$ breaking at EW scale : no D term contribution associated with $U(1)_Y$ at GUT scale
- RG induced U(1)´ - $U(1)_Y$ mixing
- non-zero D-term contribution associated with U(1)´ due to RG running
- allowing small negative lepton charges

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L_1	$q_{L_1} = 1/2$	Q_1	$q_{Q_1} = 1003/450$
L_2	$q_{L_2} = -1/2$	Q_2	$q_{Q_2} = -1447/225$
L_3	$q_{L_3} = -1/2$	Q_3	$q_{Q_3} = 983/225$
e_1^c	$q_{e_1} = 34401781/1586700$	u_1^c	$q_{u_1} = -23393609/1586700$
e_2^c	$q_{e_2} = 32815081/1586700$	u_2^c	$q_{u_2} = -30279887/1586700$
e_3^c	$q_{e_3} = 29641681/1586700$	u_2^c	$q_{u_3} = -42656147/1586700$
N_1	$q_{N_1} = -31757281/1586700$	d_1^c	$q_{d_1} = 3517617/176300$
N_2	$q_{N_2} = -31757281/1586700$	d_2^c	$q_{d_2} = 187671/7052$
N_3	$q_{N_3} = -31757281/1586700$	d_3^c	$q_{d_3} = 487027/35260$
H_u	$q_{H_u} = 35724031/1586700$	Φ	$q_\Phi = -1/3$
H_d	$q_{H_d} = -27261631/1586700$	Ξ	$q_\Xi = 28583881/793350$

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$H0$	251	$\tilde{s}_L$	748
$A0$	251	$\tilde{s}_R$	1016
$H+$	264	$\tilde{t}_1$	356
$\tilde{g}$	880	$\tilde{t}_2$	782
χ_1	134	$\tilde{b}_1$	747
χ_2	362	$\tilde{b}_2$	908
χ_3	512	$\tilde{e}_L$	313
χ_4	520	$\tilde{e}_R$	274
$\chi_1^\pm$	134	$\tilde{\mu}_L$	288
$\chi_2^\pm$	518	$\tilde{\mu}_R$	246
$\tilde{u}_L$	827	$\tilde{\tau}_1$	163
$\tilde{u}_R$	790	$\tilde{\tau}_2$	289
$\tilde{d}_L$	830	$\tilde{\nu}_{e_L}$	302
$\tilde{d}_R$	966	$\tilde{\nu}_{\mu_L}$	276
$\tilde{c}_L$	744	$\tilde{\nu}_{\tau_L}$	275

Conclusions

- AMSB: UV insensitive ⇒ high predictivity
  - mAMSB: negative slepton masses
- solving the slepton mass problem with addition of D terms: preserve UV insensitivity
- non-anomalous $U(1)'$: highly constrained model
- generation dependent $U(1)'$: can explain fermion mass hierarchy and mixing
- solutions exist, though charges are not simple