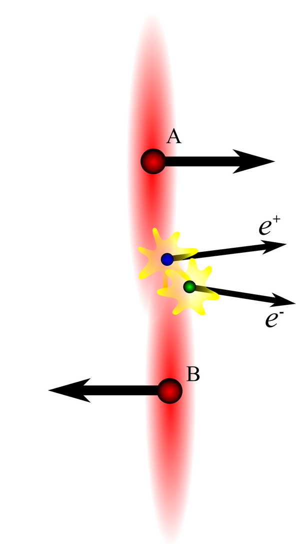

# $e^+e^-$ pair production in peripheral collisions of ultrarelativistic heavy ions


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## Abstract

*The Coulomb corrections to the cross section of $e^+e^-$ pair production in ultrarelativistic nuclear collisions are calculated in the next-to-leading approximation with respect to the parameter $L = \ln\gamma_A\gamma_B$ ($\gamma_{A,B}$ are the Lorentz factors of colliding nuclei). We found considerable reduction of the Coulomb corrections even for large $\gamma_A\gamma_B$ due to the suppression of the production of $e^+e^-$ pair with the total energy of the order of a few electron masses in the rest frame of one of the nuclei. Our result explains why the deviation from the Born result were not observed in the experiment at SPS.*

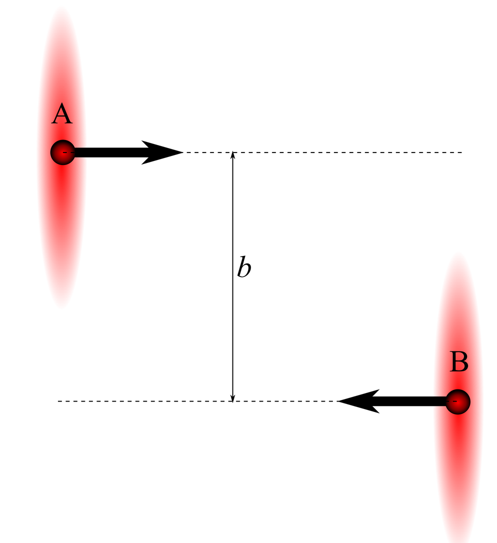

## 1. Introduction

Electron-positron pair production in ultrarelativistic nuclear collisions is investigated intensively during almost two last decades. This process is important in the problem of beam lifetime and luminocity of hadron colliders. It is also a serious background for many experiments because of its large cross section. **For heavy nuclei, the effect of higher-order terms (Coulomb corrections) of the perturbation theory with respect to the parameters $Z_A\alpha$ and $Z_B\alpha$ can be very important** ($Z_A$ and $Z_B$ are the charge numbers of the nuclei $A$ and $B$). However, **no evidence of the Coulomb corrections has been found in the SPS experiments** [1, 2]. In the set of theoretical works [3, 4, 5] a light-front approach has been elaborated which seemingly resulted in vanishing of the Coulomb corrections in the ultrarelativistic limit. This statement was considered as an explanation of the experimental results. However, it contradicted to the result obtained in Ref. [6] with the help of the Weizsäcker-Williams approximation in the leading logarithmic approximation. This contradiction has been resolved in Ref. [7]. Consistent approach of Ref. [7] results in the Coulomb corrections which coincide with those from Ref. [6].

**The absence of the Coulomb corrections in the experiments [1, 2] has remained unexplained.**

## 2. External field approximation

Since the nuclear mass is large compared to the electron mass, it is possible to treat the nuclei as sources of the external field and calculate the probability $P_n(b)$ of $n$-pair production at a fixed impact parameter $b$ between the nuclei.



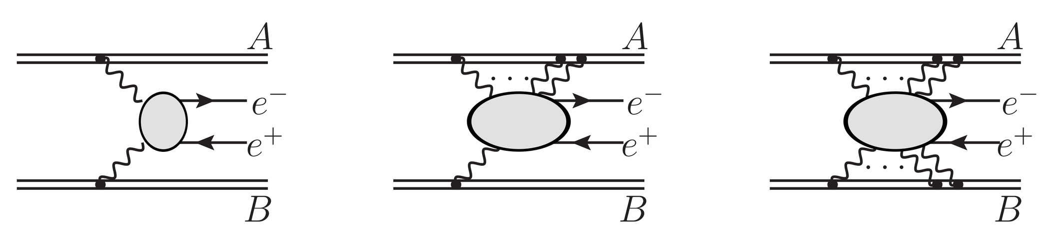

It is convenient to introduce the average number $W(b)$ of produced pairs and the number-weighted cross section $\sigma_T$ as

$$W(b) = \sum_{n=1}^{\infty} nP_n(b)\,, \quad \sigma_T = \int d^2b W(b) = \sum_{n=1}^{\infty} n\sigma_n\,, \qquad (1)$$

where $\sigma_n = \int d^2bP_n(b)$ is the cross section of $n$-pair production. The cross section $\sigma_T$ can be presented in the form:

$$\sigma_T = \sigma^0 + \sigma^A + \sigma^B + \sigma^{AB}\,, \qquad (2)$$

where $\sigma^0 \propto (Z_A\alpha)^2(Z_B\alpha)^2$ is the Born cross section, $\sigma^A$ and $\sigma^B$ are the Coulomb corrections with respect to nucleus $A$ and $B$, respectively, and $\sigma^{AB}$ is the Coulomb corrections with respect to both nuclei.




$\sigma_0$ $\quad$ $\sigma_A$ $\quad$ $\sigma_{AB}$

The cross section $\sigma^0$ is well-known *(Landau,1934;Racah,1937)*:

$$\sigma^0 = \frac{28(Z_A\alpha)^2(Z_B\alpha)^2}{27\pi m^2}\left[L^3 - 2.2\,L^2 + 3.8\,L - 1.6\right]\,, \qquad (3)$$

In the leading logarithmic approximation, the quantities $\sigma^{A,B} \propto L^2$ and $\sigma^{AB} \propto L$ were obtained in Refs. [6, 7] and in Ref. [8],respectively:

$$\sigma^{A,B}_{LO} = -\frac{28(Z_A\alpha)^2(Z_B\alpha)^2}{9\pi m^2} f(Z_{A,B}\alpha)L^2 \qquad (4)$$

$$\sigma^{AB}_{LO} = \frac{56(Z_A\alpha)^2(Z_B\alpha)^2}{9\pi m^2} f(Z_A\alpha)f(Z_B\alpha)L\,. \qquad (5)$$

## 3. Unitarity corrections

Unitarity correction $\sigma_{\rm unit}$ is the difference between the cross section $\sigma_1$ of one pair production and the number-weighted cross section $\sigma_T$:

$$\sigma_1 = \sigma_T + \sigma_{\rm unit}\,,$$

In the leading logarithmic approximation $P_n(\rho)$ is subject to Poisson distribution and

$$\sigma_{\rm unit} = -\int d^2\,bW(b)\left(1 - \mathrm{e}^{-W(b)}\right)\,. \qquad (6)$$

The unitarity correction has been numerically evaluated in Ref. [9]. It is rather small and negative, therefore can not explain the experimental $Z^2$ scaling of the cross section.

## 4. Next-to-leading contribution to $\sigma^A$

Note that $\sigma^A$, being proportional to $(Z_B\alpha)^2$, can be directly calculated as the Coulomb corrections to $\sigma_1$ with respect to the parameter $Z_A\alpha$, so that it can be represented as

$$\sigma^A = \int \left[dn_\perp(\omega, Q^2)\sigma_\perp(\omega, Q^2) + dn_\parallel(\omega, Q^2)\sigma_\parallel(\omega, Q^2)\right] \qquad (7)$$

Here $dn_\perp(\omega, Q^2)$ and $dn_\parallel(\omega, Q^2)$ are the numbers of virtual photons $\gamma^*_{\perp,\parallel}$ with the energy $\omega$ and the virtuality $-Q^2 < 0$. The quantities $\sigma_\perp(\omega, Q^2)$ and $\sigma_\parallel(\omega, Q^2)$ are the Coulomb corrections to the cross sections of the processes $\gamma^*_{\perp,\parallel}A \to e^+e^-A$.

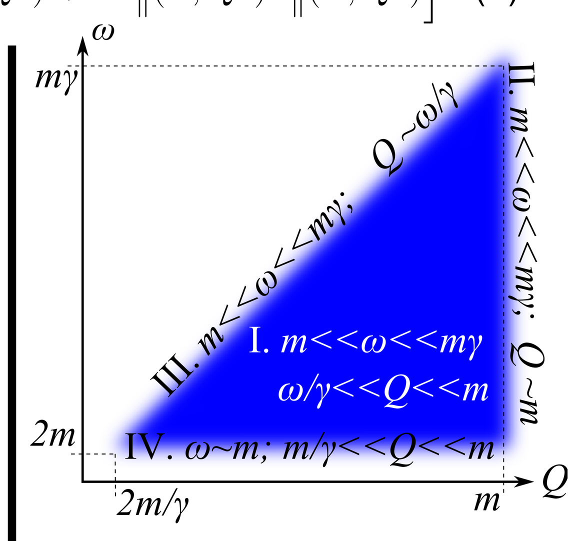



**Figure 1:** *Regions of integration*

The leading logarithmic contribution $\propto L^2$ comes from the region I, and the correction $\propto L$ comes from regions II,III,IV (see Fig.1).

## 5. Asymptotic region (I+II+III)

Since $\omega \gg m$ in regions I-III, we can use quasiclassical approximation to calculate the contribution of these regions and replace

$$\sigma_{\perp,\parallel}(\omega, Q^2) \to \sigma_{\perp,\parallel}(\infty, Q^2) \qquad (8)$$

If we adopt this replacement also in region IV, we obtain [10, 11].

$$\sigma^A_{\rm as} = -\frac{28(Z_B\alpha)^2(Z_A\alpha)^2}{9\pi m^2} f(Z_A\alpha)\left[L^2 + \frac{20}{21}L\right]\,. \qquad (9)$$

The coefficient in front of $L^1$ is of the order of unity and positive!

**If this was the final result, the Coulomb corrections should have been observed in the experiment.**

## 6. Near-threshold region (IV)

Let us represent

$$\sigma^A = \sigma^A_{\rm as} + \delta\sigma^A\,. \qquad (10)$$

Note that $\delta\sigma^A$ comes entirely from region IV (with logarithmic precision). Therefore, we can neglect the virtuality $Q^2$ of the photon and obtain

$$\delta\sigma^A = -\frac{28(Z_B\alpha)^2(Z_A\alpha)^2 G(Z_A\alpha)}{9\pi m^2} f(Z_A\alpha)L\,,$$

$$G(Z_A\alpha) = 2\int_{2m}^{\infty}\frac{d\omega}{\omega}\left[\frac{\sigma_\perp(\omega, 0)}{\sigma_\perp(\infty, 0)} - 1\right]\,. \qquad (11)$$

**The quantity $\sigma_\perp(\omega, 0) \equiv \sigma_{\gamma A}(\omega)$ is the Coulomb corrections to the cross section of $e^+e^-$ pair production by real photon in the Coulomb field**, and $\sigma_\perp(\infty, 0) = -28(Z_A\alpha)^2/9m^2$. Taking the sum of Eqs. (9) and (11), we finally obtain $\sigma^A$ in the next-to-leading approximation

$$\sigma^A = -\frac{28(Z_B\alpha)^2(Z_A\alpha)^2}{9\pi m^2} f(Z_A\alpha)\left[L^2 + \left(G(Z_A\alpha) + \frac{20}{21}\right)L\right] \qquad (12)$$

The function $G(Z\alpha)$ is shown in Fig. 2. It is seen that $G(Z\alpha)$ varies varies slowly from $-6.6$ for $Z = 1$ to $-6.14$ for $Z = 100$. The large value of $G$ leads to a big difference between $\sigma^A$ from Eq. (12) and its leading logarithmic approximation (4).

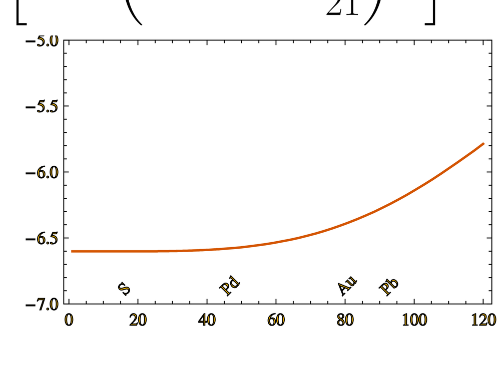



**Figure 2:** $G(Z\alpha)$ *vs* $Z$.

## 7. Results

The contribution $\delta\sigma^A$, corresponding to $G(Z_A\alpha)$ in Eq. (12) is very important. If one omits this contribution and use $\sigma^A_{\rm as}$, Eq. (9), as an approximation to $\sigma^A$, then the contribution of linear in $L$ term becomes much less important, see the dashed curve in Fig. 3.

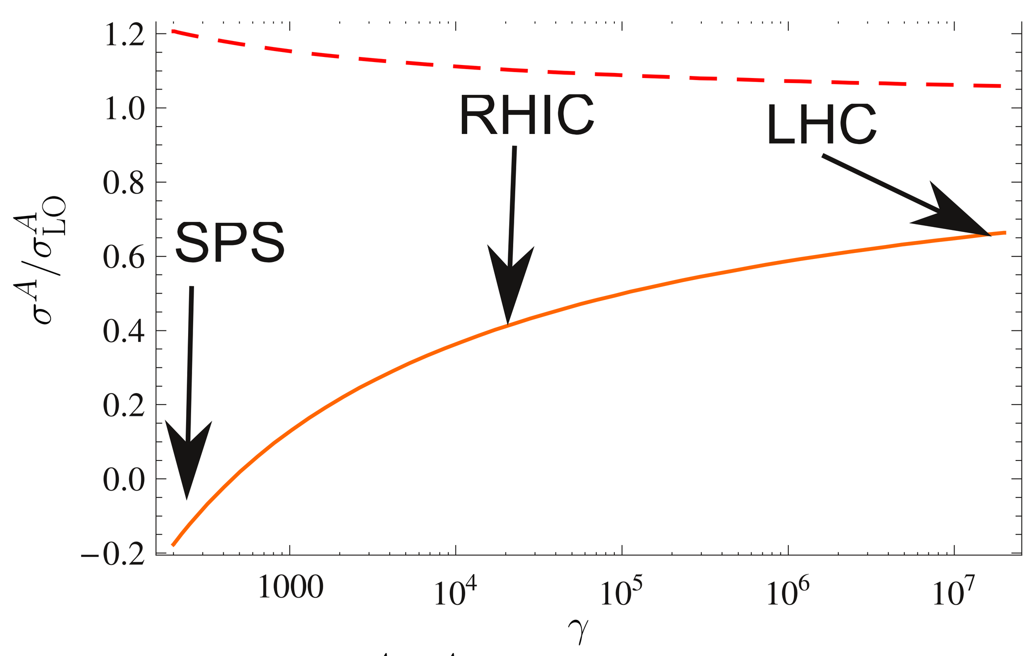



**Figure 3:** *The ratio $\sigma^A/\sigma^A_{\rm LO}$ (solid curve) as a function of $\gamma$ for $Z_A = 82$. Here $\sigma^A_{\rm LO}$, Eq. (4) is the Coulomb corrections calculated in the leading logarithmic approximation. Dashed curve shows the ratio $\sigma^A_{\rm as}/\sigma^A_{\rm LO}$.*

Note that for Pb-Pb collisions at LHC one has $\gamma \approx 1.8\times10^7$ and $\sigma^A/\sigma^A_{\rm LO} \approx 0.66$. For Au-Au collisions at RHIC one has $\gamma \approx 2.3\times10^4$ and $\sigma^A/\sigma^A_{\rm LO} \approx 0.42$. For the experiments at SPS [1, 2], the Lorentz factor was $\gamma \approx 200$. Naturally, we can not use the result (12) obtained in the logarithmic approximation in the region $\gamma \lesssim 500$ where the logarithmic correction to $\sigma^A$ becomes larger than the leading term $\sigma^A_{\rm LO}$. However, we can claim that, due to the strong compensation between the leading term and the correction, the Coulomb corrections $\sigma^A$ are much smaller than $\sigma^A_{\rm LO}$ at $\gamma \lesssim 500$. Therefore, this naturally explains why there was no evidence of the Coulomb corrections in the experiments [1, 2].

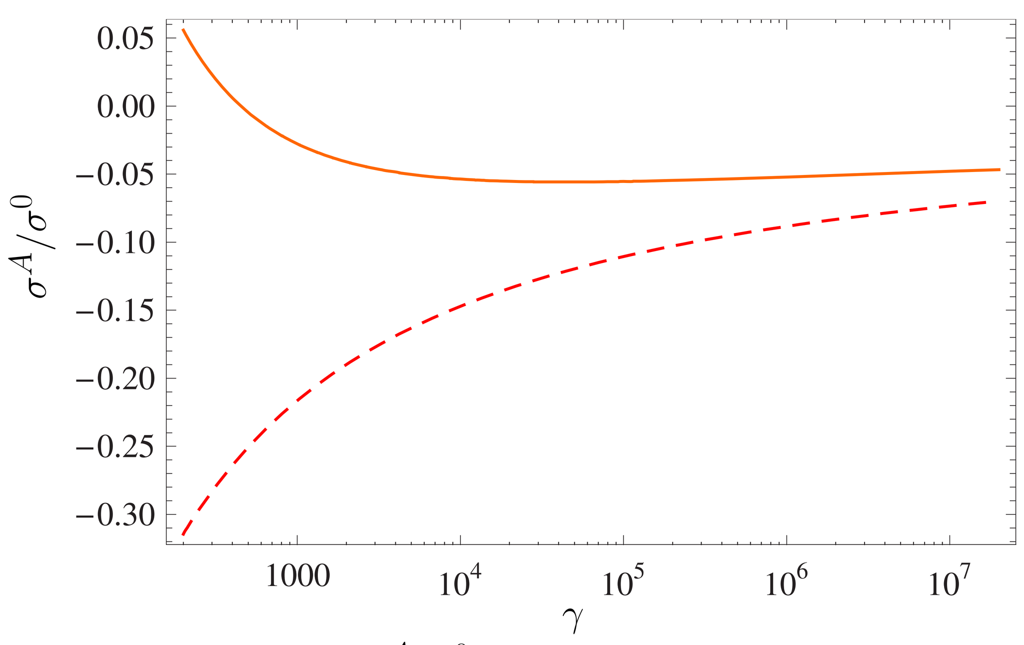



**Figure 4:** *The ratio $\sigma^A/\sigma^0$ (solid curve) as a function of $\gamma$ for $Z_A = 82$. Dashed curve shows the ratio $\sigma^A_{\rm LO}/\sigma^0$.*

## 8. Conclusion

We have calculated the Coulomb corrections $\sigma^A$ to $e^+e^-$ pair production in the next-to-leading logarithmic approximation. After the account of the next-to-leading term, the magnitude of $\sigma^A$ becomes small in comparison with the Born cross section, in contrast to the leading term $\sigma^A_{\rm LO}$. The big difference between our result and previously suggested one is due to strong suppression of the exact Coulomb corrections are in a rather wide region $2m < \omega \lesssim 20m$ near threshold. Our results, combined with $\sigma^{AB}$, Ref. [8], complete the calculation of linear in $L$ terms in the number-weighted cross section $\sigma_T$.

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