NNLL resummation for QCD cross sections

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- Resummation
- Soft anomalous dimensions
- Two-loop eikonal calculations
- NNLL resummation for top quark processes
- LHC and Tevatron phenomenology
Resummation

Soft-gluon corrections important in many processes, particularly near threshold

Needed at higher-orders for increased accuracy in theoretical predictions

Terms \( \ln^k \left( \frac{s_4}{M^2} \right) \), \( k \leq 2n - 1 \), \( s_4 \to 0 \) at threshold arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

Soft corrections exponentiate

Resummation follows from factorization

At NLL accuracy requires one-loop calculations in the eikonal approximation

New results: NNLL accuracy – two-loop calculations

Approximate NNLO cross section from expansion of resummed cross section

Many phenomenological applications:

- top pair and single top production;
- jet, direct photon, or W production at high \( p_T \);
- (charged) Higgs, squark and gluino production; etc.
Resummed cross section

Resummation follows from factorization properties of the cross section - performed in moment space

Use RGE to evolve function associated with soft-gluon emission

\[ \hat{\sigma}^{\text{res}}(N) = \exp \left[ \sum_i E_i(N) \right] H(\alpha_s) \]

\[ \times \exp \left[ \int \frac{d\mu}{\sqrt{s}} \Gamma_S^\dagger(\alpha_s(\mu)) \right] S \left( \alpha_s \left( \frac{\sqrt{s}}{N} \right) \right) \exp \left[ \int \frac{d\mu}{\sqrt{s}} \Gamma_S(\alpha_s(\mu)) \right] \]

where

\[ \Gamma_S \] is the soft anomalous dimension - a matrix in color space

and a function of kinematical invariants \( s, t, u \)

Calculate \( \Gamma_S \) in eikonal approximation

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Eikonal approximation

Feynman rules for soft gluon emission simplify

\[ \bar{u}(p) \left( -i g_s T_F^c \right) \gamma^\mu \frac{i(p + k + m)}{(p + k)^2 - m^2 + i\epsilon} \rightarrow \bar{u}(p) g_s T_F^c \gamma^\mu \frac{p + m}{2p \cdot k + i\epsilon} = \bar{u}(p) g_s T_F^c \frac{v^\mu}{v \cdot k + i\epsilon} \]

with \( p \propto v, T_F^c \) generators of SU(3)

Perform calculation in momentum space and Feynman gauge

Complete two-loop results for

- soft (cusp) anomalous dimension for \( e^+ e^- \rightarrow t\bar{t} \)
- \( s \)-channel single top production
- \( bg \rightarrow tW^- \) and \( bg \rightarrow tH^- \)
- \( t\bar{t} \) hadroproduction
Soft (cusp) anomalous dimension

One-loop eikonal diagrams

\[ \Gamma_S = \frac{\alpha_s}{\pi} \Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2} \Gamma_S^{(2)} + \cdots \]

The one-loop soft anomalous dimension, \( \Gamma_S^{(1)} \), can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

\[ \Gamma_S^{(1)} = C_F \left[ -\frac{(1 + \beta^2)}{2\beta} \ln \left( \frac{1 - \beta}{1 + \beta} \right) - 1 \right] \quad \text{with} \quad \beta = \sqrt{1 - \frac{4m^2}{s}} \]
Two-loop eikonal diagrams

Vertex correction graphs

Heavy-quark self-energy graphs
Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

\[ \Gamma_S^{(2)} = \frac{K}{2} \Gamma_S^{(1)} + C_F C_A \left\{ \frac{1}{2} + \frac{\zeta_2}{2} + \frac{1}{2} \ln^2 \left( \frac{1 - \beta}{1 + \beta} \right) \right. \]
\[ \left. - \frac{(1 + \beta^2)^2}{8\beta^2} \left[ \zeta_3 + \zeta_2 \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \frac{1}{3} \ln^3 \left( \frac{1 - \beta}{1 + \beta} \right) + \ln \left( \frac{1 - \beta}{1 + \beta} \right) \text{Li}_2 \left( \frac{(1 - \beta)^2}{(1 + \beta)^2} \right) - \text{Li}_3 \left( \frac{(1 - \beta)^2}{(1 + \beta)^2} \right) \right] \right. \]
\[ \left. - \frac{(1 + \beta^2)}{4\beta} \left[ \zeta_2 - \zeta_2 \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \ln^2 \left( \frac{1 - \beta}{1 + \beta} \right) - \frac{1}{3} \ln^3 \left( \frac{1 - \beta}{1 + \beta} \right) + 2 \ln \left( \frac{1 - \beta}{1 + \beta} \right) \ln \left( \frac{(1 + \beta)^2}{4\beta} \right) \right] \right. \]
\[ \left. - \text{Li}_2 \left( \frac{(1 - \beta)^2}{(1 + \beta)^2} \right) \right\} \]

where \( K = C_A (67/18 - \zeta_2) - 5n_f/9 \)

In terms of the cusp angle \( \gamma = \ln [(1 + \beta)/(1 - \beta)] \) we get

\[ \Gamma_S^{(1)} = C_F (\gamma \coth \gamma - 1) \]

and

\[ \Gamma_S^{(2)} = \frac{K}{2} \Gamma_S^{(1)} + C_F C_A \left\{ \frac{1}{2} + \frac{\zeta_2}{2} + \frac{\gamma^2}{2} - \frac{1}{2} \coth^2 \gamma \left[ \zeta_3 - \zeta_2 \gamma - \frac{\gamma^3}{3} - \gamma \text{Li}_2 (e^{-2\gamma}) - \text{Li}_3 (e^{-2\gamma}) \right] \right. \]
\[ \left. - \frac{1}{2} \coth \gamma \left[ \zeta_2 + \zeta_2 \gamma + \gamma^2 + \frac{\gamma^3}{3} + 2 \gamma \ln (1 - e^{-2\gamma}) - \text{Li}_2 (e^{-2\gamma}) \right] \right\} \]

\( \Gamma_S^{(2)} \) vanishes at \( \beta = 0 \), the threshold limit, and diverges at \( \beta = 1 \), the massless limit.

If one quark is massless and one is massive,

\[
\Gamma_S^{(2)} = \frac{K}{2} \Gamma_S^{(1)} + C_F C_A \left( \frac{1 - \frac{\beta}{3}}{4} \right)
\]

QCD processes

Color structure gets more complicated with more than two colored partons in the process.

Cusp anomalous dimension an essential component of other calculations.

Next, we compute two-loop soft anomalous dimensions for:

- Single top production in \( s \)-channel (also direct photon production)
- Associated top production with a \( W \) boson or a charged Higgs
- Top-antitop pair hadroproduction
s-channel single top production

One-loop eikonal diagrams

Two-loop eikonal diagrams
Soft anomalous dimension for $s$-channel single top production

\[ \Gamma^{(1)}_{S, \text{top } s-ch} = C_F \left[ \ln \left( \frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] \]

\[ \Gamma^{(2)}_{S, \text{top } s-ch} = \frac{K}{2} \Gamma^{(1)}_{S, \text{top } s-ch} + C_F C_A \frac{1 - \zeta_3}{4} \]

Associated production of a top quark with a $W^-$ or $H^-$

Two-loop eikonal diagrams (+ extra top-quark self-energy graphs)

Soft anomalous dimension for $bg \rightarrow tW^-$

\[
\Gamma^{(1)}_{S,tW^-} = C_F \left[ \ln \left( \frac{m_t^2 - t}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln \left( \frac{m_t^2 - u}{m_t^2 - t} \right)
\]

\[
\Gamma^{(2)}_{S,tW^-} = \frac{K}{2} \Gamma^{(1)}_{S,tW^-} + C_F C_A \frac{1 - \zeta_3}{4}
\]

Same analytical result for $\Gamma_S$ for $bg \rightarrow tH^-$
Top-antitop production in hadron colliders

The soft anomalous dimension matrix for \( q\bar{q} \to t\bar{t} \) is

\[
\Gamma_{S q\bar{q}} = \begin{bmatrix} \Gamma_{q\bar{q}11} & \Gamma_{q\bar{q}12} \\ \Gamma_{q\bar{q}21} & \Gamma_{q\bar{q}22} \end{bmatrix}
\]

At one loop

\[
\Gamma_{q\bar{q}11}^{(1)} = -C_F \left[ L_\beta + 1 \right] \quad \Gamma_{q\bar{q}21}^{(1)} = 2 \ln \left( \frac{u_1}{t_1} \right) \quad \Gamma_{q\bar{q}12}^{(1)} = \frac{C_F}{C_A} \ln \left( \frac{u_1}{t_1} \right)
\]

\[
\Gamma_{q\bar{q}22}^{(1)} = C_F \left[ 4 \ln \left( \frac{u_1}{t_1} \right) - L_\beta - 1 \right] + \frac{C_A}{2} \left[ -3 \ln \left( \frac{u_1}{t_1} \right) + \ln \left( \frac{t_1 u_1}{s m^2} \right) + L_\beta \right]
\]

where \( L_\beta = \frac{1+\beta^2}{2\beta} \ln \left( \frac{1-\beta}{1+\beta} \right) \) with \( \beta = \sqrt{1-4m^2/s} \)

Write the two-loop cusp anomalous dimension as \( \Gamma_{S}^{(2)} = \frac{K}{2} \Gamma_{S}^{(1)} + C_F C_A M_\beta \). Then at two loops

\[
\Gamma_{q\bar{q}11}^{(2)} = \frac{K}{2} \Gamma_{q\bar{q}11}^{(1)} + C_F C_A M_\beta \quad \Gamma_{q\bar{q}22}^{(2)} = \frac{K}{2} \Gamma_{q\bar{q}22}^{(1)} + C_A \left( C_F - \frac{C_A}{2} \right) M_\beta
\]

\[
\Gamma_{q\bar{q}21}^{(2)} = \frac{K}{2} \Gamma_{q\bar{q}21}^{(1)} + C_A N_\beta \ln \left( \frac{u_1}{t_1} \right) \quad \Gamma_{q\bar{q}12}^{(2)} = \frac{K}{2} \Gamma_{q\bar{q}12}^{(1)} - \frac{C_F}{2} N_\beta \ln \left( \frac{u_1}{t_1} \right)
\]

with \( N_\beta \) a subset of terms of \( M_\beta \)

Similar results for \( gg \to t\bar{t} \) channel
\( p \bar{p} \rightarrow t \bar{t} \) at Tevatron \( S^{1/2} = 1.96 \text{ TeV} \) \( \mu = m \)

\( \sigma (\text{pb}) \)

- **NNLO approx**
- **NLO**

\( p p \rightarrow t \bar{t} \) at LHC \( S^{1/2} = 7 \text{ TeV} \) \( \mu = m \)

\( \sigma (\text{pb}) \)

- **NNLO approx**
- **NLO**

\( p \bar{p} \rightarrow t \bar{t} \) at Tevatron \( S^{1/2} = 1.96 \text{ TeV} \) \( m = 173 \text{ GeV} \) \( \mu = m_T \)

\( \sigma (\text{pb}) \)

- **NNLO approx**
- **NLO**

\( p p \rightarrow t \bar{t} \) at LHC \( S^{1/2} = 7 \text{ TeV} \) \( m = 173 \text{ GeV} \) \( \mu = m_T \)

\( \frac{d\sigma}{dp_T} (\text{pb}/\text{GeV}) \)

- **NNLO approx**
- **NLO**

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Summary

- Soft-gluon corrections and resummation
- Two-loop calculations in eikonal approximation
- Massive quarks involve further complications
- Two-loop soft anomalous dimensions and NNLL resummation
- Application to single top production, $t\bar{t}$ production, and other processes at LHC and Tevatron energies