Decay constants of heavy mesons from QCD sum rules

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We present a sum-rule extraction of the decay constants of D, D_s, B, and B_s mesons from the two-point function of heavy-light pseudoscalar currents with the main emphasis on the uncertainties in these quantites, both related to the input QCD parameters and the intrinsic uncertainties of the method of sum rules.

A QCD sum-rule calculation of hadron parameters involves two steps:

I. One calculates the operator product expansion (OPE) series for a relevant correlator; obtains the sum rule which relates this OPE to the sum over hadronic states.

We make use of the well-known three-loop OPE for the pseudoscalar correlator in $\overline{MS}$*-scheme***.**

II. One extracts the parameters of the ground state by a numerical procedure.

NEW :

(a) Make use of a more accurate duality relation based on Borel-parameter-dependent threshold.

Allows a more accurate extraction of the decay constants and provides realistic estimates of the *intrinsic (systematic) errors* *— those related to the sum-rule extraction procedure.*

(b) Study the sensitivity of the extracted value of f_P *to the OPE parameters (quark masses, condensates,...). The corresponding error is referred to as* *OPE uncertainty, or statistical error*.

Basic object: OPE for $\Pi(p^2) = i\int dx e^{ipx}\langle 0|T\left(j_5(x)j_5^\dagger(0)\right)|0\rangle,\qquad j_5(x) = (m_Q + m)\bar{q}i\gamma_5 Q(x)$
and its Borel transform $(p^2 \to \tau)$**.**

Quark – hadron duality assumption :

$$\boxed{f_Q^2 M_Q^4 e^{-M_Q^2\tau} = \int\limits_{(m_Q+m_u)^2}^{s_{\rm eff}} e^{-s\tau}\rho_{\rm pert}(s,\alpha,\mu)\,ds + \Pi_{\rm power}(\tau,\mu) \equiv \Pi_{\rm dual}(\tau,\mu,s_{\rm eff})}$$

Only a few lowest-dimension power corrections are known, work at $\tau m_Q \le 1$ **("window").**

The "dual" mass: $\boxed{M^2_{\rm dual}(\tau) \equiv -\frac{d}{d\tau}\log\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).}$

If quark-hadron duality is implemented properly, then $M_{\rm dual}$ **should be equal to** M_Q $\to$

$s_{\rm eff}$ is a function of τ (and μ) : $s_{\rm eff}$ (τ, μ)

Taking into account the dependence of $s_{\rm eff}$ on τ
allows one to improve the accuracy of the duality approximation.

Obviously, in order to predict f_Q, we need to fix $s_{\rm eff}$.

Our new algorithm for extracting ground – state parameters when M_Q is known

(i) Consider a set of Polynomial τ-dependent Ansaetze for $s_{\rm eff}$:

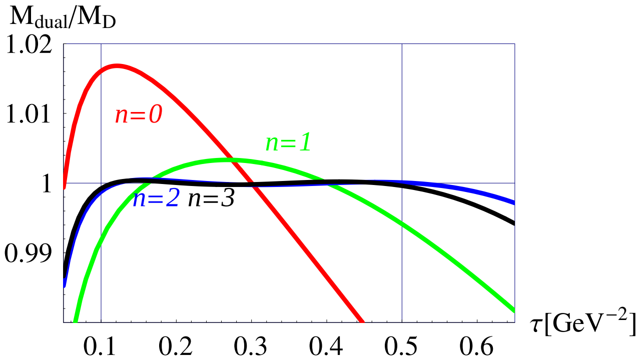
$$s_{\rm eff}^{(n)}(\tau) = \sum_{j=0}^{n} s_j^{(n)}(\tau)^j.$$

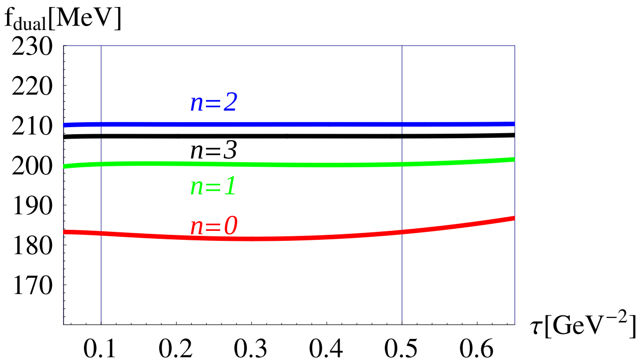
(ii) Minimize the squared difference between the “daul” mass $M^2_{\rm dual}$ and the known value M_Q^2 in the working region of τ. This gives us the parameters of the effective continuum threshold.

(iii) Making use of the obtained thresholds, calculate the decay constant.

(iv) Take the band of values provided by the results corresponding to linear and quadratic effective thresholds as the characteristic of the intrinsic uncertainty of the extration procedure.

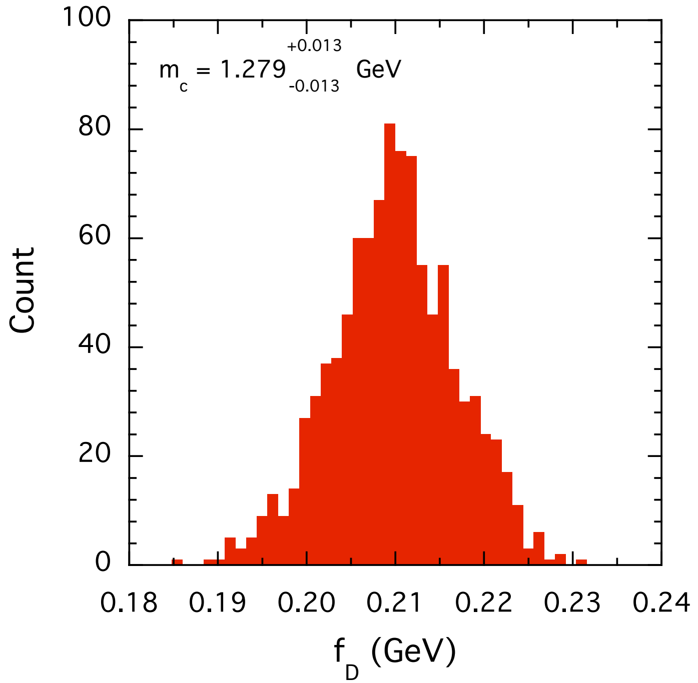
Example: D-meson

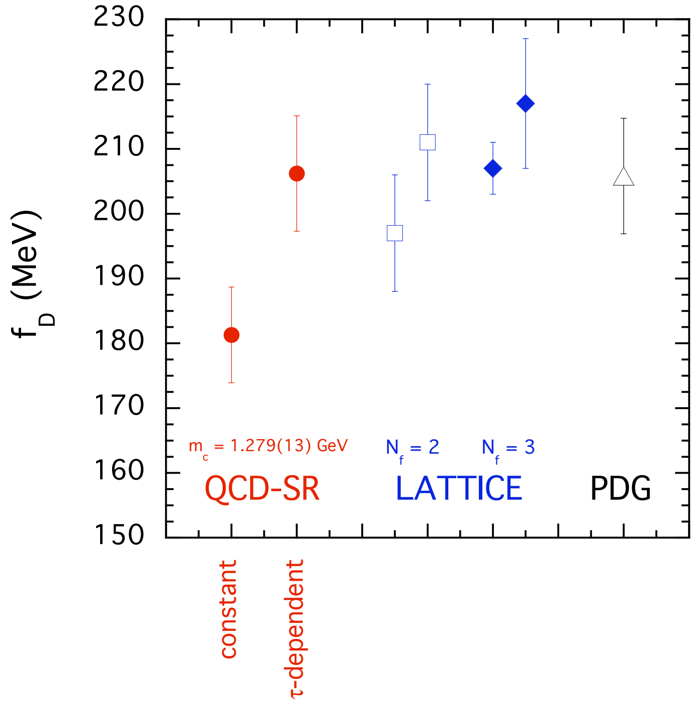




Extraction of f_D

$m_c(m_c) = 1.279 \pm 0.013\,\mathrm{GeV}, \mu = 1 - 3\,\mathrm{GeV}.$





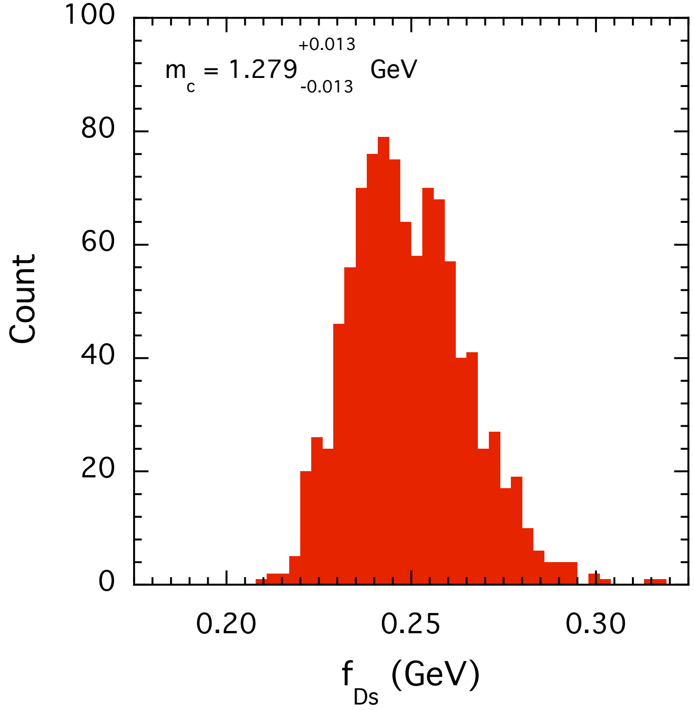
$$f_D = 206.2 \pm 7.3_{OPE} \pm 5.1_{syst}\ \mathrm{MeV}$$

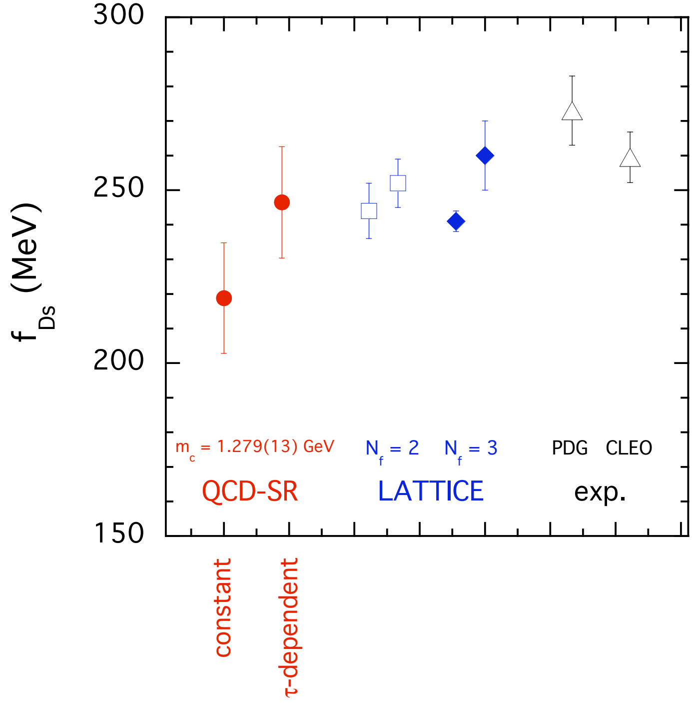
f_D (const) $= 181.3 \pm 7.4_{OPE}$ MeV

The effect of τ-dependent threshold is visible!

Extraction of f_{Ds}

$m_c(m_c) = 1.279 \pm 0.013\,\mathrm{GeV}, \mu = 1 - 3\,\mathrm{GeV}.$

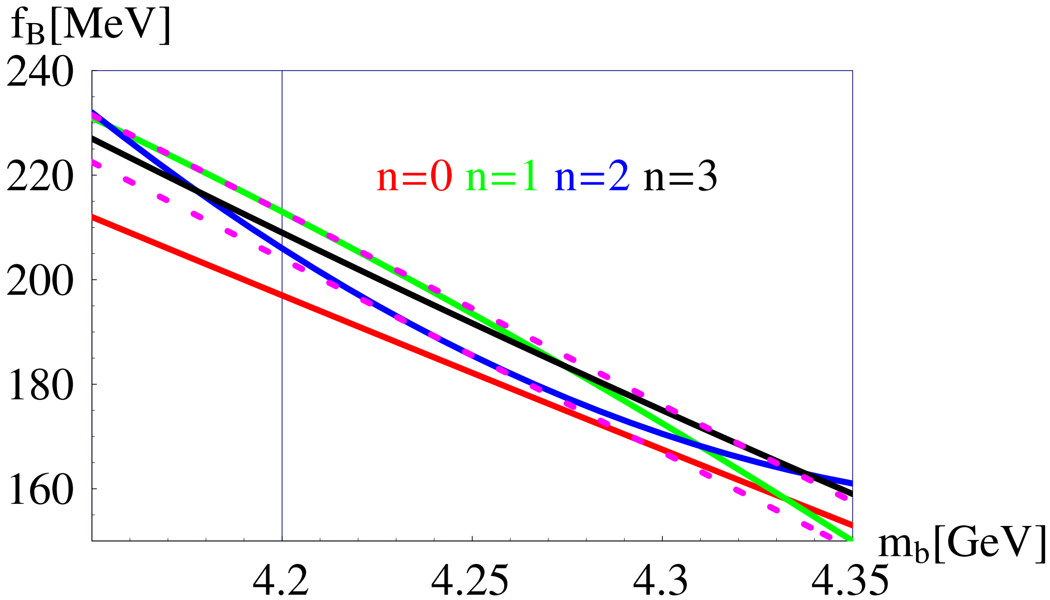




f_{Ds} (const) = 218.8 ± 16.1$_{OPE}$ MeV

$f_{D_s} = 246.5 \pm 15.7_{OPE} \pm 5_{syst}$ MeV

Extraction of f_B: a very strong sensitivity to $m_b(m_b)$

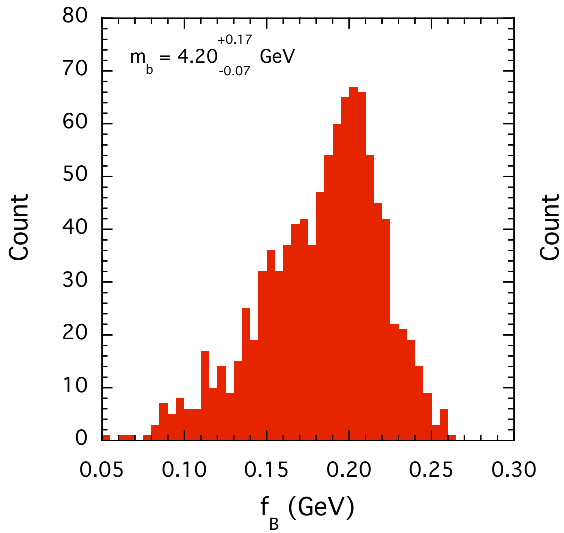


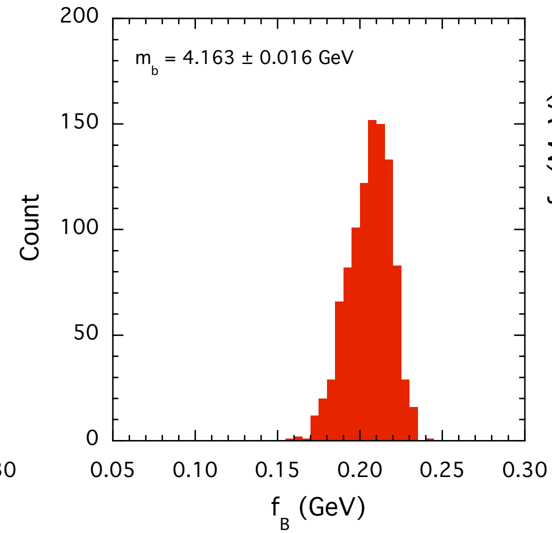
τ-dependent effective threshold:

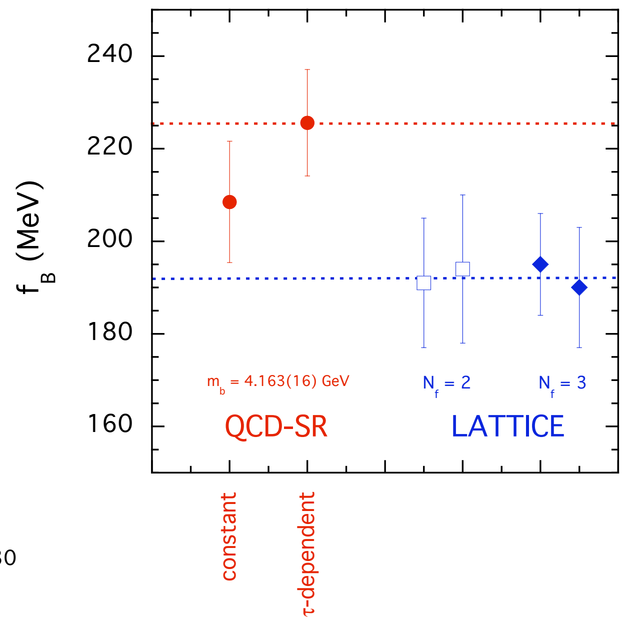
$$f_B^{\rm dual}(m_b, \langle\bar{q}q\rangle, \mu = m_b) = \left[208 \pm 4 - 37\left(\frac{m_b - \mathbf{4.2\ GeV}}{\mathbf{0.1\ GeV}}\right) + 4\left(\frac{\langle\bar{q}q\rangle^{1/3} - \mathbf{0.267\ GeV}}{\mathbf{0.01\ GeV}}\right)\right] MeV,$$

$\pm$ 100 MeV on $m_b \to \mp$ 37 MeV on f_B!

μ=2-8 GeV:







The prediction for f_B seems not feasible without a very precise knowledge of m_b

For the range $m_b(m_b) = 4.163 \pm 0.016$ GeV:

$$f_B = 225 \pm 11.3_{OPE} \pm 4_{syst} \text{ MeV}$$

$$f_{Bs} = 265 \pm 17_{OPE} \pm 5_{syst} \text{ MeV}$$

f_B (const) = $208.5 \pm 13.1_{OPE}$ MeV

f_{Bs} (const) = $245 \pm 18_{OPE}$ MeV

Conclusions

The effective continuum threshold $s_{\rm eff}$ is an important ingredient of the method which determines to a large extent the numerical values of the extracted hadron parameters. Finding a criterion for fixing $s_{\rm eff}$ poses a problem in the method of sum rules.

• τ-dependence of $s_{\rm eff}$ emerges naturally when trying to make quark-hadron duality more accurate. For those cases where the ground-state mass M_Q is known, we proposed a new algorithm for fixing $s_{\rm eff}(\tau)$. Our algorithm leads to the extraction of more accurate values of bound-state parameters than the standard algorithms used in the context of sum rules before.

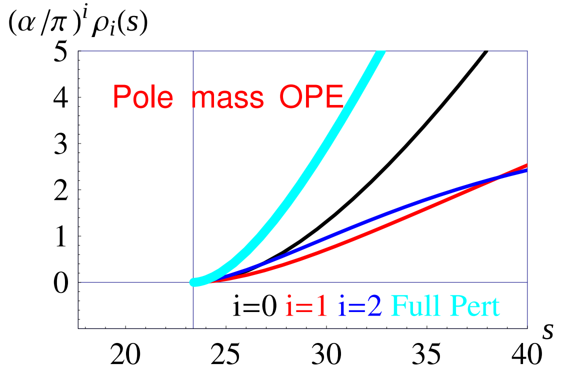
• τ-dependent $s_{\rm eff}$ is a useful concept as it allows one to probe realistic intrinsic uncertainties of the extracted parameters of the bound states.

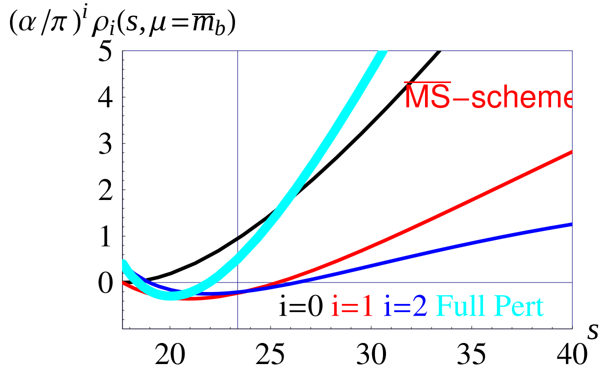
• We obtained predictions for the decay constants of heavy mesons f_Q which along with the "statistical" errors related to the uncertainties in the QCD parameters, for the first time include realistic "systematic" errors related to the uncertainty of the extraction procedure of the method of QCD sum rules.

OPE : heavy – quark pole mass or running mass?

To α_s^2-accuracy, $m_{b,pole} = 4.83$ GeV $\leftrightarrow$ $m_b(m_b) = 4.20$ GeV:

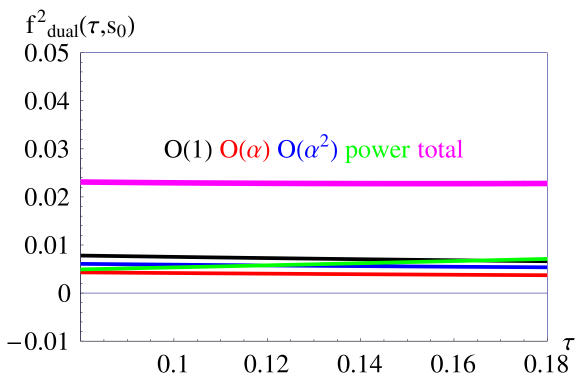
Spectral densities

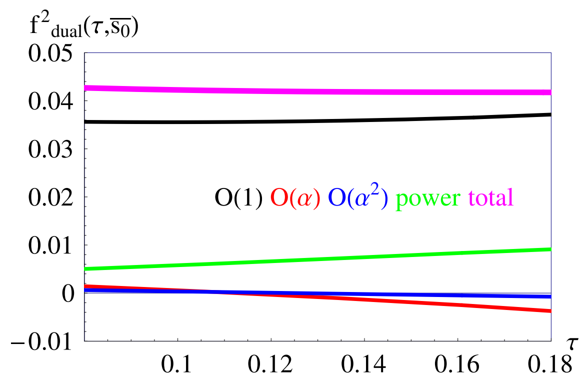




- **In pole mass scheme poor convergence of perturbative expansion**
- **In $\overline{MS}$ scheme the pert. spectral density has negative regions → higher orders NOT negligible**

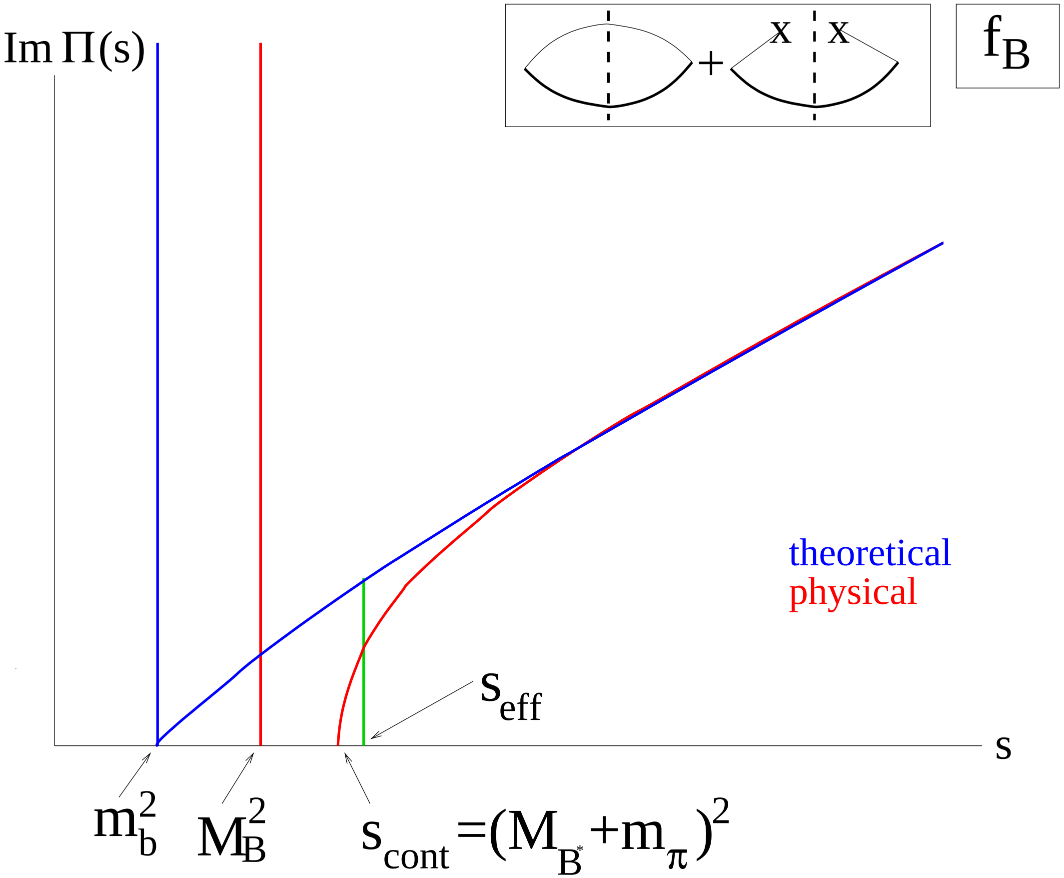
Extracted decay constant





- **Decay constant in pole mass shows NO hierarchy of perturbative contributions**
- **Decay constant in $\overline{MS}$-scheme shows such hierarchy. Numerically, f_P using pole mass $\ll$ f_P using $\overline{MS}$ mass.**

	OPE	mQ, GeV		fB, MeV
Aliev [1983]	$O(\alpha)$	pole : 4.8		130 (± 20 %)
Narison [2001]	$O(\alpha^2)$	pole : 4.7	$\overline{MS}$: 4.05	$203 \pm 23_{OPE}$
Jamin [2001]	$O(\alpha^2)$	pole : 4.83	$\overline{MS}$: 4.21 ± 0.05	$215 \pm 19_{OPE}$
Our results	$O(\alpha^2)$		$\overline{MS}$: 4.16 ± 0.016	$225 \pm 11.3_{OPE} \pm 4_{syst}$


Im Π(s)
x
x
+
f_B
theoretical
physical
s_eff
s
m_b^2
M_B^2
s_cont=(M_B* + m_π)^2