Towards a Novel Description of Flavor Dynamics in Holographic QCD

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based on 1005.2140 and ongoing work
Introduction

• There is an obvious interest to find an analytic description of the strong coupling dynamics of gauge theories, like QCD.

• By now there is a wide range of large-$N$ gauge theories whose strong coupling limit can be attacked with the AdS/CFT correspondence.

Large-$N$ QCD itself is not expected to admit a weakly curved gravitational dual, but there are theories with similar IR dynamics as QCD that have a dual weakly curved gravitational description.

It is interesting to examine what can be learnt from such descriptions.
• An important part of QCD dynamics is associated with the flavor degrees of freedom (the quarks).

• We know well how to add flavor in AdS/CFT in the quenched approximation: we add an open string sector (D-branes typically stretching in the holographic radial direction).

• However, in certain cases we lack a clear understanding of the dynamics of this sector. This impedes the discussion of fundamental questions about the flavor sector of the dual strongly coupled gauge theory, such as:

  the order parameter of chiral symmetry breaking, how one incorporates bare quark masses, the structure of the mesonic spectra etc.
A setup for Holographic QCD
the Sakai-Sugimoto model

• The Sakai-Sugimoto model is based on the following configuration of branes in type IIA string theory

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
N_c \, D_4 : & \times & \times & \times & \times & \bullet \\
N_f \, D_8 : & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
N_f \, \text{anti-D8} : & \times & \times & \times & \times & \times & \times & \times & \times & \times
\end{array}
\]

• The 4-direction is compactified with anti-periodic boundary conditions for fermions. SUSY is broken.
• Weak 't Hooft coupling: IR dynamics captured by a non-local version of the Nambu-Jona-Lasinio model  \textbf{Antonyan et al., '06} \\

• Strong 't Hooft coupling: IR dynamics captured by QCD with Nf flavors  \textbf{Sakai-Sugimoto, '04} \\

The D4-branes are replaced by a SUGRA solution (Wick rotated black D4-brane) \\

\[ ds^2 = \left( \frac{u}{R} \right)^{\frac{3}{2}} \left( -dt^2 + (dx^i)^2 + f(u)(dx^4)^2 \right) + \left( \frac{R}{u} \right)^{\frac{3}{2}} \left( \frac{du^2}{f(u)} + u^2 d\Omega_4^2 \right) \] \\

\[ e^\Phi = g_s \left( \frac{u}{R} \right)^{\frac{3}{4}}, \quad F_4 = 3\pi N_c \ell_s^3 \varepsilon_4, \quad f(u) = 1 - \frac{u_{KK}^3}{u^3} \]
This setup is controlled by three numbers: \( \frac{u_{KK}}{\ell_s}, \frac{R}{\ell_s}, N_c \).

To avoid a conical singularity at \( u_{KK} \) : \( R_4 = \frac{2R^3}{3u_{KK}^{1/3}} \).

A strong coupling cutoff must be set at \( u_{\text{max}} = g_s^{-4/3} R \).

The \( N_f \) D8-antiD8 pairs reconnect to \( N_f \) U-shaped D8’s with a modulus-dependent asymptotic separation \( L \).
• We are interested in the low-energy open string dynamics on the D8 branes

• The commonly used Dirac-Born-Infeld (DBI) action is **not** a proper effective field theory description of the dynamics in this case for the following reasons:

1. Large accelerations develop near the turning point and DBI breaks down.

2. The holographic dictionary implies that there is more to the low-energy description than just transverse scalars and gauge fields. A complex scalar field $T$ in the bi-fundamental representation of the flavor $U(1)xU(1)$ symmetry group **must** be present in the low-energy description.

The normalizable branch of $T$ captures the vev of the dual meson-like operator that acts as the *order parameter for flavor chiral symmetry breaking*. The non-normalizable branch of $T$ captures the *bare quark mass*.

Fluctuations of $T$ reproduce the sector of scalar mesons.
• Where does $T$ come from?

• It comes from the NS- sector of the open string stretching between the asymptotic D8 and anti-D8 branches. This mode has a non-trivial wavefunction in $u$ and becomes important (like a localized mode) in the vicinity of the turning point.

Its effects are closely related to the geometric reconnection (motion of transverse scalars) that captures the chiral symmetry breaking, but they also lead to a non-geometric smearing of the brane near the turning point.
A proper description of the SS model requires an extension of the DBI action that incorporates the bifundamental scalar mode $\mathbf{T}$.

Minimal progress has been achieved in this problem.

Technical complication: $\mathbf{T}$ is a mode that comes from a long open string that stretches a finite distance. It has non-local physics. Is it possible to treat such physics with a local effective action?

U-shaped (hairpin-like) branes, like the D8 flavor branes in SS, appear frequently in holographic backgrounds.

Holographic backgrounds with NSNS fluxes only are interesting cases where:
(a) we can solve string theory with alpha’-exact worldsheet methods,
(b) we can calibrate any candidate EFT description by comparison to the exact answer
(c) we can import the lessons to backgrounds with RR fluxes and learn about open string dynamics in these cases and related dynamics of strongy coupled gauge theories.
An analogue of the SS model with NSNS fluxes

- Replace the Nc D4 branes by k NS5 branes

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- The 4-direction is again compactified with anti-periodic boundary conditions for fermions. SUSY is broken.

- The D1 branes reconnect and form D1 hairpin branes in the (near-horizon) background of the NS5 branes

\[ ds^2 = -dt^2 + (dx^i)^2 + \alpha' k d\Omega_3^2 + \alpha' k (d\rho^2 + \tanh^2 \rho d\theta^2) , \quad e^\Phi = \frac{g_s}{\cosh \rho} \]
A Unified Effective Description of BPS, non-BPS and D-antiD branes

Based on *Erkal, Kutasov and Lunin, ’09*

- The TDBI action in flat space

\[ S = - \int d^{p+1}\sigma \, V(T) \sqrt{-\det(\eta_{ab} + \partial_a X^I \partial_b X^I + F_{ab} + \partial_a T \partial_b T)} \]

\[ V(T) = \frac{\tau_p}{\cosh \alpha T}, \quad \alpha = 1, \frac{1}{\sqrt{2}} \text{ bosonic, type II} \]

incorporates the tachyon dof T of non-BPS branes.

It has been derived in open string theory from first principles *Kutasov, VN, ’03*

- The real scalar field T appears in this action as an extra scalar.

  The tachyon potential V(T) appears as a T-dependent contribution to the dilaton.
Imagine an extended 11D spacetime with coordinates $X^\mu$, $T$ and metric

$$ds^2 = g_{\mu\nu}dX^\mu dX^\nu + dT^2$$

The DBI action for a p-brane in this space reads

$$S = -\int d^{p+1}\sigma \ e^{-\Phi(X,T)} \sqrt{-\det(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \partial_a T \partial_b T + F_{ab})}$$

Take a generalized dilaton of the factorized form

$$e^{-\Phi(X,T)} = e^{-\Phi(X)} V(T)$$

BPS branes, non-BPS branes and brane-antibrane pairs arise (in flat space) as different solution curves in (X,T) space.
• BPS and non-BPS branes

- DBI description recovered for normalizable excitations with correct descent relation
- TDBI description automatically recovered

TDBI basics
• **Brane-antiBrane pairs**

The TDBI action admits the following solution (Euclidean version of rolling tachyon solution)

\[
\sinh(\alpha T) = A \cos(ax)
\]

\(A\) is an arbitrary constant.

\(A=0\) gives the non-BPS brane.

\(A=\infty\) gives an array of BPS-antiBPS branes separated by the critical distance at which the DDbar tachyon \(T\) is **massless** (hence the marginality of \(A\)).
A closed ‘PAPERCLIP’ curve in (T,X) space reproduces the key features of the brane-antibrane system (dof, interactions of $T$, transverse scalars and $U(1) \times U(1)$ gauge field,...) and deals naturally with the non-local nature of $T$.

The Tachyon Paperclip
The TDBI action provides a new point of view on tachyon condensation in the brane-antibrane system: **a tachyon paperclip shrinks to zero size** (a time and space dependent solution of the TDBI *real* scalar $T$).
Let us return to the problem of interest.

To describe more efficiently the dynamics of hairpin-like branes in holographic backgrounds we should be looking for a `radially condensing’ tachyon-paperclip solution of the TDBI action: T=T(u,x).

The DBI is only a partial description of such branes.

A `condensing tachyon’ is necessarily present with a non-trivial profile.
• **D1 hairpins in the background of k NS5 branes**

In this case we can describe explicitly these branes as boundary states in worldsheet conformal field theory for any $k > 1$.

We can see explicitly how the half-winding bifundamental tachyon $T$ arises and how it controls the open string theory dynamics. We would like to reproduce these results using the TDBI effective field theory.

The starting point is the TDBI action for a non-BPS $D2$ brane that wraps the cigar in the $(\rho, \theta)$ directions

$$S = - \int dt d\rho d\theta \sinh \rho V(T) \sqrt{1 + \frac{1}{k} (\partial_{\rho} T)^2 + \frac{1}{k} \coth^2 \rho (\partial_{\theta} T)^2}$$

*(will leave the tachyon potential $V(T)$ free)*
Some of the main points that we find:

- The slope of the tachyon potential around $T=0$ is the same for all $k$ and coincides with the value of the flat space result

$$\left. \frac{1}{V(0)} \frac{d^2V}{dT^2} \right|_{T=0} = -\alpha^2, \quad \alpha = \frac{1}{\sqrt{2}}$$

- We can reproduce the asymptotic behavior of the bifundamental tachyon $T$ at $\rho \gg \rho_0$ with a tachyon-paperclip solution of the `rolling tachyon’ form

$$T \sim a + b(k)\rho + c(k) \log \cos \theta$$

and a modified (k-dependent) tachyon potential with large-$T$ behavior

$$V(T) \sim e^{-\beta T}, \quad \beta = \beta(k) > 0 \quad \left(\beta(k) = \frac{1}{\sqrt{k}}\right)$$

- There are two independent parameters controlling the leading and subleading branches of the solution.
TDBI description of Hairpin-branes

- In the vicinity of the turning point we find a $\rho$-dependent elliptical solution

$$T = A \sqrt{\rho - \rho_0} \cos \sigma + \ldots, \quad \theta = B \sqrt{\rho - \rho_0} \sin \sigma + \ldots$$

$$2kB^2 \tanh^2 \rho_0 - A^2B^2 \tanh \rho_0 + 2A^2 = 0$$

- Bootstrap methods in worldsheet CFT imply a specific relation between the boundary cosmological constant of the standard hairpin-brane and its turning point. In terms of the leading branch coefficient $\mu_T$ of $e^T$ this relation translates to

$$\log \mu_T(\rho_0) = x - z \log \sinh(yp_0), \quad y = k, \quad z = \frac{1}{2(k - 1)}$$

For $k=2$ and $V(T) = \cosh^{-1} \left( \frac{T}{\sqrt{2}} \right)$ numerical evaluation gives a rather good fit with $y=0.79$, $z=0.51$. 

TDBI description of Hairpin-branes

\[ \log(\mu_T) \]

\[ \rho_* \]
• **D8 hairpins in the Sakai-Sugimoto model**

A qualitatively similar picture is anticipated in the Sakai-Sugimoto model.

There are interesting differences, e.g. the asymptotic separation $L$ of the hairpin branches is now modulus-dependent.

Does one need a more drastic modification of the tachyon potential $V(T)$ to reproduce this feature?

Work is underway to determine appropriate tachyon-paperclip solutions, to put constraints on the tachyon potential, to determine the implications of this formalism for holographic QCD (details of holographic dictionary, bare quark mass dependence, mesonic spectra,...)