Decay constants and sigma terms from the lattice.

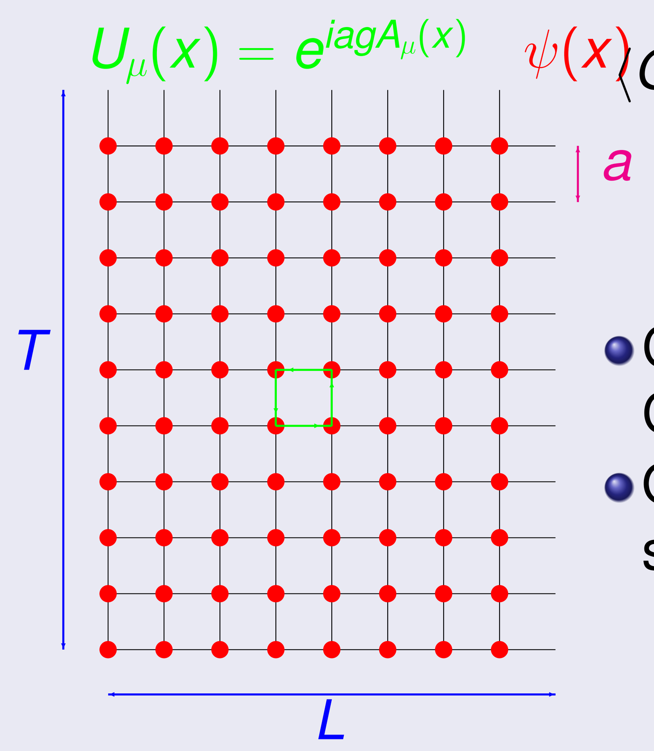
A. Ramos[1] for the Budapest-Marseille-Wuppertal Collaboration
[1] *Centre de Physique Théorique CNRS Luminy, Case 907, F-13288 Marseille Cedex 9, France*

Abstract

Thanks to the recent developments both in our understanding of lattice simulations and in computer power, lattice gauge theory can give accurate predictions of QCD with all the sources of error under control. I review recent results of the Budapest-Marseille-Wuppertal lattice collaboration: first π and K decay constants can be used to compute CKM matrix elements and check the unitarity of its first row. Second the strange content of the nucleon, for which some preliminary results based on a subset of our new dataset are presented here, is key to understand whether dark matter could be detected. I will emphasize the control of the systematics errors associated with these calculations.

Lattice QCD

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



$$\langle O\rangle = \int \mathcal{D}[U]\mathcal{D}\overline{\psi}\mathcal{D}\psi\, O(U,\overline{\psi},\psi)e^{-S_G[U]-S_F[U,\overline{\psi},\psi]}$$
$$= \int \mathcal{D}[U]\, O(U)_{\text{Wick}}e^{-S_G[U]}\det(D)$$

- Compute the integral numerically $\rightarrow$ Monte Carlo sampling of $e^{-S_G[U]}\det(D) \geq 0$.
- Observable computed averaging over samples

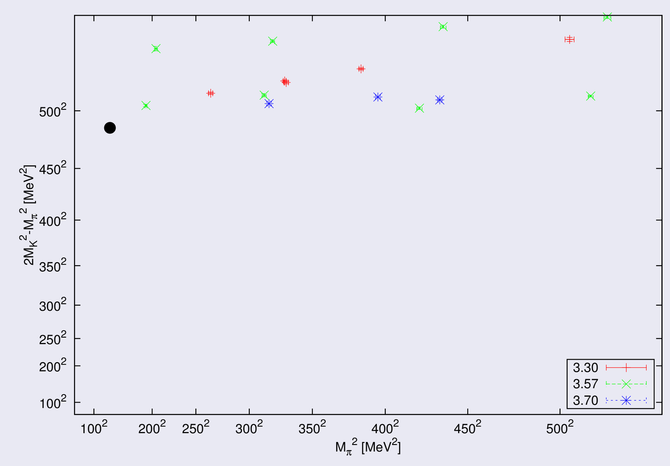
$$\langle O\rangle = \frac{1}{N_{\text{conf}}}\sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

Not a model for QCD

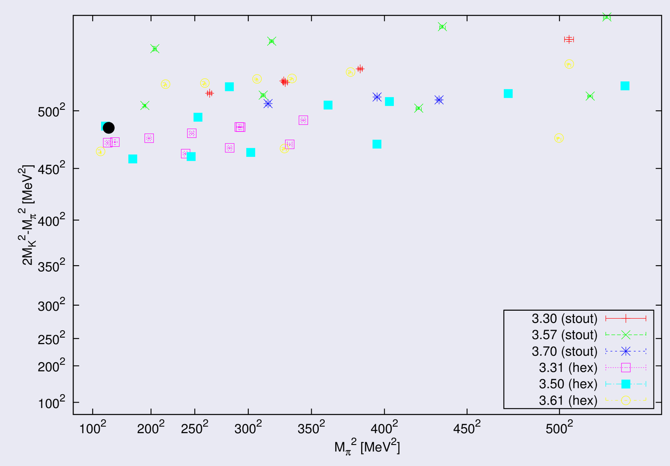
Lattice QCD IS QCD in the appropiate limit ($a \rightarrow 0$, $L \rightarrow \infty$, ...).

Tremendous progress in lasts years

- Computational cost increases dramatically when you lower quark masses.
- Only tremendous progress in lasts years both in computer power, and our understanding of lattice simulations, have made possible (recently) to simulate at the physical point.



(a) 2008 BMW dataset.



(b) 2008 + (partial)2010 BMW dataset.

- Big effort of the BMW collaboration to simulate directly at the physical point.

F_K/F_π and physics beyond the Standard Model

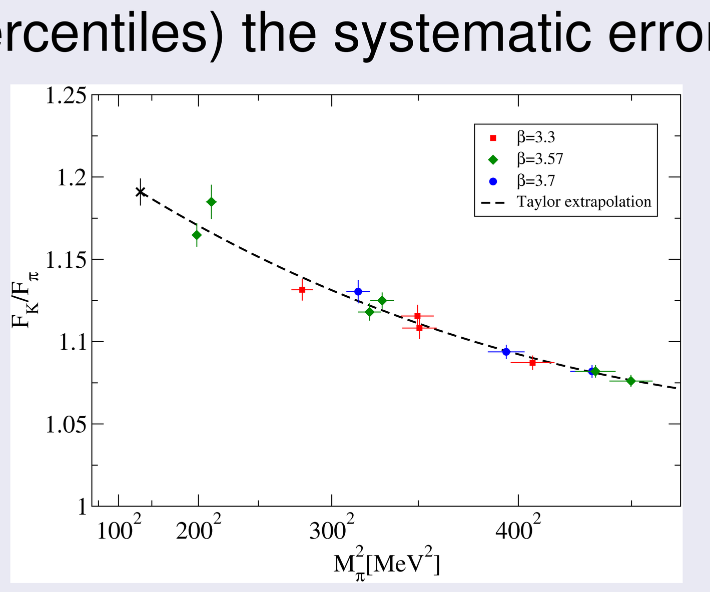
Analysis of pion and kaon decay constants can constraint physics beyond the SM.

$$\frac{\Gamma(K\rightarrow\mu\overline{\nu}_\mu)}{\Gamma(\pi\rightarrow\mu\overline{\nu}_\mu)} = \frac{|V_{us}|^2 M_K(1-m_\mu^2/M_K^2)^2}{|V_{ud}|^2 M_\pi(1-m_\mu^2/M_\pi^2)^2}\left[1+\frac{\alpha}{\pi}(C_K - C_\pi)\right]\frac{F_K}{F_\pi}$$

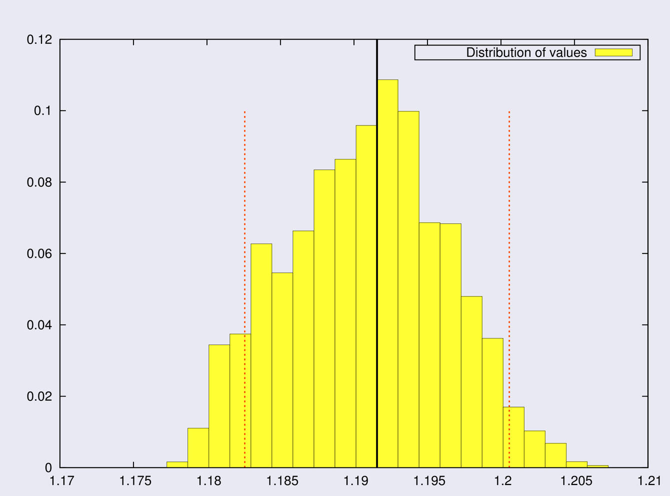
- Blue quantities are well determined experimentally.
- Conclusion: A precise determination of F_K/F_π can be used to determine $|V_{us}|^2/|V_{ud}|^2$.
- With the well known experimental value of $|V_{ud}|$, we can determine $|V_{us}|$.
- In the SM $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 1$. Any deviation from this is a (model independent) signal of physics beyond the SM.

Lattice determination of F_K/F_π with 2008 dataset [S. Dürr et. al. Phys.Rev.D81 (2010)]

- Control of systematic uncertainties: Use many different ways to obtain the final physical result.
  - Finite volume effects theoretically proportional to $e^{-M_\pi L}$. In all our ensembles $M_\pi L \gtrsim 4 \Longrightarrow$ small corrections. Use two loop chiral perturbation theory to correct the data before the fit.
  - To extrapolate to the physical values of the quark masses use a total of 7 expressions: 3 based on $SU(3)$ Chiral perturbation theory, 2 based on $SU(2)$ Chiral perturbation theory, 2 analytical expressions around $m_q \neq 0$. Also use two different cuts $M_\pi < 360$ MeV and $M_\pi < 460$ MeV.
  - Cutoff effects are partially cancelled in the ratio F_K/F_π. Continuum limit are parametrized in 3 ways: no cutoff effects (consistent with our data), as $\mathcal{O}(a)$ and as $\mathcal{O}(a^2)$.
  - Excited states are taken into account using 18 different fitting intervals for the correlators.
  - We set the scale in 2 ways: at the physical values of M_π and M_K with either M_Ω or M_Ξ.
- This leads to a total of $18\times 2\times 2\times 7\times 3 = 1512$ different analyses. We weigh them by the quality of fit. The typical result of our analysis (median) gives our final result. The spread of the results (16 and 84 percentiles) the systematic error.



(c) One of the 1512 analysis.



(d) Distribution of the 1512 analysis weighted by the fit quality.

Results

- $F_K/F_\pi = 1.192(7)_{\text{stat}}(6)_{\text{sys}}$. $|V_{us}| = 0.2256(18)$.
- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 1.0001(9)$.

Nucleon strange content and sigma term.

- Nucleon sigma term

$$\sigma_{\pi N} = \hat{m}\langle N(p)\mid(\overline{u}u+\overline{d}d)(0)\mid N(p)\rangle = \hat{m}\frac{\partial M_N}{\partial\hat{m}}$$

- Nucleon strange content

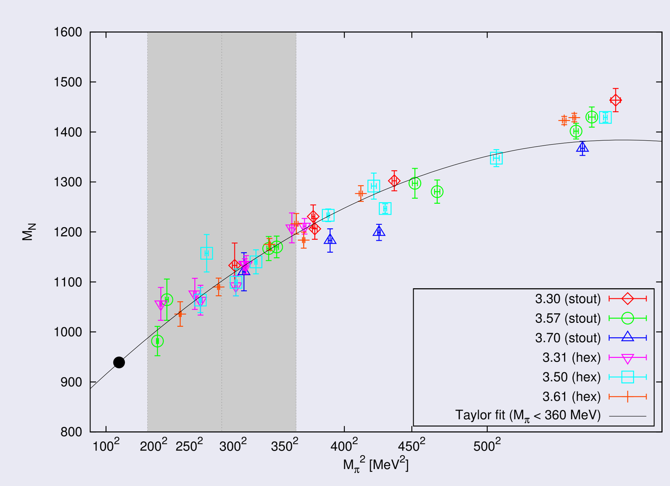
$$y = \frac{2\langle N\mid\overline{s}s\mid N\rangle}{\langle N\mid\overline{u}u+\overline{d}d\mid N\rangle} = \left[2m_s\frac{\partial M_N}{\partial m_s}\right]\Big/\left[\hat{m}\frac{\partial M_N}{\partial\hat{m}}\right]$$

- Relevant for:
  - Hadron spectrum
  - Detection of dark matter
  - The quark mas ratio $m_s/\hat{m}$
  - $\pi - N$ and $K - N$ scattering amplitudes

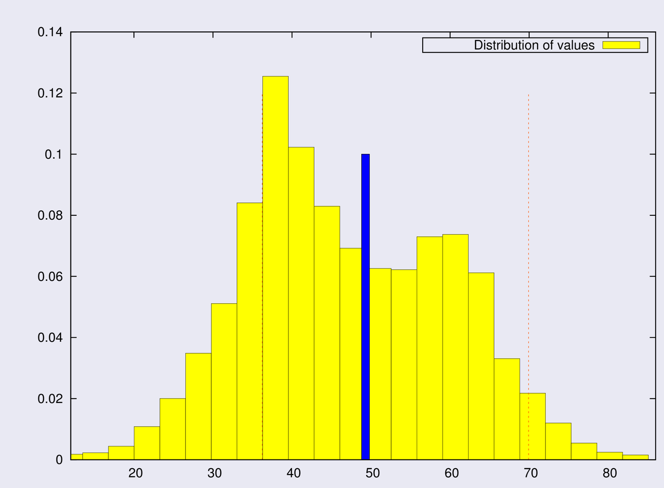
Lattice determination of the Nucleon strange content and sigma term

Part of a joint project BMW + Regensburg [S. Dürr (Lattice 2010)]. Preliminary determination with 2008 dataset + subset of 2010 dataset [A. Ramos (Lattice 2010)].

- Control of systematic uncertainties: Use many different ways to obtain the final physical result.
  - Finite volume effects theoretically proportional to $e^{-M_\pi L}$. In all our ensembles $M_\pi L \gtrsim 4 \Longrightarrow$ small corrections. Data consistent with no cutoff effects.
  - For the preliminary analysis we extrapolate to the physical values of the quark masses only with 2 expressions based on analytical expansions around $m_q \neq 0$. A Chiral perturbation theory analysis will be included in the final result [S. Dürr (Lattice 2010)]. Also use two different cuts $M_\pi < 360$ MeV and $M_\pi < 460$ MeV.
  - Cutoff effects absent in our data within statistical errors.
  - Excited states are taken into account using 144 different fitting intervals for the correlators.
  - Since we are not interested in the value of the masses themselves, but only in the sigma term, we set the scale with M_π, M_K and M_N.
- This leads to a total of $144\times 2\times 2 = 576$ different analyses. We weigh them by the quality of fit. The typical result of our analysis (median) gives our final result. The spread of the results (16 and 84 percentiles) the systematic error.



(e) One of the 576 analysis.



(f) Distribution of the 576 analysis weighted by the fit quality.

Preliminary results (2008 dataset + subset of 2010 dataset)

- $\sigma_{\pi N} = 49(10)_{\text{stat}}(11)_{\text{sys}}$ MeV
- $y = 0.08(7)_{\text{stat}}(4)_{\text{sys}}$