Status of Higher Order QCD Calculations

Aude Gehrmann-De Ridder
QCD at High Energy Colliders

- QCD: successful theory of strong interactions
- QCD is omnipresent in high energy collisions

QCD effects
- initial state: parton distributions
- final state: jets
- hard scattering matrix elements with multiple radiation

Detailed understanding of QCD mandatory for
- Interpretation of collider data
- Precision studies
- Searches for new physics
Expectations at LHC

- LHC brings new frontiers in energy and luminosity
- Production of short-lived heavy states (Higgs, SUSY, ...)
  - detected through their decay products
  - yield multi-particle final states involving jets, leptons, $\gamma$, $\slashed{E}_T$
- Search for new effects in multi-particle final states
  - typically involving jets
  - need to understand signal and background processes
- Require precise predictions: NLO

Example: SUSY signature:
$3j + \slashed{E}_T$
Large production rates for Standard Model processes
- jets
- top quark pairs
- vector bosons
Allow precision measurements
- masses
- couplings
- parton distributions
Require precise theory: NNLO
Outline

- Multiparticle production at NLO
- Precision observables at NNLO
NLO Multiparticle Production

- Why NLO?
  - reduce uncertainty of theory prediction
    - reliable normalization and shape
  - accounts for effects of extra radiation
  - jet algorithm dependence

- Require two principal ingredients (here: pp → 3j)
  - one-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all 2 → 2 processes
      - known for many 2 → 3 processes
      - current frontier 2 → 4: major challenge
  - tree-level matrix elements
    - implicit poles from soft/collinear emission
Combining virtual and real emission
  - extract process-independent implicit poles from real emission
    - residue subtraction (S. Frixione, Z. Kunszt, A. Signer)
    - dipole subtraction (S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi)
    - antenna subtraction
      (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover; A. Daleo, T. Gehrmann, D. Maitre, M. Ritzmann, AG)

Automated subtraction tools
  - residue method: MadFKS (R. Frederix, S. Frixione, F. Maltoni, T. Stelzer)

Bottleneck up to now: one-loop multileg matrix elements
NLO: One-loop multi-leg amplitudes

- General structure

\[ A = \sum_i d_i \text{Box}_i + \sum_i c_i \text{Triangle}_i + \sum_i b_i \text{Bubble}_i + \sum_i a_i \text{Tadpole}_i + R \]

- One-loop scalar integrals known analytically
  (K. Ellis, G. Zanderighi; A. Denner, S. Dittmaier)

- Task: compute integral coefficients

- Challenges
  - complexity: number of diagrams, number of scales
  - stability: linear dependence among external momenta

- Enormous progress using two approaches
  - traditional: Feynman diagram based
  - unitarity based: reconstruct integral coefficients from cuts

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NLO multi-leg: traditional approach

- Based on one-loop Feynman diagrams
  - contain high-rank tensor integrals
  - reduced to basis integrals: with analytical (A. Denner, S. Dittmaier) or semi-numerical (GOLEM: T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, C. Schubert) approach

- Successfully applied in first complete $2 \rightarrow 4$ calculation:
  $$ pp \rightarrow t\bar{t}bb $$
  (A. Bredenstein, A. Denner, S. Dittmaier, S. Pozzorini)
  see talk by S. Dittmaier

- and in many $2 \rightarrow 3$ processes
NLO multi-leg: unitarity-based method

- **Generalized unitarity**
  - apply multi-particle cuts: one or more loop propagators on-shell (Z. Bern, L. Dixon, D. Dunbar, D. Kosower, R. Britto, F. Cachazo, B. Feng; P. Mastrolia; D. Forde)
  - result: integral coefficients are products of tree-level amplitudes evaluated at complex momenta

- **Reduction at integrand level** (OPP: G. Ossola, C. Papadopoulos, R. Pittau)

- **Rational terms not determined by unitarity**
  - Special recursion relations (C. Berger et al.)
  - Feynman diagram approach (OPP)
  - D-dimensional unitarity (R. Ellis, W. Giele, Z. Kunszt, K. Melnikov)

- **Algorithmic procedure: can be automated**
Automating NLO calculations

- Virtual corrections: implementations
  - semi-numerical form factor decomposition: GOLEM
    (T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, T. Reiter)

- unitarity and multi-particle cuts: BlackHat

- reduction at integrand level: CutTools (G. Ossola, C. Papadopoulos, R. Pittau)

- generalized D-dimensional unitarity: Rocket (W. Giele, G. Zanderighi)

- generalized D-dimensional unitarity: Samurai
  (P. Mastrolia, G. Ossola, T. Reiter, F. Tranmontano)

- several more packages in progress
  (A. Lazopoulos; W. Giele, Z. Kunszt, J. Winter; K. Melnikov, M. Schulze)
### The Les Houches Wish List (2010)

<table>
<thead>
<tr>
<th>process wanted at NLO</th>
<th>background to</th>
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<tbody>
<tr>
<td>1. ( pp \rightarrow VV + \text{ jet} )</td>
<td>( t\bar{t}H ), new physics</td>
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<tr>
<td></td>
<td>Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi</td>
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<tr>
<td>2. ( pp \rightarrow H + 2 \text{ jets} )</td>
<td>( H ) in VBF</td>
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<tr>
<td></td>
<td>Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier</td>
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<tr>
<td>3. ( pp \rightarrow t\bar{t}bb )</td>
<td>( t\bar{t}H )</td>
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<td>Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek</td>
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<tr>
<td>4. ( pp \rightarrow t\bar{t} + 2 \text{ jets} )</td>
<td>( t\bar{t}H )</td>
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<tr>
<td></td>
<td>Bevilacqua, Czakon, Papadopoulos, Worek</td>
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<tr>
<td>5. ( pp \rightarrow VVb\bar{b} )</td>
<td>VBF ( \rightarrow H \rightarrow VV ), ( t\bar{t}H ), new physics</td>
</tr>
<tr>
<td>6. ( pp \rightarrow VV + 2 \text{ jets} )</td>
<td>VBF ( \rightarrow H \rightarrow VV )</td>
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<tr>
<td></td>
<td>VBF: Bozzi, Jäger, Oleari, Zeppenfeld</td>
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<tr>
<td>7. ( pp \rightarrow V + 3 \text{ jets} )</td>
<td>new physics</td>
</tr>
<tr>
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<td>Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi</td>
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<tr>
<td>8. ( pp \rightarrow VVV )</td>
<td>SUSY trilepton</td>
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<tr>
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<td>Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau</td>
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<tr>
<td>9. ( pp \rightarrow b\bar{b}bb )</td>
<td>Higgs, new physics</td>
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</table>

Feynman diagram methods now joined by unitarity based methods.

L. Dixon

CERN HO10
NLO multileg: $W^\pm + 3j$, $Z^0 + 3j$

- Calculations of $W^\pm + 3j$
  - Rocket (R.K. Ellis, K. Melnikov, G. Zanderighi)

- excellent description of Tevatron data
  - moderate corrections
  - precise predictions
  - rich phenomenology

- Calculation of $Z^0 + 3j$ (Blackhat + Sherpa)

- Ongoing: $W^\pm + 4j$ (Blackhat + Sherpa)
  (see talk by D. Kosower)
Where are NNLO corrections needed?

- **Processes measured to few per cent accuracy**
  - $e^+e^- \rightarrow 3$ jets
  - 2+1 jet production in deep inelastic scattering
  - hadron collider processes:
    - jet production
    - vector boson (+jet) production
    - top quark pair production

- **Processes with potentially large perturbative corrections**
  - Higgs or vector boson pair production

- **Require NNLO corrections for**
  - meaningful interpretation of experimental data
  - precise determination of fundamental parameters
What is known to NNLO?

- **fully inclusive observables**
  - total cross sections: R-ratio, Drell-Yan and Higgs production
  - structure functions in deep inelastic scattering
  - evolution of parton distributions
  - Higgs production in vector boson fusion (P. Bolzoni, F. Maltoni, S. Moch, M. Zaro)

- **single differential observables**
  -rapidity distribution in Drell-Yan process
  (C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello)

- **fully differential observables**
  - colourless high mass system including decays
  - jet production
NNLO calculations

- Require three principal ingredients (here: pp → 2j)
  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless 2 → 2 processes
  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from soft/collinear emission
      - usually known from NLO calculations
  - tree-level matrix elements
    - implicit poles from two partons unresolved
      - known from LO calculations

- Challenge: combine contributions into parton-level generator
- need method to extract implicit infrared poles
NNLO calculations

- **Solutions**
  - sector decomposition: expansion in distributions, numerical integration (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello; M. Czakon)
  - subtraction: add and subtract counter-terms: process-independent approximations in all unresolved limits, analytical integration
    - several well-established methods at NLO
    - NNLO for specific hadron collider processes:
      - $q_T$ subtraction
        (S. Catani, M. Grazzini)
      - NNLO for $e^+e^-$ processes:
        antenna subtraction
        (T. Gehrman, E.W.N. Glover, AG)
Higgs boson production at NNLO

- Dominant production process: gluon fusion
- exclusive calculations to NNLO, including H decay
  - using sector decomposition (C. Anastasiou, K. Melnikov, F. Petriello)
  - using $q_T$-subtraction (S. Catani, M. Grazzini)
- Application: Higgs at Tevatron
  - $H \rightarrow WW \rightarrow l\nu l\nu$
    - all distributions to NNLO (C. Anastasiou, G. Dissertori, M. Grazzini, F. Stöckli, B. Webber)
    - cuts on jet activity
    - neural-network output to NNLO
Vector boson production at NNLO

- Fully exclusive calculations
- parton-level event generator
  - using sector decomposition (K. Melnikov, F. Per trìello)
  - using q_T subtraction (S. Catani, L. Cieri, G. Ferrera, D. de Florian, M. Grazzini)
- including vector boson decay
- allowing arbitrary final-state cuts
- Application: lepton charge asymmetry (S. Catani, G. Ferrera, M. Grazzini)
- small NNLO corrections
- determine quark distributions

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ICHEP 2010
Jet production at NNLO: $e^+e^-$ collisions

- Two calculations of NNLO corrections to $e^+e^- \rightarrow 3$ jets
  - using antenna subtraction (T. Gehrmann, E.W.N. Glover, G. Heinrich, AG; S. Weinzierl)
  - as parton-level event generator
  - allow evaluation of event shapes and jet rates
- Improved description of data with reduced scale uncertainty
- One per cent for three-jet rate
- Use to extract $\alpha_s$ from LEP data:

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020 \text{(exp)} \pm 0.0015 \text{(th)}$$
NNLO jet cross sections at hadron colliders

- two-loop matrix elements known for
  - two-jet production
  - vector-boson-plus-jet production (T. Gehrmann, E. Remiddi)
  - (2+1) jet production in DIS (T. Gehrmann, E.W.N. Glover)
- antenna subtraction formalism at NNLO: with radiators in initial state

![Diagram of NNLO jet cross sections with labels: final-final, initial-final, initial-initial]
NNLO jet cross sections at hadron colliders

- First implementation of antenna subtraction
  - $gg \rightarrow 4g$ subtraction constructed and tested (E.W.N. Glover, J. Pires)

- Integration of antenna functions
  - final-final antennae known
  - initial-final antennae derived recently: sufficient for (2+1) jets in DIS (A. Daleo, T. Gehrmann, G. Luisoni, AG)
  - initial-initial in progress (R. Boughezal, M. Ritzmann, AG)

- Top pair production at NNLO
  - In progress (see talk of R. Bonciani)
Conclusions and Outlook

- QCD is crucial for the success of LHC physics
  - interpretation of collider data
  - searches for new physics
  - precision studies

- Particle theory is getting ready
  - impressive progress in automated multiparticle NLO cross sections
  - high precision NNLO calculations for fully differential observables in benchmark processes are in progress