Outline

- Introduction
- $D^0$ mixing in dimension 6 & 7
- SU(3)$_F$ breaking in higher dimensions
- Conclusions
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- Introduction

- $D^0$ mixing in dimension 6 & 7

- $SU(3)_F$ breaking in higher dimensions

- Conclusions
recent status of $D^0$ mixing: experiment

mixing parameters: $x = \frac{\Delta M}{\Gamma}$, $y = \frac{\Delta \Gamma}{2\Gamma}$

- HFAG average 2010
  - BaBar, Belle, CDF, CLEO
    $$x = (0.59 \pm 0.20) \%$$
    $$y = (0.80 \pm 0.13) \%$$

- new BaBar result
  - hep-ex/1004.5053
    $$x = \left(1.6 \pm 2.3^{\text{stat.}} \pm 1.2^{\text{syst.}} \pm 0.8^{\text{model}}\right) \cdot 10^{-3}$$
    $$y = \left(5.7 \pm 2.0^{\text{stat.}} \pm 1.3^{\text{syst.}} \pm 0.7^{\text{model}}\right) \cdot 10^{-3}$$
recent status of $D^0$ mixing: theory

**exclusive approach**

- Falk, Grossman, Ligeti and Petrov (’02); Falk, Grossman, Ligeti, Nir, & Petrov (’04)

  - sum over intermediate hadronic states
  - **LIMITATIONS:** needs a lot of decay amplitudes & strong phases at high precision
  - e.g. estimate contribution from $SU(3)_F$ breaking in phase space
  - assuming the absence of cancellations between
    - $SU(3)_F$ breaking from phase space & matrix elements
    - different $SU(3)_F$ multipletts
  - **TYPICALLY…** $y = O(1\%)$ considered as natural

**inclusive approach**

- Georgi (’92); Ohl, Ricciardi, & Simmons (’93); Bigi & Uraltsev (’01); Golowich, Pakvasa, & Petrov (’07)

  - OPE of $\Delta C = 2$ effective Hamiltonian
  - **LIMITATIONS:** relies on $m_c \gg \Lambda$ and quark-hadron duality
  - **TYPICALLY…** $x, y \lesssim 10^{-3}$
neutral meson oscillations

\[ i \frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left( \hat{M} - \frac{i}{2} \hat{\Gamma} \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} \]
neutral meson oscillations

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Heavy quark expansion (HQE)

expand the decay width matrix as a series of local operators of increasing dimension (series expansion in \( \Lambda/m_c \))

\[ \hat{\Gamma} \propto \text{Im} \int d^4x \langle T \mathcal{H}(x) \mathcal{H}(0) \rangle = \sum_{\text{dim } n = 0}^{\infty} \left( \frac{\Lambda}{m_c} \right)^n G_n \langle Q_n \rangle \]
neutral meson oscillations

\[ i \frac{\partial}{\partial t} \left( \begin{array}{c} D^0(t) \\ \bar{D}^0(t) \end{array} \right) = \left( \hat{M} - \frac{i}{2} \hat{\Gamma} \right) \left( \begin{array}{c} D^0(t) \\ \bar{D}^0(t) \end{array} \right) \]

Heavy quark expansion (HQE)
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diagonalisation $\leadsto$ stationary states

\[ (\Delta M)^2 - \frac{1}{4} (\Delta \Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2, \quad \Delta M \Delta \Gamma = 4|M_{12}| |\Gamma_{12}| \cos \phi \]

\[ \phi = \text{arg} \left( -\frac{M_{12}}{\Gamma_{12}} \right) \]
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$\Gamma_{12}$ in the HQE

Heavy quark expansion of $\Gamma_{12}$

$$\Gamma_{12} = \Gamma_0 + \left(\frac{\Lambda}{m_c}\right)^2 \Gamma_2 + \left(\frac{\Lambda}{m_c}\right)^3 \Gamma_3 + \left(\frac{\Lambda}{m_c}\right)^4 \Gamma_4 + \ldots$$

QCD: 
$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{\pi} \Gamma_i^{(1)} + \ldots$$

- $D = 7$ corrections $\Gamma_4^{(0)}$
  - Beneke, Buchalla, & Dunietz ('96); Dighe, Kim, Hurth, & Yoshikawa ('02)

- $O(\alpha_s)$ QCD corrections $\Gamma_3^{(1)}$ to the $D = 6$ diagrams
  - Beneke, Buchalla, Greub, Lenz, & Nierste ('99); Ciuchini, Franco, Lubicz, Mescia, & Tarantino ('03); Beneke, Buchalla, Lenz, & Nierste ('03)

- $D = 8$ corrections $\Gamma_5^{(0)}$
  - Badin, Gabbiani, & Petrov ('07)
SU(3)\textsubscript{F} symmetry and GIM mechanism I

\begin{align*}
B^0_{d,s} : \quad \Gamma_{12} &= -\left( \lambda_c^2 \Gamma_{12}\text{cc} + 2 \lambda_c \lambda_u \Gamma_{12}\text{uc} + \lambda_u^2 \Gamma_{12}\text{uu} \right) \\
D^0 : \quad \Gamma_{12} &= -\left( \lambda_s^2 \Gamma_{12}\text{ss} + 2 \lambda_s \lambda_d \Gamma_{12}\text{ds} + \lambda_d^2 \Gamma_{12}\text{dd} \right)
\end{align*}

\lambda_q \equiv V_{qs}^* V_{qb} (B^0_s) - \lambda_q \equiv V_{cq}^* V_{uq} (D^0)

- CKM unitarity: \( \lambda_d + \lambda_s + \lambda_b = 0 \)

\begin{align*}
\Gamma_{12}(B^0) &= -\lambda_c^2 \left( \Gamma_{12}\text{cc} - 2 \Gamma_{12}\text{uc} + \Gamma_{12}\text{uu} \right) + 2 \lambda_c \lambda_t \\
\Gamma_{12}(D^0) &= -\lambda_s^2 \left( \Gamma_{12}\text{ss} - 2 \Gamma_{12}\text{ds} + \Gamma_{12}\text{dd} \right) + 2 \lambda_s \lambda_b
\end{align*}

\sim \text{ reveals effects of CKM hierarchy and flavour symmetry}
SU(3)$_F$ symmetry and GIM mechanism II

CKM hierarchy:

\[ V_{\text{CKM}} = \begin{pmatrix}
\end{pmatrix} \]

GIM suppression

CKM hierarchy and residual SU(3)$_F$ symmetry require charm oscillations to be very slow.

- $\Gamma_{12}(B^0_d) \simeq -\lambda^6$
- $\Gamma_{12}(B^0_s) \simeq -\lambda^4$
- $\Gamma_{12}(D^0) \simeq -\lambda^2$

\[ \bar{z}^2 = \times m_c^4 / m_b^4 \times m_c^4 / m_b^4 \times m_s^4 / m_c^4 \]

\[ \bar{z} = + \lambda^6 \times m_c^2 / m_b^2 \times m_c^2 / m_b^2 \times m_s^2 / m_c^2 - \lambda^6 - \lambda^4 - \lambda^{10} \times 1 \times 1 \times 1 \]

$\lambda = \sin \theta_C \simeq 0.2255$
charm mixing in $D = 6, 7$ – numerical results

\[
\Gamma_{12}(D^0) = -\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \left( \Gamma_{12}^{sd} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}
\]

\[
10^7\Gamma_{12} = -14.0083 + 0.0009i \\
-6.65 - 15.7i \\
0.28 - 0.29i \\
= -20.4 - 16.0i
\]

$\text{1st term: } O(\lambda^{9.0})$ 
$\text{2nd term: } O(\lambda^{8.9})$ 
$\text{3rd term: } O(\lambda^{11})$

$\text{MB, A. Lenz, J. Riedl, J. Rohrwild. JHEP 03 (2010), 009.}$

$\frac{\text{HQE } D = 6, 7 \text{ (SM)}}{\text{HFAG}} = 10^{3...4}$
charm mixing in $D = 6, 7$ – numerical results

$$
\Gamma_{12}(D^0) = -\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) + 2\lambda_s \lambda_b \left( \Gamma_{12}^{sd} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}
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10^7 \Gamma_{12} & = -14.0083 + 0.0009 i \\
& \quad -6.65 - 15.7 i \\
& \quad 0.28 - 0.29 i \\
\Rightarrow \quad & -20.4 - 16.0 i
\end{align*}

1st term: $O(\lambda^{9.0})$
2nd term: $O(\lambda^{8.9})$
3rd term: $O(\lambda^{11})$

\section*{the weak phase in $\Gamma_{12}$}
- folklore... set $\lambda_b \sim 0$
  \[\Downarrow\]
  negligible phase in $\Gamma_{12}$: \[\arg \lambda_s^2 \sim 10^{-4}\]
- keeping 2nd & 3rd term:
  \[\arg \Gamma_{12} = 0.5 \ldots 2.6\]

\[\text{HQE } D = 6, 7 \text{ (SM)} \quad 10^3 \ldots 4\]

\[\text{HFAG} \]
What is the dominant contribution to $\Delta \Gamma$?
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beyond HQE?
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- HQE series does not converge?
  - $1/m_c$: charm not ‘heavy’
  - QCD $\propto \alpha_s(m_c)$ large
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- Important contributions not captured by the HQE approach?
  - Long-distance dynamics (violation of quark hadron duality)
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**Within HQE**

- Important contributions not captured by the HQE approach?
  - $\uparrow \downarrow$
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**within HQE**

- in the SM: SU(3)$_F$ breaking effects, enhancing
  $$\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}$$
  - higher orders of the HQE
  - MB, Braun, Lenz, Nierste, & Prill (‘10)

- violating $3 \times 3$ CKM unitarity
  - factor $O(10)$ with a 4th generation:
  - MB, Lenz, Riedl, & Rohrwild (‘09)

- right-handed charged currents
  - Golowich, Hewett, Pakvasa, & Petrov (‘07)
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### beyond HQE?

- HQE series does not converge?
  - $1/m_c$: charm not ‘heavy’
    - $D = 7$-terms $\lesssim 50\%$
  - QCD $\propto \alpha_s(m_c)$ large
    - $O(\alpha_s)$-terms $\lesssim 50\%$ in $D = 6$

- important contributions not captured by the HQE approach?
  - $\uparrow$
  - long-distance dynamics (violation of quark hadron duality)

### within HQE

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  $$\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}$$
  - higher orders of the HQE
    - MB, Braun, Lenz, Nierste, & Prill ('10)
  - new physics...
    - violating $3 \times 3$ CKM unitarity
      - $\uparrow$ factor $O(10)$ with a 4th generation
        - MB, Lenz, Riedl, & Rohrwild ('09)
    - right-handed charged currents
      - $\uparrow$ Golowich, Hewett, Pakvasa, & Petrov ('07)
What is the dominant contribution to $\Delta \Gamma$?

**beyond HQE?**

- invoke meson lifetimes to test the applicability of the HQE approach to the charm sector
  - HQE series does not converge?
    - $1/m_c$: charm not ‘heavy’
      - $D = 7$-terms $\lesssim 50\%$
    - QCD $\propto \alpha_s(m_c)$ large
      - $O(\alpha_s)$-terms $\lesssim 50\%$ in $D = 6$
  - important contributions not captured by the HQE approach?
    - long-distance dynamics (violation of quark hadron duality)

**within HQE**

- in the SM: SU(3)$_F$ breaking effects, enhancing
  $$\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}$$
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    - MB, Braun, Lenz, Nierste, & Prill ('10)
  - new physics...
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    - right-handed charged currents
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a consistency test for the HQE

- not affected by GIM ⇒ HDO & NP effects small
- ideal testing ground for the HQE

work in progress!
a consistency test for the HQE

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naïve estimate for deviation from LO/HQE:
lifetime-ratios of $D^0$, $D^+$, and $D_s^+$ mesons

- set $\Gamma = \Gamma_0 (c) (1 + \delta)$ and use $\Pi_{fs}(D^+) = \Pi_{fs}(D_s^+)$

$$
\frac{\tau (D^+)}{\tau (D^0)} \equiv \frac{\Gamma_0 (c) 1 + \delta (D^0)}{\Gamma_0 (c) 1 + \delta (D^+)} = \frac{1 + \delta (D^0)}{1 + \delta (D^+)} \simeq 2.5
$$

$$
\frac{\tau (D_s^+)}{\tau (D^0)} \equiv \frac{\Gamma_0 (c) 1 + \delta (D^0)}{\Gamma_0 (c) 1 + \delta (D_s^+)} = \frac{1 + \delta (D^0)}{1 + |V_{us}/V_{ud}|^2 \delta (D^+)} \simeq 1.2
$$

$\delta (D^0) = +17\%$

$\delta (D^+) = -53\%$

hadronic uncertainties cancel!
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breaking SU(3)$_F$ flavour interference

- origin of SU(3)$_F$ suppression in $D = 6, 7$:

$$-\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \left( \Gamma_{12}^{ds} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}$$
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- one mass insertion to break flavour symmetry
- 2$^{nd}$ one to compensate chirality flip

$D \geq 6$
breaking SU(3)$_F$ flavour interference

- origin of SU(3)$_F$ suppression in $D = 6, 7$:

$$- \lambda_s^2 (\Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd}) + 2\lambda_s\lambda_b (\Gamma_{12}^{ds} - \Gamma_{12}^{dd}) - \lambda_b^2 \Gamma_{12}^{dd}$$

- one mass insertion to break flavour symmetry
- 2$^\text{nd}$ one to compensate chirality flip

- break 1 order of SU(3)$_F$ interference by cutting one internal line

- dominance of $D = 9, 12$ of the HQE

> Georgi (’92), Ohl, Ricciardi, & Simmons (’93), Bigi & Uraltsev (’01)
factorisation of 6-quark operators

- estimate matrix elements of 6-quark operators

SU(3)$_F$ breaking in higher dimensions

\[ D \geq 6 \]

\[ D \geq 9 \]

\[ D \geq 12 \]
factorisation of 6-quark operators

- estimate matrix elements of 6-quark operators
- factorisation approximation: intermediate state quark fields saturated with vacuum

$D \geq 6$

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factorisation of 6-quark operators

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- factorisation approximation: intermediate state quark fields saturated with vacuum

\[ \langle \bar{q}q \rangle \delta \Gamma \]

can be expanded in terms of 4-quark operators

\[ \hat{Q} \]

\[ \hat{Q}_S \]

sea quark & gluon content modelled with the vacuum condensate, neglect higher excitations in the meson state

\[ MB, V.M. Braun, A. Lenz \]
SU(3)$_F$ breaking in higher dimensions

**factorisation of 6-quark operators**

- Estimate matrix elements of 6-quark operators
- Factorisation approximation: intermediate state quark fields saturated with vacuum
- Sea quark & gluon content modelled with the vacuum condensate, neglect higher excitations in the meson state
- ⟨\bar{q}q⟩ correction $\delta \Gamma_{12}$ can be expanded in terms of 4-quark operators

$$\hat{Q} = (\bar{u} \, c)_{V-A} \otimes (\bar{u} \, c)_{V-A}$$

$$\hat{Q}_S = (\bar{u} \, c)_{S+P} \otimes (\bar{u} \, c)_{S+P}$$
diquark condensate intermediate states

- quark condensate background

\[
\langle \bar{q} q \rangle \quad = \quad \langle 0 \mid : q(x) \otimes \bar{q}(0) : \mid 0 \rangle = -\frac{\langle \bar{q} q \rangle}{4N_c} \times 1_c \left( 1_D - \frac{i m_d}{d} \chi \right)
\]
**SU(3)_F breaking in higher dimensions**

**Diquark condensate intermediate states**

- **Quark condensate background**

  \[
  \langle \bar{q}q \rangle = \langle 0 \mid : q(x) \otimes \bar{q}(0) : \mid 0 \rangle = -\frac{\langle \bar{q}q \rangle}{4N_c} \times \mathbb{1}_c \left( 1_D - \frac{i m}{d} \gamma \right)
  \]

- **SU(3)_F breaking**

  \[
  \delta \Gamma_{12} = -\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s \lambda_b \left( \Gamma_{12}^{ds} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}
  \]

  \[
  Z^{3/2} \quad \text{and} \quad \sqrt{Z}
  \]

\[\text{16 / 20 July 24, 2010 Markus Bobrowski – Charm mixing in the SM Institut für Theoretische Teilchenphysik (TTP)}\]
Diquark condensate intermediate states

- Quark condensate background

\[
\langle \bar{q}q \rangle = \langle 0 | : q(x) \otimes \bar{q}(0) : | 0 \rangle = -\frac{\langle \bar{q}q \rangle}{4N_c} \times 1_c \left( 1_D - \frac{im}{d} \gamma^5 \right)
\]

- SU(3)_F breaking has to overcome generic suppression \( \sim 4\pi\alpha_s \frac{\langle ss \rangle}{m_c^3} \sim 0.3 \)

\[
\delta \Gamma_{12} = -\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \left( \Gamma_{12}^{ds} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}
\]

\( \Rightarrow \) Diquark condensate removes one power of \( m_s/m_c \)
SU(3)$_F$ breaking in higher dimensions

diquark condensate intermediate states

\begin{tabular}{cccc}
D_1 & D_2 & D_3a & D_3b \\
D_4a & D_4b & D_5a & D_5b \\
D_6a & D_6b & D_7a & D_7b \\
D_8a & D_8b & D_9a & D_9b \\
D_{10} & D_{11}a & D_{11}b & D_{12} \\
D_{13}a & D_{13}b & & \\
\end{tabular}
SU(3)$_F$ breaking in higher dimensions

numerical results
numerical results

- GIM cancellations softer in $D = 10/12$

\[ \Gamma_{12} = -\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \]

\[ \delta \Gamma_{12} = -\lambda_s^2 \delta \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \delta \left( \Gamma_{12}^{ds} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \]

\[ \Gamma_{dd}^{12} \]

\[ \Gamma_{ss}^{12} = 1.908 \pm 0.036 (\pm 1.9 \%) \]

\[ \Gamma_{sd}^{12} = 1.935 \pm 0.018 (\pm 0.9 \%) \]

\[ \Gamma_{dd}^{12} = 1.962 \]

\[ y = \left( 0.86 \pm 7.3 \right) \times 10^{-6} \]
numerical results

- GIM cancellations softer in $D = 10/12$

\[
\Gamma_{12} = -\lambda_s^2 \left( \Gamma_{ss}^{12} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \left( \Gamma_{12}^{ds} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}
\]

\[
\delta \Gamma_{12} = -\lambda_s^2 \delta \left( \Gamma_{ss}^{12} - 2\Gamma_{12}^{ds} + \Gamma_{12}^{dd} \right) + 2\lambda_s\lambda_b \delta \left( \Gamma_{12}^{ds} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \delta \Gamma_{12}^{dd}
\]

- flavour symmetry breaking:

\[
\Gamma_{12}^{ss} = 1.908 + 0.036 \quad (+1.9\%)
\]

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\Gamma_{12}^{sd} = 1.935 + 0.018 \quad (+0.9\%)
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\]

\[
\Gamma_{12}^{12} = -\lambda_b^2 \left( \Gamma_{ds}^{12} - \Gamma_{dd}^{12} \right) - \lambda_b^2 \\
\delta \Gamma_{12}^{12} = -\lambda_b^2 \left( \Gamma_{ds}^{12} - \Gamma_{dd}^{12} \right) - \lambda_b^2
\]

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\Gamma_{12}^{dd} = 1.962
\]
**numerical results**

- GIM cancellations softer in $D = 10/12$

\[
\Gamma_{12} = -\lambda_s^2 \frac{(\Gamma_{ss}^{12} - 2\Gamma_{ds}^{12} + \Gamma_{dd}^{12})}{1.15 z^2} + 2\lambda_s \lambda_b \Gamma_{12}^{dd} - \frac{\lambda_b^2}{1.96} \times 13
\]

\[
\delta \Gamma_{12} = -\lambda_s^2 \frac{\delta \left(\Gamma_{ss}^{12} - 2\Gamma_{ds}^{12} + \Gamma_{dd}^{12}\right)}{0.43 z^2} + 2\lambda_s \lambda_b \Gamma_{12}^{dd} - \frac{\lambda_b^2}{0.19 z^2} \times 0.66
\]

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\Gamma_{ss}^{12} = 1.908 + 0.036 \quad (+1.9\%)
\]
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\Gamma_{sd}^{12} = 1.935 + 0.018 \quad (+0.9\%)
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\[
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\[
y = (0.86 + 7.3) \cdot 10^{-6} \times 8.5
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> MB, Braun, Lenz, Nierste, Prill ('10)
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**numerical results**

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\delta \Gamma_{12} = -\lambda_s^2 \left( \Gamma_{ss}^{12} - 2\Gamma_{ds}^{12} + \Gamma_{dd}^{12} \right) + 2\lambda_s\lambda_b
\]

\[\times 13\]

- flavour symmetry breaking:

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\]

- weak phase in the absorptive part: $\arg \Gamma_{12} \approx 3\%$

\[
y = (0.86 + 7.3) \cdot 10^{-6} \times 8.5
\]

MB, Braun, Lenz, Nierste, Prill ('10)
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- Introduction
- $D^0$ mixing in dimension 6 & 7
- SU(3)$_F$ breaking in higher dimensions
- Conclusions
summary & outlook

- options for the dominant contribution to $\Gamma_{12}$:
  - QCD: flavour symmetry breaking in higher orders of the HQE
  - QCD beyond HQE (QHD)
  - new physics

- $D = 10/12$ enhanced by a factor $\mathcal{O}(10) \implies SU(3)_F$ breaking mechanism works

- weak phase $\arg \Gamma_{12} = \mathcal{O}(1\%)$ is not unnatural in the SM

- useful next steps:
  - calculate meson lifetimes to check HQE
  - other condensate topologies, such as $\langle \bar{q} q \rangle$, $\langle q G \rangle$, $\langle G G \rangle$
    $\sim$ enhancement $\times \mathcal{O}(100)$?
  - calculate $M_{12}$
backup slides
stationary state labels

- stationary neutral meson eigenstates can be labeled as
  - heavy/light
  - short/long
  - predominantly CP even/odd

\[ \Gamma_{\text{CP}} = \sum_f N_f |\langle f | D_\pm \rangle|^2 = \Gamma \pm \text{Re} \Gamma_{12} \]

\[ D_+ = D_{\text{sh}} \]
\[ D_- = D_{\text{lg}} \]

Dunietz, Fleischer, & Nierste ('01)

In accordance with Belle measurement

\[ y_{\text{CP}} = \frac{\Gamma (D^0 \to K^+ K^-)}{\Gamma (D^0 \to K^- \pi^+)} - 1 > 0 \]

Starič & al. ('07); Bergmann, Grossman, Ligeti, Nir, & Petrov ('00)