A review of proposed
Mass Measurement Techniques for the Large Hadron Collider

(more details in the recent review arXiv:1004.2732)

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What mass reconstruction techniques am I supposed to talk about?

• In this talk
  – am not interested in fully visible final states as standard mass reconstruction techniques apply
  – will only consider new particles of unknown mass decaying (at least in part) into invisible particles of unknown mass and other visibles.

• Have been asked to say something about “kinks” in transverse and stransverse masses
Types of Technique

- Missing momentum (ptmiss)
- M_eff, H_T
- s Hat Min
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT ("sub-system")
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Max Likelihood / Matrix Element
Types of Technique

Vague conclusions

- Missing momentum (ptmiss)
- $M_{\text{eff}}, H_T$
- $s$ Hat Min
- $M_{\text{TGEN}}$
- $M_{T2} / M_{CT}$
- $M_{T2}$ (with “kinks”)
- $M_{T2} / M_{CT}$ (parallel / perp)
- $M_{T2} / M_{CT}$ (”sub-system”)
- “Polynomial” constraints

Specific conclusions

- Multi-event polynomial constraints
- Whole dataset variables
- Max Likelihood / Matrix Element
Types of Technique

Robust

- Missing momentum (ptmiss)
- M_eff, H_T
- s Hat Min
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT ("sub-system")
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Max Likelihood / Matrix Element

Fragile
The balance of benefits

Few assumptions → Vague conclusions → Robust

Many assumptions → Specific conclusions → Fragile
More Realistic Hadron Collider

Proton 1

Remnant 1

ISR

Proton 2

Remnant 2

ISR

UE / MPI

Visible

Invisible
transverse variables without baggage

\[ |\vec{p}_T| \]  
(also known as ET\text{miss}, PT\text{miss}, missing energy, missing momentum etc)

\[ M_{\text{est}} = \sum_i |\vec{p}_{T,i}| + |\vec{p}_T| \]  
(also known as Meff, or the effective mass)

\[ H_T = E_{T(2)} + E_{T(3)} + E_{T(4)} + |\vec{p}_T| \]

\[ E_T = E \sin \theta \]

(There are no standard definitions of \( M_{\text{est}} \) and \( H_T \) authors differ in how many jets are used etc.)

All have some sensitivity to the overall mass scales involved, but interpretation requires a model and more assumptions.
Observable $M_{\text{est}}$

\[ M_{\text{est}} = \sum_i |p_{T,i}| + |\not{p}_T| \]

sometimes correlates with property of model $M_{\text{eff}}$
defined by

\[ M_{\text{eff}} = \left( M_{\text{susy}} - \frac{M_X^2}{M_{\text{susy}}} \right) \]

but correlation is model dependent
\[ \sqrt{\hat{S}}_{\text{min}} \]

seeks to bound the invariant mass of the interesting part of the collision

\[ \sqrt{\hat{S}}_{\text{min}} = \left( E^2 - P_Z^2 \right)^{\frac{1}{2}} + \left( \rho_T^2 + M_{\text{invis}}^2 \right)^{\frac{1}{2}} \]

Without ISR / MPI

\[ \text{ET} \rightarrow \text{b} \overline{\text{b}} l^+ l^- \text{E}_T \]

From arXiv:0812.1042
With ISR & MPI etc

\[ \tilde{g}\tilde{g} \rightarrow 4j + E_T \quad (m_\chi = 100 \text{ GeV}) \] (a)

\hspace{1cm} HS, HS+ISR, HS+MPI, HS+ISR+MPI

\[ \frac{1}{N \cdot dN / ds_{\text{min}}^{1/2}} \text{ (GeV}^{-1}) \]

From arXiv:0812.1042
Transverse variables are less sensitive to ISR (this is both good & bad)

Though dependence on ISR is large, it is calculable and may offer a good test of our understanding. See arXiv:0903.2013 and 1006.0653
What about (transverse) variables designed to measure the masses of individual particles?
A popular new-physics scenario

Proton 1

Remnant 1

Upstream Momentum

Visible

Invisible

Proton 2

Remnant 2
Example:
We have two copies of this:

One copy could be just as relevant!
Can get a long way just using the (full) transverse mass!
Recall the W transverse mass

\[ m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - p_e \cdot p_\nu) \]

- Transverse mass in $W \rightarrow e\nu$
- Observable $m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - p_e \cdot p_\nu)$
- Extremize, subject to constraints
- Minimum at $m_T = m_e + m_\nu$
- Maximum at $m_T = m_W$
W transverse mass: why used?

- In every event $m_T < m_W$ if the W is on shell
- In every event $m_T$ is a lower bound on $m_W$
- There are events in which $m_T$ can saturate the bound on $m_W$.

The above properties motivate $m_T$ in W mass measurements.
But outside standard model

• Don’t usually know mass of invisible final state particle!
  • (neutralino?)

So for new physics need:

• Chi parameter “χ” to represent the hypothesized mass of invisible particle
Chi parameter “$\chi$”
(mass of “invisible” final state particle)
is EVERYWHERE!

(most commonly on x-axis of many 2D plots which occur later)
Reminder:

We define the “full” transverse mass in terms of “$\chi$”, a hypothesis for the mass of the invisible particle, since it is unknown.

\[ m_T^2(\hat{A}) = m_{vis}^2 + \hat{A}^2 + 2(E_{Tvis}E_{Tmiss} | \mathbf{p}_{Tvis} : \mathbf{p}_{Tmiss}) \]

where

\[ E_{Tvis}^2 = m_{vis}^2 + p_{Tvis}^2 \]

and

\[ E_{Tmiss}^2 = \hat{A}^2 + p_{Tmiss}^2 \]
Schematically, all we have guaranteed so far is the picture below:

- Since “χ” can now be “wrong”, some of the properties of the transverse mass can “break”:
  - $m_T(\chi)$ max is no longer invariant under transverse boosts! (except when $\chi=m_B$)
  - $m_T(\chi)<m_A$ may no longer hold! (however we always retain: $m_T(m_B) < m_A$)
It turns out that one actually gets things more like this:

$m_T(\chi)$

$m_A$

$m_B$

$m_B$
In fact, we get this **very nice result**:

The “full” transverse mass curve is the boundary of the region of (mother, daughter) masses consistent with the observed event!

Minimal Kinematic Constraints and $m(T2)$, Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: [arXiv:0810.5178 [hep-ph]]
$m_T(\chi)$

$\chi$
Event 2 of 8

\[ m_T(\chi) \]

Diagram showing \( m_A \), \( m_B \), and \( m_B \) with arrows indicating the direction of \( \chi \) and \( \Pi \).
Event 3 of 8

\[ m_T(\chi) \]

\[ m_A \]

\[ m_B \]

A → ? → B
Event 4 of 8

\[ m_T(\chi) \]

Diagram showing a graph with axes labeled \( m_A \) and \( m_B \), and a curve labeled \( m_B \) leading to a yellow circle labeled ? with arrows pointing to A and B.
Event 5 of 8

$m_T(\chi)$

$m_A$

$m_B$

$m_B$

ucleon

A

B
Event 6 of 8

$m_T(\chi)$

$m_A$

$m_B$

$m_B$

A

B
Event 7 of 8

$m_T(\chi)$

$m_A$

$m_B$

$m_B$

? -> A

A -> B
Event 8 of 8

\[ m_T(\chi) \]

- \( m_A \)
- \( m_B \)

\( \chi \)
Overlay all 8 events

$m_T(\chi)$

$m_A$

$m_B$

$m_B$

$\chi$

A

B
Overlay many events

\[ m_T(\chi) \]

\[ m_A \]

SPT large pt

CASE 2

arXiv: 0711.4008

CASE 2
Here is a transverse mass “KINK”!

\[ m_T(\chi) \]

\[ m_A \]

\[ m_B \]

\[ \chi \]
Alternatively, look at $M_T$ distributions for a variety of values of $\chi$.

Each curve has a different value of $\chi$.

Where is the kink now?
What causes the kink?

• Two entirely independent things can cause the kink:
  – (1) Variability in the “visible mass”
  – (2) Recoil of the “interesting things” against Upstream Transverse Momentum

• Which is the dominant cause depends on the particular situation … let us look at each separately:
Kink cause 1: Variability in visible mass

- $m_{\text{Vis}}$ can change from event to event
- Gradient of $m_T(\chi)$ curve depends on $m_{\text{Vis}}$
- Curves with low $m_{\text{Vis}}$ tend to be “flatter”
Kink cause 1: Variability in visible mass

- $m_{\text{Vis}}$ can change from event to event
- Gradient of $m_T(\chi)$ curve depends on $m_{\text{Vis}}$
- Curves with high $m_{\text{Vis}}$ tend to be “steeper”
Kink cause 2:
Recoil against Upstream Momentum
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM opposite to visible momenta tend to be “flatter”
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM parallel to visible momenta tend to be “steeper”
MT works for:

What do we do in events with a pair of decays?
MT2 : the stransverse mass

For a pair of decays

one can generalize $m_T$ to $m_{T2}$

(“Transverse” mass to “Stransverse” mass)

$$m_{T2}(\tilde{A}) = \min_{\text{splittings}} \left( \max[m_T(\tilde{A}; \text{side1}); m_T(\tilde{A}; \text{side2})] \right)$$

MT2 distribution over many events:

MT2 endpoint structure is weaker than MT (due to more missing information in the event)
MT2 (like MT) is also a mass-space boundary

The MT2(\chi) curve is the boundary of the region of (mother, daughter) mass-space consistent with the observed event!

Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD)
MT2 and MT behave in exactly the same way as each other, and consequently they share the same kink structure.

Somewhat surprisingly, MT and MT2 kink-based methods are the only (*) methods that have been found which can in principle determine the mass of the invisible particles in short chains! (see arXiv:0810.5576)

(*) There is evidence (Alwall) that Matrix Element methods can do so too, though at the cost of model dependence and very large amounts of CPU.
This should worry you …
Are kinks observable?

Expect KINK only from UTM Recoil (perhaps only from ISR!)

Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.

arXiv: 0711.4008
More hopeful news …..

“Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF” (arXiv:0911.2956 and PRD) reports that the mT2 measurement of the top-mass has the “smallest systematic error” in that channel.

Top-quark physics is an important testing ground for mT2 methods, both at the LHC and at the Tevatron.
Not all proposed new-physics chains are short!

(more details in arXiv:1004.2732)
If chains a longer use “edges” or “Kinematic endpoints”.

Plot distributions of the invariant masses of what you can see.
What is a kinematic endpoint?

- Consider $M_{LL}$
Dilepton invariant mass distribution

\[ (m_{ll}^{\text{max}})^2 = \frac{\left( m_0^2 - m_{\tilde{\chi}_2^0}^2 \right) \left( m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2 \right)}{m_{\tilde{\ell}_R}^2} \]

This is the Endpoint!
What about these invariant masses?
Some extra difficulties – may not know order particles were emitted

Might therefore need to define:

\[ m_{ql}^{\text{high}} = \max[m_{ql+}, m_{ql-}] \]
\[ m_{ql}^{\text{low}} = \min[m_{ql+}, m_{ql-}] \]

There are many other possibilities for resolving problems due to position ambiguity. For example, compare hep-ph/0007009 with arXiv:0906.2417
Kinematic Edges

(a) $m_{h_1}$ (GeV)
(b) $m_{h_2}$ (GeV)
(c) $m_{h_3}$ (GeV)
(d) $m_{h_4}$ (GeV)
(e) $m_{h_5}$ (GeV)
(f) $m_{h_6}$ (GeV)
(g) $m_{h_7}$ (GeV)
(h) $m_{h_8}$ (GeV)

(a1) $m_{h_1}$ (GeV)
(b1) $m_{h_2}$ (GeV)
(c1) $m_{h_3}$ (GeV)
(d1) $m_{h_4}$ (GeV)
(e1) $m_{h_5}$ (GeV)
(f1) $m_{h_6}$ (GeV)
(g1) $m_{h_7}$ (GeV)
(h1) $m_{h_8}$ (GeV)
<table>
<thead>
<tr>
<th>Related edge</th>
<th>Kinematic endpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l^+l^-$ edge</td>
<td>$(m_{l_l}^{\text{max}})^2 = (\tilde{\xi} - \tilde{l})(\tilde{l} - \tilde{\chi})/\tilde{l}$</td>
</tr>
</tbody>
</table>
| $l^+l^-q$ edge         | $(m_{l_lq}^{\text{max}})^2 = \left\{\begin{array}{l} \max \left[(\tilde{\xi} - \tilde{\chi})(\tilde{\xi} - \tilde{\chi}), (\tilde{l} - \tilde{\xi})(\tilde{l} - \tilde{\xi}), (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\xi})\right] \\
\text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and } \tilde{\xi}^2\tilde{\chi} < \tilde{q}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}_1})^2. \end{array}\right.$ |
| $Xq$ edge              | $(m_{Xq}^{\text{max}})^2 = X + (\tilde{q} - \tilde{\xi}) \left[\xi + X - \tilde{\chi} + \sqrt{(\xi - X - \tilde{\chi})^2 - 4X\tilde{\chi}}\right]/(2\tilde{\xi})$ |
| $l^+l^-q$ threshold    | $(m_{l_lq}^{\text{min}})^2 = \left\{\begin{array}{l} \frac{2\tilde{l}(\tilde{q} - \tilde{\xi})(\tilde{\xi} - \tilde{\chi}) + (\tilde{q} + \tilde{\xi})(\tilde{\xi} - \tilde{l})(\tilde{l} - \tilde{\chi})}{-(\tilde{q} - \tilde{\xi})\sqrt{(\tilde{\xi} + \tilde{l})^2(\tilde{l} + \tilde{\chi})^2 - 16\tilde{\xi}\tilde{l}\tilde{\chi}}} \end{array}\right.$ |
| $l^\pm_{\text{near}q}$ edge | $(m_{l_{l_{\text{near}q}}}^{\text{max}})^2 = (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/\tilde{\xi}$ |
| $l^\pm_{\text{far}q}$ edge | $(m_{l_{l_{\text{far}q}}}^{\text{max}})^2 = (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/\tilde{l}$ |
| $l^\pm q$ high-edge    | $(m_{l_{l_{\text{high}q}}}^{\text{max}})^2 = \max \left[(m_{l_{l_{\text{near}q}}}^{\text{max}})^2, (m_{l_{l_{\text{far}q}}}^{\text{max}})^2\right]$ |
| $l^\pm q$ low-edge     | $(m_{l_{l_{\text{low}q}}}^{\text{max}})^2 = \min \left[(m_{l_{l_{\text{near}q}}}^{\text{max}})^2, (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/(2\tilde{l} - \tilde{\chi})\right]$ |
| $M_{T2}$ edge          | $\Delta M = m_{\tilde{\chi}_1} - m_{\tilde{\chi}_1}^0$ |

Table 4: The absolute kinematic endpoints of invariant mass quantities formed from decay chains of the types mentioned in the text for known particle masses. The following shorthand notation has been used: $\tilde{\chi} = m_{\tilde{\chi}_1}, \tilde{l} = m_{\tilde{l}}, \tilde{\xi} = m_{\tilde{\xi}}, \tilde{q} = m_{\tilde{q}}$ and $X$ is $m_{\tilde{h}}$ or $m_{\tilde{\omega}}$ depending on which particle participates in the “branched” decay.
Different parts of model space behave differently: $m_{\text{QLL}}^{\text{max}}$

Where are the big mass differences?

\[
\left( m_{ll_q}^{\text{max}} \right)^2 = \max \left[ \frac{(\hat{q} - \hat{z})(\hat{z} - \hat{x})}{\hat{z}}, \frac{(\hat{q} - \hat{l})(\hat{l} - \hat{x})}{\hat{l}}, \frac{(\hat{q} - \hat{\bar{x}})(\hat{\bar{x}} - \hat{l})}{\hat{\bar{x}}\hat{l}} \right]
\]

except for the special case in which $\hat{l}^2 < \hat{q}\hat{x} < \hat{z}^2$ and $\hat{x}^2 \hat{x} < \hat{q}\hat{l}^2$ where one must use $(m_{\hat{q}} - m_{\tilde{x}_1})^2$. 

hep-ph/0007009
Solve all edge position for masses!
Over (or “just”) constrained events

Even if there are invisible decay products, events can often be fully reconstructed if decay chains are long enough (or if events contain pairs of sufficiently long identical chains, e.g. as above with massless invisibles).

Left: case considered in hep-ph/9812233
Small collections of under-constrained events can be over-constrained!

- For example (hep-ph/0312317) quintuples of events of the form:

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{arrow-diagram.png}}
\end{array}
\]

are exactly constrained

- Similarly pairs of events of the form:

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{arrow-diagram.png}}
\end{array}
\]

(arXiv:0905.1344) are exactly constrained.
Not time to talk about many things

- Parallel and perpendicular MT2 and MCT
- Subsystem MT2 and MCT methods
- Solution counting methods (eg arXiv:0707.0030)
- Hybrid Variables
- Phase space boundaries (arXiv:0903.4371)
- Cusps and Singularity Variables (Ian-Woo Kim)
- and many more!

And in 20 minutes I have only scratched the surface of the variables that have been discussed. Even the recent review of mass measurement methods arXiv:1004.2732 makes only a small dent in 50+ pages. However it provides at least an index …
Let’s stop here!
Extras if time ...
Other MT2 related variables (1/3)

- **MCT** ("Contralinear-Transverse Mass")
  (arXiv:0802.2879)
  - Is equivalent to MT2 in the special case that there is no missing momentum (and that the visible particles are massless).
  - Proposes an interesting multi-stage method for measuring additional masses
  - Can be calculated fast enough to use in ATLAS trigger.
Other MT2 related variables (2/3)

- **MTGEN** ("MT for GENeral number of final state particles") (arXiv:0708.1028)
  - Used when
    - each “side” of the event decays to MANY visible particles (and one invisible particle) and
    - it is not possible to determine which decay product is from which side … all possibilities are tried

- **Inclusive or Hemispheric MT2** (Nojirir + Shimizu) (arXiv:0802.2412)
  - Similar to MTGEN but based on an assignment of decay product to sides via hemisphere algorithm.
  - Guaranteed to be $\geq$ MTGEN
Other MT2 related variables (3/3)

• **M2C** ("MT2 Constrained") [arXiv:0712.0943](https://arxiv.org/abs/0712.0943) (wait for v3 ... there are some problems with the v1 and v2 drafts)

• **M2CUB** ("MT2 Constrained Upper Bound")

• There is a sense in which these two variables are really two sides of the same coin.
  – if we could re-write history we might name them more symmetrically
  – I will call them $m_{Small}$ and $m_{Big}$ in this talk.
\[ m_{\text{Small}} \quad \text{and} \quad m_{\text{Big}} \]

• Basic idea is to combine:
  
  – MT2

• with
  
  – a di-lepton invariant mass endpoint measurement (or similar) providing:

\[ \Delta = M_A - M_B \]

(or \( M_Y - M_N \) in the notation of their figure above)
“Best case”
(needs SPT, i.e. large recoil PT)
Both $m_{\text{Big}}$ and $m_{\text{Small}}$ are found.
“Typical ZPT case” (no $m_{\text{Big}}$ is found)
“Possible ZPT case” (neither \( m_{\text{Big}} \) nor \( m_{\text{Small}} \) is found)*

* Except for conventional definition of \( m_{\text{Small}} \) to be \( \Delta \) in this case.
“Possible SPT case”
(no $m_{\text{Small}}$ is found)*

* Except for conventional definition of $m_{\text{Small}}$ to be $\Delta$ in this case.
What \( m_{\text{Small}} \) and \( m_{\text{Big}} \) look like, and how they determine the parent mass

Here is the true value of the parent mass … determined nicely
Outcome:

- $m_{\text{Big}}$ provides the first potentially-useful event-by-event upper bound for $m_A$
  - (and a corresponding event-by-event upper bound for $m_B$ called $m_{\chi UB}$)
- $m_{\text{Small}}$ provides a new kind of event-by-event lower bound for $m_A$ which incorporates consistency information with the dilepton edge
- $m_{\text{Big}}$ is always reliant on SPT (large recoil of interesting system against “up-stream momentum”) – cannot ignore recoil here!