Towards the continuum limit of the lattice Landau-gauge gluon and ghost propagators

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Abstract: We present recent results for the Landau gauge gluon and ghost propagators both in $SU(2)$ and $SU(3)$ pure gauge theory for lattice sizes up to $112^4$ corresponding to physical volumes up to $(15.8 \text{ fm})^4$. Considerable attention is paid to finite-volume, finite-size and Gribov copy effects. We employ a gauge-fixing method that combines a simulated annealing algorithm with finalizing overrelaxation. In the infrared region $q^2 \leq 0.01 \text{ GeV}^2$ we find the gluon propagator to become flat as a function of $q^2$. The ghost dressing function seems to tend to a constant value in the deep infrared, while running coupling $\alpha_s$ goes to zero for $q^2 \to 0$. In $SU(2)$ case we study transition to continuum limit using sequence of lattices with growing $L$ keeping physical volume fixed.
Introduction

- Nonperturbative studies of Landau gauge gluon and ghost propagators

\[ D^{ab}_{\mu\nu} = \delta^{ab} \left( \delta_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) \frac{Z(q^2)}{q^2}, \quad G^{ab} = \delta^{ab} \frac{J(q^2)}{q^2} \]

with continuum Dyson-Schwinger (DS) or Funct. Renorm. Group (FRG) Eqs. and within the lattice approach hopefully will provide consistent results.

- DS and FRG Eqs. have conformal solution
  [von Smekal, Hauck, Alkofer '98; Zwanziger '02; Lerche, von Smekal '02]

\[ J(q^2) \propto (q^2)^{\alpha_{gh}} \quad \text{and} \quad Z(q^2) \propto (q^2)^{\alpha_{gl}} \]

with \( \alpha_{gl} + 2\alpha_{gh} = 0 \),

\[ D(q^2) = \frac{Zq^2}{q^2} \to 0, \quad J(q^2) \to \infty \quad \text{for} \quad q^2 \to 0. \]

It is argued to be unique, when DS combined with FRG Eqs. [Fischer, Pawlowski, '07]

- DS Eqs. provide also a decoupling solution
  [Boucaud et al. '05 - '07; Aguilar et al. '04 - '08]

- We present some further steps towards IR limit for the Landau gauge gluon and ghost propagators in quenched QCD on very large lattices.
SU(3)
In order to fix the Landau gauge we apply a gauge transformation \( g(x) \) to link variables \( U_{x,\mu} \in SU(3) \) such that the gauge functional is maximized

\[
F_U[g] = \sum_{x,\mu} \frac{1}{3} \Re \text{Tr} \ g U_{x,\mu}.
\]

⇒ For \( A_\mu(x+\hat{\mu}/2) := (1/2i g_0) \left( U_{x,\mu} - U_{x,\mu}^\dagger \right) \) traceless
this is equivalent to \( \Delta_\mu A_\mu = 0 \),
⇒ but not unique: Gribov copies,
⇒ search for global maxima -
fundamental modular region (FMR).

Standard prescription:

i) \( g(x) \) taken with periodic b.c.’s,

ii) maximize \( F_U[g] \) with overrelaxation (OR) method.

Drawbacks of OR:

i) substantial slowing down of OR convergence
with increasing lattice extension \( L \),

ii) its possibilities to find global maximum of \( F_U[g] \)
are strongly limited.
Simulated annealing: the principle

- Simulated annealing (SA) is a “stochastic optimization method” – here with the statistical weight \( W[g] \propto \exp\{F_U[g]/T\} \) – allowing quasi-equilibrium tunnelings through functional barriers, in the course of a ”temperature” \( T \) decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, ”tied” to the (initially chosen) local maximum).

- Control parameters at hand:
  
  i) \( N_{iter}, T_{max} \) and \( T_{min} \),

  ii) schedule for temperature steps \( T_i, i = 1, ..., N_{iter} \) can be optimized.

\[ \Rightarrow \] The larger \( N_{iter} \) the higher the local maxima, \( N_{iter} \to \infty \Rightarrow \) global maximum.

\[ \Rightarrow \] Schedule in practice: \( T_{max} = 0.45, T_{min} = 0.01, \]
\( N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3) \) with tiny (larger) \( T \)-steps close to \( T_{max} \) (close to \( T_{min} \)).
Lattice Faddeev-Popov operator can be written in terms of the (gauge-fixed) link variables $U_{x,\mu}$ as

$$M_{xy}^{ab} = \sum_{\mu} A_{x,\mu}^{ab} \delta_{x,y} - B_{x,\mu}^{ab} \delta_{x+\mu,y} - C_{x,\mu}^{ab} \delta_{x-\mu,y}$$

with

$$A_{x,\mu}^{ab} = \Re \text{Tr} \left[ \{T^a, T^b\} (U_{x,\mu} + U_{x-\mu,\mu}) \right],$$
$$B_{x,\mu}^{ab} = 2 \cdot \Re \text{Tr} \left[ T^b T^a U_{x,\mu} \right],$$
$$C_{x,\mu}^{ab} = 2 \cdot \Re \text{Tr} \left[ T^a T^b U_{x-\mu,\mu} \right]$$

and $T^a$, $a = 1, \ldots, 8$ being the (hermitian) generators of the $\text{su}(3)$ Lie algebra satisfying $\text{Tr} [T^a T^b] = \delta^{ab}/2$.

The ghost propagator is given by

$$G^{ab} = \sum_{x,y} \left< e^{-ik \cdot (x-y)} [M^{-1}]_{x,y}^{ab} \right>$$

$M$-inversion with conjugate gradient method and plane wave sources.
**Gauge fixing: SA vs. OR**

$SU(3)$ ghost propagator for $\beta = 5.70$, $L = 56$

\[ G(q^2) \]

\[ q^2 [\text{GeV}^2] \]

\[ 0 \quad 1 \]

\[ 0 \quad 50 \quad 100 \]

Influence of Gribov copies clearly visible, but seems to be moderate
⇒ Weak estimator’s dependence on MC configuration.

⇒ Finite-size effects of ghost propagator are very small and do not agree with finite-volume DS results [Fischer, Pawlowski '07].

⇒ No power-like asymptotics visible, i.e. differs from DS conformal solution with $\alpha_{gh} \approx 0.595$.

⇒ Lattice evidence for IR-regular ghost dressing function in agreement with the regular, decoupling DS solutions.
Gluon propagator and running coupling

\[ \beta = 5.7 \]

Flattening is clearly seen.

Results seem to support plateau hypothesis with \( \alpha_{gl} = 1 \) and \( \alpha_{gl} + 2\alpha_{gh} \neq 0 \).

No IR fixed point seen for running coupling

\[ \alpha_s(q^2) = \frac{g_0^2}{4\pi} J^2(q^2) Z(q^2). \]
Conclusions and Questions

• Our lattice results seem to support the decoupling DS solution for the Landau gauge gluon and ghost propagators and to contradict the conformal one.

• Gribov copy effects seem to be moderate, but are still visible for the ghost propagator. **Open question**: Influence of enlargement of gauge orbits (e.g. with $Z(N)$ flips) and its influence on the finite-size behaviour.

• Weaknesses of the lattice approach:
  – in the IR the continuum limit not under control,
  – BRST invariance not properly treated
  – choice of the potential $A_{\mu}$ not unique,
  – choice of the boundary conditions not unique (here always periodic).

• Rôle of zero-momentum modes? Can they be suppressed by proper choice of $A_{\mu}$ and/or boundary conditions with non-periodic gauge transformations as shown in the $U(1)$ case? [Bogolubsky et al., ’00]
SU(2)
Sources of distortions are:

⇒ Finite-volume effects

⇒ Finite-size effects

⇒ Gribov copy effects.

⇒ Zero-momentum modes.
Simulated annealing: the principle

- Simulated annealing (SA) is a “stochastic optimization method” – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ – allowing quasi-equilibrium tunnelings through functional barriers, in the course of a ”temperature” $T$ decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, ”tied” to the (initially chosen) local maximum).

- Control parameters at hand:
  i) $N_{iter}, T_{max}$ and $T_{min}$,
  ii) schedule for temperature steps $T_i, i = 1, ..., N_{iter}$ can be optimized.

$\Rightarrow$ The larger $N_{iter}$ the higher the local maxima, $N_{iter} \rightarrow \infty \quad \Rightarrow \quad$ global maximum.

$\Rightarrow$ Schedule in practice: $T_{max} = 1.1$ for $SU(2)$, $T_{min} = 0.01$, $N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3)$ with smaller (larger) $T$-steps close to $T_{max}$ (close to $T_{min}$).
$SU(2)$ gluon, nonrenormalised

$SU(2)$ gluon propagator for $L = 40, 56, 80, 112$, $\beta = 2.3$

⇒ Finite-volume effects are small for $L \geq 56$
**SU(2) Gluon propagators**

*SU(2)* nonrenormalized gluon propagator for fixed physical volume, various $L = 40, 56, 80, 112$

 Finite-size effects are small for $L \geq 56$ and $\beta \geq 2.4$

⇒ We are close to continuum limit!
Gauge fixing: \textbf{SA vs OR}

\textit{SU}(3) ghost propagator for $\beta = 5.70$, $L = 56$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{su3_ghost_propagator}
\caption{Ghost propagator \textit{SU}(3) for $\beta = 5.70$, $L = 56$.}
\end{figure}

\textit{SU}(2) gluon propagator for $\beta = 2.3$, $L = 80$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{su2_gluon_propagator}
\caption{Gribov ambiguity for gluon propagator \textit{SU}(2) for $\beta = 2.3$, $L = 80$.}
\end{figure}

⇒ Influence of Gribov copies clearly visible, \textbf{NEW!} : for gluon the effect is already seen when comparing \textit{SA vs OR}, without applying flip gauge transformation!
⇒ Finite-size effects for $\beta=2.4$ are small!

⇒ Require renormalization !.

⇒ Again no power-like asymptotics visible, i.e. differs from DS scaling solution.

⇒ Again clearly seen plateau of ghost dressing function in agreement with the decoupling DS solutions
We check for gluon in $SU(2)$ "multiplicative renormalisation" see [Bloch et al,'03]

$\Rightarrow$ Multiplicative renormalisation seems to hold!

$\Rightarrow$ No IR fixed point seen for $SU(2)$ running coupling

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi} J^2(q^2) Z(q^2).$$
References