A study of the b-quark fragmentation function
with the DELPHI detector at LEP I
and an averaged distribution obtained at the Z pole

To be submitted to Eur. Phys. J. C

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b-Fragmentation = process by which b-quarks organize themselves into hadrons (strong interaction, $\Delta t \sim 10^{-24}$ sec)

Defined within event generators: $z = \frac{(E+p_{\parallel})_B}{(E+p)_b}$, $(0<z<1)$

Directly measurable by experiments:

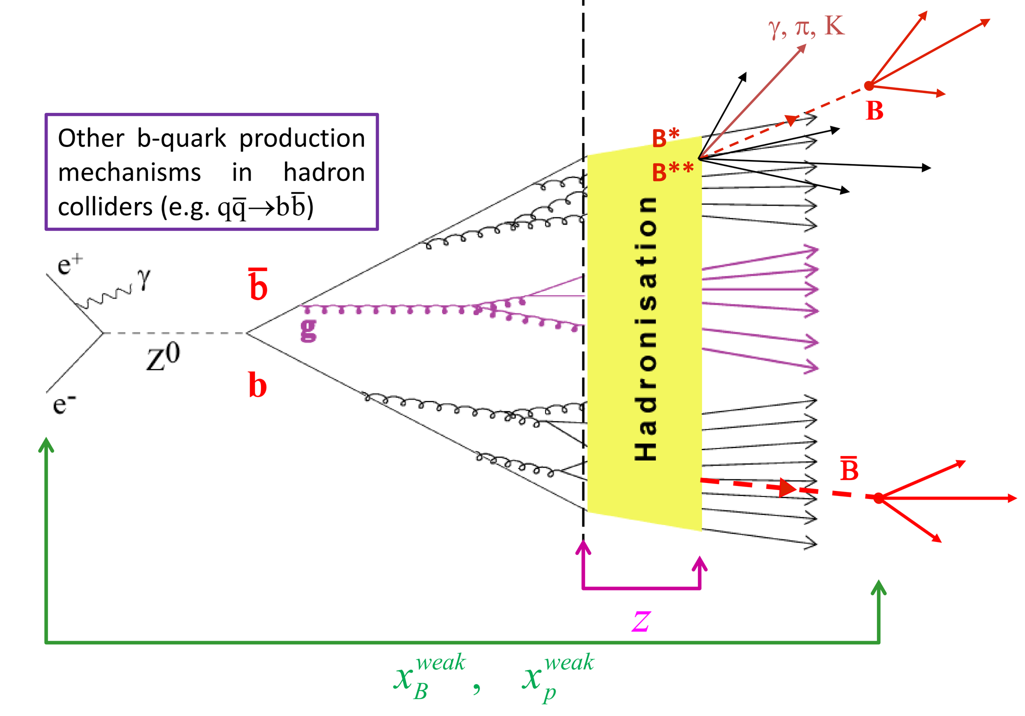
$$x_B^{weak} = \frac{E_B^{weak}}{E_{beam}}, \quad \left(x_{\min} < x_B^{weak} < 1 \;;\; x_{\min} = \frac{2m_B}{\sqrt{s}}\right)$$

$$x_p^{weak} = \frac{p_B^{weak}}{p_{B,\max}^{weak}} = \frac{\sqrt{x_B^{weak\,2} - x_{\min}^2}}{\sqrt{1-x_{\min}^2}}, \quad \left(0 < x_p^{weak} < 1\right)$$

Simple transformation

Fragmentation function: probability density function of z, x_B^{weak}, x_p^{weak} …

Perturbative - gluon radiation (calculable in QCD) | Non-perturbative - hadronisation (non-calculable: usually described by a model)



Hadronisation model: a parameterisation of the non-perturbative component

(parameterises how energy is shared between the parent b-quark after perturbative gluon radiation and the final state b-hadron)

DELPHI measurement of the fragmentation function

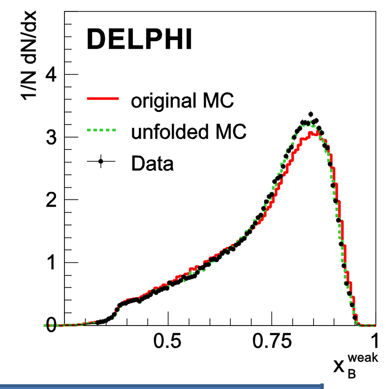
Combination of results of the x_B^{weak} distribution from two independent analyses, using different approaches.

Regularised unfolding

- *Unfolds* from the observed distribution in data the underlying fragmentation function. The two differ from each other due to effects of resolution, acceptance and backgrounds.

$$g\left(x_{B,rec}^{weak}\right) = \int R\left(x_{B,rec}^{weak}; x_B^{weak}\right) f\left(x_B^{weak}\right) dx_B^{weak} + b\left(x_{B,rec}^{weak}\right)$$

- Uses *Neural Networks* to reconstruct the b-hadron energy.

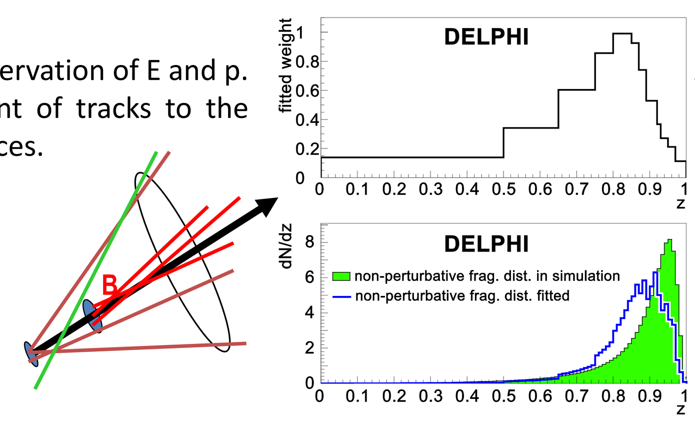


Weighted fitting

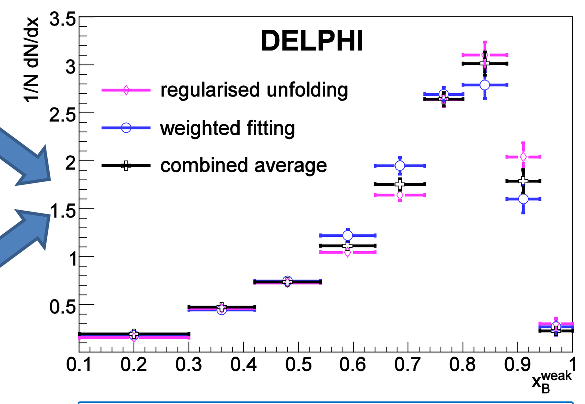
- Jet energy: fit imposing conservation of E and p.
- b-hadron energy: assignment of tracks to the primary and secondary vertices.

$E_B = E_{jet} - E_{primary\ vertex}$

- *Apply weights* to the z distribution in simulation.
- Fit weights to obtain best agreement between p_B in data and simulation.



The results are independent of any initial assumption regarding the actual shape of the underlying fragmentation distribution in simulation.



Average value of the DELPHI distribution:

$$\left\langle x_B^{weak} \right\rangle = 0.699 \pm 0.011$$

Averaged distribution obtained at the Z pole

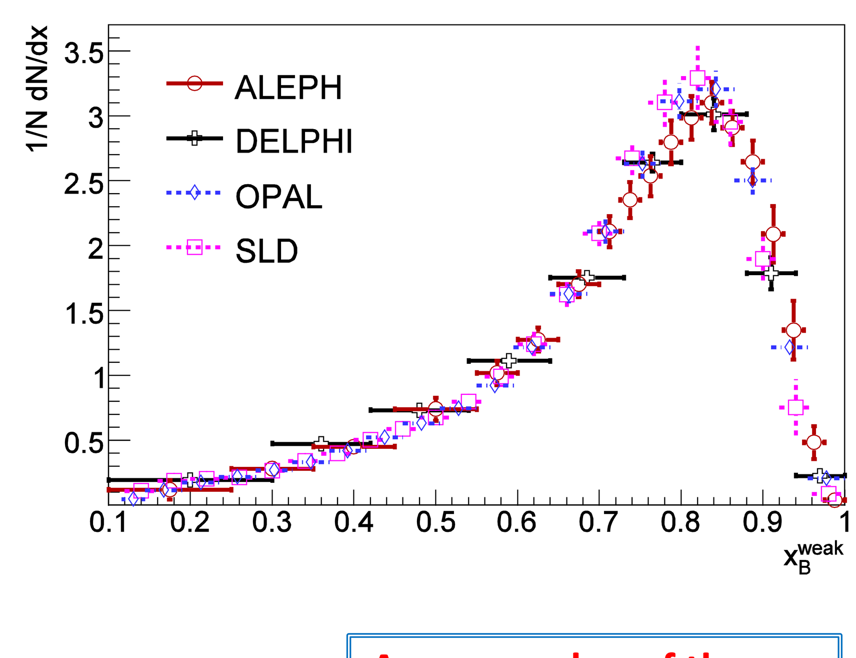
Each of the 4 measurements of the fragmentation distribution used a different choice of binning and has a different number of effective degrees of freedom. To obtain a combined distribution, a global fit was done using the parameterization:

$$p_0\left(p_1 x^{p_2}(1-x)^{p_3} + (1-p_1)x^{p_4}(1-x)^{p_5}\right),$$

and cutting away non-significant degrees of freedom of the individual error matrices. The fit result:

	x_B^{weak}	x_p^{weak}
p_1	$12.97^{+0.77}_{-0.71}$	$12.50^{+0.82}_{-0.76}$
p_2	$2.67^{+0.15}_{-0.14}$	$2.63^{+0.17}_{-0.15}$
p_3	$2.29^{+0.19}_{-0.17}$	$2.05^{+0.19}_{-0.18}$
p_4	$1.45^{+0.28}_{-0.22}$	$1.31^{+0.24}_{-0.20}$
p_5	$0.663^{+0.035}_{-0.036}$	0.664 ± 0.036

Uncertainties for x_B^{weak} (x_p^{weak}) are rescaled by 1.24 (1.37) to account for the dispersion of measurements, mainly between ALEPH and SLD.

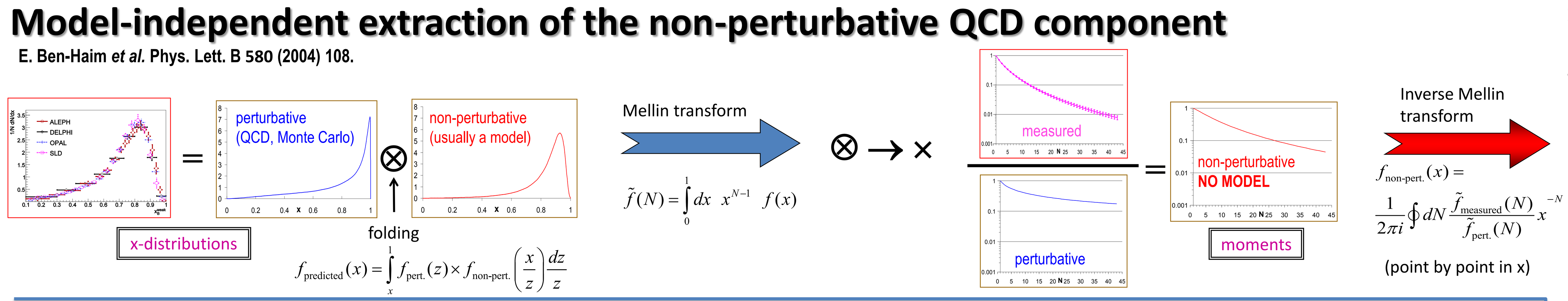


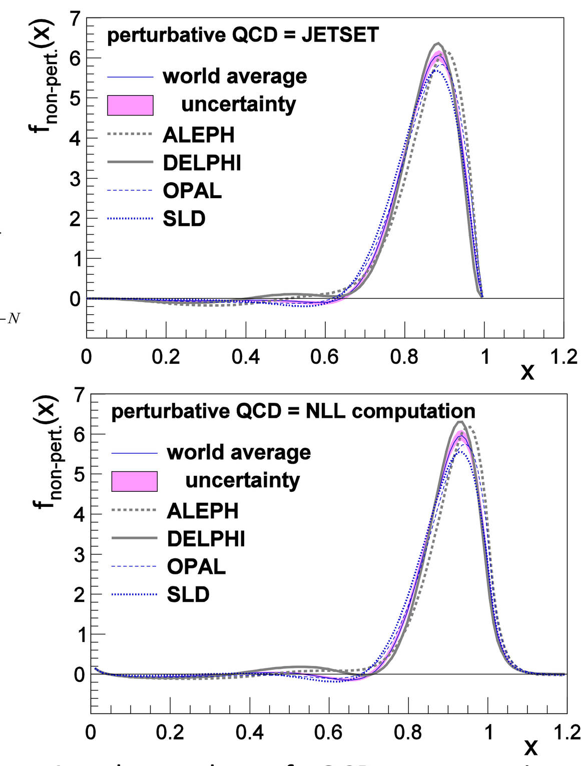
Average value of the combined distribution:

$$\left\langle x_B^{weak} \right\rangle = 0.7092 \pm 0.0025$$

Model-independent extraction of the non-perturbative QCD component

E. Ben-Haim *et al.* Phys. Lett. B 580 (2004) 108.





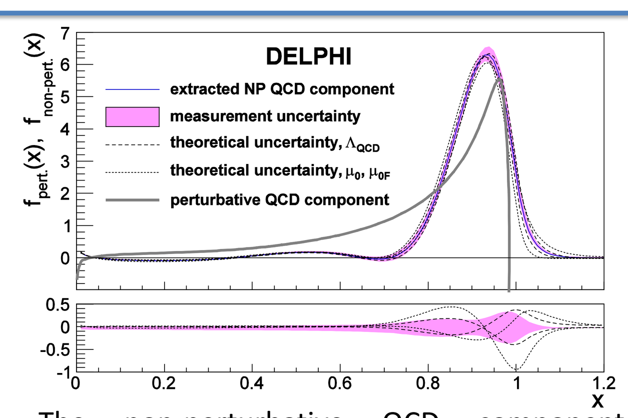
As the order of QCD computation increases, the non-perturbative peak is displaced to higher x.
The low-x region indicates that hard gluon radiation is not well accounted for in the perturbative component.

The non-perturbative QCD frag. function obtained by this method is **directly extracted from data**, and **hadronisation-model independent**.

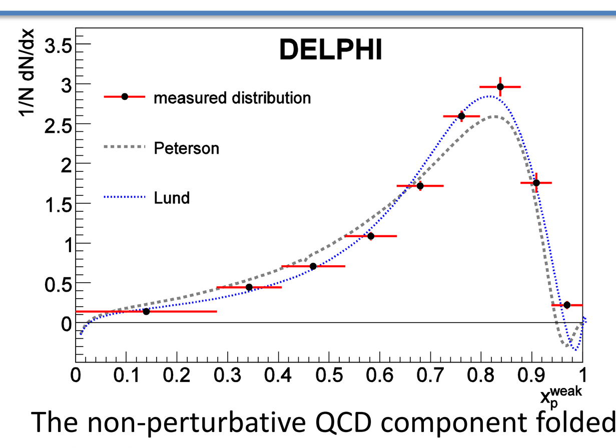
It **strongly depends on the perturbative** component used in the extraction.

The non-perturbative QCD component **may be used** in studies of b-hadron production **in other experimental environments than LEP (e.g. hadron colliders)**, provided that it is used jointly **with the same perturbative framework** as the one used for its extraction.

When extracted with the NLL perturbative QCD computation, the non-perturbative component shows a "non-physical" behaviour: it has to be extended to $x > 1$. This is related to the break-down of theory near threshold ($x \sim 1$), where the NLL perturbative component becomes negative.
Folding the two components together results in the physical measured fragmentation function.
Folding the non-physical perturbative component with a physical non-perturbative one (e.g. hadronisation model) results in a non-physical product.



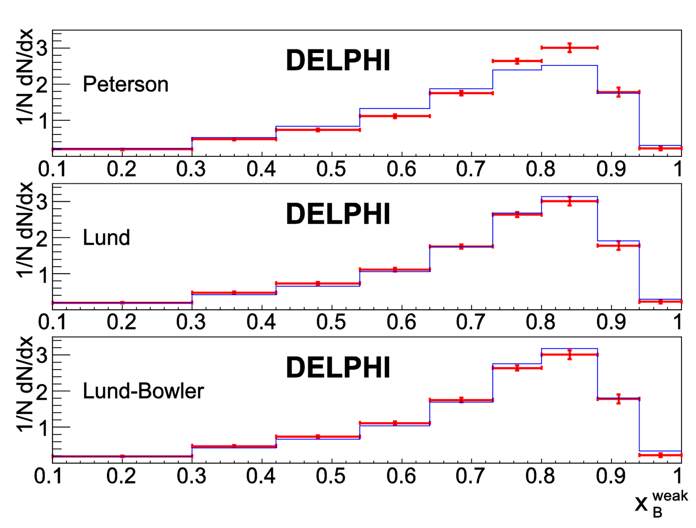
The non-perturbative QCD component extracted from DELPHI's result. Experimental and theoretical uncertainties are shown.



The non-perturbative QCD component folded with hadronisation models does not reproduce the measurement.

Fits to hadronisation models

The DELPHI measurement was compared with expectations from different non-perturbative hadronisation models within a Monte Carlo simulation (PYTHIA 6.156). **Only the Lund and Lund-Bowler models give reasonable descriptions of the data**, the Lund ansatz being favoured.

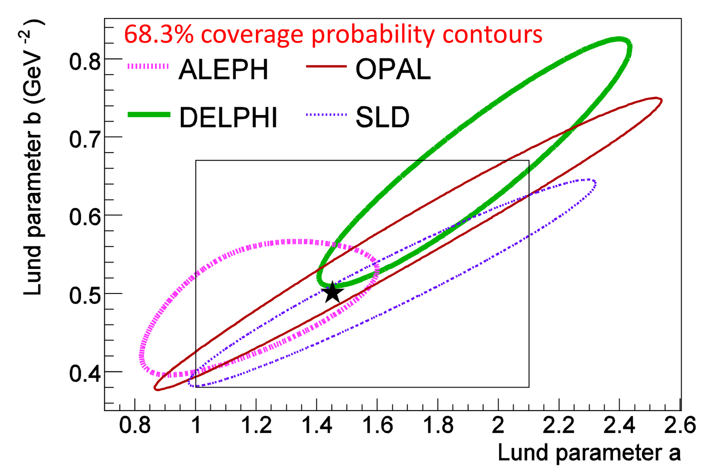


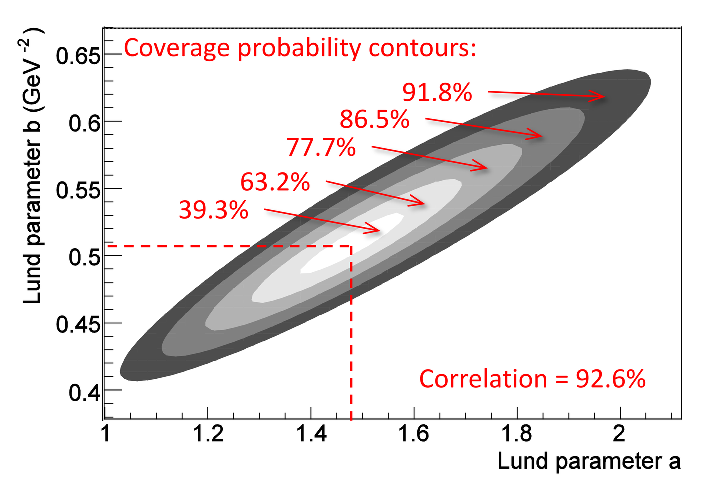
Model	Parameters	χ^2/NDF	Correlation
Peterson $\left[\frac{1}{x}\left(1-\frac{1}{x}-\frac{\epsilon_b}{1-x}\right)^{-2}\right]$	$\epsilon_b = (4.06^{+0.46}_{-0.41}) \times 10^{-3}$	55.8/6	—
Lund $\left[\frac{1}{x}(1-x)^a \exp\left(-\frac{bm_{b\perp}^2}{x}\right)\right]$	$a = 1.84^{+0.23}_{-0.21}$ $b = 0.642^{+0.073}_{-0.063}$ GeV^{-2}	9.8/5	92.2%
Lund-Bowler $\left[\frac{1}{x^{1+r_Q bm_{b\perp}^2}}(1-x)^a \exp\left(-\frac{bm_{b\perp}^2}{x}\right)\right]$ $(r_Q = 1)$	$a = 1.04^{+0.14}_{-0.12}$ $b = 3.08^{+0.45}_{-0.39}$ GeV^{-2}	20.7/5	85.6%

A global fit of the Lund and Lund-Bowler models parameters has been done using measurements from ALEPH, DELPHI, OPAL and SLD. The χ^2 minimised in this study was the sum of χ^2 corresponding to the four results.
The fit clearly favours the Lund model over the Lund-Bowler one.
Results obtained by this approach within a Monte Carlo simulation were found to be similar to the ones obtained by comparing the resulting integral of the folding product:

$$f_{\text{predicted}}(x) = \int_x^1 f_{\text{pert.}}(z) \times f_{\text{non-pert.}}^{\text{model}}\left(\frac{x}{z}\right)\frac{dz}{z}$$

in each bin of the measured function.





Result for the world average Lund parameters to use in PYTHIA 6.156:

$$a = 1.48^{+0.11}_{-0.10}$$

$$b = 0.509^{+0.024}_{-0.023}\ \text{GeV}^{-2}$$

This result is expected to be valid in experimental environments other than LEP. It would be fruitful to check how it fits data in the LHC and the TeVatron.

References:
Experimental result from ALEPH: A. Heister *et al., ALEPH Collaboration, Phys. Lett. B512 (2001) 30.*
Experimental result from OPAL: G. Abbiendi *et al., OPAL Collaboration, Eur.Phys.J. C29 (2003) 463.*
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NLL perturbative QCD computation: M. Cacciari and S. Catani, Nucl. Phys. B617 (2001) 253.
See also PhD thesis: E. Ben-Haim, *The b Quark Fragmentation Function, from LEP to TeVatron,* http://publication.lal.in2p3.fr/2004/lal04124.pdf