Refining Geometric Scaling

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Contents:

• Geometric Scaling in Deep Inelastic Scattering, Theory
• Geometric Scaling in Deep Inelastic Scattering, Phenomenology
• Scaling Fits to $F_2$ data

Work done in collaboration with Christophe Royon
Geometric Scaling: Original observation

Plot of $\sigma(\gamma^* p)$ as a function of $e^\tau = Q^2 \left(\frac{x}{x_0}\right)^\lambda$

Stasto, Golec-Biernat, Kwiecinski, (2001)
• Balitsky-Kovchegov equation for dipole amplitude

\[
\frac{\partial T}{\partial Y} = \alpha_S \left[ \chi(-\partial_L)T - T^2 \right]
\]  

(1)

\( \chi \): BFKL kernel, \( L = \log \frac{Q^2}{\Lambda_{QCD}} \), \( \alpha_S \sim 1/L \), \( Y = \log \frac{1}{x} \)

BFKL equation, if saturation term \( T^2 \) missing

• \( \alpha_S \) constant:

– Saturation \( \Rightarrow \) SGK Geometric Scaling

– \( \tau = L - \lambda Y \), independent from initial conditions

– Scaling called “Fixed coupling” in the following
Geometric Scaling in Deep Inelastic Scattering, Theory (II)

- Extension of Balitsky-Kovchegov equation

\[
\frac{\partial T}{\partial Y} = \alpha_s(Q^2) \left[ \chi(-\partial_L)T - T^2 \right]
\]

- \(\alpha_s\) running:
  - \(\alpha_s \sim 1/L = 1/\log[Q^2/\Lambda_{QCD}]\)

- Scaling
  - Running coupling I: \(\tau = L - \lambda \sqrt{Y}\) \hspace{2cm} Munier, R.P., (2003)
  - Running coupling II: \(\tau = L - \lambda Y/L\) \hspace{2cm} G. Beuf, (2008)
Stochastic Balitsky-Kovchegov equation

\[ \frac{\partial T}{\partial Y} = \alpha_s(Q^2) \left[ \chi(-\partial_L)T - T^2 + \kappa \sqrt{\alpha_s^2 T} \nu(L, Y) \right] \]

Noise strength \( \kappa \sim \) Pomeron Loop Coupling

- \( \nu \) is the Gaussian “noise”: fluctuating number of gluons \( \text{Munier (2005)} \)
- \( T^2 \): Gluon Merging \( v.s. \) Gluon Splitting : \( \kappa \sqrt{\alpha_s^2 T} \nu(L, Y) \)
- Diffusive scaling: \( \tau = (L - \lambda Y)/\sqrt{Y} \) \( \text{Hatta,Iancu,Marquet,Soyez,Triantafyllopoulos, (2006)} \)
- NB: There can be additional parameters: take \( Y - Y_0, \ L = \log Q^2/Q_0^2 \)

Constraint: \( \tau \) positive in the “dilute” phase-space \( Q^2 > 3 \text{ Gev}^2 \)
• How to compare different scalings?
  – DIS cross section $\sim F_2/Q^2$ depends on only one $\tau$ variable or not?
    (for given $\tau$)
  – The form of the $\tau$ dependence is not known and an estimator is needed on how data points ($F_2$ for instance) depend only on $\tau$ or not

• Method:
  – i) Normalise data sets $v_i = \log(\sigma_i)$, and $u_i = \tau_i(\lambda)$, $0 < v_i, u_i < 1$
    \[ \text{NB: use of log [cross section]} \]
  – ii) Order the scalings in $u_i$
  – iii) Define the quality factor:
    \[ QF(\lambda) = \left[ \sum_i \frac{(v_i - v_{i-1})^2}{(u_i - u_{i-1})^2 + \epsilon^2} \right]^{-1} \]
    \[ \epsilon \] needed when two data points have the same $x$ and $Q^2$ for $F_2$, $\epsilon^2 = 0.0001$
  – iv) Fit $\lambda$ to maximise $QF$: $QF$ is large when data close to scaling
Scaling tests in DIS using $F_2$

- Combined $F_2$ measurements from H1/ZEUS (small error bars)
- Cuts on data: $4 \leq Q^2 \leq 150$ GeV$^2$, $x \leq 10^{-2}$, $y \leq 0.6$:
  - Stay in perturbative domain, avoid valence quark domination
  - Avoid high $y$ region where $F_L$ is large:
- After all cuts: 117 data points
Comparison of different scalings

- Value of parameters and QF for \( Q^2 \geq 4 \text{ GeV}^2 \)
- FC, RC1 and RC2 favoured, DS disfavoured

**FC**: \( \tau = \log Q^2 - \lambda \log(1/x) \)

**RC1**: \( \tau = \log Q^2 - \lambda \sqrt{\log(1/x)} \)

**RC2**: \( \tau = \log(\frac{Q^2}{0.2^2}) - \lambda \frac{\log(1/x)}{\log(Q^2/0.2^2)} \)

**DS**: \( \tau = \frac{\log Q^2/\sqrt{\log 1/x}}{\log(1/x)} - \lambda \log(1/x) \)

<table>
<thead>
<tr>
<th>scaling</th>
<th>parameter</th>
<th>1/QF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Coupling</td>
<td>( \lambda = 0.31 )</td>
<td>150.2</td>
</tr>
<tr>
<td>Running Coupling I</td>
<td>( \lambda = 1.61 )</td>
<td>137.9</td>
</tr>
<tr>
<td>Running Coupling II</td>
<td>( \lambda = 2.76 )</td>
<td>124.3</td>
</tr>
<tr>
<td>Diffusive Scaling</td>
<td>( \lambda = 0.31 )</td>
<td>210.7</td>
</tr>
</tbody>
</table>

NB: Additional parameters \( (Q_0, Y_0) \) do not improve much Scaling Quality
Scaling plot

Example: scaling plot for RCI:

\[ \tau > 0 \text{ after definition of } Q_0 \text{ (scaling } \tau = \log \frac{Q^2}{Q_0^2} - \lambda \log \left( \frac{1}{x} \right) \)
Quality Factors

Differences in $1/Q_F$ for FC, RCI, RCII, DS

![Graph showing differences in $1/Q_F$ for various coupling types and HERA Data.]
Fits to HERA data

- Fit to HERA data inspired by RCI:
  all data above $Q^2 = 4G eV^2$ should be in the dilute regime (full saturation expected only at very low $Q^2$)

- Scale $\tau$ and expression for DIS cross section:

  $$ \tau = \log \left( \frac{Q^2}{Q_0^2} \right) - \lambda \sqrt{\log(1/x) - Y_0} $$
  $$ \sigma = N \exp(-\alpha \tau) \exp \left( -\beta \tau^{3/2} / (\log 1/x - Y_0)^{1/4} \right) $$

- Fit formula deduced (Gregory Soyez) from the dipole amplitude with saturation with the tail of the Airy function solution of Balitsky Kovchegov equation

  - Fit using 6 parameters: $\lambda, \alpha, \beta, Q_0, Y_0, N$
  - Explicit moderate scaling violation: $(\log 1/x - Y_0)^{1/4}$ term: fits performed with and without this predicted moderate scaling violation.
Fit results

• Fit variables:

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \sqrt{\log\left(\frac{1}{x}\right) - Y_0}
\]

\[
\sigma = N \exp(-\alpha \tau) \exp\left(\frac{-\beta \tau^{3/2}}{\left(\log \frac{1}{x} - Y_0\right)^{1/4}}\right)
\]

• Fit I: \(\chi^2 = 130.1\) for 117 points, \(\chi^2/dof = 1.2\)

• Fit II: \(\chi^2 = 119.0\) without the scaling violation term:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit I</th>
<th>Fit II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda)</td>
<td>1.54 ± 0.02</td>
<td>1.54 ± 0.02</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.34 ± 0.01</td>
<td>0.18 ± 0.01</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.24 ± 0.01</td>
<td>0.18 ± 0.01</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>0.079 ± 0.01</td>
<td>0.064 ± 0.01</td>
</tr>
<tr>
<td>(Y_0)</td>
<td>-1.46 ± 0.02</td>
<td>0.50 ± 0.02</td>
</tr>
<tr>
<td>(N)</td>
<td>0.51 ± 0.01</td>
<td>0.72 ± 0.01</td>
</tr>
</tbody>
</table>
**Fit results**

- Good description of HERA low $Q^2$ and low $x$ reduced cross section data
- Fit does not describe the reduced cross section at high $y$: 
  $$\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$$: needs a model of $F_L$, in progress

![Graph showing HERA data and fits for various $Q^2$ values](image-url)
Fit extrapolation at low $Q^2$

- Leads to a fair description of data at lower $Q^2$
- Need a parameterisation of $F_L$ to describe high $y$ data
- Need a description in the saturated region to describe very low $Q^2$ data: only description in the “dilute” regime so far
Fit extrapolation at high $Q^2$

Leads to a fair description of data at higher $Q^2$, except at high $x$ (needs valence quark contribution)
Comparison with other fits

• Formula for RCII:

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \frac{\log(1/x) - Y_0}{\log\left(\frac{Q^2}{Q_0^2}\right)}
\]

\[
\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^3}{2\left(\log 1/x - Y_0\right)^{1/4}}\right)
\]

- \(\chi^2 = 190.4\), worse description than for RCI

• Formula for FC:

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \log\left(\frac{1}{x}\right)
\]

\[
\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^2}{\log 1/x - Y_0}\right)
\]

- \(\chi^2 = 156.4\), worse than RCI,
- \(\chi^2 = 230.5\) without the scaling violation term
**Conclusion**

- **Different scalings studied in $F_2$ data**: fixed coupling, running coupling I and II, diffusive scaling
  - Fixed coupling, running coupling I and II lead to a good description of data using the QF formalism
  - Diffusive scaling disfavoured
- **Fit of $F_2$ data using BK theory**: parameterised with or without moderate scaling violations
  - Fits disfavour RCII and FC
  - RCI favoured, leads to a good description of low $Q^2$, low $x$ $F_2$ data
- **Outlook**: fits of lower $Q^2$ data in the saturation region; fits of high $y$ data including $F_L$ parameterisation; comparison with numerical solution of BK equation with $\alpha_S$ running