



Neutrino Physics

Alain Blondel

1. What are neutrinos and how do we know ?
2. Standard Model neutrino properties
3. Neutrino transitions and neutrino mass
4. neutrino oscillations and *CP* violation
5. on-going and future neutrino experiments on oscillations
6. what is the origin of neutrino masses?
7. neutrino-less double-beta experiments
8. See-saw, «sterile» neutrinos
9. Conclusions

Preamble

Neutrino Physics is a very vast domain. Historically and still today it is carried out across several disciplines of physics

- neutrino beams (SBL, LBL): neutrino oscillations and cross-sections
- beam dump and flavour factories (heavy neutrino searches in D and B decays)
- high energy colliders (tau physics, heavy neutrino searches in W and Z decays)
- nuclear physics ($0\nu\beta\beta$ decay, neutrino mass measurements)
- astrophysics (solar neutrinos, atmospheric neutrinos) cosmology (neutrino mass)
- also a tool for astrophysics (multimessenger astronomy, ICECUBE, km³net)

That neutrinos have mass is a very clear sign of physics beyond the SM.

Neutrinos are different because they are neutral ($\nu \leftrightarrow \bar{\nu}$ transition is allowed, but BSM)

The physics of massive neutrinos attracts considerable interest by its profound potential implications on the primordial universe and its evolution, as well as its wide range of experimental methods.

The bi-annual 2018 Neutrino conference gathered over 800 participants, which is not very different from the 1100 participants of ICHEP18.

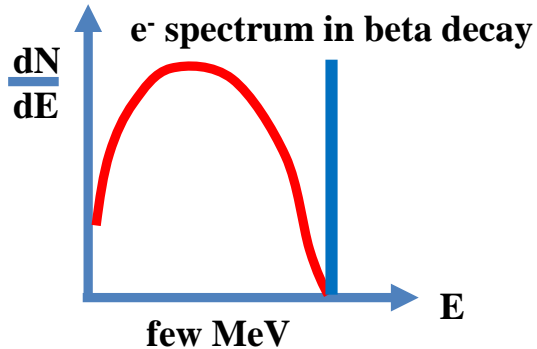
The organization of neutrino physics, however, spans several domains, distributed across different organizational and support frameworks: nuclear physics; astro-particle physics; astronomy; non-accelerator and accelerator physics.

I will address only selected points.

1. The three families of active neutrinos

Consider ${}^6\text{He}^{++} \rightarrow {}^6\text{Li} \bar{\nu}_e e^-$

$Q = 3.5078 \text{ MeV}$ $T/2 \approx 0.8067 \text{ s}$



930

Neutrinos: *the birth of the idea*

Pauli's letter of the 4th of December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that **there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle** and which further differ from light quanta in that they do not travel with the velocity of light. **The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses.** The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

I agree that my remedy could seem incredible because one should have seen those neutrons very earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's well better not to think to this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge.

Unfortunately, I cannot appear in Tübingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant

. W. Pauli

Wolfgang Pauli

Neutrinos: *direct detection*

The anti-neutrino coming from the nuclear reactor interacts with a proton of the target, giving a positron and a neutron.

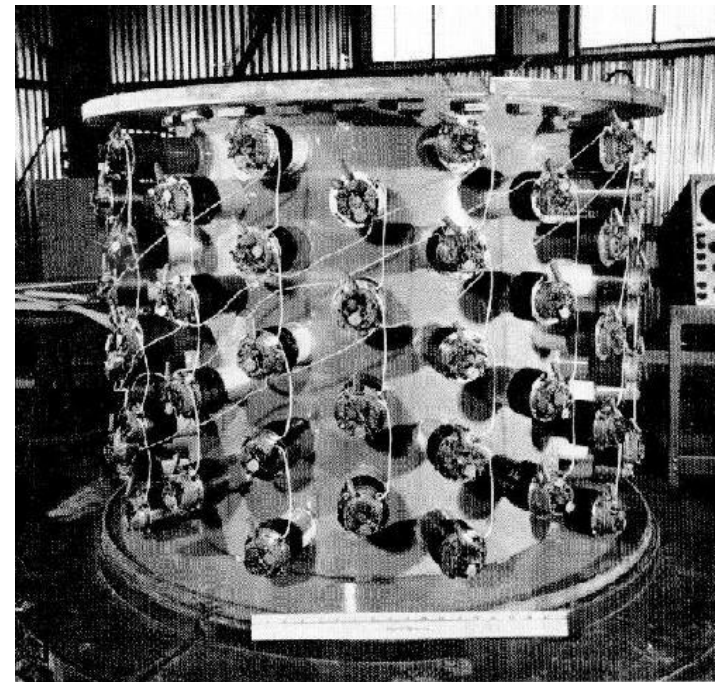
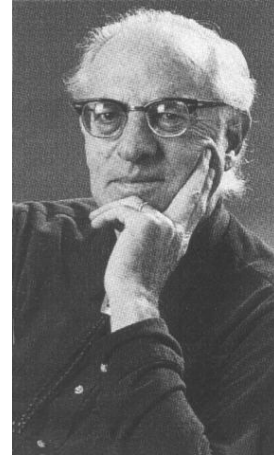


The positron annihilates with an electron of target and gives two simultaneous photons ($e^+ + e^- \rightarrow \gamma\gamma$).

The neutron slows down before being eventually captured by a cadmium nucleus, that gives the emission of 2 photons about 15 microseconds after those of the positron.

All those 4 photons are detected and the 15 microseconds identify the "neutrino" interaction.

The target is made of about 400 liters of water mixed with cadmium chloride



4-fold delayed coincidence

1956 Parity violation in Co beta decay: electron is left-handed (C.S. Wu et al)

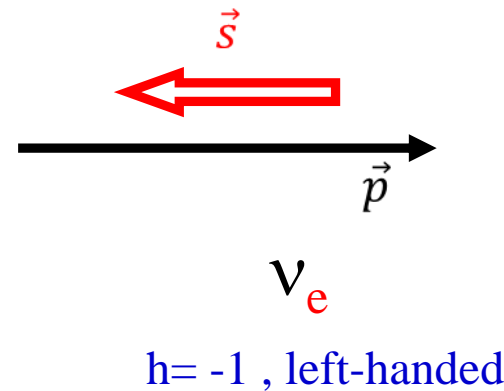
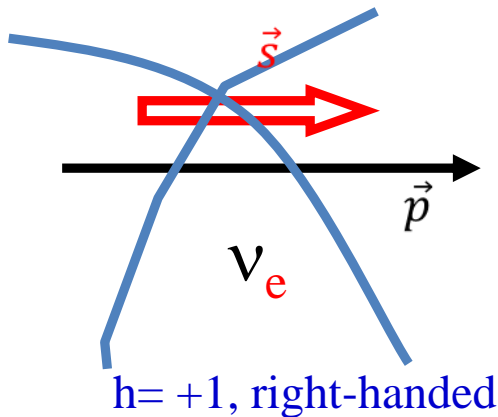
1957 Neutrino helicity measurement

M. Goldhaber et al Phys.Rev.109(1958)1015

neutrinos have negative helicity

(If massless fermion this is the same as left-handed *chirality*)

$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}| \cdot |\vec{p}|}$$



**1959 Ray Davis established that
(anti) neutrinos from reactors do not interact with chlorine to produce argon**

reactor : $n \rightarrow p \text{ } e^- \text{ } \nu_e \text{ or } \bar{\nu}_e ?$

these ν_e **don't do** $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$

they do this: $\bar{\nu}_e + p \rightarrow e^+ + n$

they are **anti-neutrinos!**

Introduce a lepton number which is

+1 for e^- and ν_e

and

-1 for e^+ and $\bar{\nu}_e$

which is observed to be conserved in weak/EM/Strong interactions

Neutrinos

the properties

1960

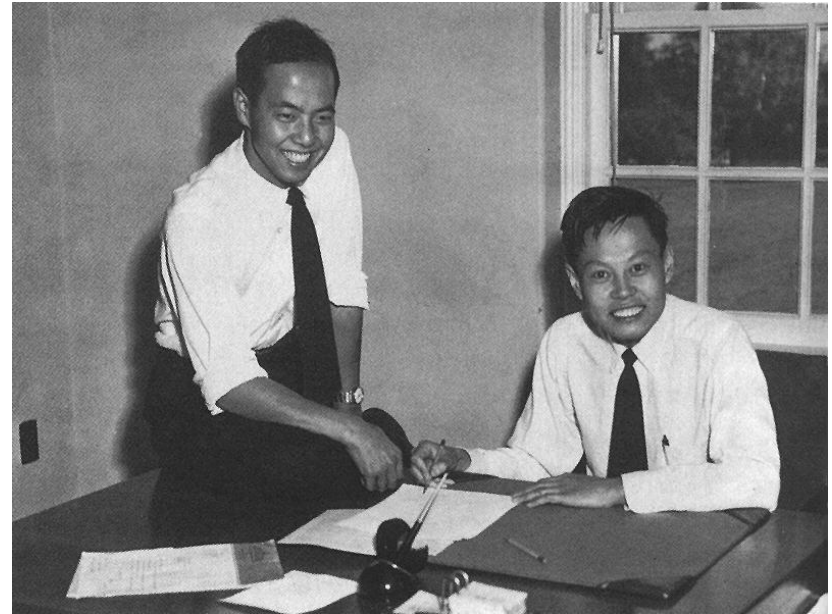
In 1960, Lee and Yang realized that if a reaction like

$$\mu^- \rightarrow e^- + \gamma$$

is not observed, this is because two types of neutrinos exist ν_μ and ν_e

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

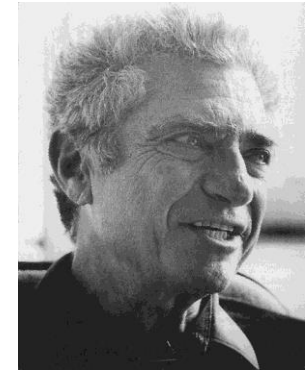
otherwise $\mu^- \rightarrow e^- + \nu + \bar{\nu}$
has the same Quantum
numbers as $\mu^- \rightarrow e^- + \gamma$



Lee and Yang

Two Neutrinos

1962

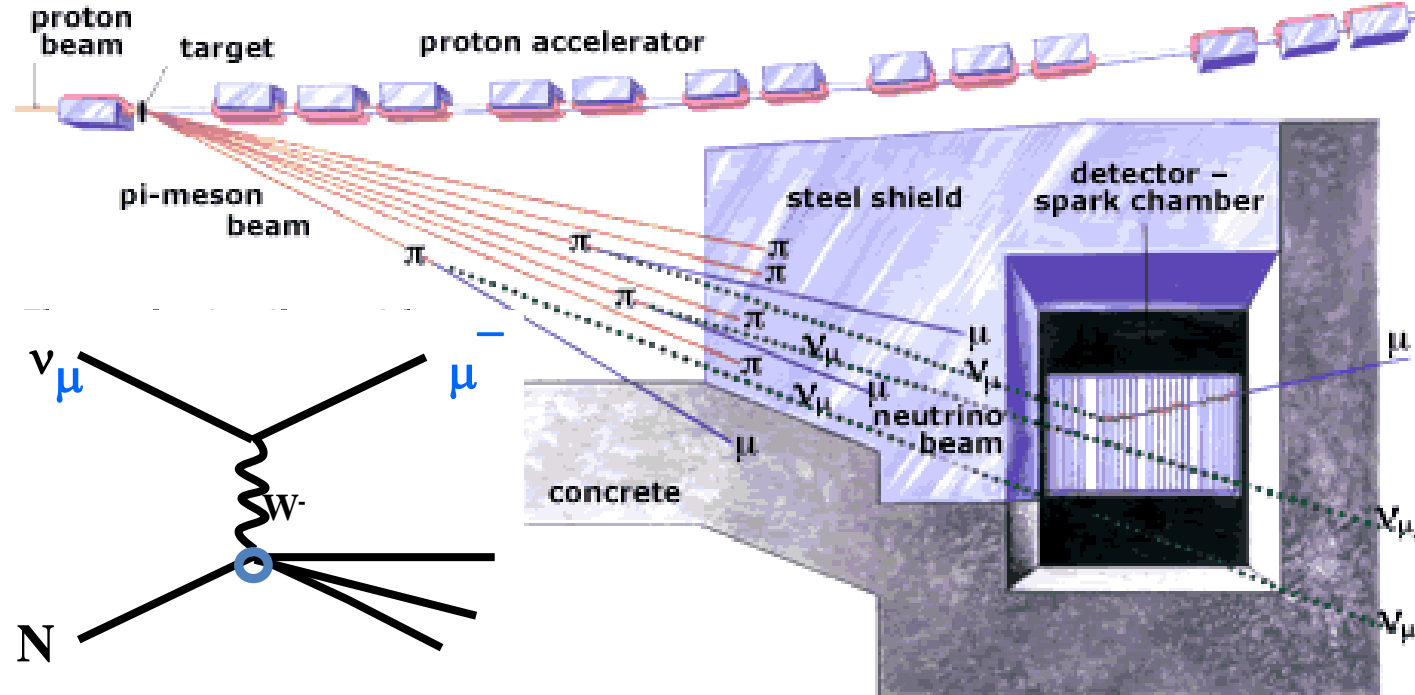


AGS Proton Beam

Schwartz

Lederman

Steinberger



Neutrinos from
 π -decay only
produce muons
(not electrons)

when they interact
in matter

hadrons

Neutrinos

the weak neutral current

Gargamelle Bubble Chamber
CERN

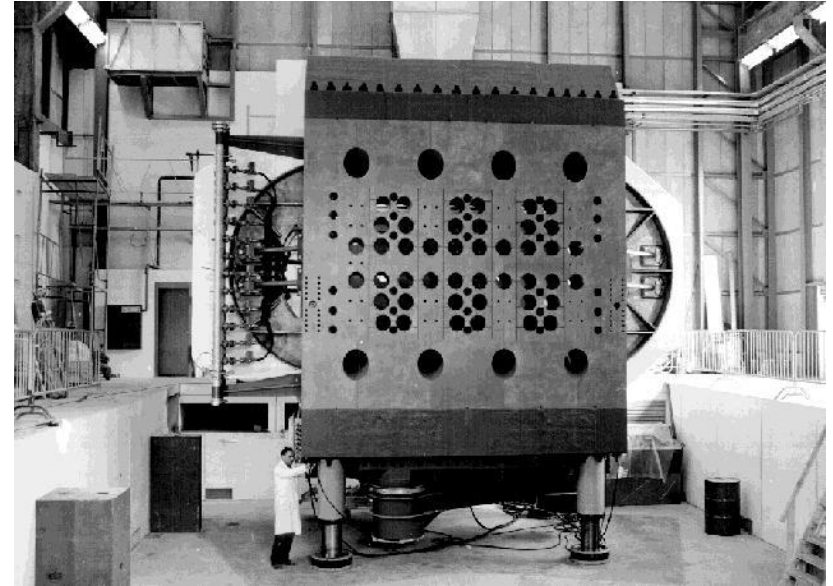
Discovery of weak neutral current

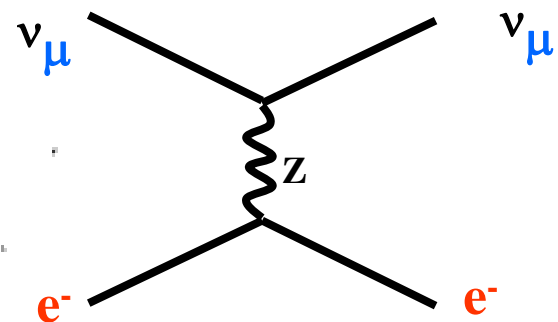
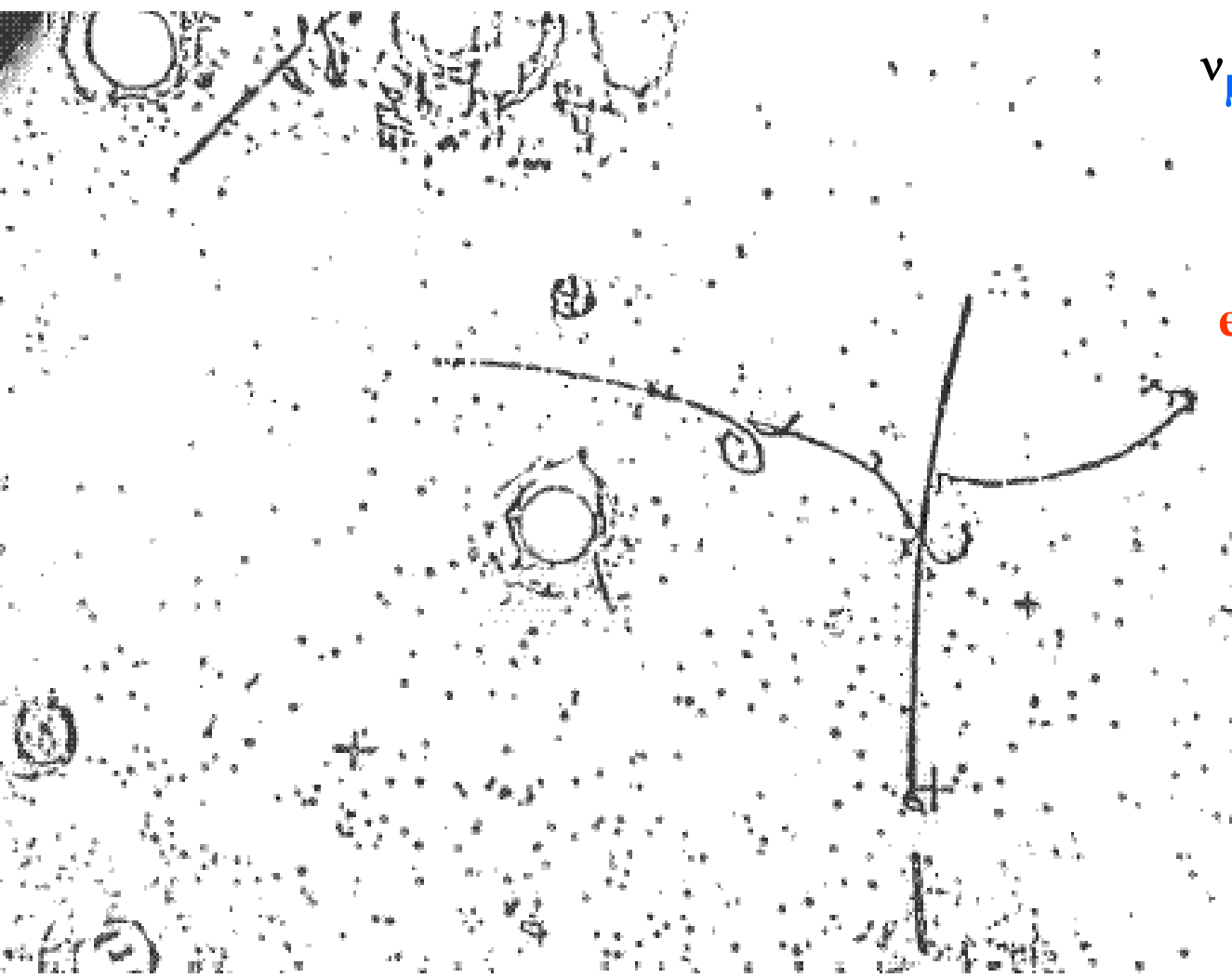
$$\nu_{\mu} + e \rightarrow \nu_{\mu} + e$$

$$\nu_{\mu} + N \rightarrow \nu_{\mu} + X \text{ (no muon)}$$

previous searches for neutral currents had been performed in particle decays
(e.g. $K^0 \rightarrow \mu\mu$) leading to extremely stringent limits (10^{-7} or so)

early neutrino experiments had set their trigger on final state (charged) lepton!





elastic scattering of neutrino
off electron in the liquid

1973 Gargamelle

Experimental birth of the Standard Model

The tau neutrino discovery

- was not simple
- ν_τ symbol appears in 1977 (but still could be some $\nu_\mu \nu_e$ combination)
- tau neutrino appears as established particle in PDG 1982
dubbed 'indirect' as it requires combination of measurement of
tau lifetime and negative tau appearance in ν_μ/ν_e beams
- further measurements of the tau lifetime,
- the observation of $W \rightarrow \tau \nu_\tau$ in the UA1 experiment at the Sp \bar{p} S and
- strong negative tau appearance by emulsion expt in ν_μ/ν_e beam E531 @Tevatron
→ ν_τ was solidly established by 1986
- is a great example of the complementarity between
collider experiments and neutrino beam experiments
- tau neutrino CC (and NC) couplings measured at LEP at permil precision level.
- ν_τ N CC interactions observed by DONUT in 2000
- see my presentation at the conference on the history of the neutrinos
<http://neutrinohistory2018.in2p3.fr/>
or [arxiv:1812.11362v2](https://arxiv.org/abs/1812.11362v2) for more details and comments.

In 1985 the observation of the W decay $W \rightarrow \tau \nu_\tau$ was reported.

5. EXPERIMENTAL EVIDENCE FOR THE HEAVY LEPTON DECAY $W \rightarrow \tau \nu_\tau$

With the observation of the $W \rightarrow \tau \nu$ decay, the 'programme' on the leptonic decay channels of the IVB is complete.

In the case of a $W \rightarrow \tau \nu$ event where the τ decays in the hadronic mode, what we measure is a jet including charged tracks and the corresponding energy deposition in some calorimeter cells (both hadronic and electromagnetic). The measured jet represents the charged and neutral π 's of ν from the W decay and that from the τ decay. Therefore, events with missing transverse energy and one trigger jet were selected in the data recorded during the 1983 runs (corresponding to an integrated luminosity of 1.2 nb⁻¹).

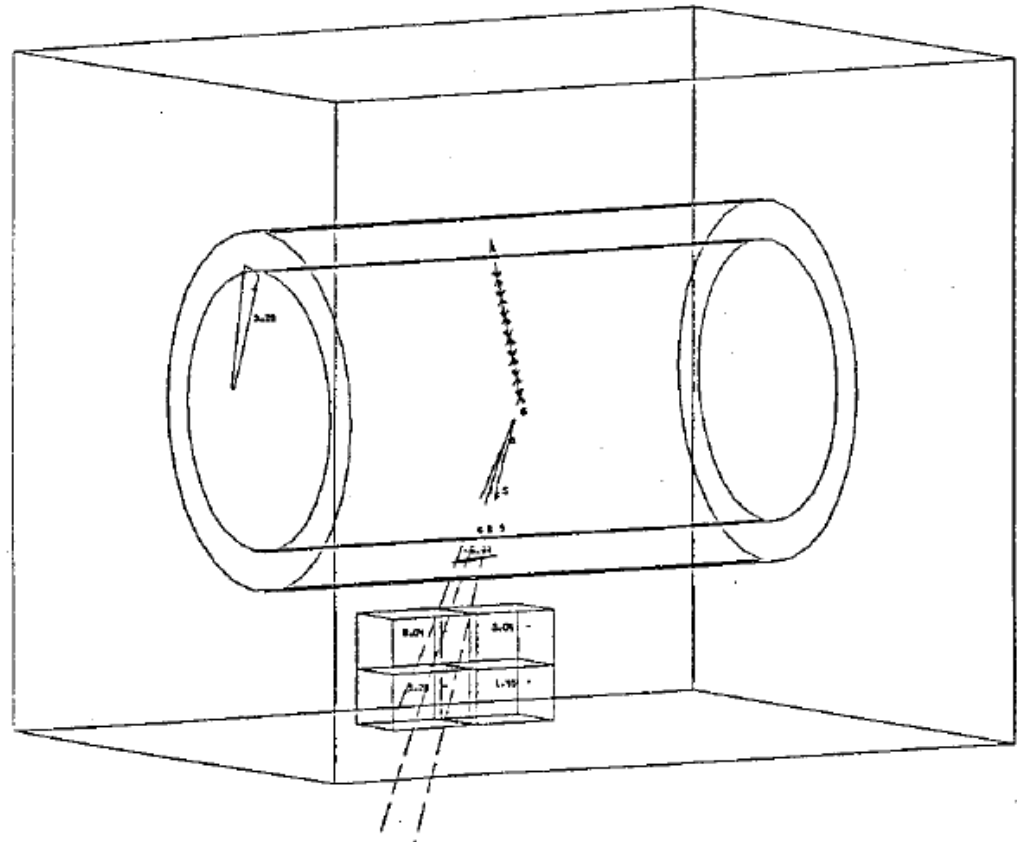
As a consequence of the experimental results, the $W \rightarrow \tau \nu$ rate is expected to be abundant. Almost half of the τ 's decay into one charged lepton (without neutrals and 38% with) and a neutrino¹³. This suggests a clear signature with a reasonable rate: an isolated high- p_T track of a hadronic type and some missing transverse energy. In this sense,

yes.... and it is also the first time that a tau neutrino is observed, that is not produced in tau decay!

1985CERN-EP/85-29
5 March 1985 W^{\pm} AND Z^0 PRODUCTION IN THE UA1 EXPERIMENT
AT THE CERN PROTON-ANTIPROTON COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

11109/247



UA1 observation of $W \rightarrow \tau \nu_{\tau}$

- low mass jet of 3 charged tracks
- missing transverse momentum

← Mass not restricted to W mass.

$$\Gamma(\tau^+ \nu)/\Gamma(e^+ \nu)$$

VALUE	EVTS	DOCUMENT ID	TECN	COMMENT
$1.02 \pm 0.20 \pm 0.12$	32	ALBAJAR	89 UA1	$E_{cm}^{p\bar{p}} = 546.630 \text{ GeV}$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$1.02 \pm 0.20 \pm 0.10$	32	ALBAJAR	87 UA1	Repl. by ALBAJAR 89

$$\Gamma_5/\Gamma_1$$

by 1987 the CC coupling of the tau is established to equal that of the electron to $\pm 20\%$

W decay is precisely what we use to define the neutrino flavours.

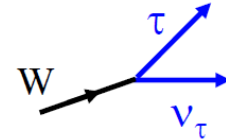
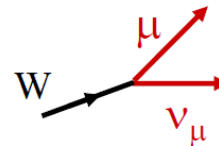
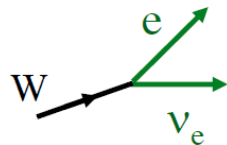
e.g. B. Kayser,
VIIth Pontecorvo School, 2017

The Neutrino Flavors

There are three flavors of charged leptons: e , μ , τ

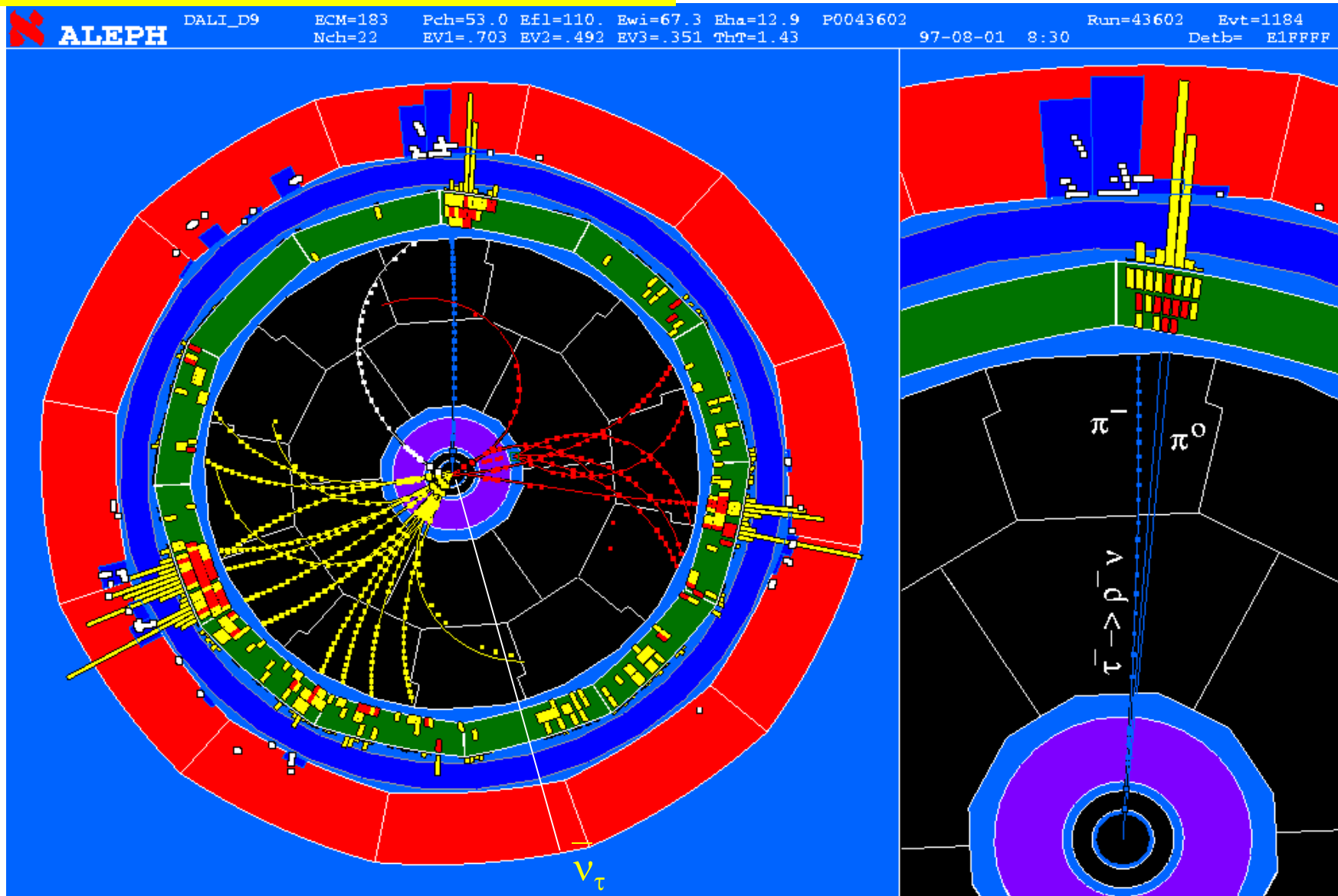
There are three known flavors of neutrinos: ν_e , ν_μ , ν_τ

We *define* the neutrinos of specific flavor, ν_e , ν_μ , ν_τ ,
by W boson decays:



the existence of the three W decay modes with similar branching ratios
establishes the tau and its neutrino as a new sequential heavy lepton doublet

kinematic reconstruction of two tau neutrinos



Observation of tau-neutrino in ALEPH at LEP (183 GeV E_{cm})
 $e^+e^- \rightarrow W^+ W^- \rightarrow (\text{hadrons})^+ + \tau^- \nu_\tau$

LEP saw several 1000's
 of those in the 90's.

23/01/2019

Neutrino Weak Couplings

present value of the tau neutrino weak couplings in tau decays:

$$\frac{g_\tau}{g_\mu} = 1.0010 \pm 0.0015 ; \frac{g_\tau}{g_e} = 1.0029 \pm 0.0015$$

NB -- the product $g_e \cdot g_\mu$ is extracted from the muon decay $\rightarrow G_F$
-- the ratio g_e/g_μ can be extracted from the ratio

So we can derive the ratio $R_\pi = \frac{\pi \rightarrow e \nu}{\pi \rightarrow \mu \nu}$

$$R_\pi = (m_e/m_\mu)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2 = \begin{array}{l} 1.2351(2) \cdot 10^{-4} \text{ (theory)} \\ 1.230(4) \cdot 10^{-4} \text{ (exp)} \end{array}$$

DONUT Collaboration

K. Kodama¹, N. Ushida¹, C. Andreopoulos², N. Saoulidou²,
G. Tzanakos², P. Yager³, B. Baller⁴, D. Boehnlein⁴,
W. Freeman⁴, B. Lundberg⁴, J. Morfin⁴, R. Rameika⁴,
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T. Nakano¹⁰, K. Niwa¹⁰, N. Nonaka¹⁰, K. Okada¹⁰,
O. Sato¹⁰, T. Akdogan¹¹, V. Paolone¹¹, C. Rosenfeld¹¹,
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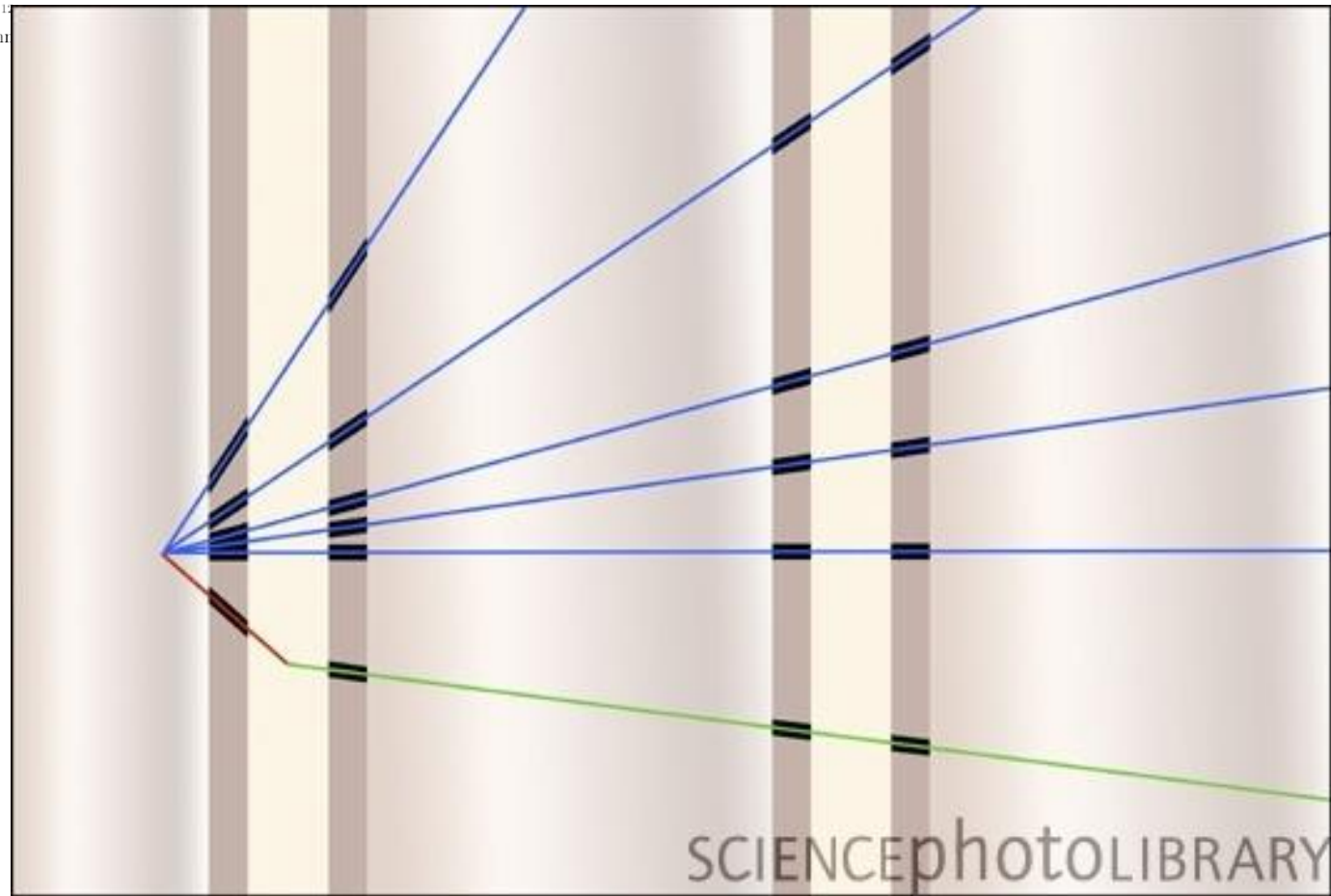
¹³ Tufts University, Medford, Massachusetts 02155

December 14, 2000

Beautiful observation
of neutrino interactions
producing taus!

there is 'small print'...

Observation of Tau Neutrino Interactions DONUT



Tau Neutrino interaction in DONUT experiment (Fermilab) 2000

1989: LEP determined the number of active neutrinos

final value (1995)

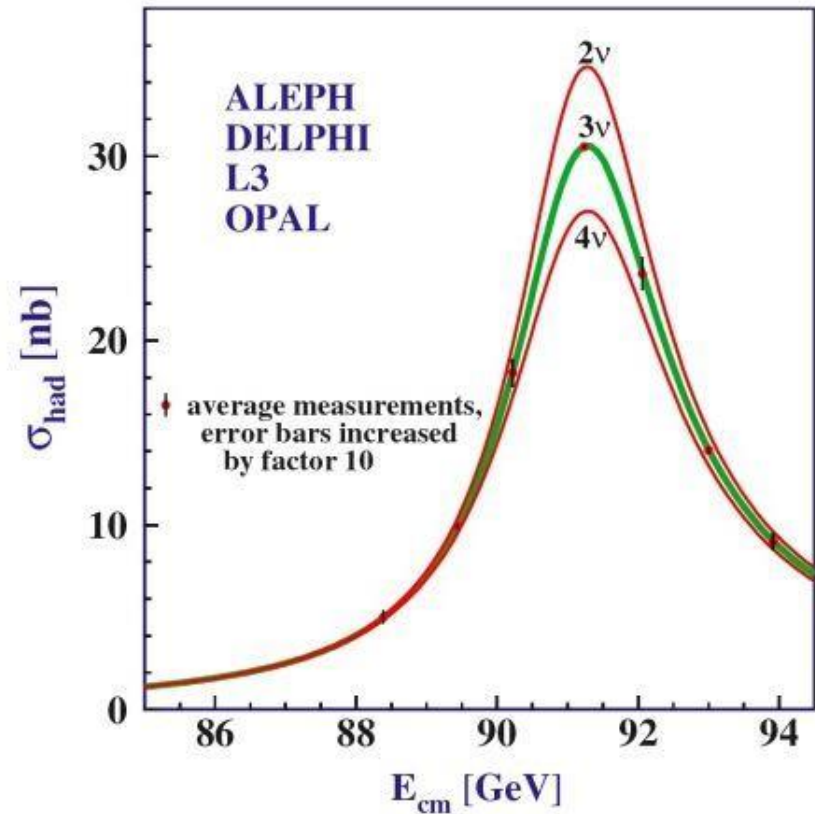
$$N_\nu = 2.984 \pm 0.008$$

Phys.Rept.427:257-454,2006

This is determined from the Z line shape scan and dominated by the measurement of the hadronic cross-section at the Z peak maximum →

The dominant systematic error is the theoretical uncertainty on the Bhabha cross-section (0.06%) which represents an error of ± 0.0046 on N_ν

Improving on N_ν by more than a factor 2 would require a large effort to improve on the Bhabha cross-section calculation!



Neutrinos

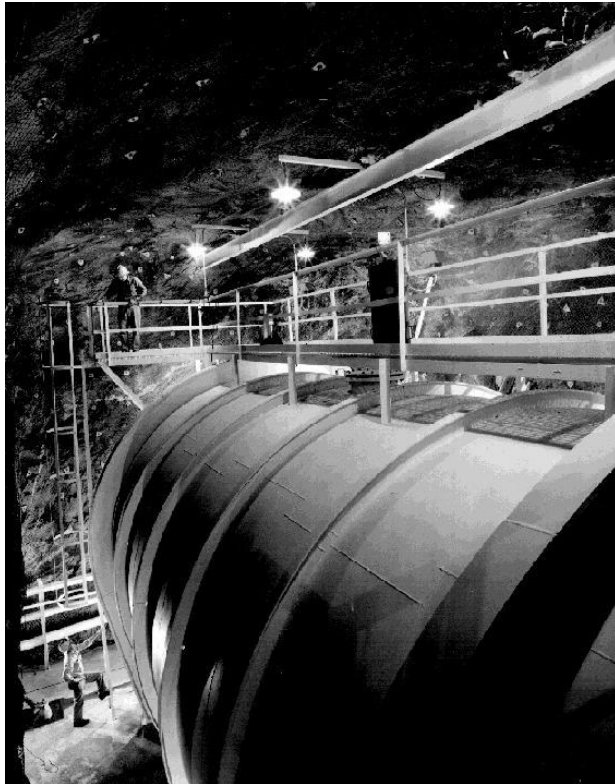
astrophysical neutrinos

Ray Davis

since ~1968



Homestake Detector



Solar Neutrino Detection 600 tons of chlorine.

- Detected neutrinos $E > 1\text{MeV}$
- fusion process in the sun

solar : $pp \rightarrow pn \ e^+ \ \nu_e$ (then D gives He etc...)

these ν_e do $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$

they are **neutrinos**

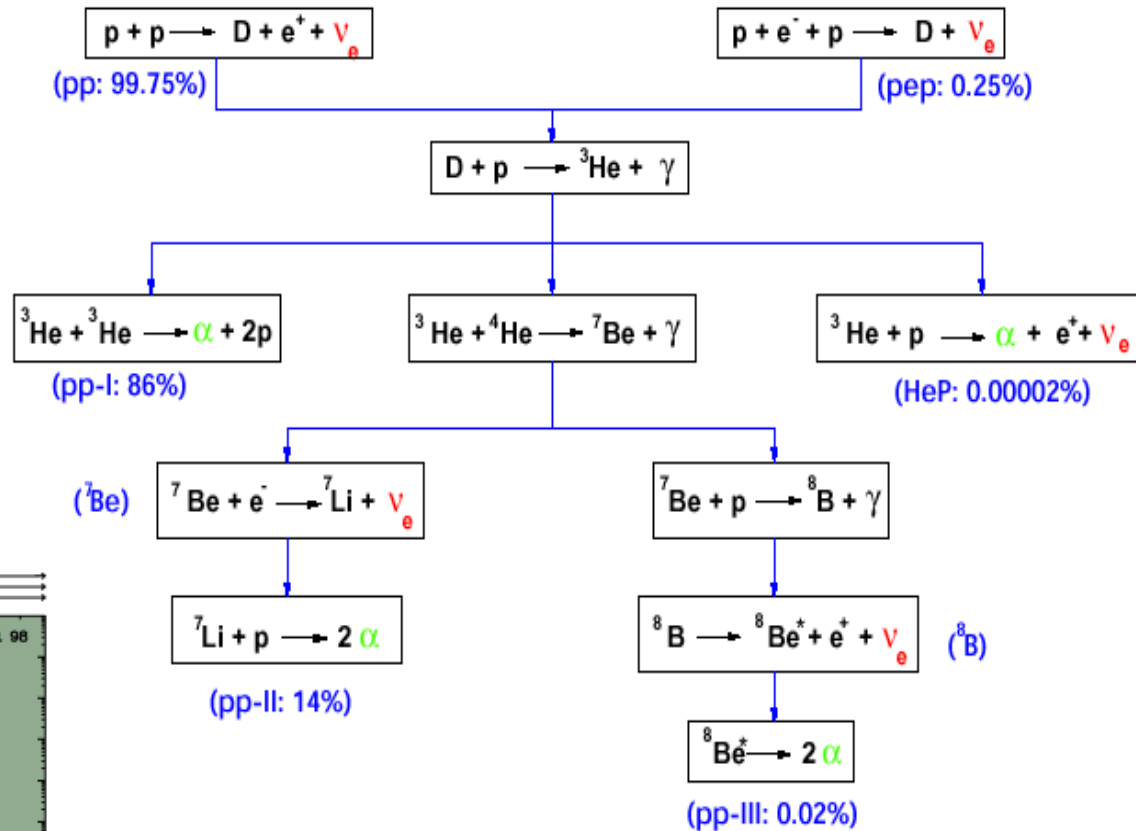
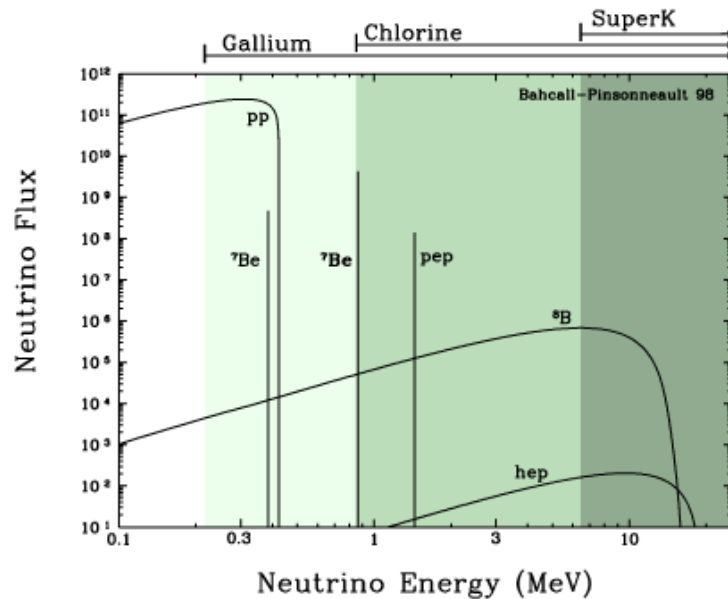
- The rate of neutrinos detected is
three times less than predicted!

solar neutrino ‘puzzle’ since 1968-1975!

solution: 1) solar nuclear model is wrong or 2) neutrino oscillate

ν_e solar neutrinos

Sun = Fusion reactor
Only ν_e produced
Different reactions
Spectrum in energy



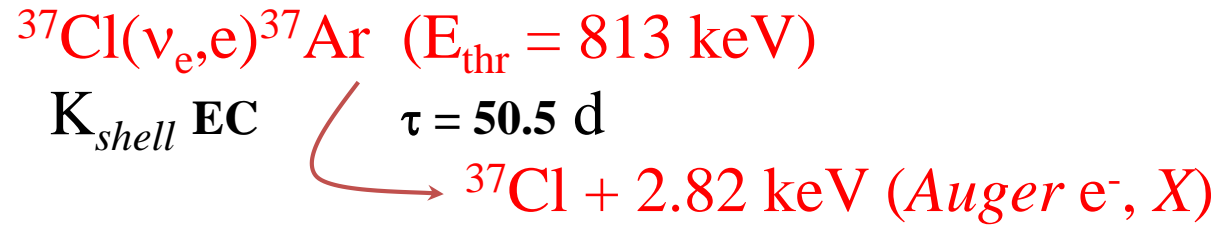
Counting experiments vs
flux calculated by SSM

BUT ...



The Pioneer: Chlorine Experiment

The interaction



ν Signal Composition:
(BP04+N14 SSM+ ν osc)

pep+hep	0.15 SNU	(4.6%)
^7Be	0.65 SNU	(20.0%)
^8B	2.30 SNU	(71.0%)
CNO	0.13 SNU	(4.0%)
Tot	3.23 SNU	$\pm 0.68 \text{ } 1\sigma$

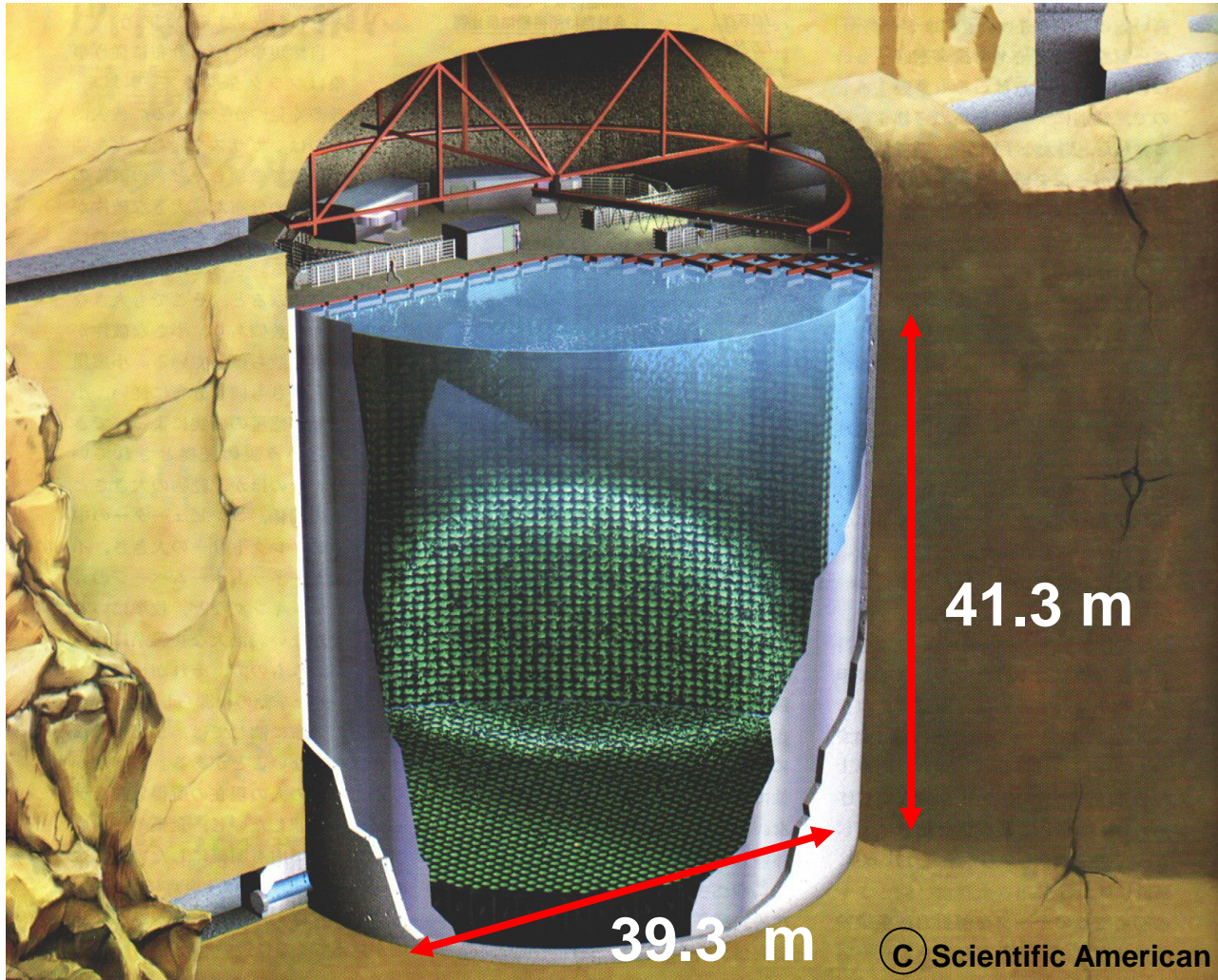
Expected Signal
(BP04 + N14)

8.2 SNU $+1.8_{-1.8} 1\sigma$

S.N.U. = Solar Neutrino Unit

(electron-) neutrino flux producing 10^{-36} captures per target atom per second

Super-K detector



Water Cerenkov
detector

50000 tons of
pure light
water

≈ 10000 PMTs

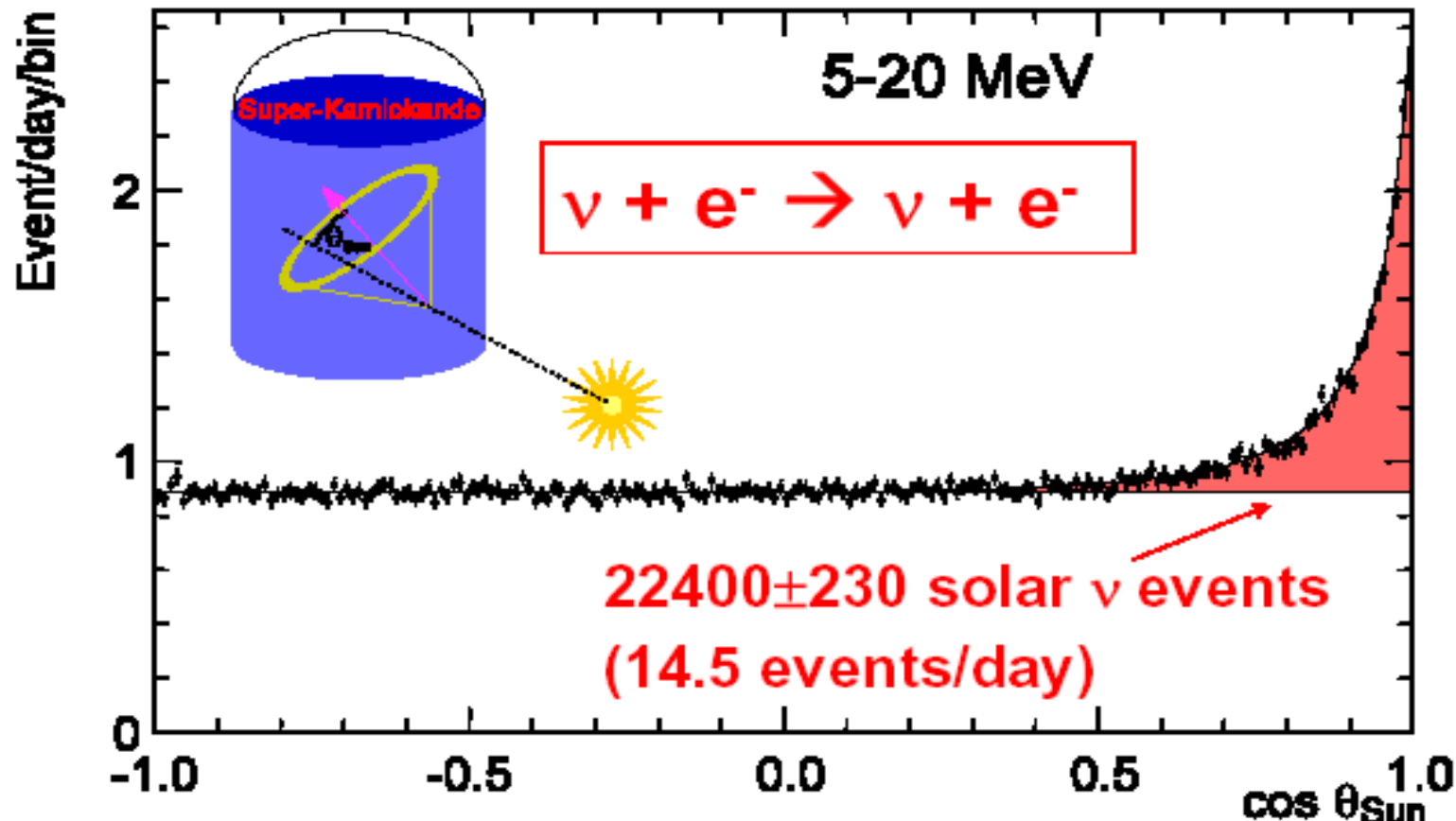
41.3 m

39.3 m

© Scientific American

Super-Kamiokande-I solar neutrino data

May 31, 1996 – July 13, 2001 (1496 days)



^8B flux : $2.35 \pm 0.02 \pm 0.08$ [$\times 10^6 / \text{cm}^2 / \text{sec}$]

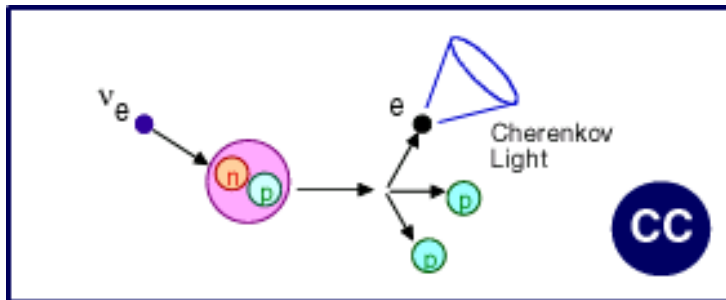
$$\frac{\text{Data}}{\text{SSM(BP2004)}} = 0.406 \pm 0.004 \begin{matrix} +0.014 \\ -0.013 \end{matrix}$$

(Data/SSM(BP2000) = $0.465 \pm 0.005 \begin{matrix} +0.016 \\ -0.015 \end{matrix}$)

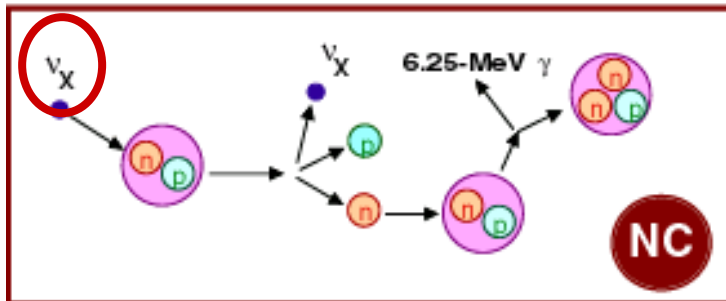
SNO detector

Aim: measuring non ν_e neutrinos in a pure solar ν_e beam

How? Three possible neutrino reaction in heavy water:

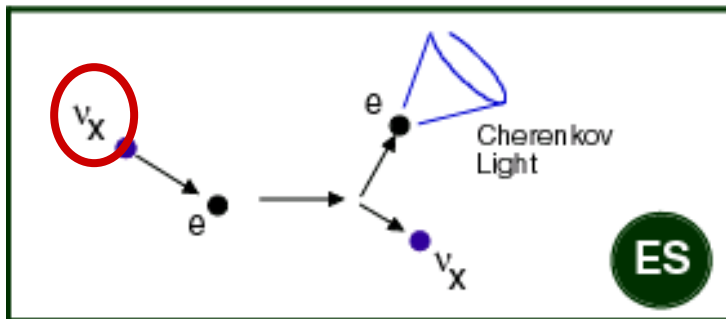


only ν_e



equally

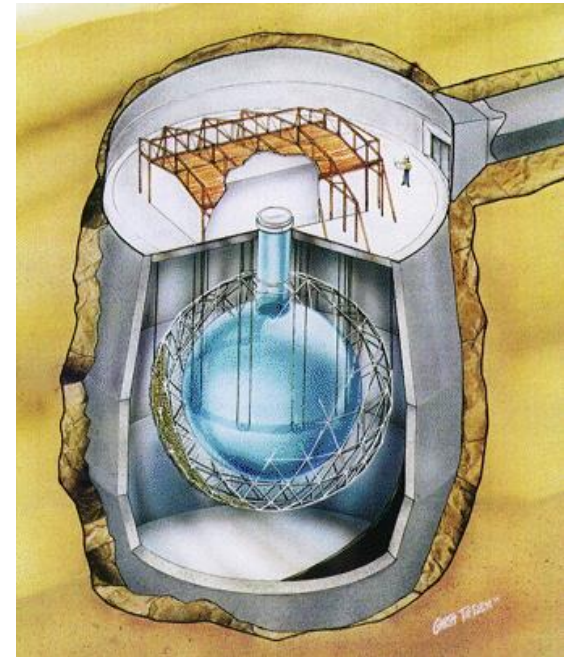
$\nu_e + \nu_\mu + \nu_\tau$



in-equally

$\nu_e +$

$0.1 (\nu_\mu + \nu_\tau)$



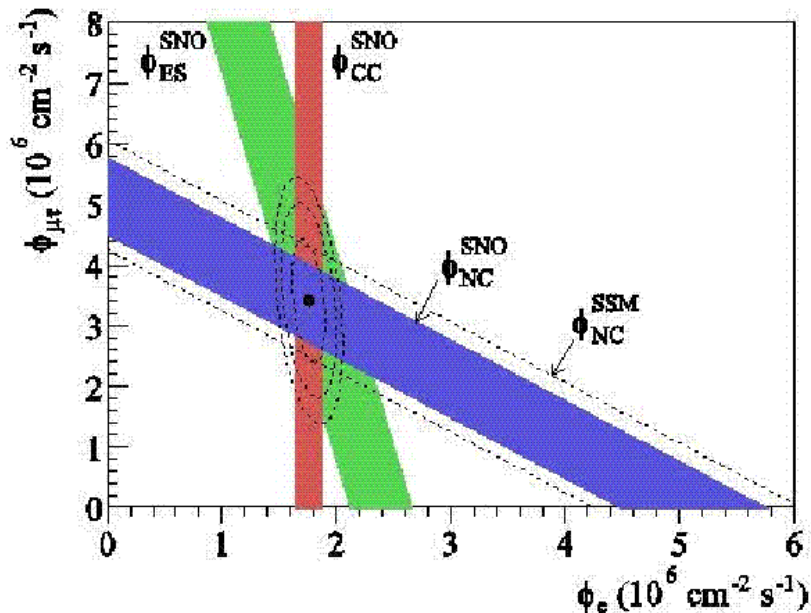
1000 ton of D₂O

12 m diam.

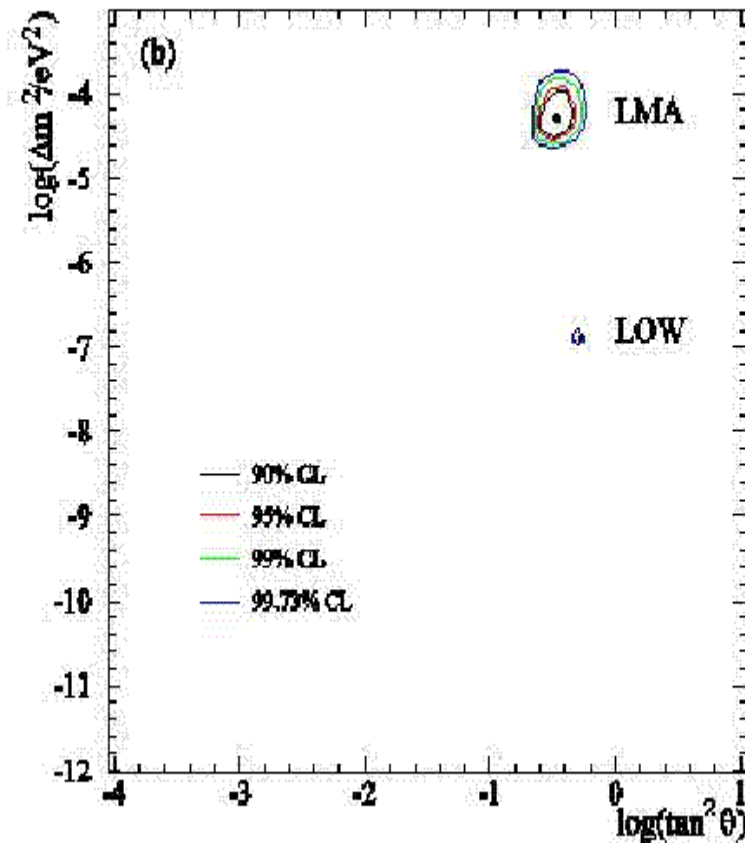
9456 PMTs

Physics Implication Flavor Content

$$\Phi_{\text{ssm}} = 5.05^{+1.01}_{-0.81} \quad \Phi_{\text{sno}} = 5.09^{+0.44+0.46}_{-0.43-0.43}$$



Combining All Experimental and Solar Model information



Strong evidence of flavor change

Charged current events are depleted (reaction involving electron neutrinos)

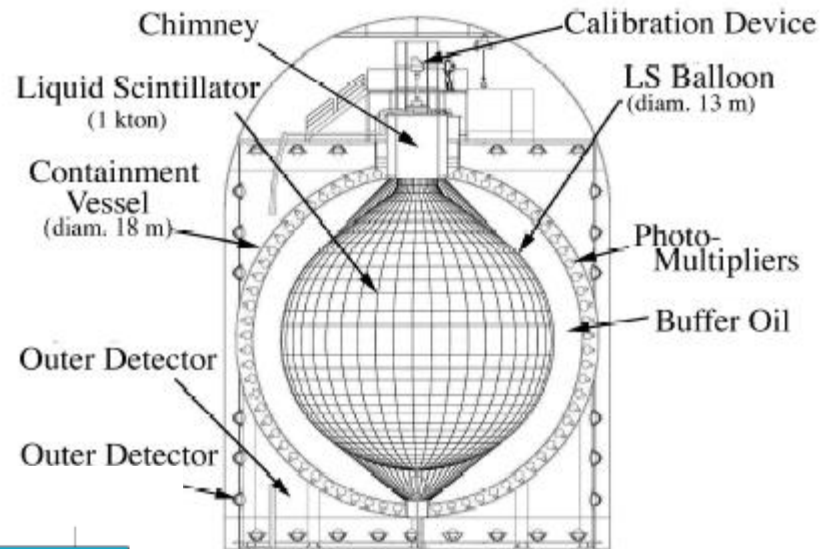
Neutral current reaction agrees with Solar Model (flavour blind)

SSM is right, neutrinos oscillate!

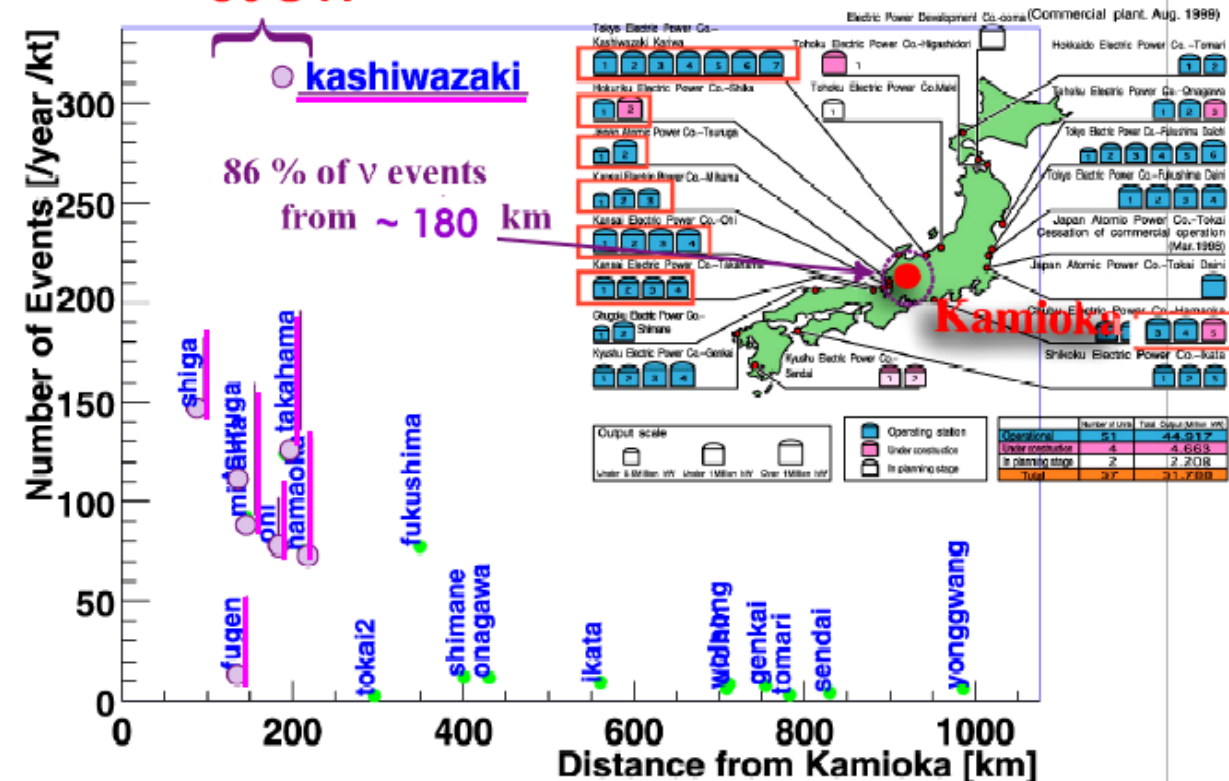
Kamland 2002-2006

20 % of world nuclear power

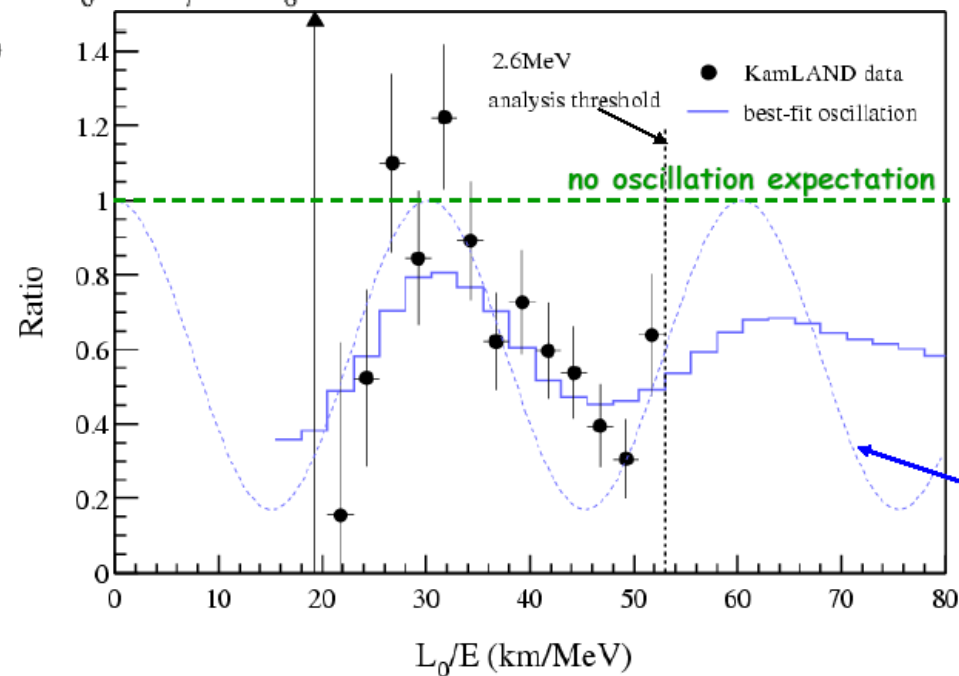
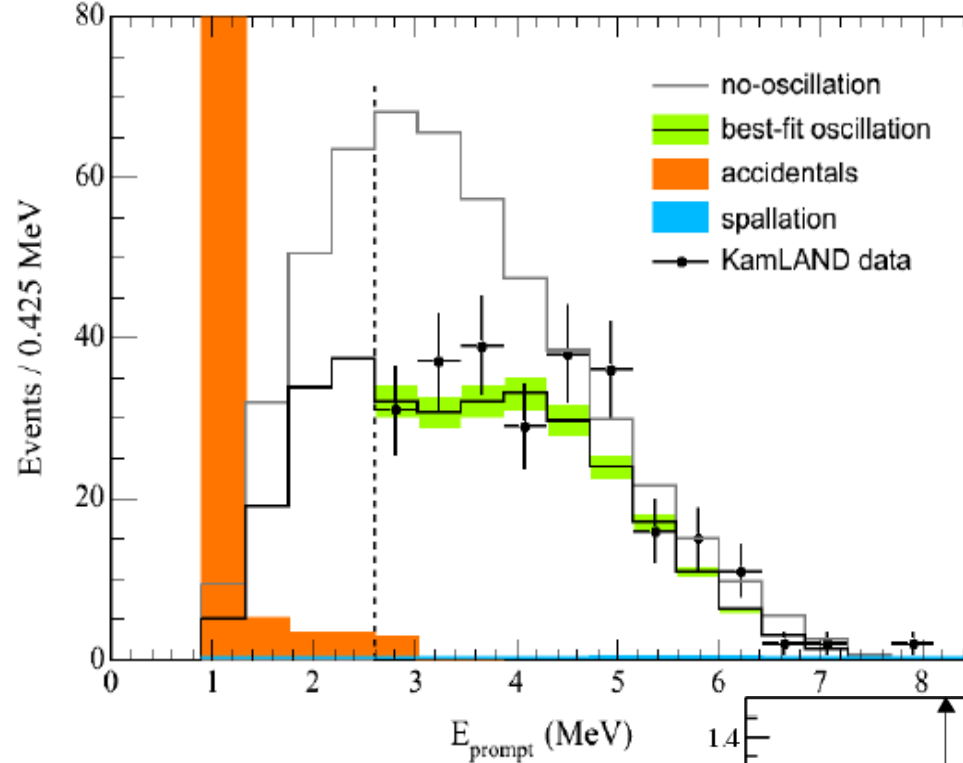
~80GW



Nuclear Power Stations in Japan

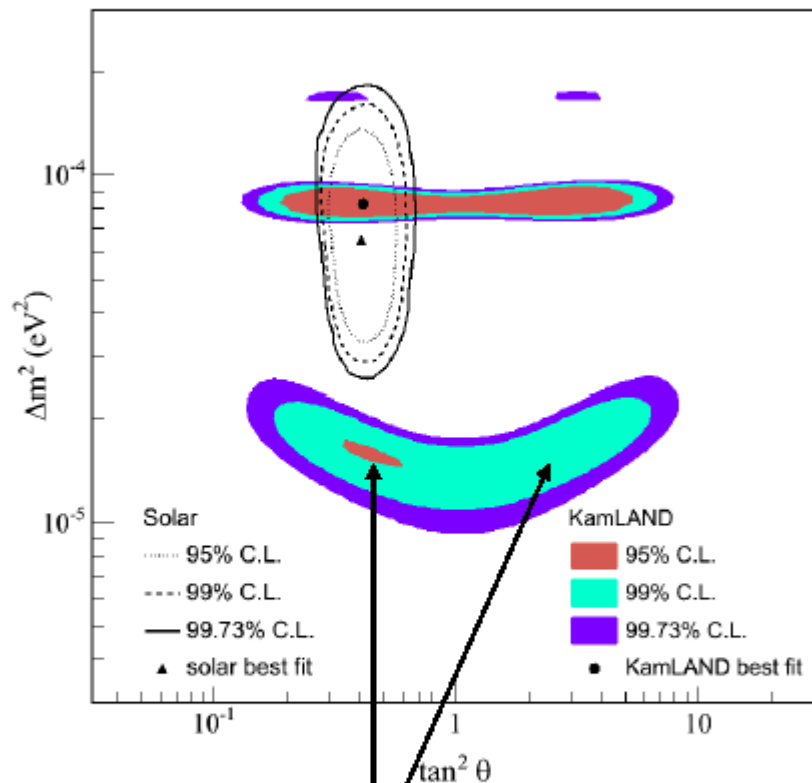


Kamland 2004



Hypothetical
single 180km
baseline
experiment

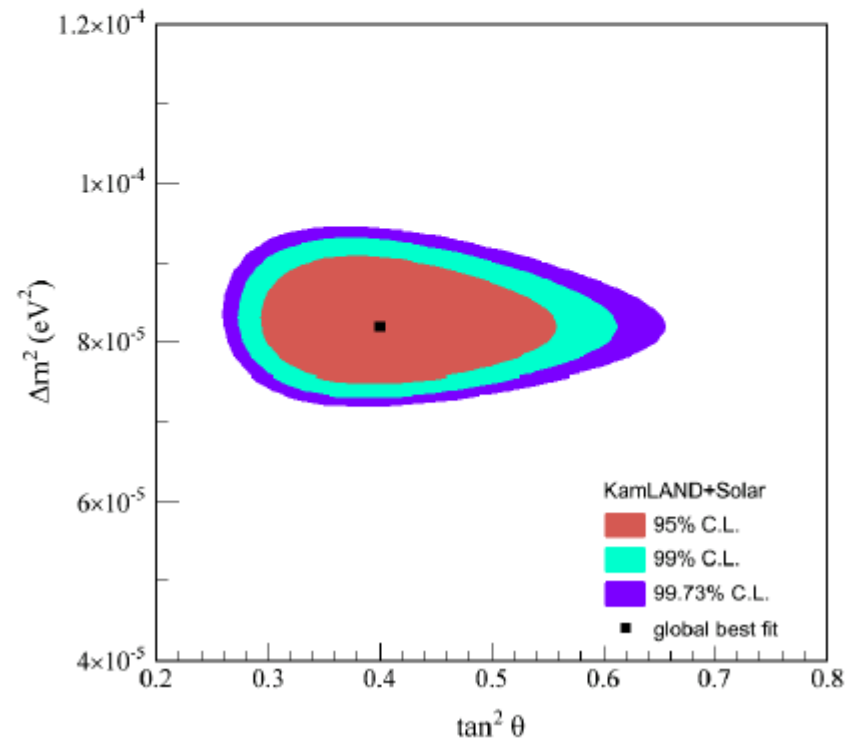
Kamland 2004



Includes (small) matter effects

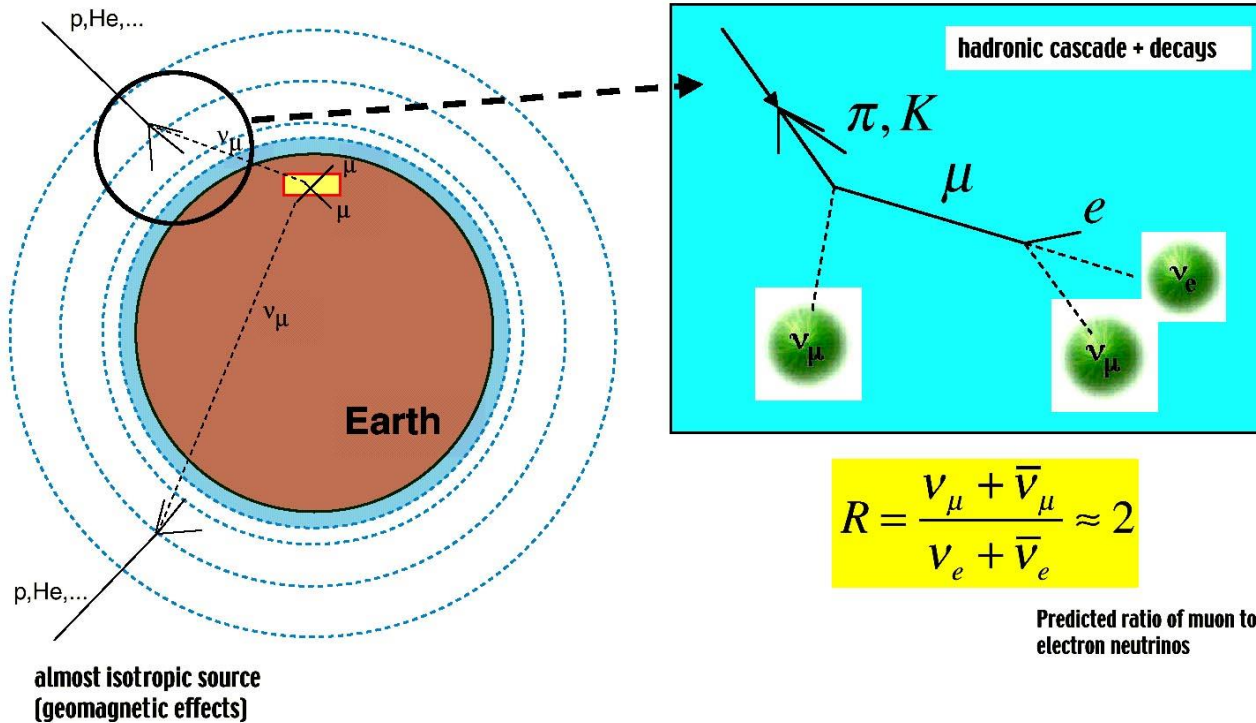
$$\Delta m_{12}^2 = 8.2^{+0.6}_{-0.5} \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \theta_{12} = 0.40^{+0.09}_{-0.07}$$

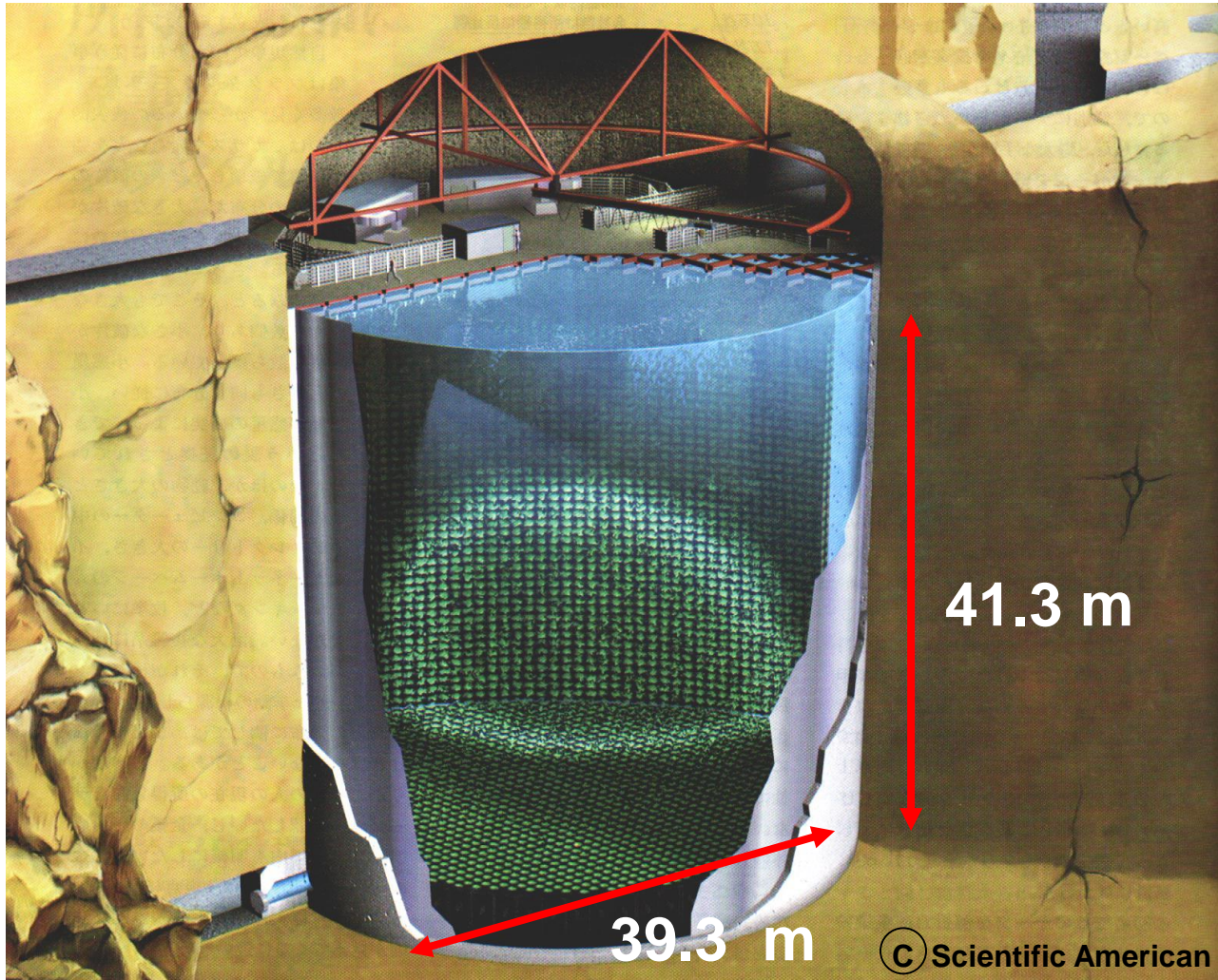


Atmospheric Neutrinos

Path length from ~20km to 12700 km



Super-K detector



Water Cerenkov
detector

50000 tons of
pure light
water

≈ 10000 PMTs

41.3 m

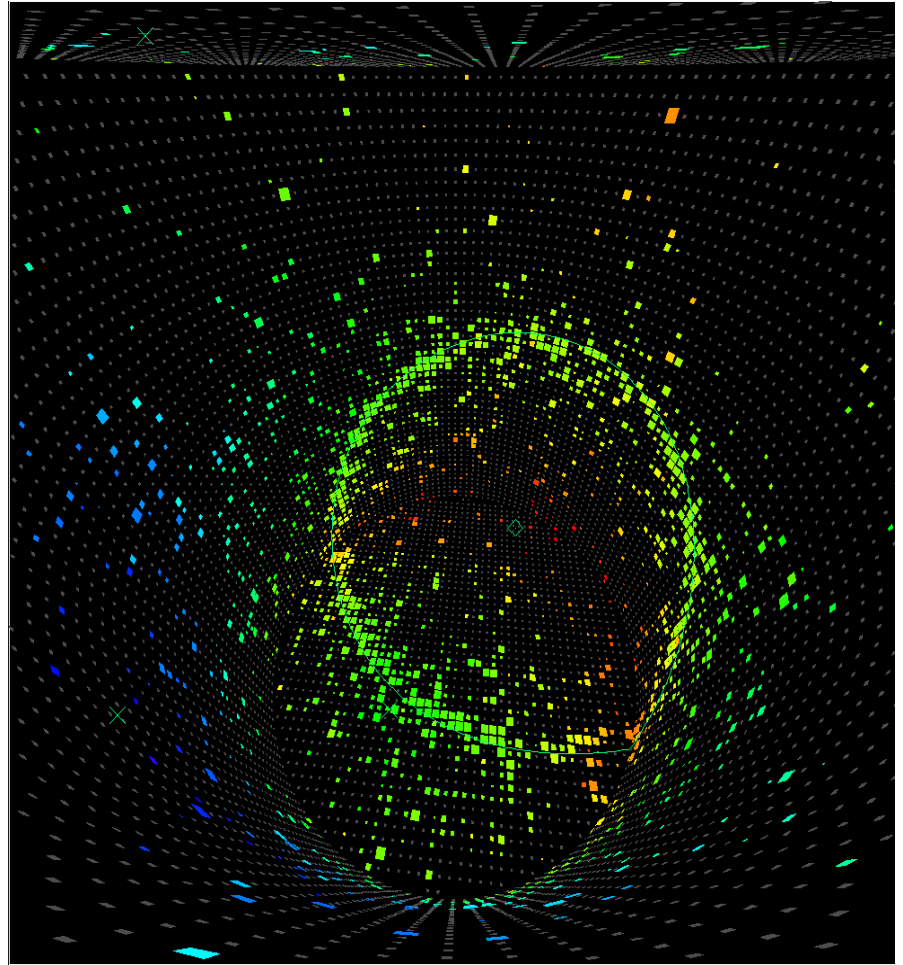
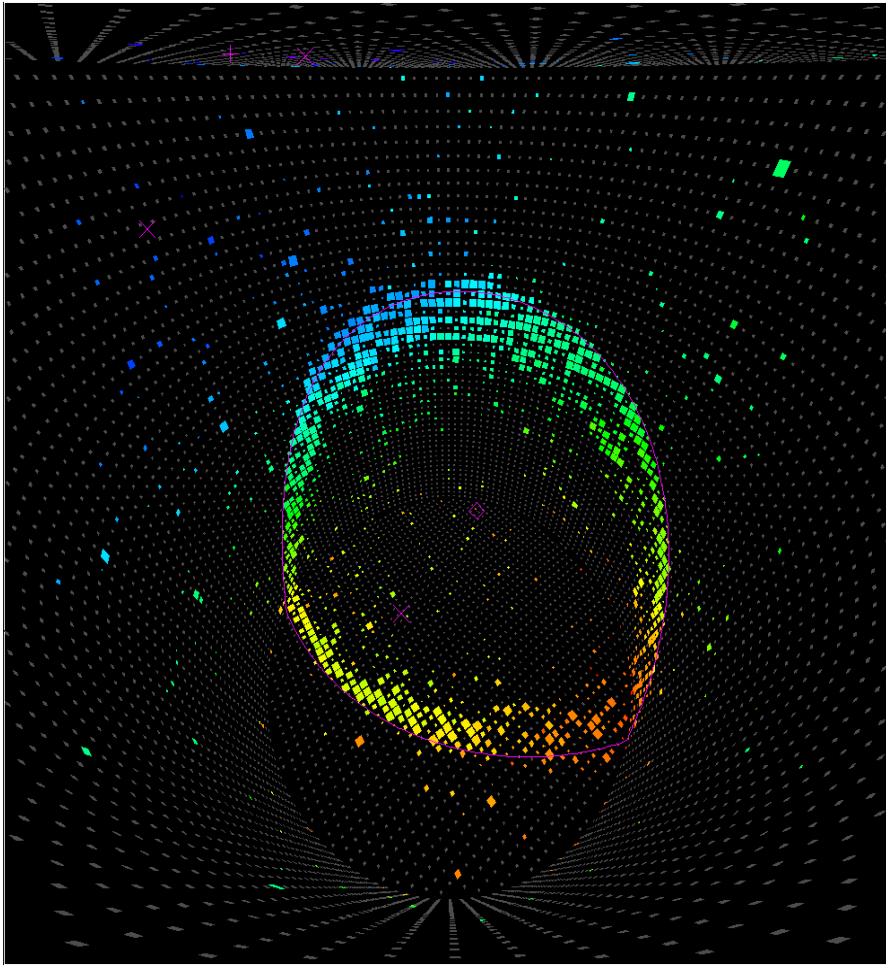
39.3 m

© Scientific American

μ/e Background Rejection

e/mu separation directly related to granularity of coverage.

Limit is around 10^{-3} (mu decay in flight) SKII coverage OKOK, less maybe possible

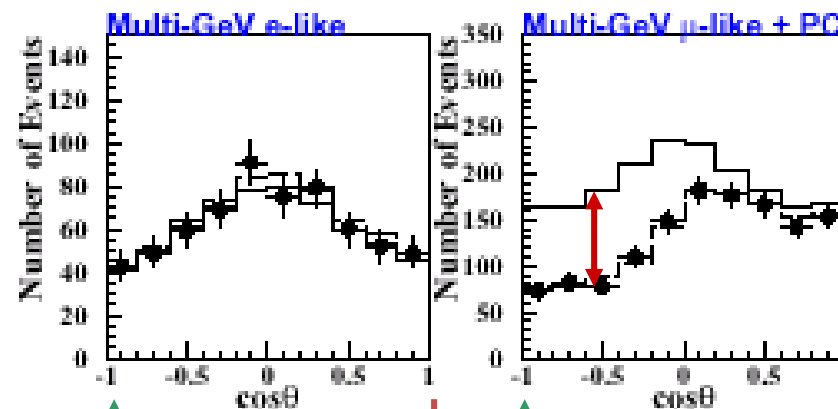
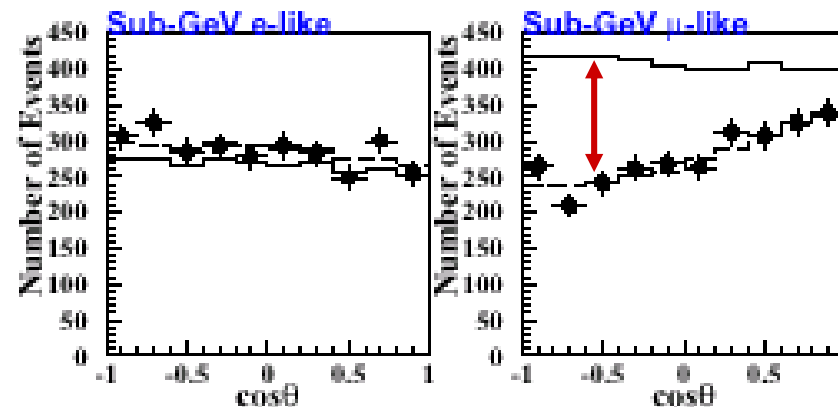
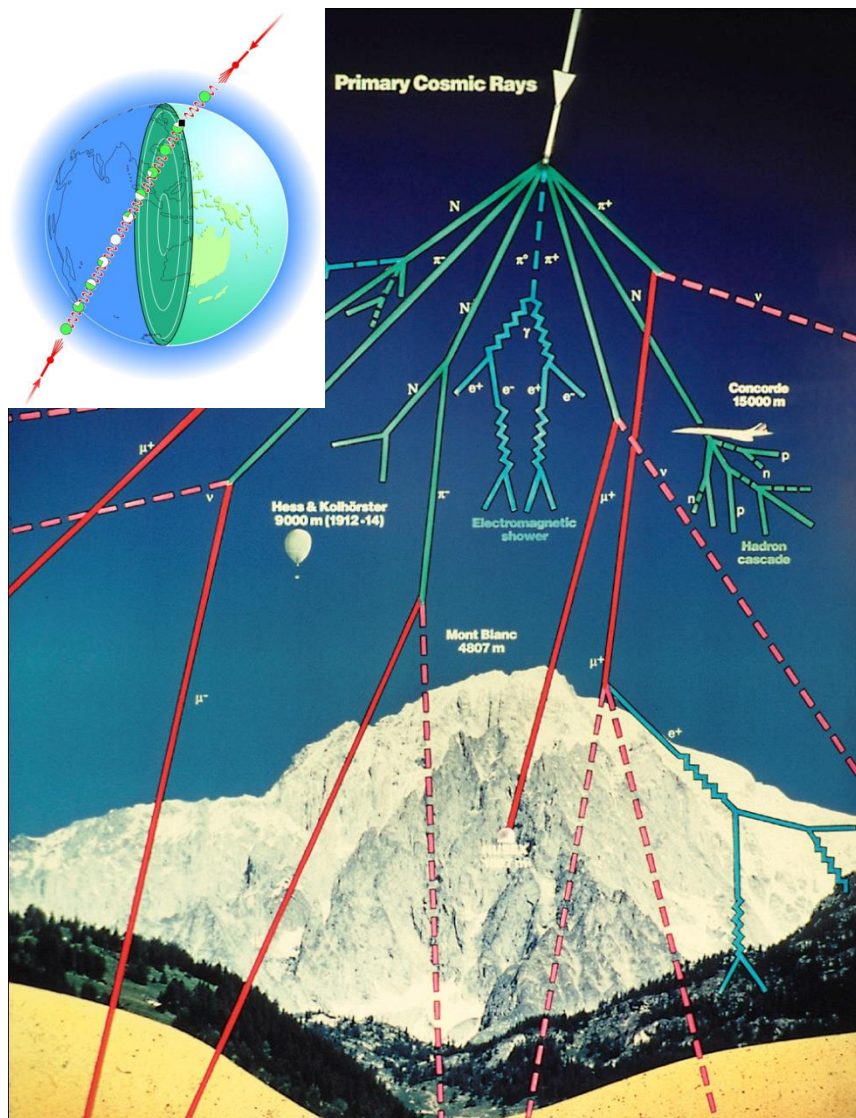


Atmospheric ν : up-down asymmetry

Super-K results

ν_e

ν_μ



up

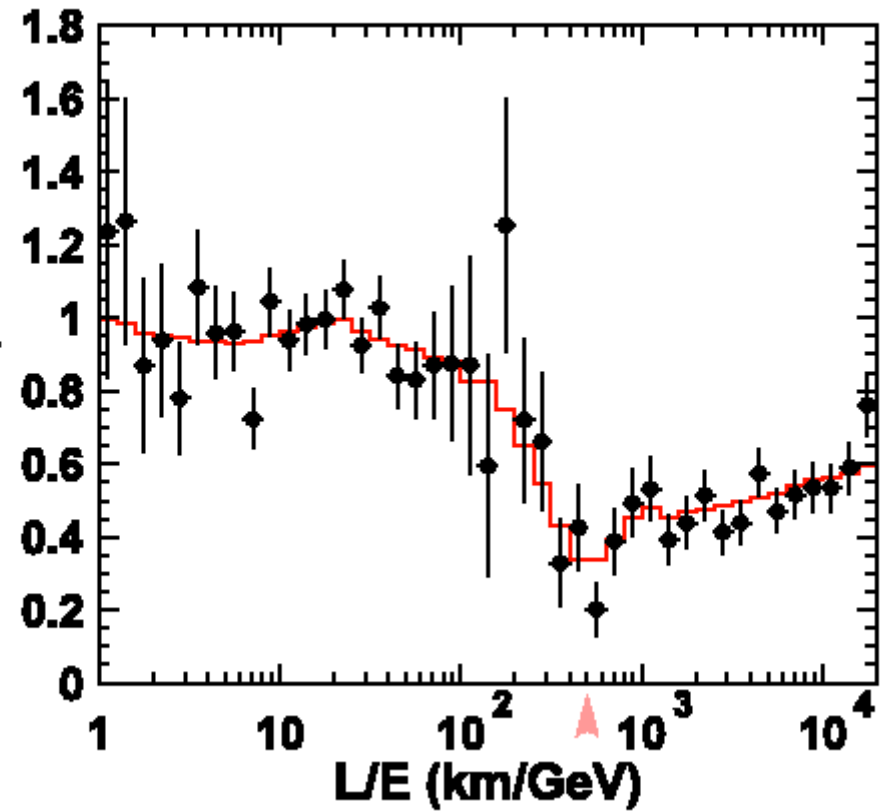
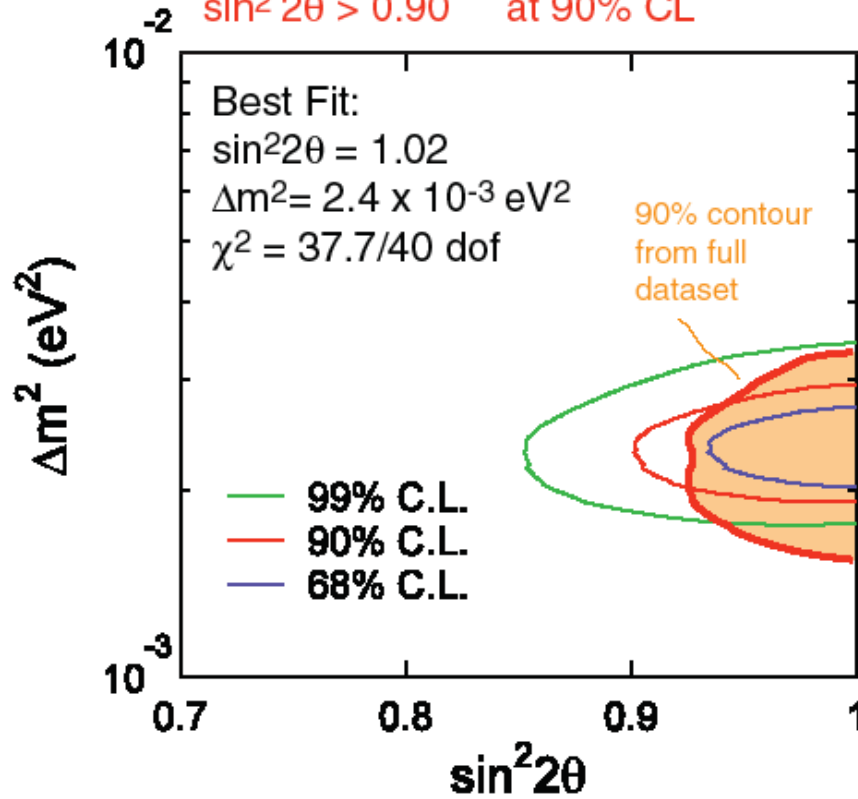
down

Atmospheric Neutrinos

SuperKamiokande *First Results*

$$1.9 \times 10^{-3} \text{ eV}^2 < \Delta m^2 < 3.0 \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta > 0.90 \quad \text{at 90\% CL}$$





The Nobel Prize in Physics 2015
Takaaki Kajita, Arthur B. McDonald

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The Nobel Prize in Physics 2015



Photo © Takaaki Kajita

Takaaki Kajita

Prize share: 1/2



Photo: K. MacFarlane.
Queen's University
/SNOLAB

Arthur B. McDonald

Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

A massless particle cannot be seen to transform

$$\tau_{\text{lab}} = \tau_{\text{particle}} \gamma = \tau_{\text{particle}} \frac{E}{m} \rightarrow \infty$$

neutrino definitions

the **electron** neutrino is present in association with an **electron** (e.g. beta decay)

the **muon** neutrino is present in association with a **muon** (pion decay)

the **tau** neutrino is present in association with a **tau** ($W \rightarrow \tau \nu$ decay)

these **flavor-neutrinos** are not (as we know now) quantum states of well defined **mass** (neutrino mixing)

the **mass-neutrino** with the highest **electron** neutrino content is called ν_1

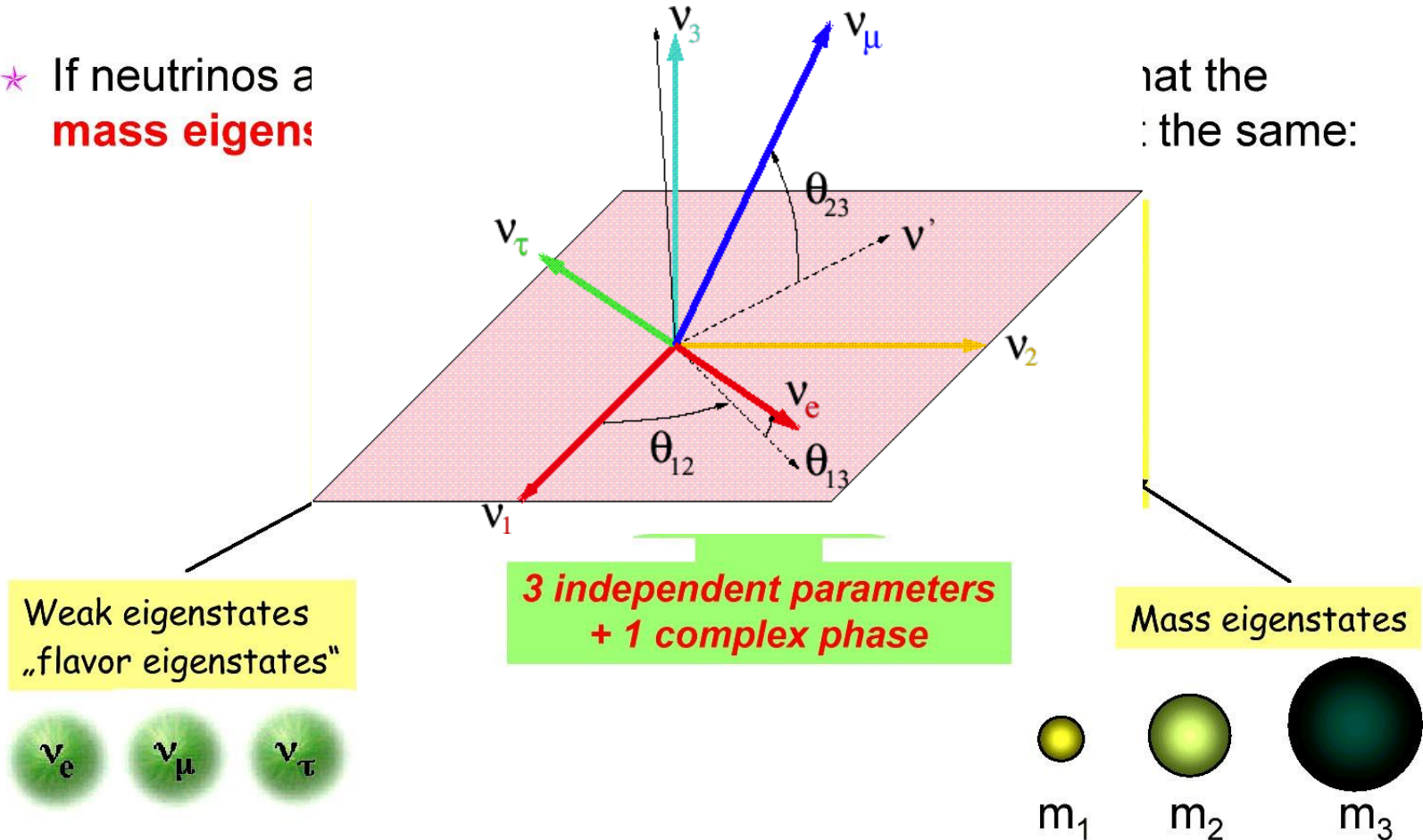
the **mass-neutrino** with the next-to-highest **electron** neutrino content is ν_2

the **mass-neutrino** with the smallest **electron** neutrino content is called ν_3

Lepton Sector Mixing

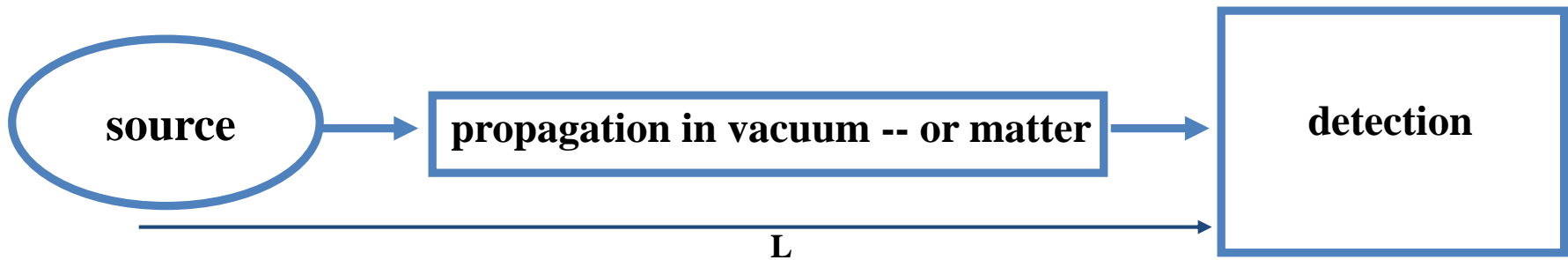
★ If neutrinos are
mass eigenstates

that the
: the same:



Pontecorvo 1957

Neutrino Oscillations (Quantum Mechanics lesson 5)



weak interaction
produces
'flavour' neutrinos

e.g. pion decay $\pi \rightarrow \mu \nu$

$$|\nu_{\mu}\rangle = \alpha |\nu_1\rangle + \beta |\nu_2\rangle + \gamma |\nu_3\rangle$$

Energy (i.e. mass) eigenstates
propagate

$$|\nu(t)\rangle = \alpha |\nu_1\rangle \exp(i E_1 t) + \beta |\nu_2\rangle \exp(i E_2 t) + \gamma |\nu_3\rangle \exp(i E_3 t)$$

$$t = \text{proper time} \propto L/E$$

α is noted $U_{1\mu}$

β is noted $U_{2\mu}$

γ is noted $U_{3\mu}$ etc....

weak interaction: (CC)

$$\nu_{\mu} N \rightarrow \mu^- X$$

or $\nu_e N \rightarrow e^- X$

or $\nu_{\tau} N \rightarrow \tau^- X$

$$P(\nu_{\mu} \rightarrow \nu_e) = |\langle \nu_e | \nu(t) \rangle|^2$$

Oscillation Probability

★ The case with two neutrinos:

→ A mixing angle: θ

→ A mass difference:

$$\Delta m^2 = m_2^2 - m_1^2$$

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

★ The oscillation probability is:

Δm^2 en eV^2

L en km

E en GeV

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(1.27 \Delta m^2 \frac{L}{E} \right)$$

where L = distance between source and detector

E = neutrino energy

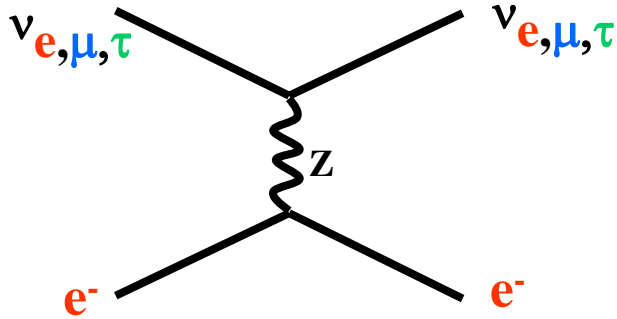
Hamiltonian = $E = \sqrt{p^2 + m^2} = p + m^2 / 2p$

for a given momentum, eigenstate of propagation in free space are the mass eigenstates!

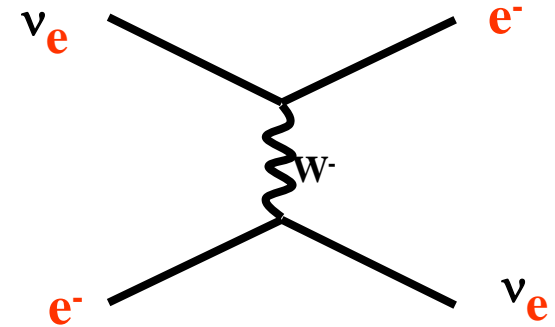
To complicate things further:

matter effects

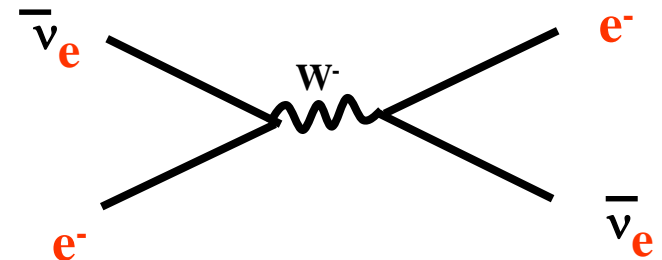
elastic scattering of (anti) neutrinos on electrons



all neutrinos and anti neutrinos do this equally



only electron neutrinos



only electron anti- neutrinos

These processes add a forward amplitude to the Hamiltonian, which is proportional to the number of electrons encountered to the Fermi constant and to the neutrino energy.

The Z exchange is diagonal in the 3-neutrino space

this does not change the eigenstates

The W exchange is only there for electron neutrinos

It has opposite sign for neutrinos and anti-neutrinos (s vs t-channel exchange)

$$D = \pm 2\sqrt{2} G_F n_e E_\nu$$

THIS GENERATES A FALSE CP VIOLATION

$$D = \pm 2\sqrt{2} G_F n_e E_\nu$$

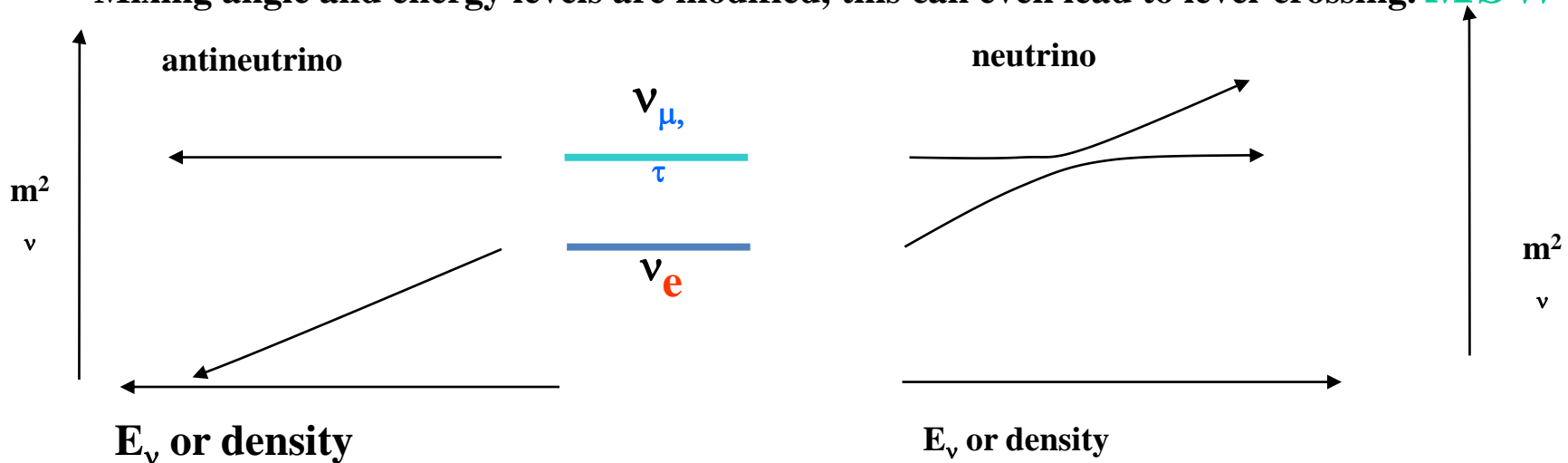
$$\mathbf{H}_{\text{flavour base}} = U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} D & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This is how YOU can solve this problem:
write the matrix,
diagonalize,
and evolve using,

$$i \frac{\partial \psi}{\partial t} = H \psi$$

This has the effect of modifying the eigenstates of propagation!

Mixing angle and energy levels are modified, this can even lead to level-crossing. *MSW effect*



oscillation is further suppressed

resonance... enhances oscillation

oscillation is enhanced for neutrinos if $\Delta m^2_{1x} > 0$, and suppressed for antineutrinos

oscillation is enhanced for antineutrinos if $\Delta m^2_{1x} < 0$, and suppressed for neutrinos

since **T** asymmetry uses neutrinos it is not affected

General framework and status:

1. We know that there are **three** families of active, light neutrinos (*LEP*)
2. **Solar** neutrino oscillations are **established** (*Homestake+Gallium+Kam+SK+SNO*)
3. **Atmospheric** neutrino ($\nu_\mu \rightarrow \nu_\tau$) oscillations are **established** (*IMB+Kam+SK+ K2K*)
4. At that frequency, ($\nu_\mu \rightarrow \nu_e$) oscillations, small (5%) have been observed (T2K, NOvA) and ν_e disappearance has been measured (Daya Bay, Reno, Double Chooz)

This allows a consistent picture with 3-family oscillations preferred:

LMA: $\theta_{12} \sim 30^\circ$ $\Delta m_{12}^2 \sim 8 \cdot 10^{-5} \text{eV}^2$, $\theta_{23} \sim 45^\circ$ $\Delta m_{23}^2 \sim \pm 2.5 \cdot 10^{-3} \text{eV}^2$, $\theta_{13} < \sim 9^\circ$

with several unknown parameters though 2018 revealed hints of CPV and NH.

=> an **exciting** experimental program for at least 20 years *)

including **leptonic CP & T violations**.

5. There are unexplained phenomena interpreted as possible higher frequency oscillation (LSND miniBooNe, reactors) but they are inconsistent with excellent disappearance experiments (MINOS, MINOS+, ICECUBE and DayaBay) so sterile neutrino explanation is ruled out, but **further investigation will be performed with time-sensitive experiments (SBN,**

*)to set the scale: **CP violation in quarks** was discovered in 1964

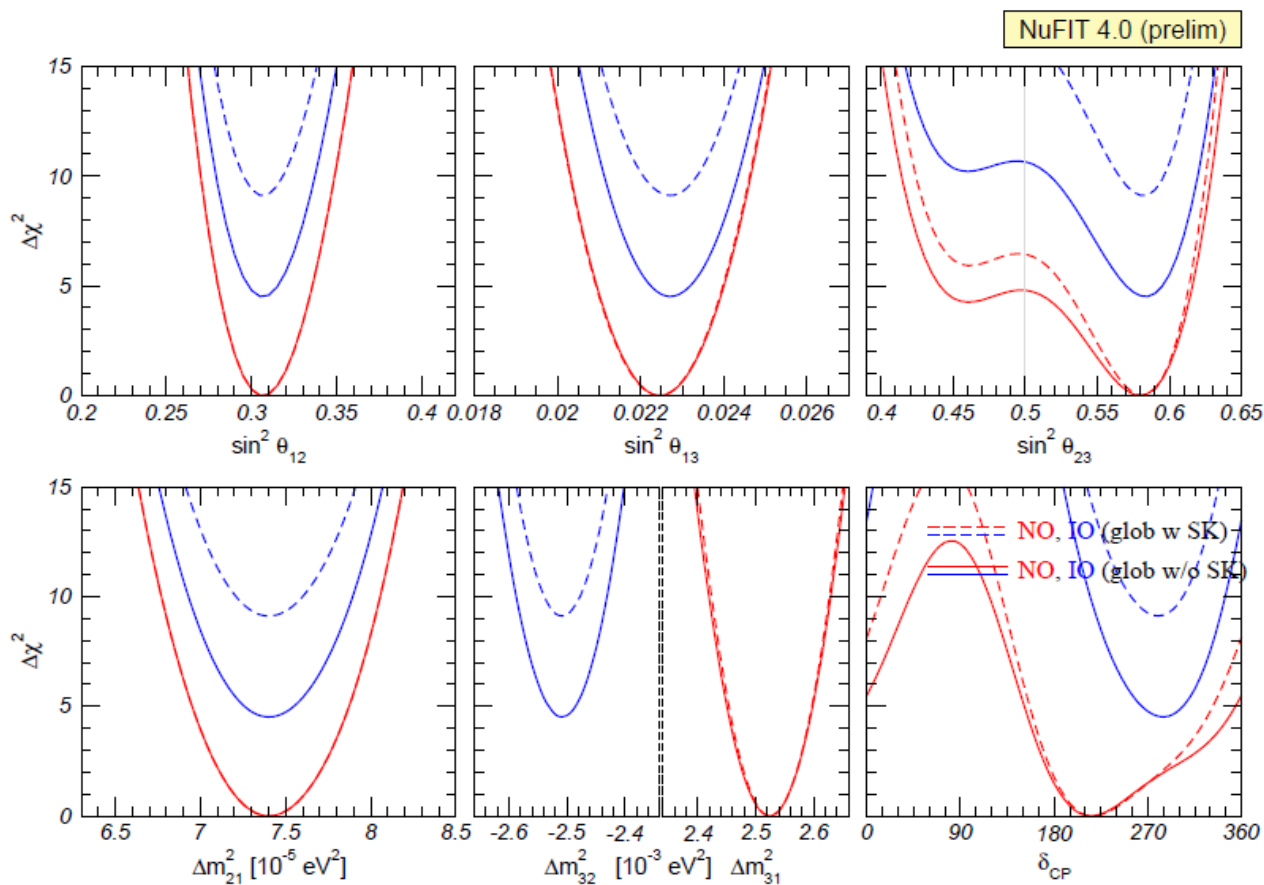
and there is still an important program (K0pi0, B-factories, Neutron EDM, BTeV, LHCb..) to go on for 10 years...i.e. a total of ~50 yrs.

and we have not discovered leptonic CP yet!

3 ν Flavour Parameters: Status OCT 2018

Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Hernandez-Cabezudo, Maltoni, Schwetz, MCG-G PRELIMINARY



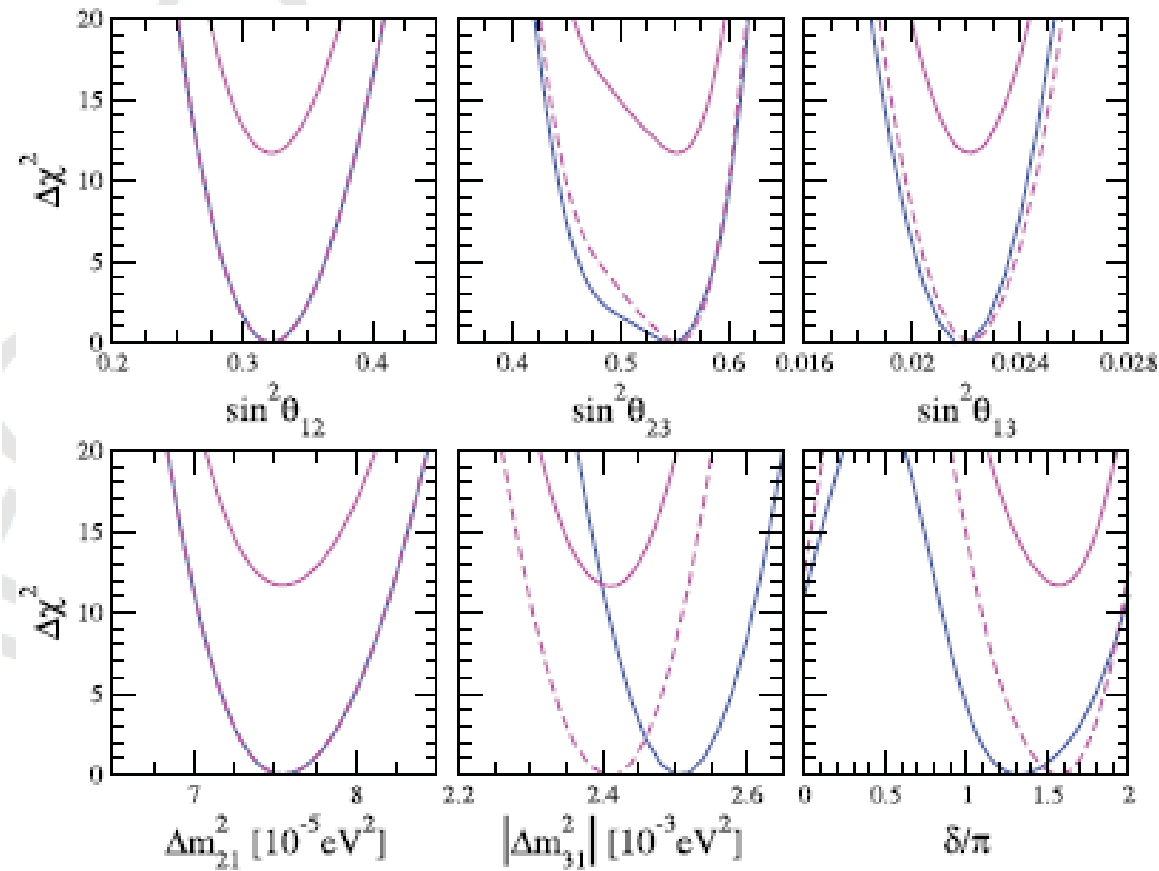
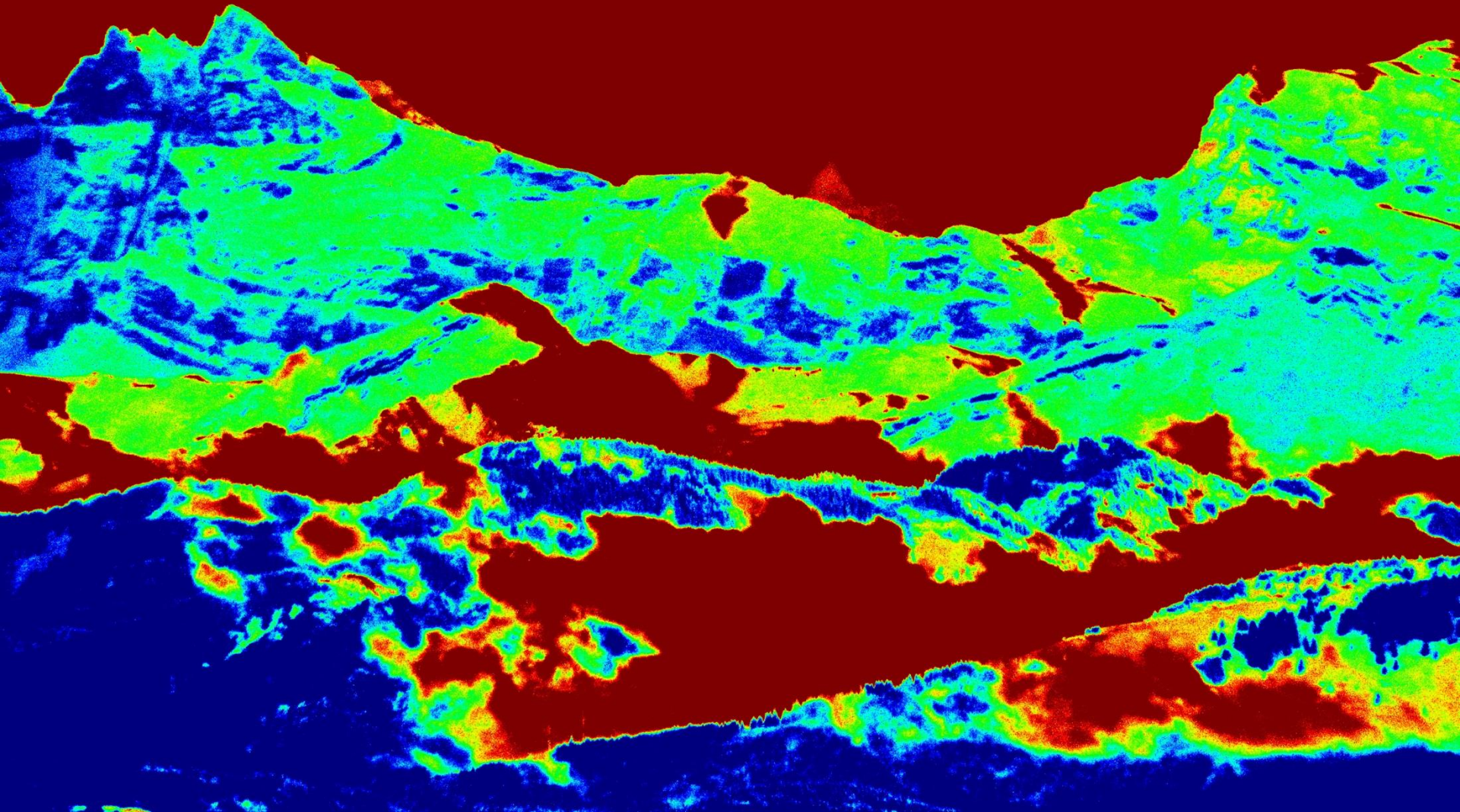


Fig. 7. Summary of neutrino oscillation parameters, 2018. Blue lines correspond to NO and magenta lines to IO. The $\Delta\chi^2$ -profiles for inverted ordering are plotted with respect to the minimum for this neutrino mass ordering (dashed) as well as with respect to the global minimum (solid lines).

Inverted hierarchy is excluded at 3.4σ and $\delta=0$ at $\sim 3\sigma$, $\delta=\pi$ at $\sim 2\sigma$
arXiv:1708.01186v2



The Search for the Right-Handed Neutrinos

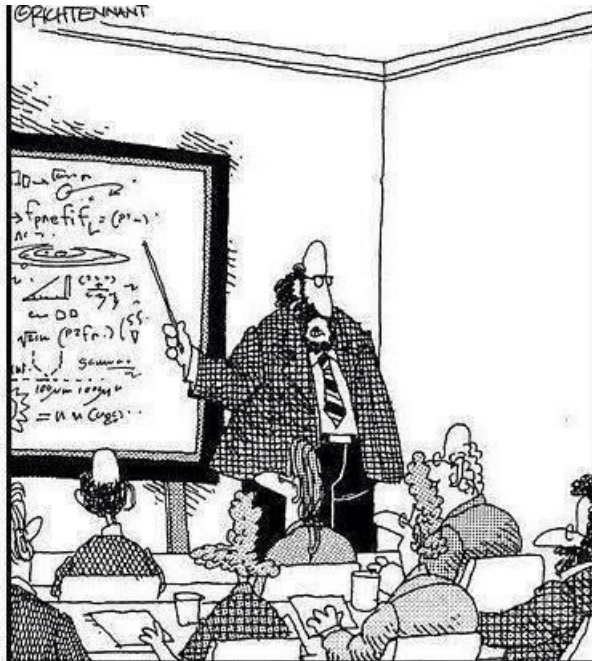


Electroweak eigenstates

$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L$	$\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L$	$\begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$	$(e)_R \ (\mu)_R \ (\tau)_R$	Q= -1
			$(\nu_e)_R \ (\nu_\mu)_R \ (\nu_\tau)_R$	Q= 0

I = 1/2

I = 0



"Along with 'Antimatter,' and 'Dark Matter,' we've recently discovered the existence of 'Doesn't Matter,' which appears to have no effect on the universe whatsoever."

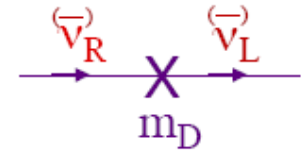
Right handed neutrinos
are singlets
no weak interaction
no EM interaction
no strong interaction

can't produce them
can't detect them
-- so why bother? --

Also called 'sterile'

Adding masses to the Standard model neutrino 'simply' by adding a Dirac mass term (Yukawa coupling)

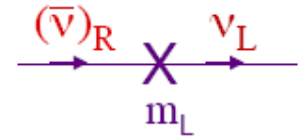
$$m_D \bar{\nu}_L \nu_R \quad m_D \bar{\nu}_L \nu_R$$



implies adding a right-handed neutrino (new particle)

No SM symmetry prevents adding then a term like

$$m_M \bar{\nu}_R^c \nu_R$$



and this simply means that a neutrino turns into a antineutrino

It is perfectly conceivable ('natural'?) that both terms are present.

Dirac mass term + Majorana mass term → 'see-saw'

B. Kayser, the physics of massive neutrinos (1989)

Mass eigenstates

See-saw type I :

$$\mathcal{L} = \frac{1}{2} (\bar{\nu}_L, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$M_R \neq 0$$

$$m_D \neq 0$$

Dirac + Majorana

mass terms

$$\tan 2\theta = \frac{2m_D}{M_R - 0}$$

$$\ll 1$$

$$m_\nu = \frac{1}{2} \left[(0 + M_R) - \sqrt{(0 - M_R)^2 + 4m_D^2} \right]$$

$$\simeq -m_D^2/M_R$$

$$M = \frac{1}{2} \left[(0 + M_R) + \sqrt{(0 - M_R)^2 + 4m_D^2} \right]$$

$$\simeq M_R$$

general formula

if $m_D \ll M_R$

$$M_R = 0$$

$$m_D \neq 0$$

Dirac only, (like e- vs e+):

\uparrow m	ν_L	ν_R	$\bar{\nu}_L$	$\bar{\nu}_R$
$I_{\text{weak}} =$	1/2	0	1/2	0

4 states of equal masses

Some have $I=1/2$ (active)

Some have $I=0$ (sterile)

$$M_R \neq 0$$

$$m_D = 0$$

Majorana only

\uparrow m	ν_L	$\bar{\nu}_R$
$I_{\text{weak}} =$	1/2	1/2

2 states of equal masses

All have $I=1/2$ (active)

$$M_R > m_D \neq 0$$

see-saw

Dirac + Majorana

\uparrow m	ν	N	$\bar{\nu}$	\bar{N}
$I_{\text{weak}} =$	1/2	0	1/2	0

dominantly:

4 states, 2 mass levels

m_1 have $\sim I=1/2$ (\sim active)

m_2 have $\sim I=0$ (\sim sterile)

Manifestations of right handed neutrinos

one family see-saw

:

$$\theta \approx (m_D/M)$$

$$m_\nu \approx \frac{m_D^2}{M}$$

$$m_N \approx M$$

$$\nu = \nu_L \cos\theta - N^c_R \sin\theta$$

$$N = N_R \cos\theta + \nu_L^c \sin\theta$$

what is produced in W, Z decays is:

$$\nu_L = \nu \cos\theta + N \sin\theta$$

ν = light mass eigenstate
 N = heavy mass eigenstate
 $\neq \nu_L$, active neutrino
 which couples to weak interaction
 and $\neq N_R$, which does'nt.

-- mixing with active neutrinos leads to various observable consequences
 -- if very light (eV), possible effect on neutrino oscillations (see talks later today)

-- if in keV region (dark matter), monochromatic photons from galaxies with $E = m_N/2$

-- possibly measurable effects at High Energy

If N is heavy it will decay in the detector (not invisible)

→ PMNS matrix unitarity violation and deficit in Z «invisible»

width

→ Higgs, Z, W visible exotic decays $H \rightarrow \nu_i \bar{N}_i$ and $Z \rightarrow \nu_i \bar{N}_i$, $W \rightarrow$

$l_i \bar{N}_i$

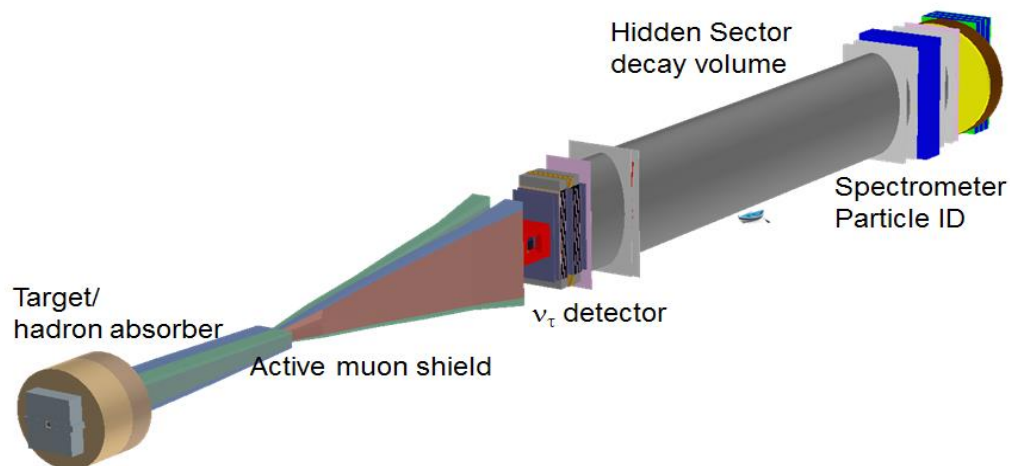
→ also in K, charm and b decays via $W^* \rightarrow l_i^\pm \bar{N}$, $N \rightarrow l_j^\pm$

with any of six sign and lepton flavour combination

Experiment	PS191	NuTeV	CHARM	SHiP
Proton energy (GeV)	19.2	800	400	400
Protons on target ($\cdot 10^{19}$)	0.86	0.25	0.24	20
Decay volume (m^3)	360	1100	315	1780
Decay volume pressure (bar)	1 (He)	1 (He)	1 (air)	10^{-6} (air)
Distance to target (m)	128	1400	480	80-90
Off beam axis (mrad)	40	0	10	0

Next generation heavy neutrino search experiment SHIP

- focuses on neutrinos from charm to cover 0.5 – 2 GeV region
 - uses beam dump to reduce background from neutrino interactions from pions and Kaons and bring the detector as close as possible to source.
 - increase of beam intensity and decay volume
- status: proposal, physics report and technical report exist. R&D phase approved at CERN

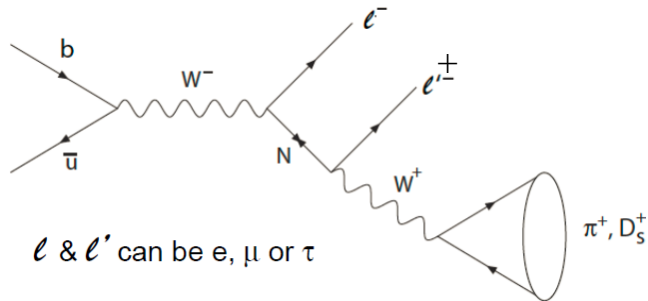


[arXiv:1504.04855](https://arxiv.org/abs/1504.04855)

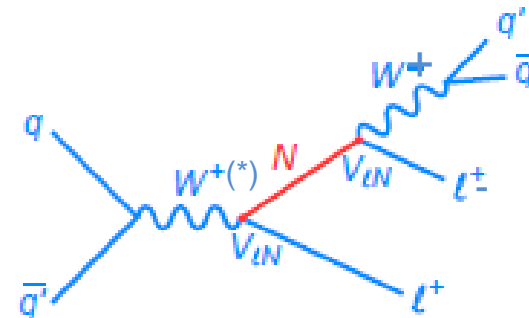
[arXiv:1504.04956](https://arxiv.org/abs/1504.04956)

Search for heavy right-handed neutrinos in collider experiments.

B factories

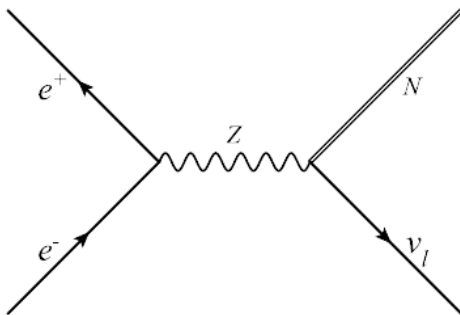


Hadron colliders

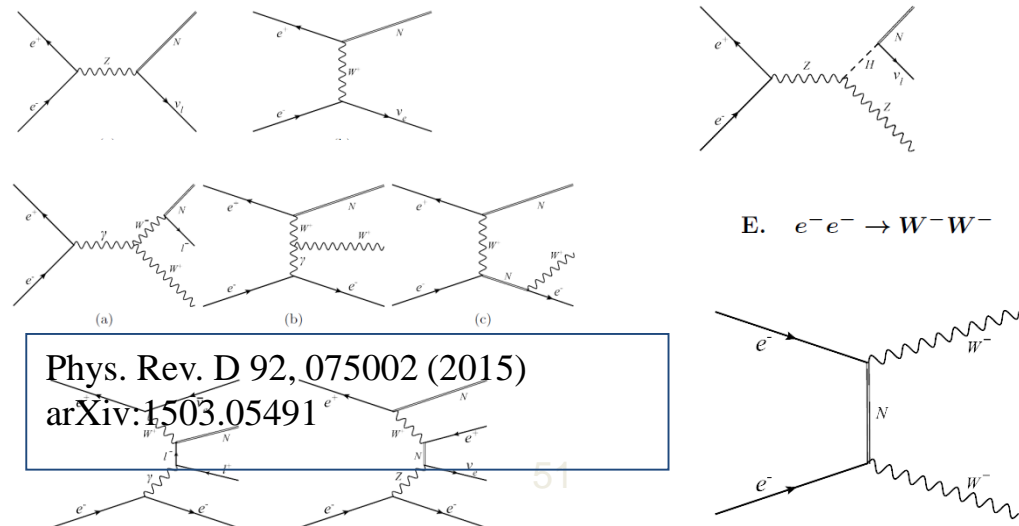


Z factory (FCC-ee, Tera-Z)

arXiv:1411.5230

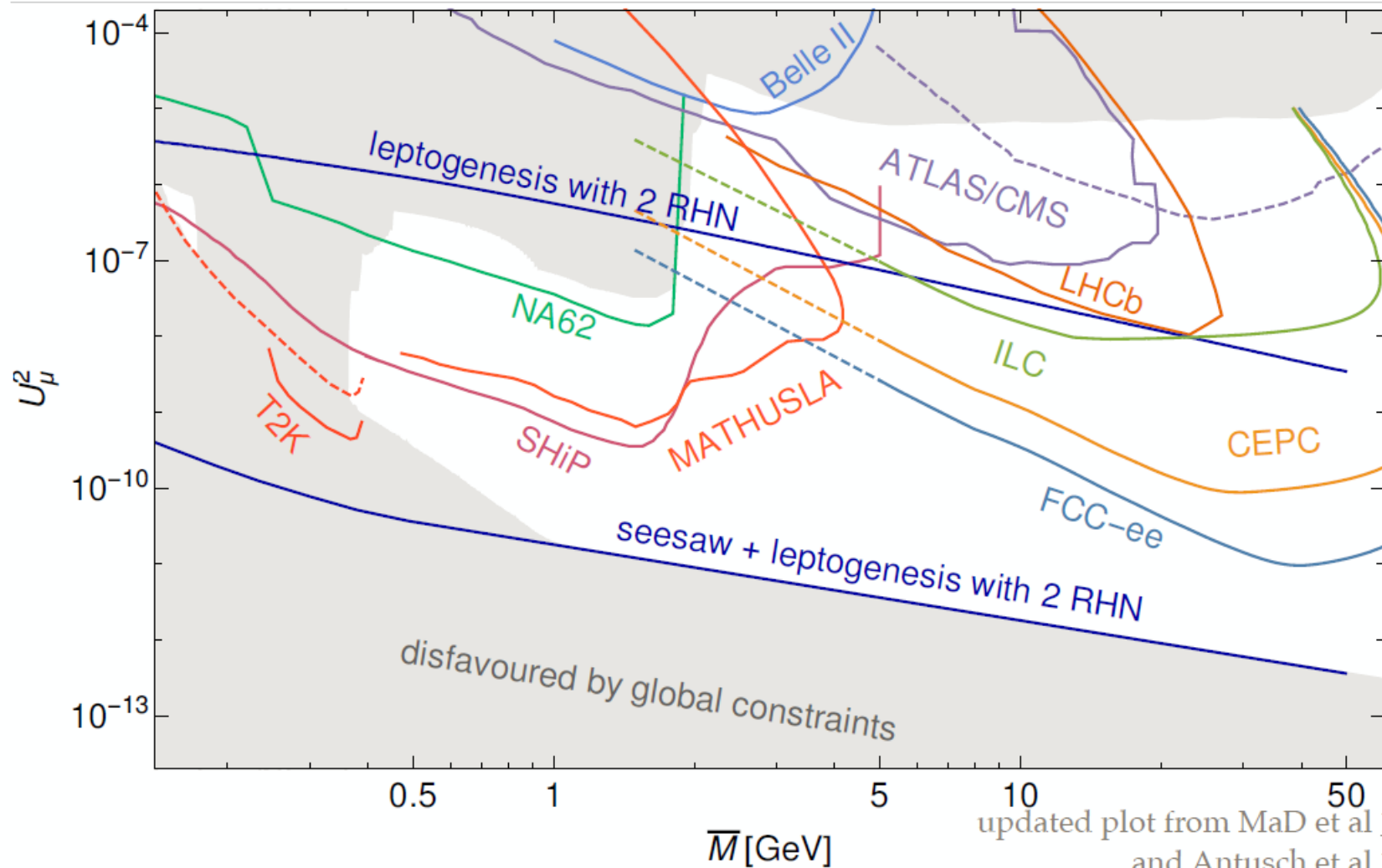


HE Lepton Collider (LEP2, CEPC, CLIC, FCC-ee, ILC)



Alain Blondel

Constraints and Future Searches



updated plot from MaD et al [1609.09069](#)
and Antusch et al [1710.03744](#)
cf. also Cai et al [1711.02180](#)

Future Circular Collider Study - SCOPE

CDR and cost review for the next ESU (2018)

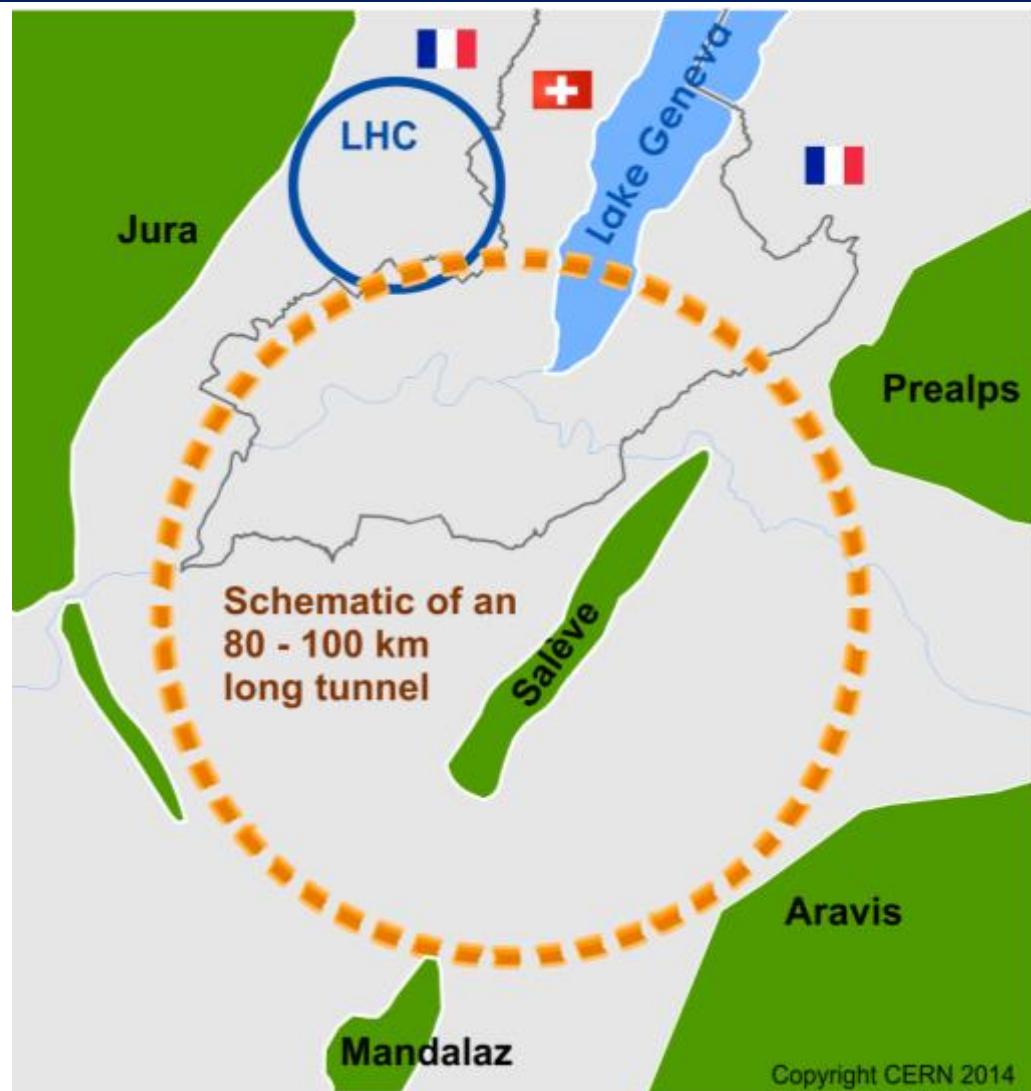
Forming an international collaboration to study:

- *pp*-collider (*FCC-hh*)

$\sim 16 \text{ T} \Rightarrow 100 \text{ TeV } pp \text{ in } 100 \text{ km}$

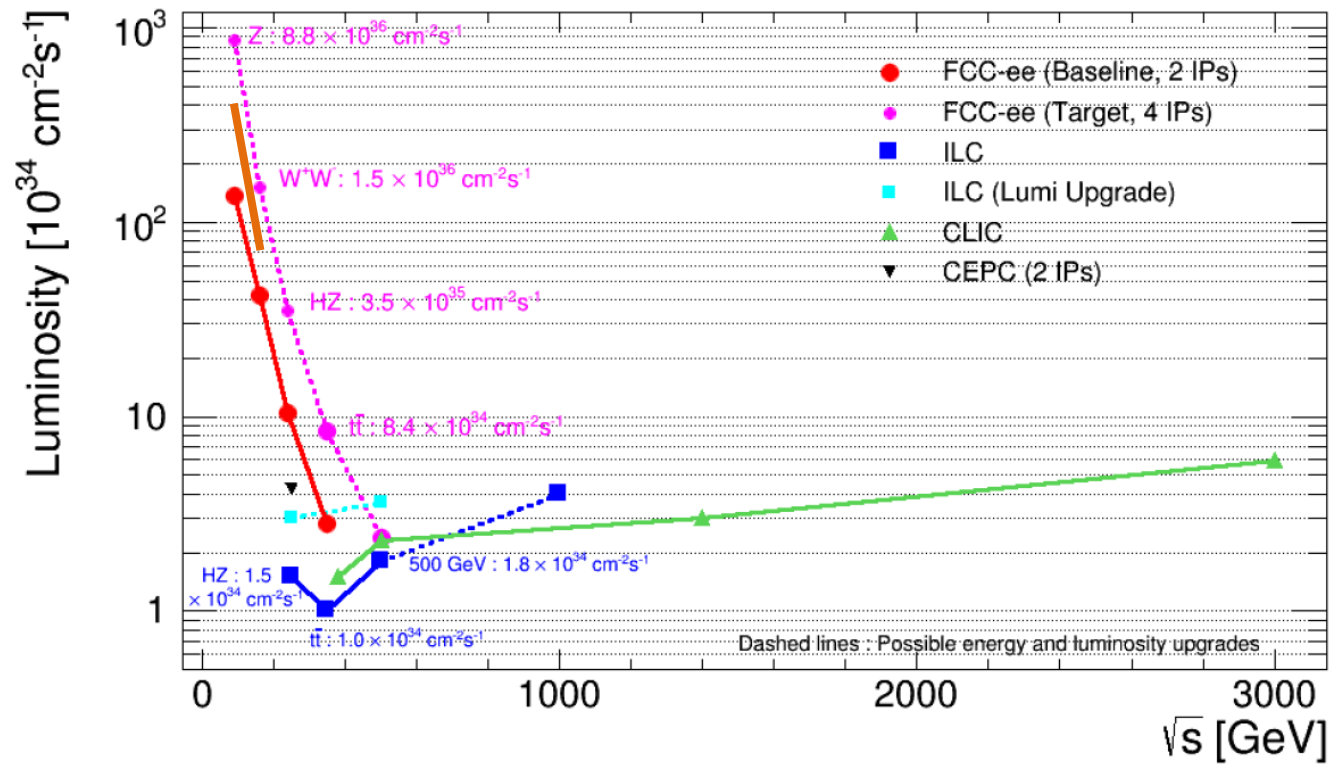
→ ultimate goal defining infrastructure requirements

- e^+e^- collider (*FCC-ee*)
as potential first step
ECM=90-400 GeV
- p - e (*FCC-he*) option
- 80-100 km infrastructure in Geneva area



FCC-ee highest possible luminosity from Z to tt by exploiting b-factory technologies:

- separate e- and e+ storage rings
- very strong focussing: $\beta^*y = 1 - 2$ mm (target, baseline -- work in progress!)
- top-up injection
- crab-waist crossing

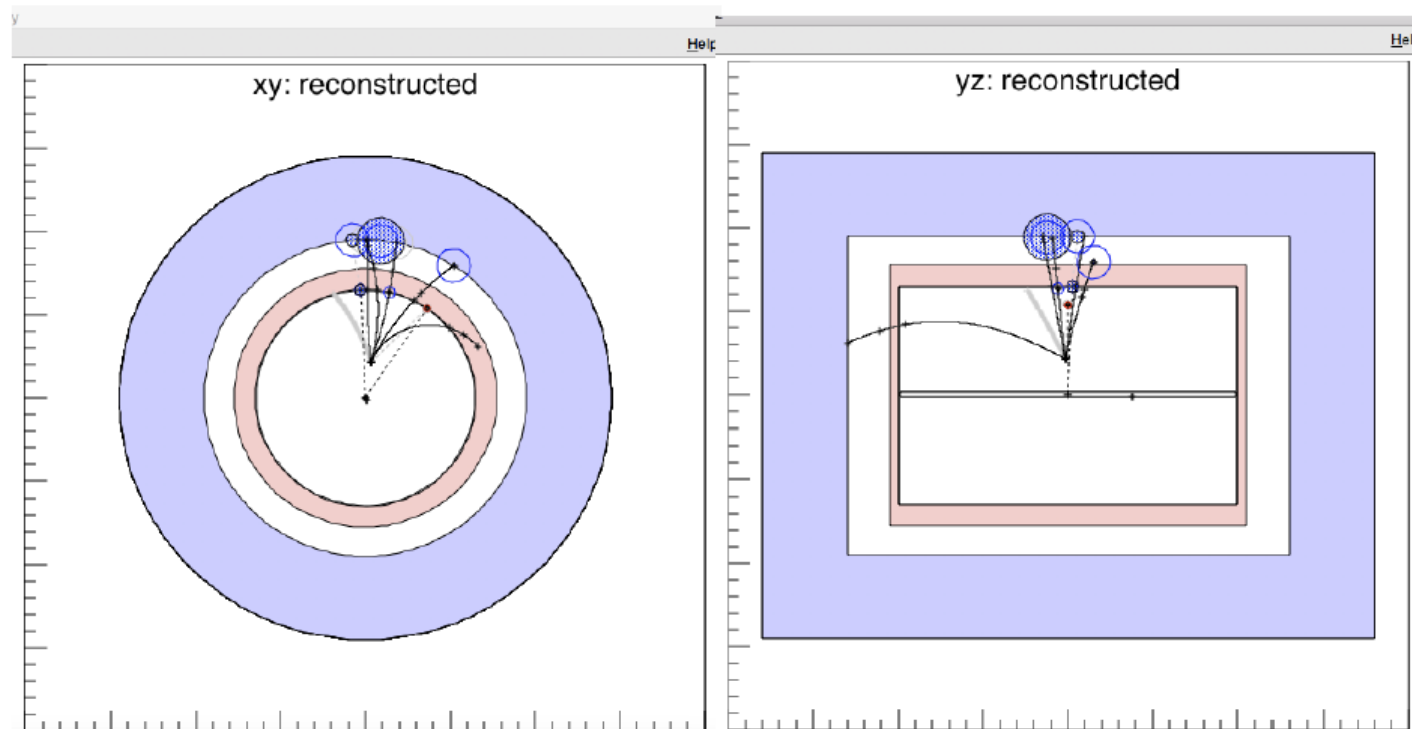


Event statistics :

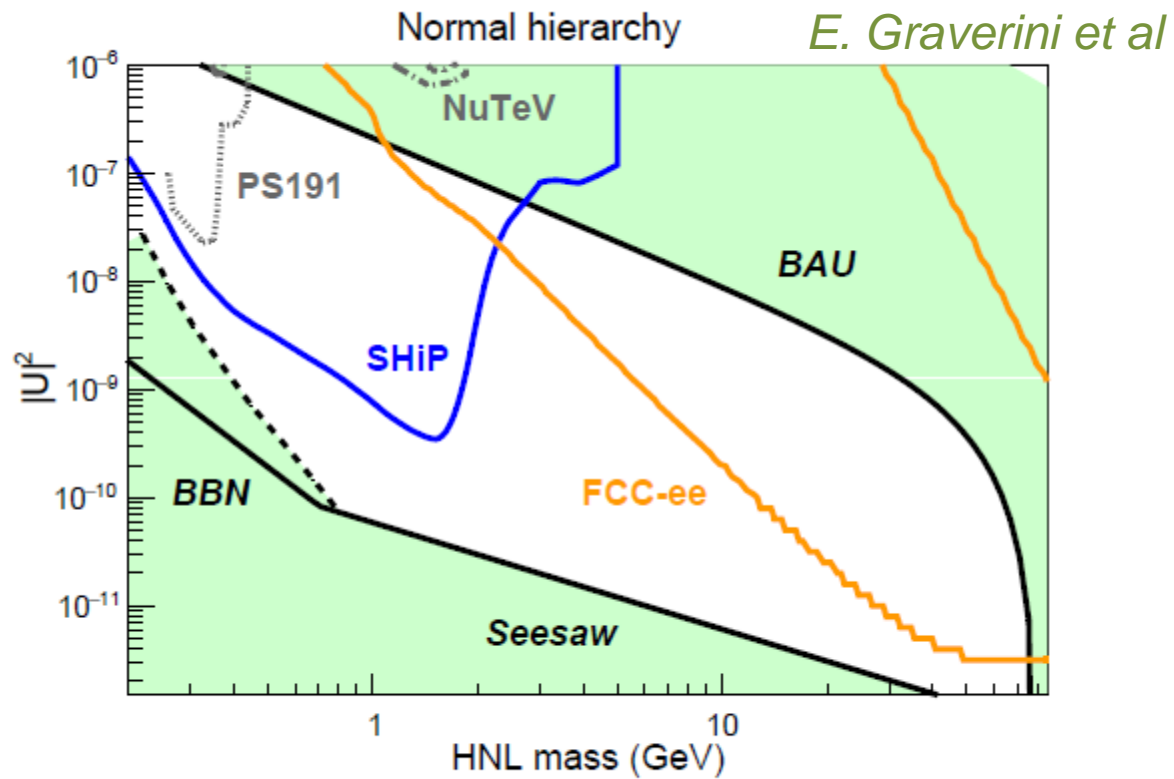
Z peak	$E_{\text{cm}} : 91 \text{ GeV}$	$5 \cdot 10^{12}$	$e^+e^- \rightarrow Z$	LEP x 10^5
WW threshold	$E_{\text{cm}} : 161 \text{ GeV}$	10^8	$e^+e^- \rightarrow WW$	LEP x $2 \cdot 10^3$
ZH threshold	$E_{\text{cm}} : 240 \text{ GeV}$	10^6	$e^+e^- \rightarrow ZH$	Never done
tt threshold	$E_{\text{cm}} : 350 \text{ GeV}$	10^6	$e^+e^- \rightarrow t\bar{t}$	Never done

Direct discovery of right-handed neutrinos

Simulation of heavy neutrino decay in a FCC-ee detector



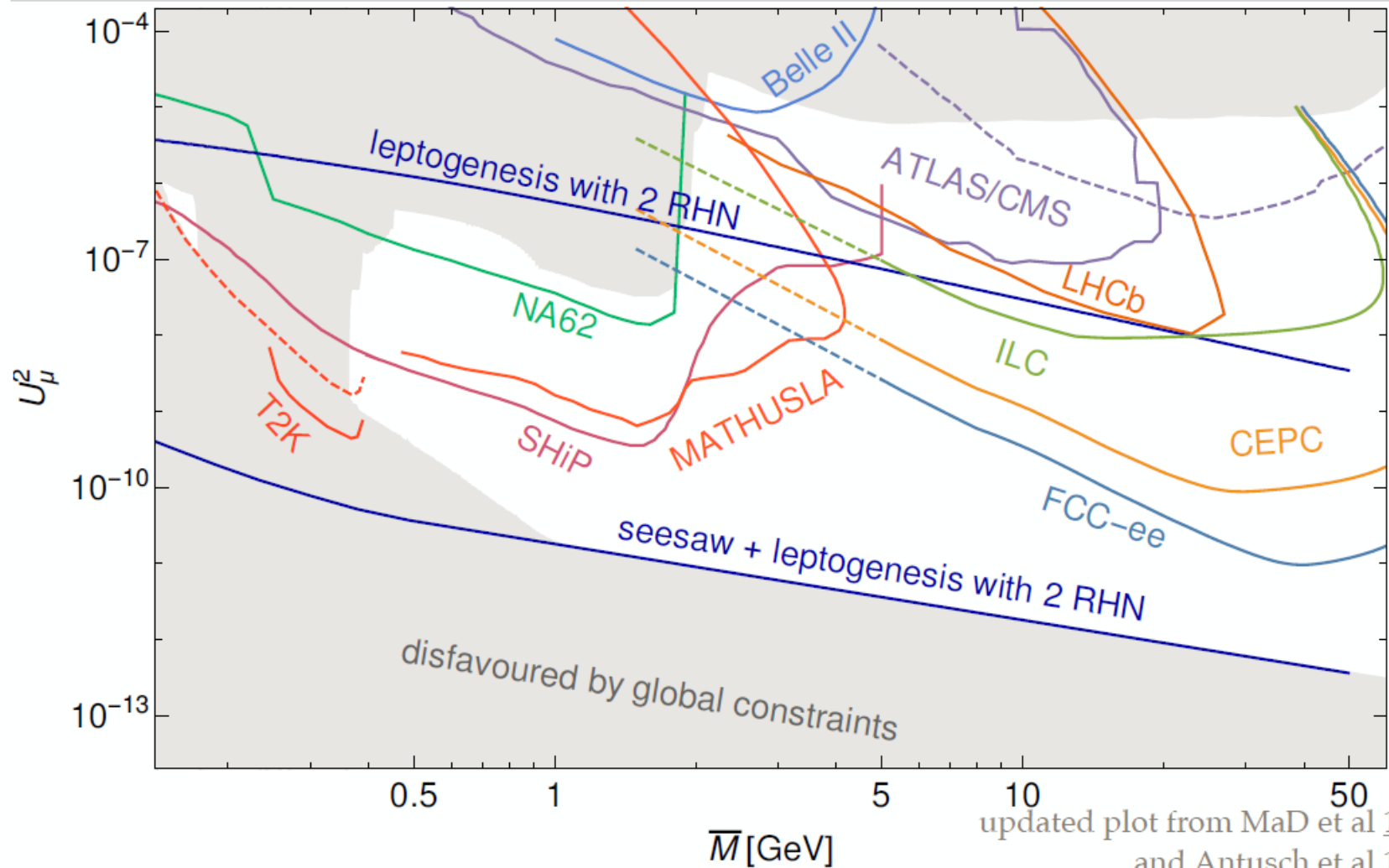
For the very small mixing typical of Type-I see-saw, the RH neutrino is very close to sterile ($|U|^2 = m_\nu/M \geq 10^{-12}$ for $M=50$ GeV)



(a) Decay length $500 \mu\text{m}$ to 2 m

with $5 \cdot 10^{12} Z$

Constraints and Future Searches



updated plot from MaD et al [1609.09069](#)
and Antusch et al [1710.03744](#)
cf. also Cai et al [1711.02180](#)

Marco Drewes, should be upgraded for full FCC-ee statistics

Final comments on statistics:

whether one uses frequentist or Bayesian approach, the results normally do not vary much and if they do one should probably use the worst.

HOWEVER: the main question is to make sure one does not misevaluate what one knows (information) or does not know

Two quotes to conclude:

- OK to use a parametric fit to a well known problem (ν oscillation, Z line shape etc.)
- It is however not recommended (i.e. should be forbidden really) to fit some data with a convenient but arbitrary or unsure or model-dependent function (i.e. fit looks good) and act as if the error matrix of the fit represents the uncertainty on the fit data. It does not, -- and this can go very wrong!

- in 1984 the UA1 collaboration was observing monojets.

Is supersymmetry found? G. Altarelli explained that this was probably the combination of $W \rightarrow \tau \nu_\tau$ events and $Zg \rightarrow \nu \nu + \text{jet}$

he also :

Called on the particle physics community (theorists and experimentalists) to stop wishful thinking about new physics and to start a serious, quantitative background evaluation

food for thought:

what result would one get if one measured the mass of a ν_e (in K-capture for instance)?

what result would one get if one measured the mass of a ν_μ (in pion decay)?

Is energy conserved when neutrinos oscillate?

Why do neutrinos oscillate and quarks do not?

food for thought: (simple)

what result would one get if one measured the mass of a ν_e (in K-capture for instance)?

what result would one get if one measured the mass of a ν_μ (in pion decay)?

Is energy conserved when neutrinos oscillate?

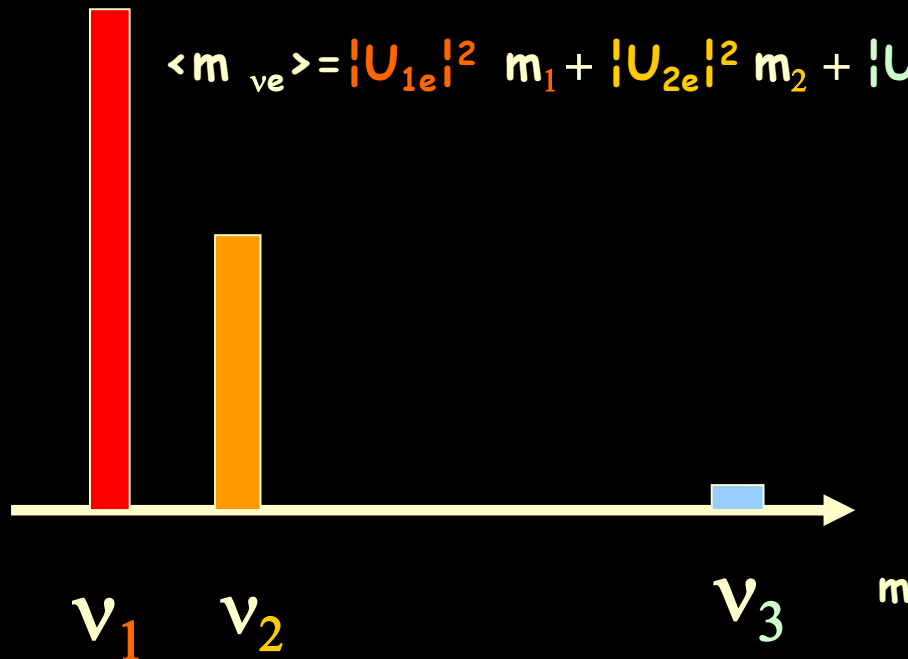
ν_e

would measure a distribution with three values of mass with the following probabilities

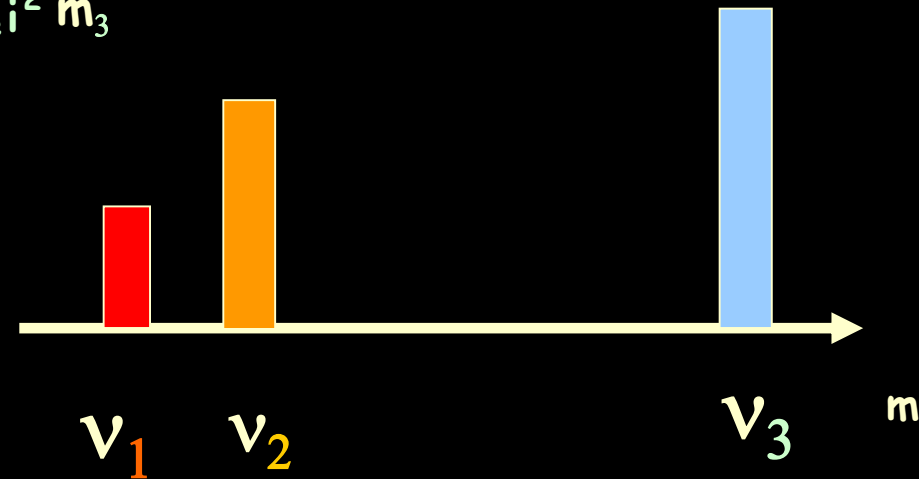
$|U_{1e}|^2$ $|U_{2e}|^2$

$|U_{3e}|^2$

$$\langle m_{\nu_e} \rangle = |U_{1e}|^2 m_1 + |U_{2e}|^2 m_2 + |U_{3e}|^2 m_3$$



ν_μ



Is energy conserved when neutrinos oscillate?

Energy (i.e. mass) eigenstates
propagate

$$\begin{aligned} |\nu(t)\rangle = & U_{1e} |\nu_1\rangle \exp(i E_1 t) \\ & + U_{2e} |\nu_2\rangle \exp(i E_2 t) \\ & + U_{3e} |\nu_3\rangle \exp(i E_3 t) \end{aligned}$$

$$P(\nu_1) = |U_{1e}|^2$$

$$P(\nu_2) = |U_{2e}|^2$$

$$P(\nu_3) = |U_{3e}|^2$$

are conserved during propagation



Why do neutrinos oscillate?

take $\pi \rightarrow \mu \nu$ decay $M = m_\pi$ $m_1 = m_\mu$ $m_2 = m_\nu$

muon momentum:

$$\frac{p}{c} = \frac{M^2 - m_1^2 - m_2^2}{2M}$$

variation of muon momentum upon neutrino mass and mass differences

$$\frac{\delta p_\mu}{c} = \left(\frac{p_\mu}{c} \right)_{m_\nu=0} - \left(\frac{p_\mu}{c} \right)_{m_\nu=m_0}, \quad \frac{\delta' p_\mu}{c} = \left(\frac{p_\mu}{c} \right)_{m_\nu=m_0} - \left(\frac{p_\mu}{c} \right)_{m_\nu=m'_0}$$

$$\frac{\delta p_\mu}{c} = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} - \frac{m_\pi^2 - m_\mu^2 - m_0^2}{2m_\pi} = \frac{m_0^2}{2m_\pi}$$

$$1.4 \times 10^{-14} \text{ MeV}/c$$

for $m_\nu = 2 \text{ eV}/c^2$

$$\frac{\delta' p_\mu}{c} = \frac{m_\pi^2 - m_\mu^2 - m_0^2}{2m_\pi} - \frac{m_\pi^2 - m_\mu^2 - m_0'^2}{2m_\pi} = -\frac{\Delta m^2}{2m_\pi}$$

$$8.9 \times 10^{-18} \text{ MeV}/c$$

for $\Delta m_\nu^2 = 2 \cdot 10^{-3} (\text{eV}/c^2)^2$



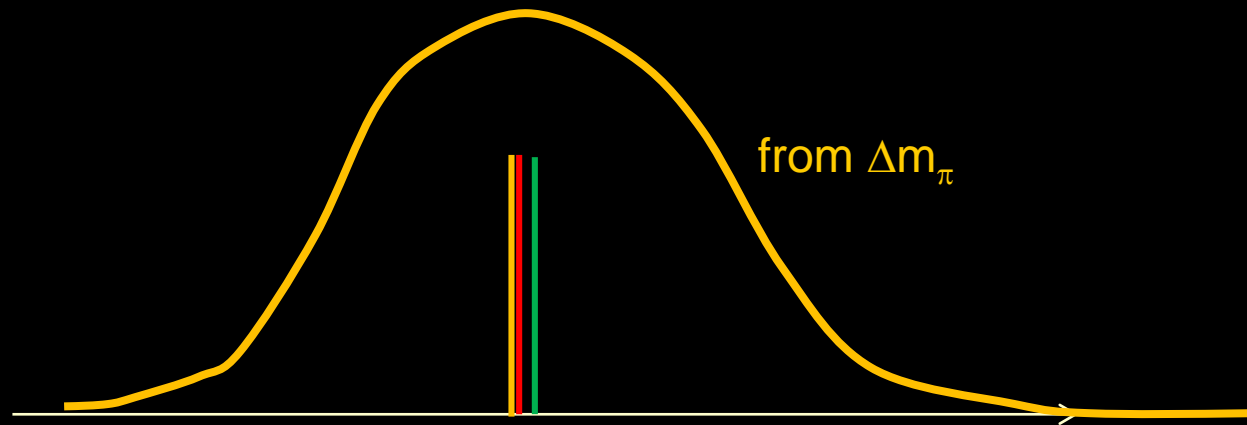
However we need to take into account the width of the pion since it decays with a life time of 26ns or $c\tau=7.8\text{m}$ ($\hbar c = 200 \text{ MeV}\cdot\text{fm}$)

$$\Delta m_\pi = \hbar/\tau \sim 4 \cdot 10^{-14} \text{ MeV}/c^2 \rightarrow \Delta p_\mu \sim 3 \cdot 10^{-14} \text{ MeV}/c$$

→ the uncertainty due to the pion decay width is much larger than the difference in momentum between the neutrino mass eigenstates.

This is the same relationship that ensures that interference happens between light coming from different holes. (can't tell which hole the light went through)

Neutrinos oscillate for the fundamental quantum reason that the width of the decaying parent makes it impossible to tell the neutrino species by measuring its mass from kinematics.



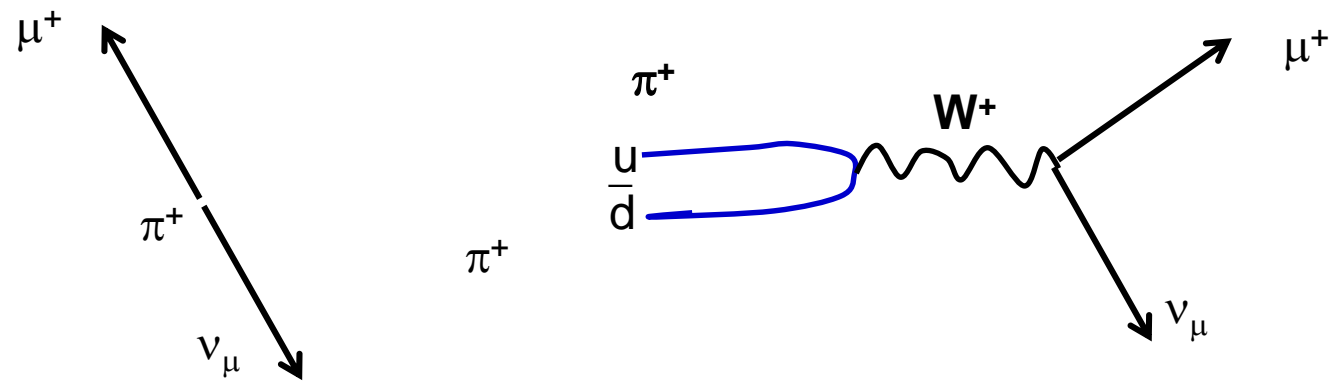
much amplified: the central value of $p_\mu(\nu 1)$, $p_\mu(\nu 2)$, $p_\mu(\nu 3)$ distribution



Unrelated Preamble

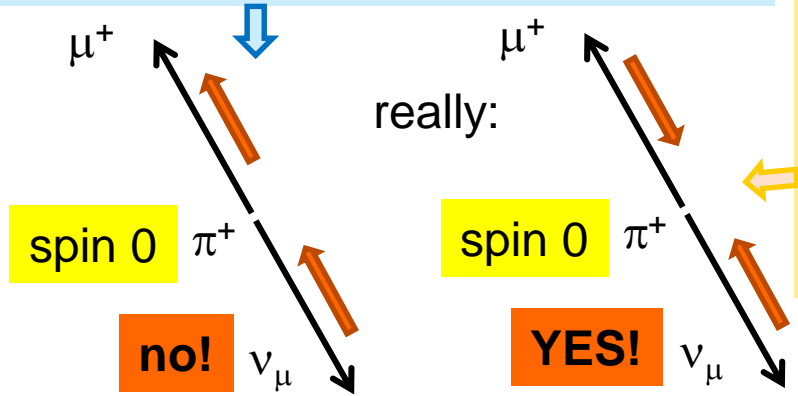
Why do pions decay into $\pi^+ \rightarrow \mu^+ \nu_\mu$ much much more than into $\pi^+ \rightarrow e^+ \nu_e$?

Imagine the π decay at rest. (obviously the decay fraction is Lorentz invariant)



momenta are equal and opposite: $(P_{\mu,\nu})^2 = (m_\pi^2 - m_\mu^2 - m_\nu^2)/2 m_\pi$
How are the spins? The μ^+ and ν_μ originate from weak interaction
→ μ^+ is right-handed and ν_μ is left-handed ... however the pion has spin 0

If helicity and chirality were identical
we would have violation of angular momentum conservation!



However they are not.
 $|R\rangle, |L\rangle$ chirality states; $|+\rangle, |-\rangle$ helicity states
 $|L\rangle = |-\rangle + m/E |+\rangle$
 $|R\rangle = |+\rangle + m/E |-\rangle$
thus the decay rate is proportional to $||\langle R|-\rangle||^2 = (m_\mu/E_\mu)^2$
Also multiply by the phase space factor
proportional to $(P_\mu)^2 = (m_\pi^2 - m_\mu^2 - m_\nu^2)/2 m_\pi$



However they are not.

$|R\rangle, |L\rangle$ chirality states; $|+\rangle, |-\rangle$ helicity states

$$|L\rangle = |-\rangle + \frac{m}{E} |+\rangle$$

$$|R\rangle = |+\rangle + \frac{m}{E} |-\rangle$$

thus the decay rate is proportional to

$$||\langle R|-\rangle||^2 = (m_\mu/E_\mu)^2$$

Also multiply by the phase space factor

$$\text{proportional to } (P_\mu)^2 = (m_\pi^2 - m_\mu^2 - m_\nu^2)/2 m_\pi$$

So we can derive the ratio $R_\pi = \frac{\pi \rightarrow e \nu}{\pi \rightarrow \mu \nu}$

$$R_\pi = (m_e/m_\mu)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2 = \begin{array}{l} 1.2351(2) \cdot 10^{-4} \text{ (theory)} \\ 1.230(4) \cdot 10^{-4} \text{ (exp)} \end{array}$$



LA MECANIQUE QUANTIQUE DES OSCILLATIONS DE NEUTRINOS

On traitera d'abord un système à deux neutrinos pour simplifier

Propagation dans le vide: on écrit le Hamiltonien pour une particule relativiste
(NB il y a là une certaine incohérence car la mécanique quantique relativiste utilise des méthodes différentes.
Dans ce cas particulièrement simple les résultats sont les mêmes.)

On se rappellera du 4-vecteur relativiste Energie Impulsion

$$\begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Dont la norme est par définition la masse (invariant relativiste)
et s'écrit

$$(mc^2)^2 = E^2 - (pc)^2$$

D'où l'énergie:

$$E = \sqrt{(pc)^2 + (mc^2)^2} \approx pc \left(1 + \frac{(mc^2)^2}{2(pc)^2}\right) = pc + \frac{m^2 c^4}{2pc}$$

On considère pour simplifier encore le cas de neutrinos dont la quantité de mouvement est connue ce qui fait que le Hamiltonien va s'écrire ainsi dans la base des états de masse bien définie:

$$H = pc \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{c^4}{2pc} \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$



LA MECANIQUE QUANTIQUE DES OSCILLATIONS DE NEUTRINOS

Pour le cas de deux neutrinos, dans la base des états de masse bien définie:

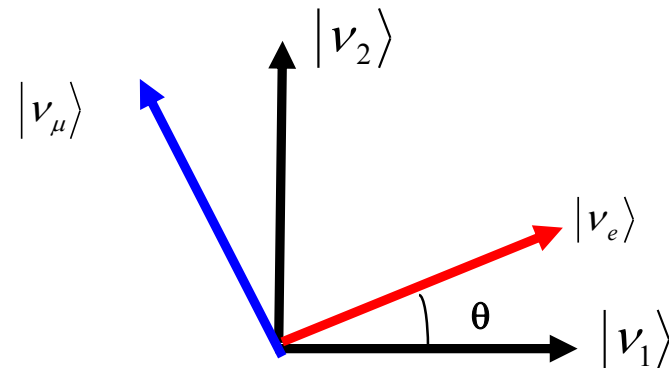
$$H = pc \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{c^4}{2pc} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

L'évolution dans le temps des états propres $|\nu_1\rangle$ et $|\nu_2\rangle$ s'écrit:

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{iE_1 t/\hbar} \quad |\nu_2(t)\rangle = |\nu_2\rangle e^{iE_2 t/\hbar}$$

Cependant les neutrinos de **saveur bien définie** sont des vecteurs orthogonaux de ce sous espace de Hilbert à deux dimensions, mais différents des neutrinos de masse bien définie: $|\nu_e\rangle$ $|\nu_\mu\rangle$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



L'évolution dans le temps s'écrit maintenant

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 e^{iE_1 t/\hbar} \\ \nu_2 e^{iE_2 t/\hbar} \end{pmatrix} = e^{iE_1 t/\hbar} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 e^{i(E_2 - E_1)t/\hbar} \end{pmatrix}$$



LA MECANIQUE QUANTIQUE DES OSCILLATIONS DE NEUTRINOS

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = e^{iE_1 t / \hbar} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 e^{i(E_2 - E_1)t / \hbar} \end{pmatrix}$$

Si nous partons maintenant au niveau de la source ($t=0$) avec un état $|\nu_e\rangle$
et que nous allons détecter des neutrinos à une distance L (soit à un temps L/c plus tard) la probabilité
Quand on observe une interaction de neutrino d'observer une interaction produisant un **electron** ou un **muon**
seront donnés par le calcul de

$$P_e(|\nu_e(t)\rangle) = \left\| \langle \nu_e | \nu_e(t) \rangle \right\|^2$$

$$P_\mu(|\nu_e(t)\rangle) = \left\| \langle \nu_\mu | \nu_e(t) \rangle \right\|^2$$

$$P_e(|\nu_e(t)\rangle) = \left\| \langle \nu_e | \nu_e(t) \rangle \right\|^2 = \left\| \cos \theta \langle \nu_e | \nu_1 \rangle + \sin \theta \langle \nu_e | \nu_2 \rangle e^{i(E_2 - E_1)t / \hbar} \right\|^2$$

$$P_e(|\nu_e(t)\rangle) = (\cos^2 \theta + \sin^2 \theta e^{-i(E_2 - E_1)t / \hbar})(\cos^2 \theta + \sin^2 \theta e^{+i(E_2 - E_1)t / \hbar})$$



$$\begin{aligned}
 P_e(|\nu_e(t)\rangle) &= \|\langle \nu_e | \nu_e(t) \rangle\|^2 = \|\cos\theta \langle \nu_e | \nu_1 \rangle + \sin\theta \langle \nu_e | \nu_2 \rangle e^{i(E_2-E_1)t/\hbar}\|^2 \\
 P_e(|\nu_e(t)\rangle) &= (\cos^2\theta + \sin^2\theta e^{-i(E_2-E_1)t/\hbar})(\cos^2\theta + \sin^2\theta e^{+i(E_2-E_1)t/\hbar}) \\
 P_e(|\nu_e(t)\rangle) &= \cos^4\theta + \sin^4\theta + \cos^2\theta \sin^2\theta (e^{+i(E_2-E_1)t/\hbar} + e^{-i(E_2-E_1)t/\hbar}) \\
 P_e(|\nu_e(t)\rangle) &= \cos^4\theta + \sin^4\theta + \cos^2\theta \sin^2\theta (2\cos((E_2-E_1)t/\hbar)) \\
 P_e(|\nu_e(t)\rangle) &= \cos^4\theta + \sin^4\theta + 2\cos^2\theta \sin^2\theta - 2\cos^2\theta \sin^2\theta (1 - \cos((E_2-E_1)t/\hbar)) \\
 P_e(|\nu_e(t)\rangle) &= 1 - \sin^2 2\theta \sin^2(1/2(E_2-E_1)t/\hbar) \\
 P_e(|\nu_e(t)\rangle) &= 1 - \sin^2 2\theta \sin^2(1/2(E_2-E_1)t/\hbar) \\
 P_\mu(|\nu_e(t)\rangle) &= \sin^2 2\theta \sin^2(1/2(E_2-E_1)t/\hbar)
 \end{aligned}$$

En utilisant:

$$1 - \cos x = 2 \sin^2 x / 2,$$

$$2 \sin x \cos x = \sin 2x$$



On a donc trouvé:

$$P_e(|\nu_e(t)\rangle) = 1 - \sin^2 2\theta \sin^2(1/2(E_2 - E_1)t/\hbar)$$

$$P_\mu(|\nu_e(t)\rangle) = \sin^2 2\theta \sin^2(1/2(E_2 - E_1)t/\hbar)$$

mélange

oscillation

Le terme d'oscillation peut être reformulé:

$$E = pc + \frac{m^2 c^4}{2pc}$$

$$E_2 - E_1 = \frac{(m_2^2 - m_1^2)c^4}{2pc} = \frac{\Delta m_{12}^2 c^4}{2pc}$$

$$\frac{\Delta m^2 c^4}{4p\hbar c} t = \frac{\Delta m^2 c^4}{4pc\hbar c} ct = \frac{\Delta m^2 c^4}{4\hbar c} \frac{L}{E}$$



Les unités pratiques sont

Les énergies en GeV

Les masses mc^2 en eV

Les longueurs en km...

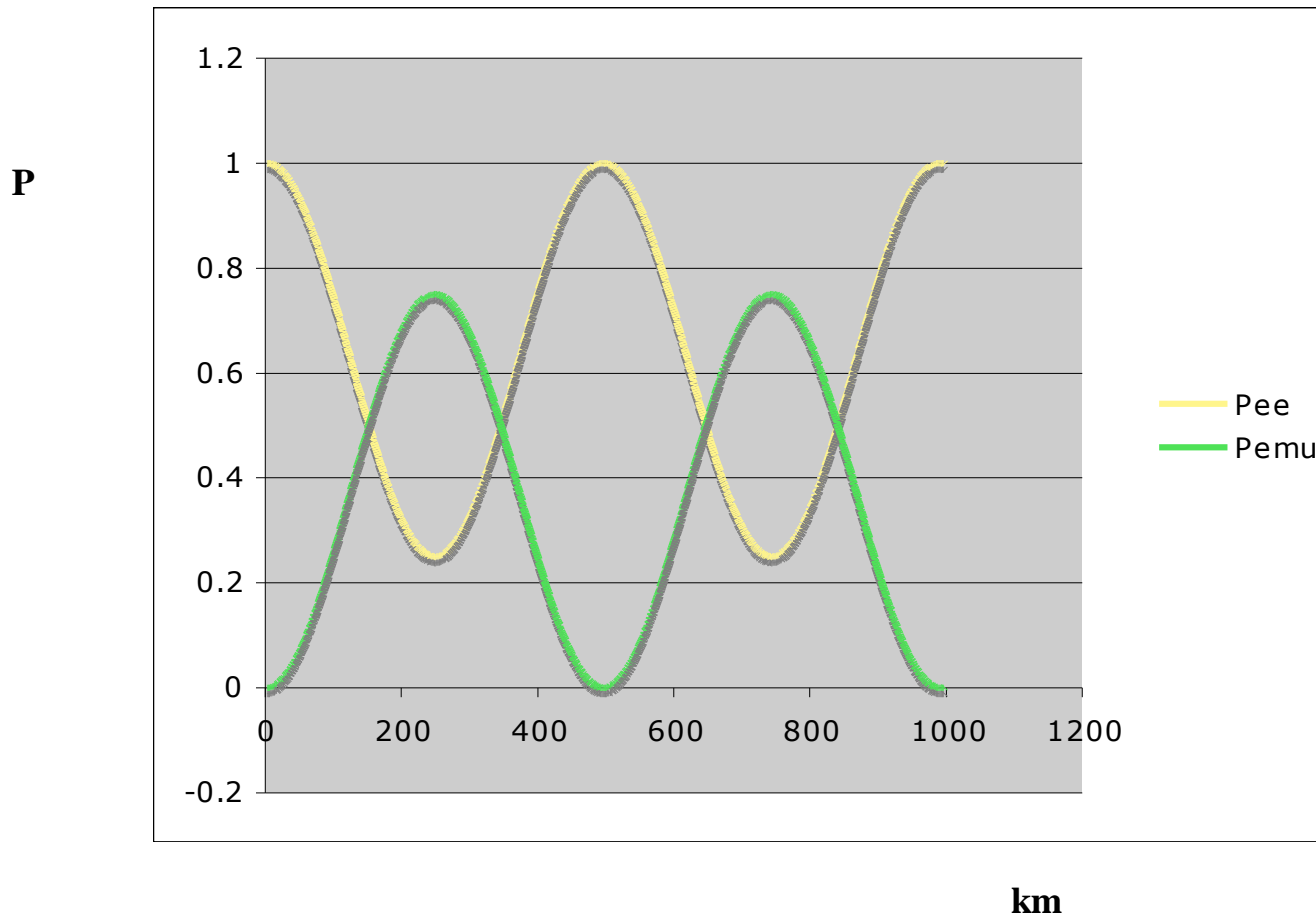
On trouve alors en se souvenant que

$$\hbar c = 197 \text{ MeV} \cdot \text{fm}$$

$$P_e(|\nu_e(t)\rangle) = 1 - \sin^2 2\theta \sin^2(1.27 \Delta m_{12}^2 L / E)$$

$$P_\mu(|\nu_e(t)\rangle) = \sin^2 2\theta \sin^2(1.27 \Delta m_{12}^2 L / E)$$





Exemple de probabilité en fonction de la distance à la source pour
 $E = 0.5 \text{ GeV}$,
 $\Delta m^2_{12} = 2.5 \cdot 10^{-3} (\text{eV}/c^2)^2$

