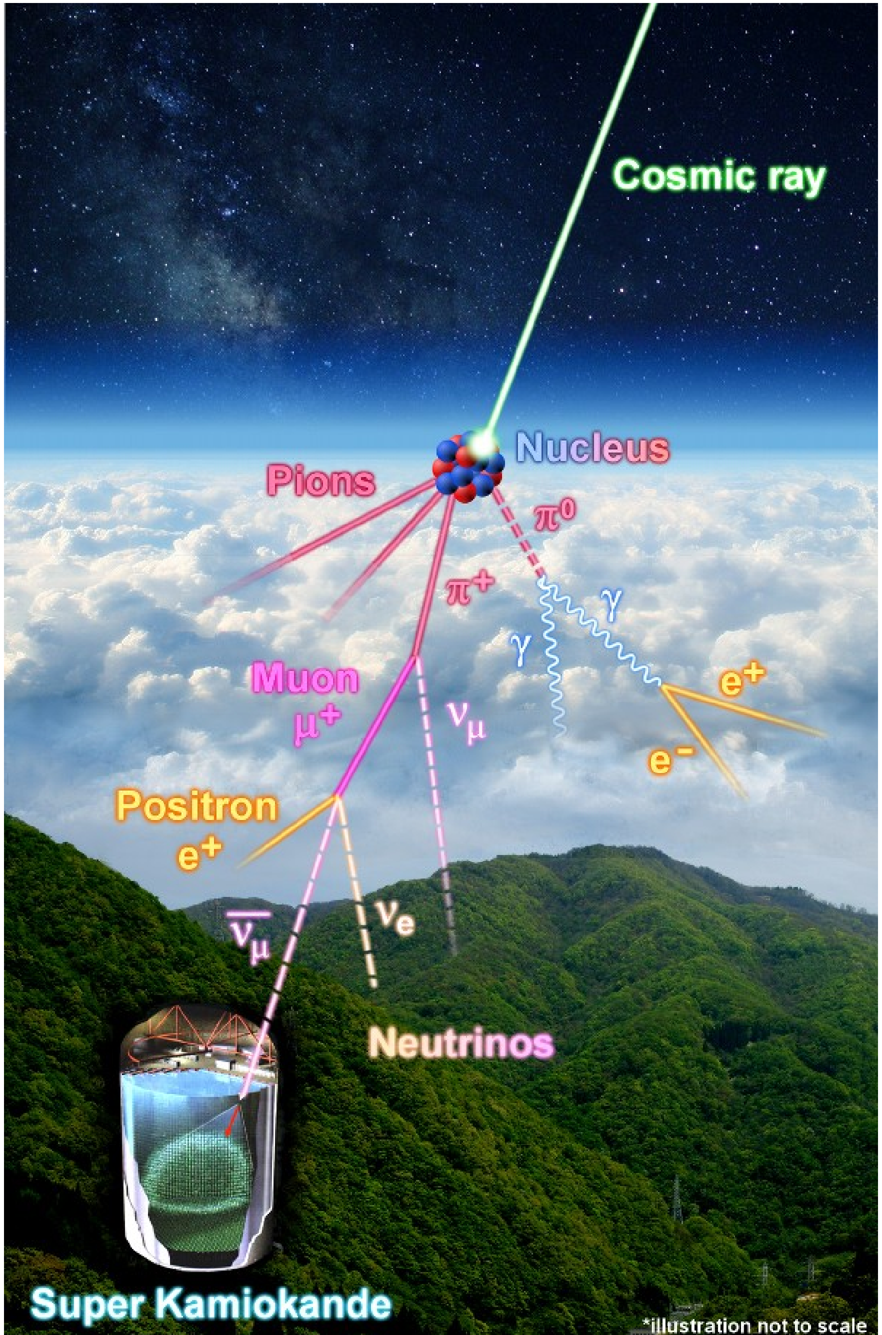
Statistical issues in atmospheric neutrino experiments

C. Bronner
January 23rd, 2019

KAMIOKA OBSERVATORY

Atmospheric neutrinos

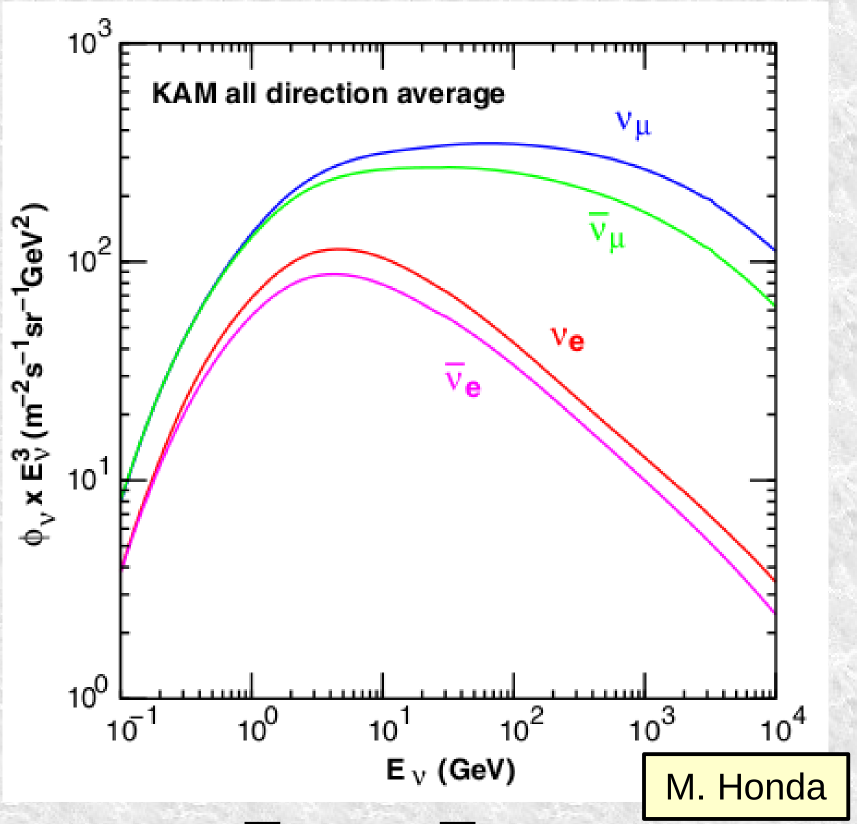


(illustration from F. Blaszczyk)

Atmospheric neutrinos are produced in the decays of secondary particles coming from interactions of cosmic rays in the atmosphere

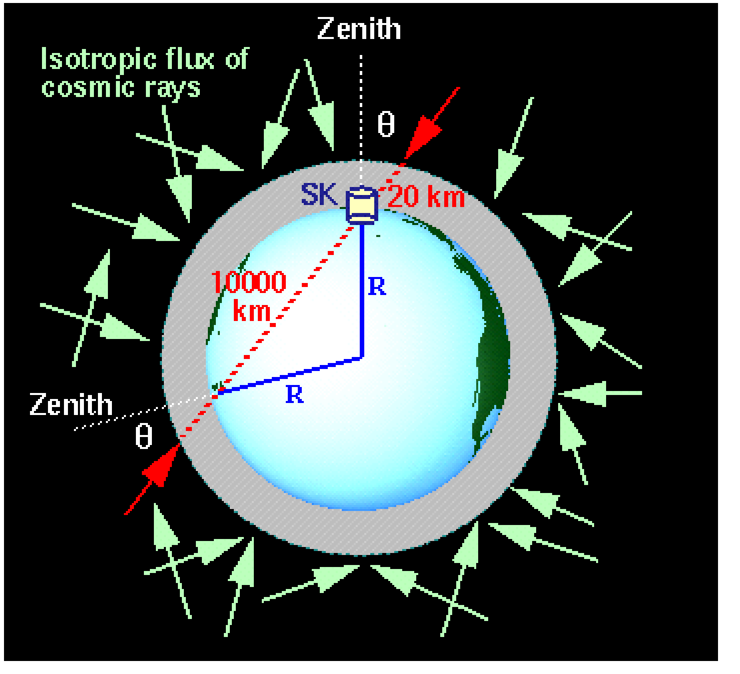
- Flux is not as well controlled as with beam neutrinos, due to uncertainties on:
  - primary cosmic ray flux and composition
  - hadronic interactions
  - atmosphere model, seasonal variations, geomagnetic effect, …
- But free neutrino source and always available

Atmospheric neutrinos
Interest for oscillation measurements



ν_μ, $\bar{\nu}_\mu$, ν_e, $\bar{\nu}_e$ over 5 decades in energy

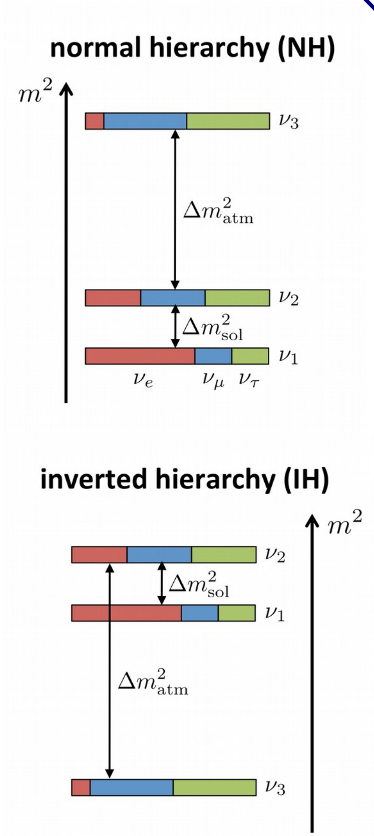
L from 10 to 13000km



- Large range of neutrino energies and propagation lengths
- Oscillations dominated by $\nu_\mu \rightarrow \nu_\tau$
- Large statistics allow to study sub-dominant effects

Neutrino oscillation
Open questions

Mass hierarchy:
$m_3 > m_2, m_1$?



PDG 2017 summary table

Parameter	best-fit	3σ
Δm^2_{21} $[10^{-5}$ eV $^2]$	7.37	6.93 – 7.96
$\Delta m^2_{31(23)}$ $[10^{-3}$ eV $^2]$	2.56 (2.54)	2.45 – 2.69 (2.42 – 2.66)
$\sin^2\theta_{12}$	0.297	0.250 – 0.354
$\sin^2\theta_{23}$, $\Delta m^2_{31(32)} > 0$	0.425	0.381 – 0.615
$\sin^2\theta_{23}$, $\Delta m^2_{32(31)} < 0$	0.589	0.384 – 0.636
$\sin^2\theta_{13}$, $\Delta m^2_{31(32)} > 0$	0.0215	0.0190 – 0.0240
$\sin^2\theta_{13}$, $\Delta m^2_{32(31)} < 0$	0.0216	0.0190 – 0.0242
δ/π	1.38 (1.31)	2σ: (1.0 - 1.9) (2σ: (0.92-1.88))

Octant of θ_{23}:
$\theta_{23} > \pi/4$?
$\theta_{23} < \pi/4$?

Violation of CP symmetry in neutrino oscillations?

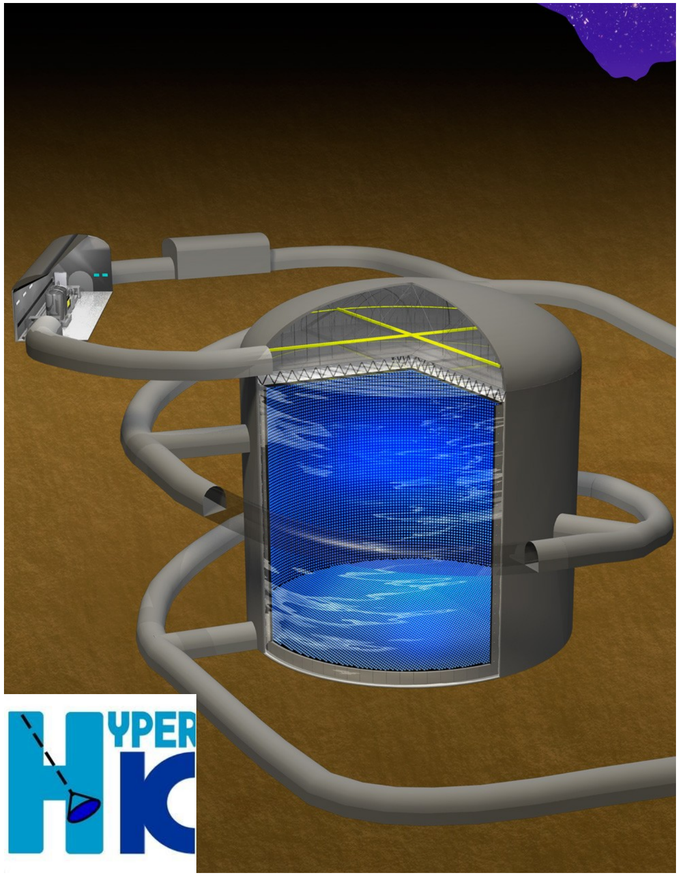
Next generation atmospheric neutrino experiments

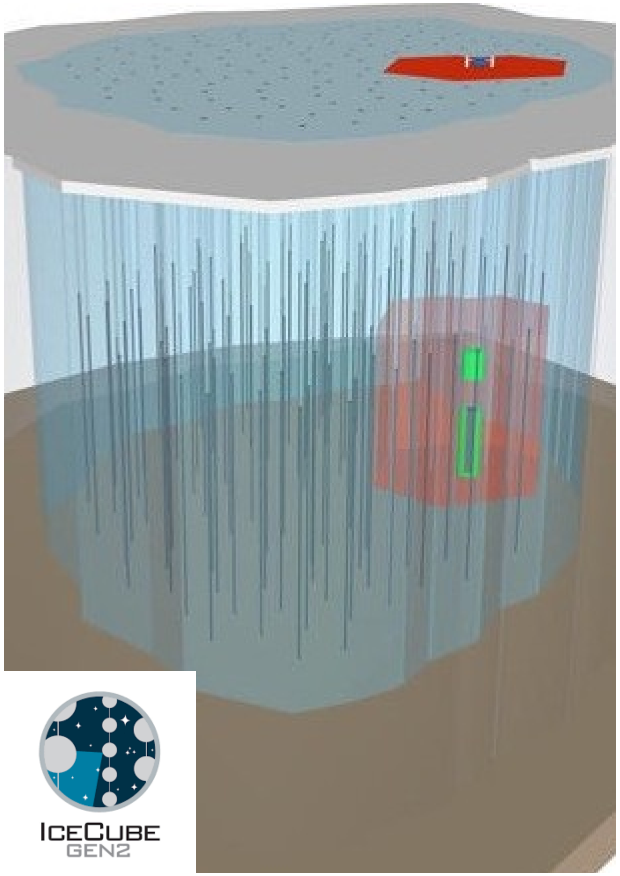
Determination of the mass hierarchy will be one of the main physics goals of the next generation of experiments studying atmospheric neutrinos

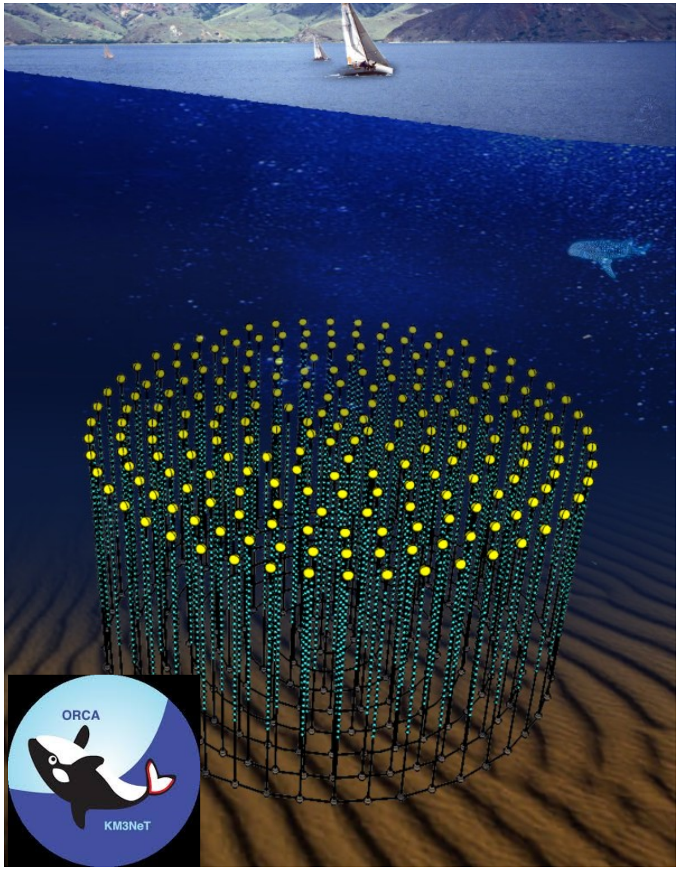
<u>Water Cerenkov</u>
Hyper-Kamiokande

<u>Instrumented ice</u>
IceCube gen2 (PINGU)

<u>Instrumented deep sea</u>
KM3NET (ORCA)



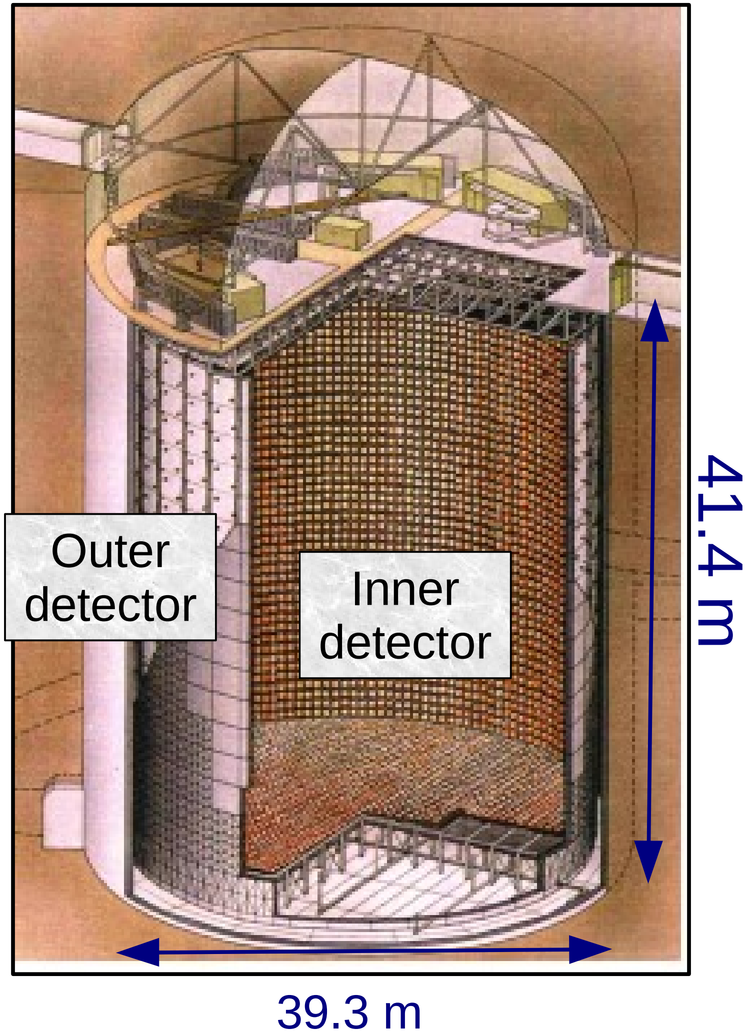


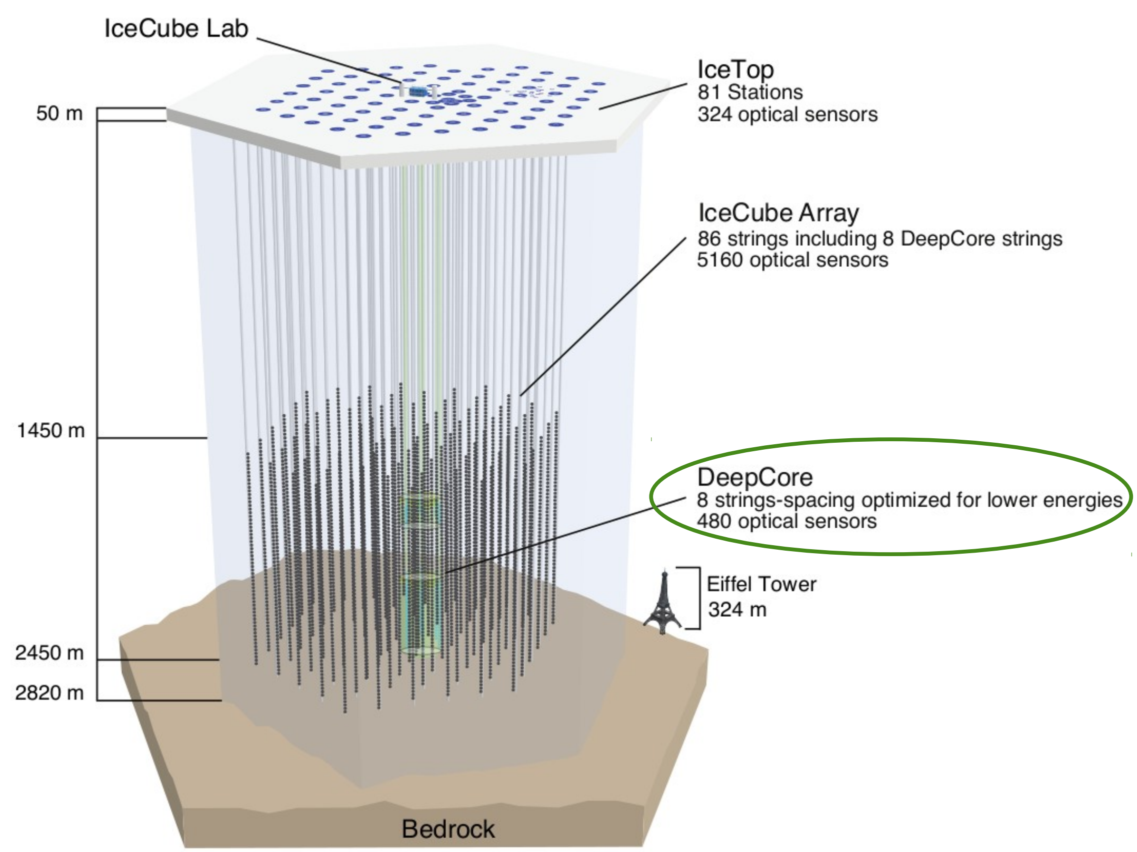


Current experiments

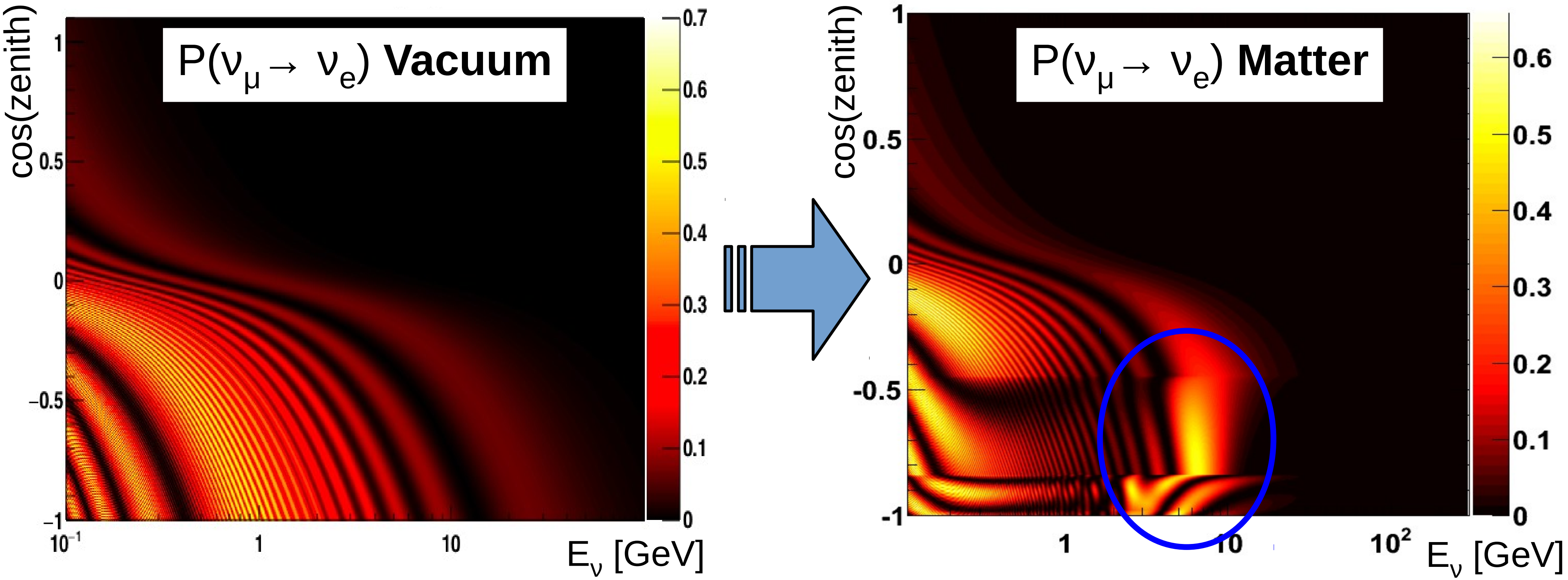
- 50 kt (22.5 kt fiducial) water Cherenkov detector
- 1000m overburden
- Operational since 1996

Super-Kamiokande and IceCube DeepCore already looking for the mass hierarchy using atmospheric neutrinos





Determining the mass hierarchy
Matter effects



Presence of a resonance driven by θ_{13} induced matter effects between 2 and 10 GeV, only for ν in NH and $\bar{\nu}$ in IH

(also some sensitivity in $P(\nu_\mu \to \nu_\mu)$ with increased ν_μ disappearance in NH for neutrinos going through the Earth's core)

Issue #1: Significance for the mass hierarchy

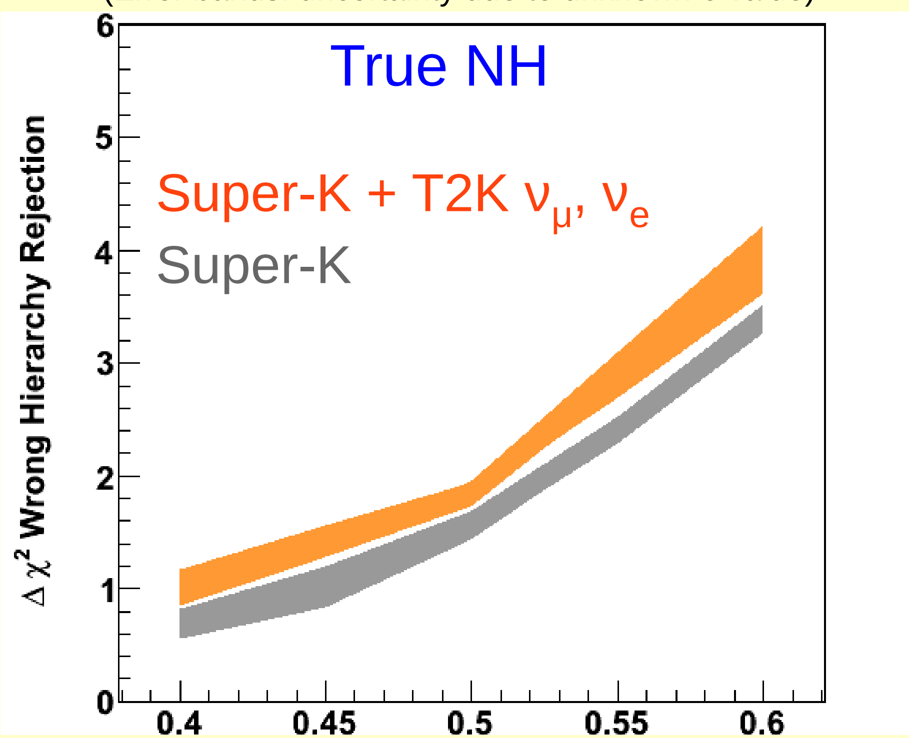
Mass hierarchy significance Problems

As is well known, cannot simply compute MH significance as square root of $\Delta\chi^2$ → need studies with pseudo-experiments

Additional problems: current experiments have limited sensitivities, and distribution of test statistics for toy experiments depend of true values assumed for unknown parameters

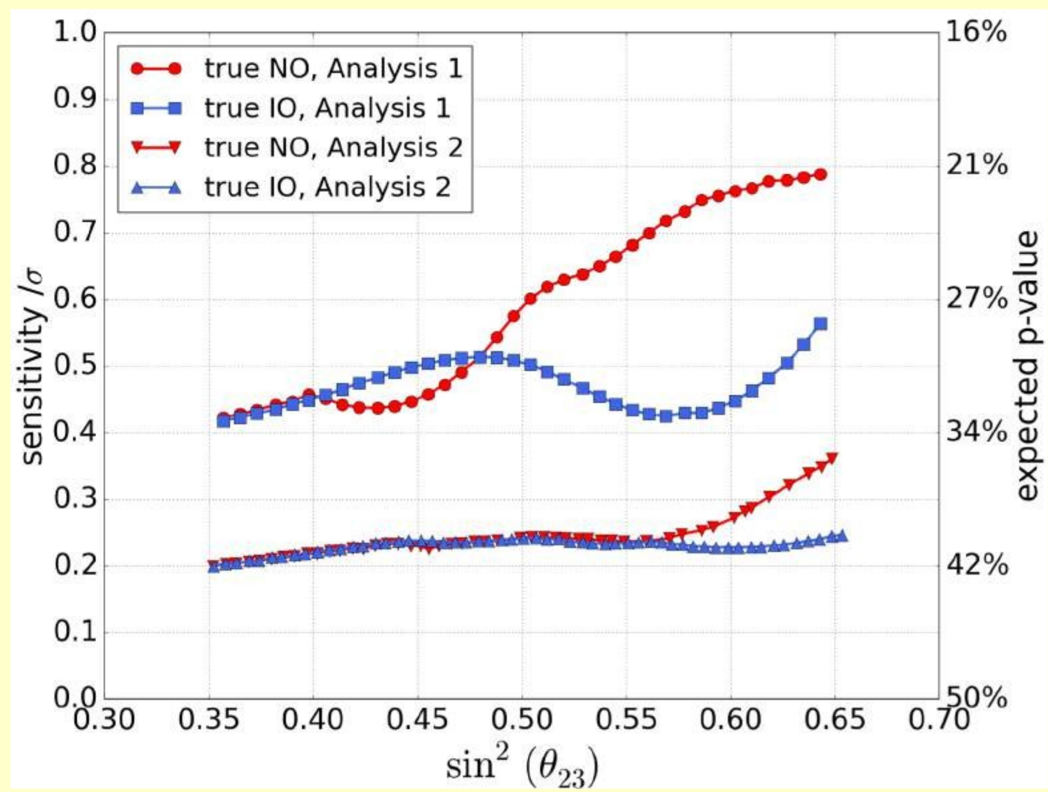
Super-Kamiokande

(Error bands: uncertainty due to unknown δ value)



$\sin^2(\theta_{23})$

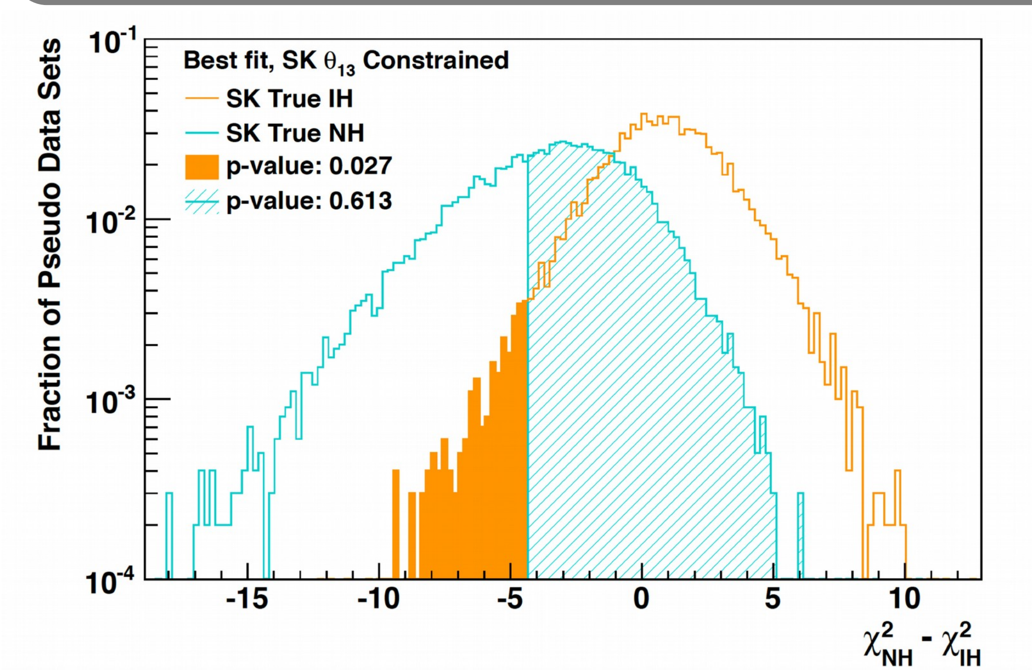
IceCube DeepCore

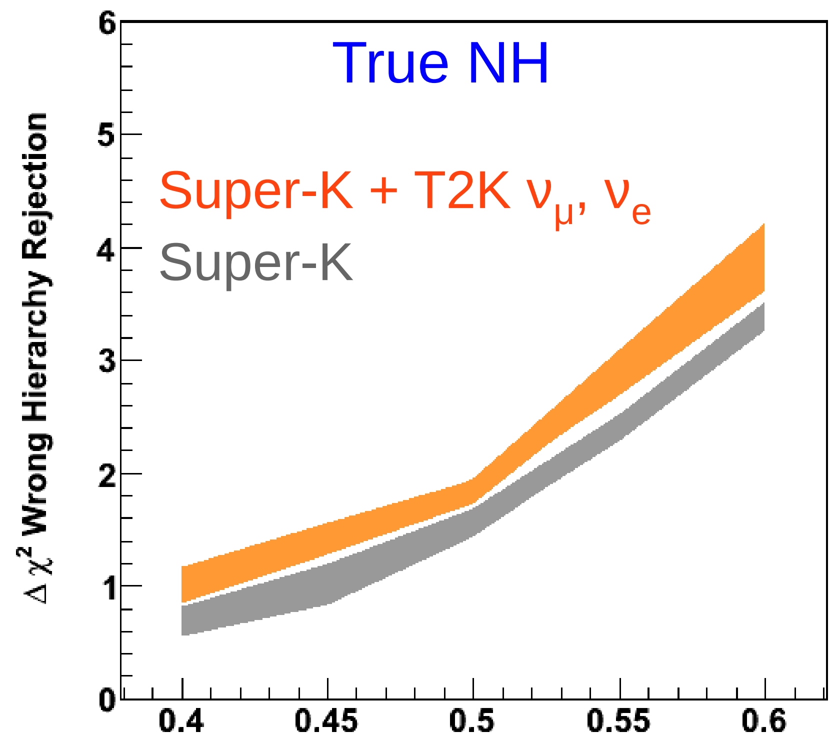


Mass hierarchy significance
Low sensitivity: Super-K case

Concerns that we might report larger exclusion of an hypothesis than we should be able to

- Expected distributions of the test statistics for the 2 hypotheses have significant overlap
- Found in data fit preference for NH larger than expected



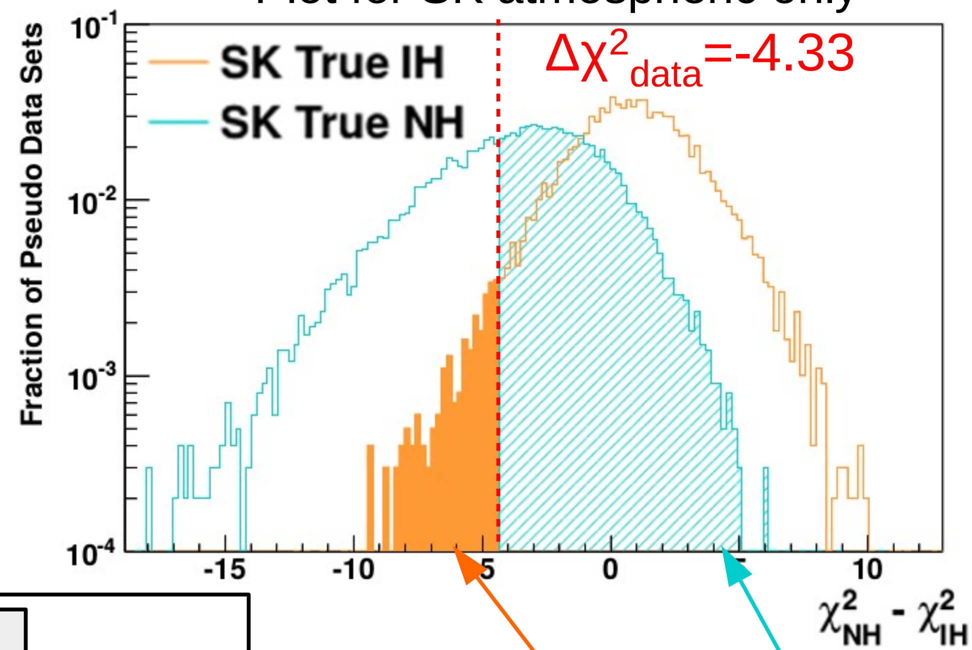


Analysis	$\Delta\chi^2$ (IH-NH)
Super-K only	-4.33
SK + T2K model	-5.27

Mass hierarchy significance Super-K results

- Used CLs to report significance: not truly frequentist, but conservative
- Computed p-values and CLs for lower/upper edges of the 90% CL intervals for $\sin^2(\theta_{23})$ and δ
- Quoted a range of CLs-based significance in the paper

Plot for SK atmospheric only



$$CL_s = \frac{p_0(IH)}{1 - p_0(NH)}$$

P-values and CLs for IH exclusion

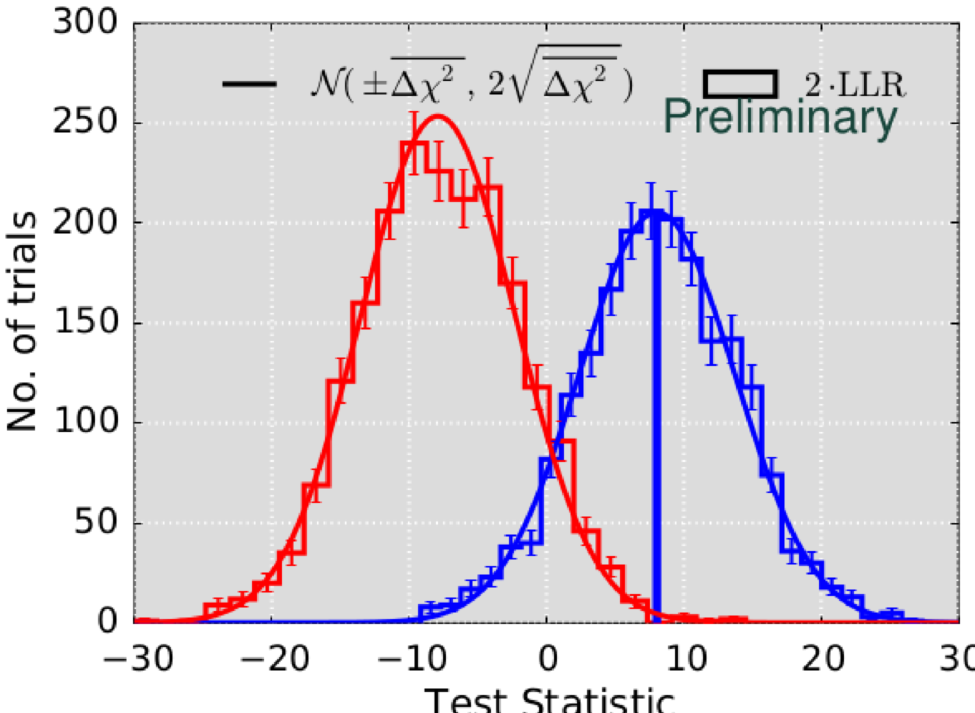
P-values	Lower	Best fit	Upper
SK only	0.012	0.027	0.020
SK+T2K model	0.004	0.023	0.024

CLs	Lower	Best fit	Upper
SK only	0.181	0.070	0.033
SK+T2K model	0.081	0.075	0.056

PRD 97, 072001 (2018)

Mass hierarchy significance IceCube case

- IceCube DeepCore results on MH in preparation, plan to use CLs as well (personal communication with IceCube)
- In the past, have been using two different methods to estimate sensitivity for next generation project PINGU (see talk by J. Hignight at previous PHYSTAT-nu)



Log Likelihood Ratio method

Similar to what SK uses for p-values, replacing data by median value of test statistics in true MH

Δχ² method

Use predictions at best fit (Asimov dataset like)

$$\overline{\Delta\chi^2} = \min_{p\in WO} \sum_i \left(\frac{\mu_i^{TO}(p_0) - \mu_i^{WO}(p)}{\sigma_i} \right)^2$$

$$\Delta\chi^2 = Gauss(\pm\overline{\Delta\chi^2}, 2\sqrt{\overline{\Delta\chi^2}})$$

Potential computing challenges for all next generation experiments:

- Larger significance requires more pseudo-data to be evaluated properly
- Systematics likely to matter and be complex, preventing from using faster approximations

Issue #2: Predictions with limited amount of MC

MC predictions for fits

Fits done by comparing observation to prediction in each bin (e.g: Super-K case)

Expected nb evts in bin n

Observed nb of evts in bin n

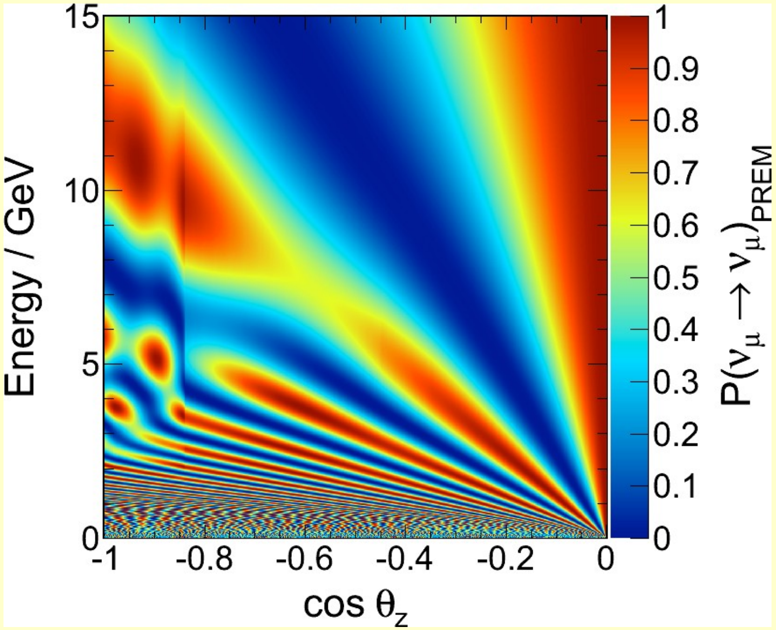
Pull for syst. i

$$\chi^2 = 2\sum_n \left(E_n - \mathcal{O}_n + \mathcal{O}_n \ln \frac{\mathcal{O}_n}{E_n} \right) + \sum_i \left(\frac{\epsilon_i}{\sigma_i} \right)^2$$

Predictions:

- Generate MC for a standard set of values of the parameters
- Apply weights to MC events for other values of the parameters

P_{osc} for atm ν: need to sample 2D space with fast variations



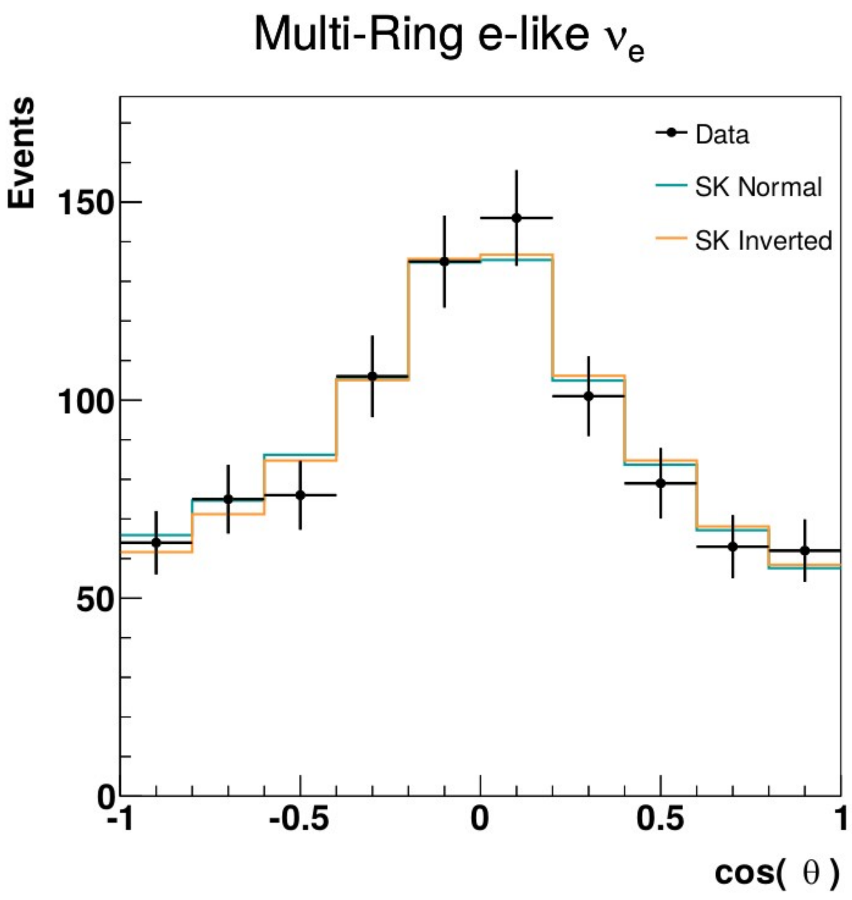
(plot from J.Hignight @PHYSTAT-nu)

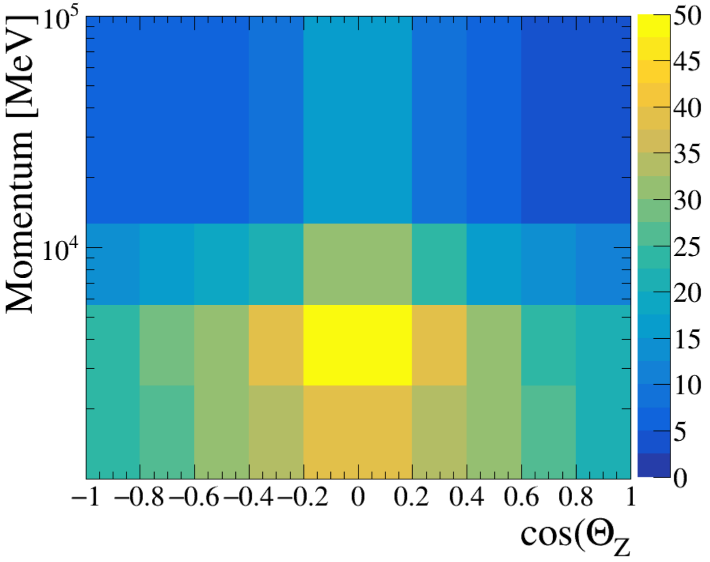
Generating MC takes time:

- Propagate many photons in ice/water
- Apply complex reconstruction/event selections
  → usually limited in the amount of MC we can produce

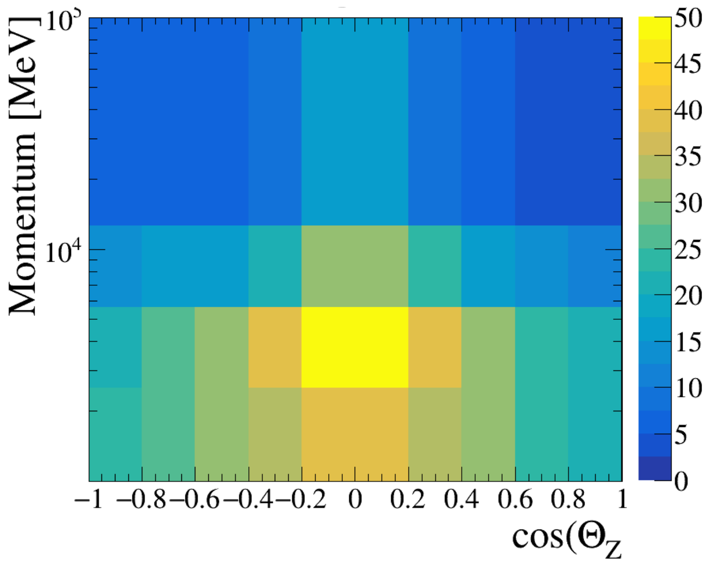
MC Statistical error and MH (SK)

- Sensitivity to the MH coming from sum of small contributions from many bins
- Differences between predictions for both MH quite small
- How precisely do we need to know expected number of events in each bin?





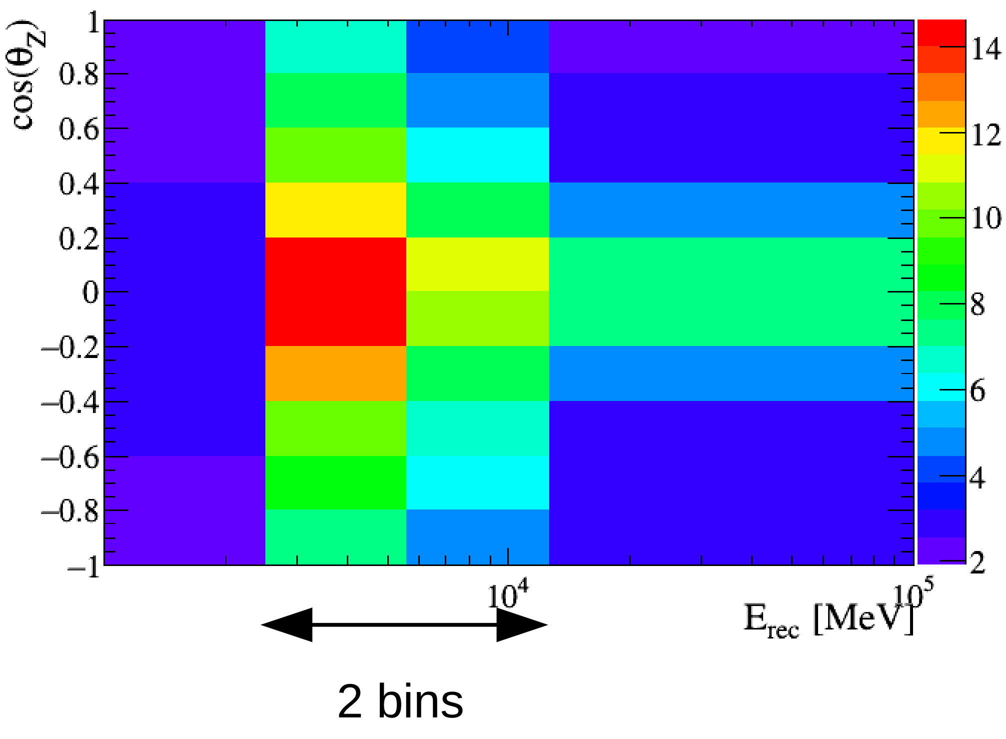
NH

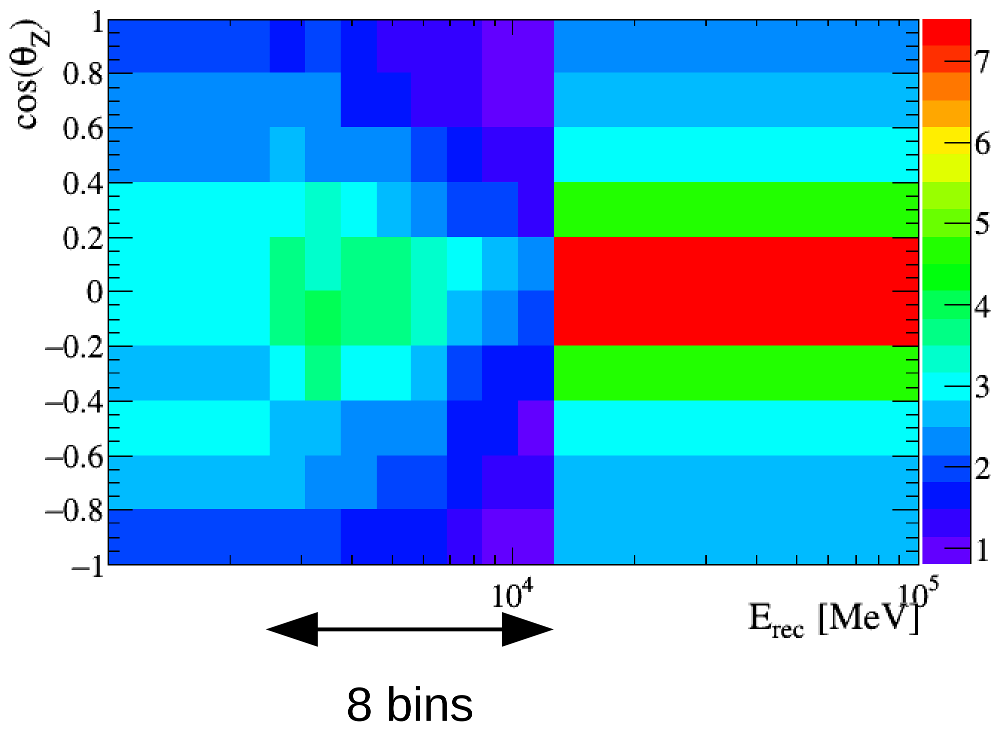


IH

MC Statistical error and MH (SK) Binning

- Assuming we can reconstruct neutrino energy well enough, could hope to increase sensitivity to MH with finer binning
- Tried to look at sensitivity with different number of bins in the resonance region for samples sensitive to MH



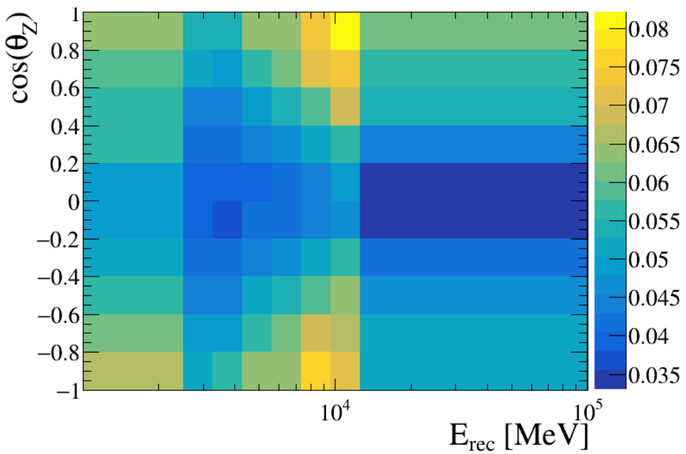


More bins → less MC events per bin → need more MC?

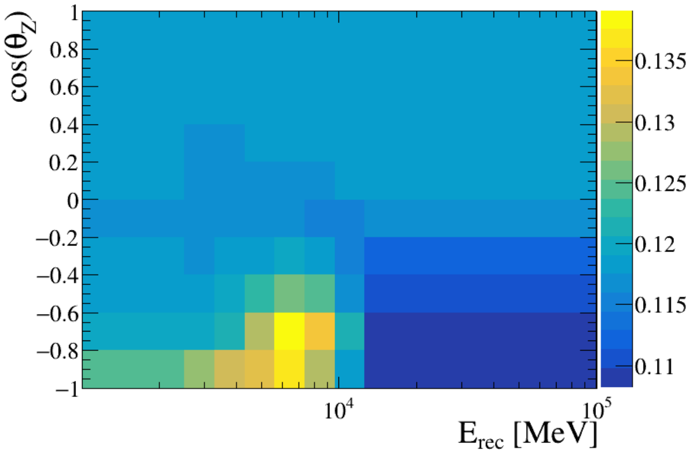
MC Statistical error and MH (SK) Metric?

- Rule of thumb 10x more MC than expected number of events does not seem useful here
- Can compute MC statistical error, but what should it be compared to?
- What would be an acceptable value in each case?

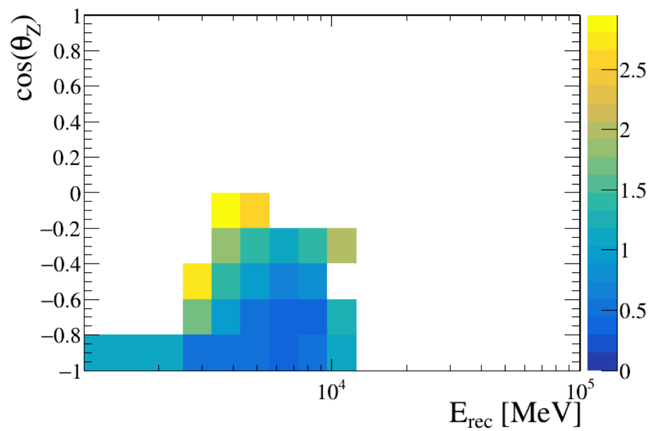
Number of events in bin?



Square root of nb of events in bin?

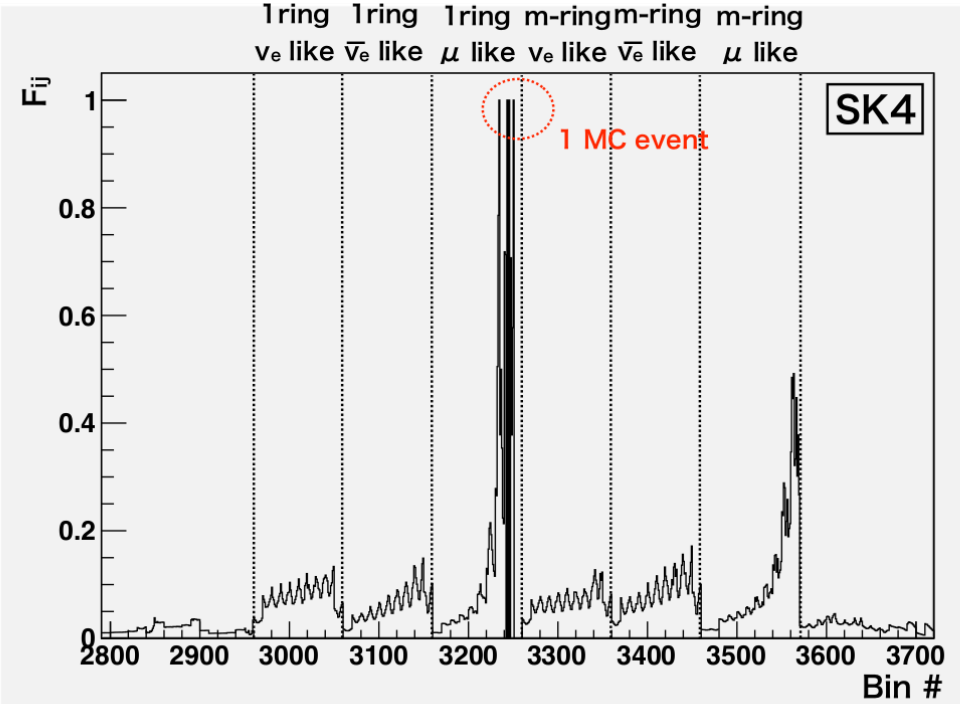


Difference between NH and IH predictions?



MC Statistical error and MH (SK) Tests

- Tried to add additional systematic parameters for MC statistical error in important bins
- Found almost no difference in the MH sensitivity
- MC statistical error does not matter in this analyis?
- Or need shape error rather than bin by bin?



$\Delta\chi^2$ value at best fit point $\Delta\chi^2 = |\chi^2(nh) - \chi^2(ih)|$

w/o MC stat	w/ MC stat (> 15% NH-IH diff)	w/ MC stat (> 10% NH-IH diff)	w/ MC stat (> 5% NH-IH diff)
4.03	4.01	4.00	3.98

$\Delta\chi^2$ value at best fit point $\Delta\chi^2 = |\chi^2(nh) - \chi^2(ih)$

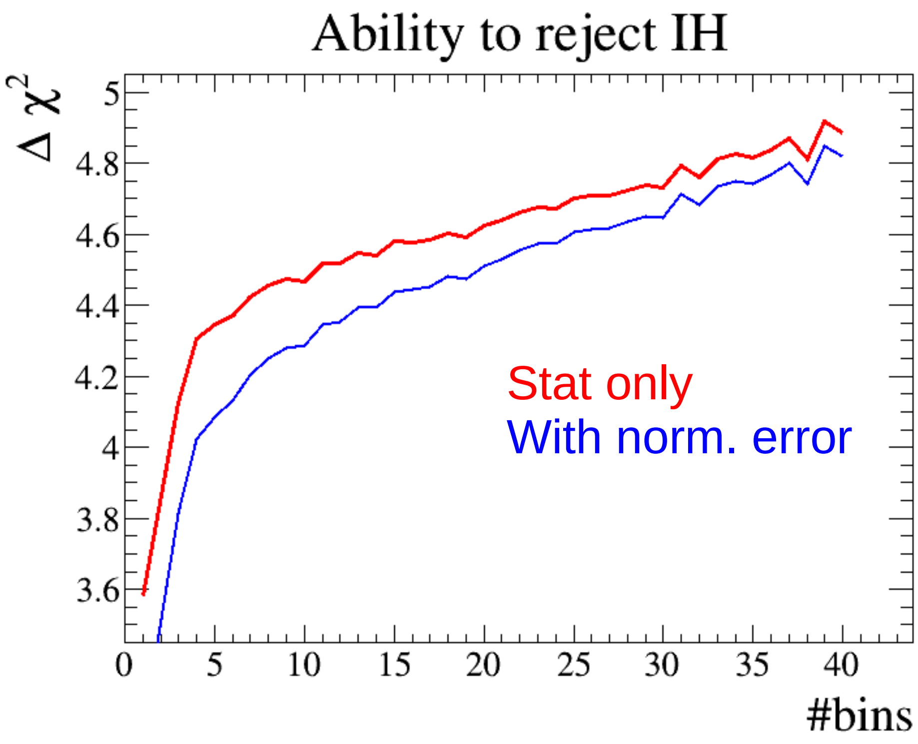
w/o MC stat	w/ MC stat (> 0.01 χ^2 contribution)
4.03	3.98

MC statistics and resolution

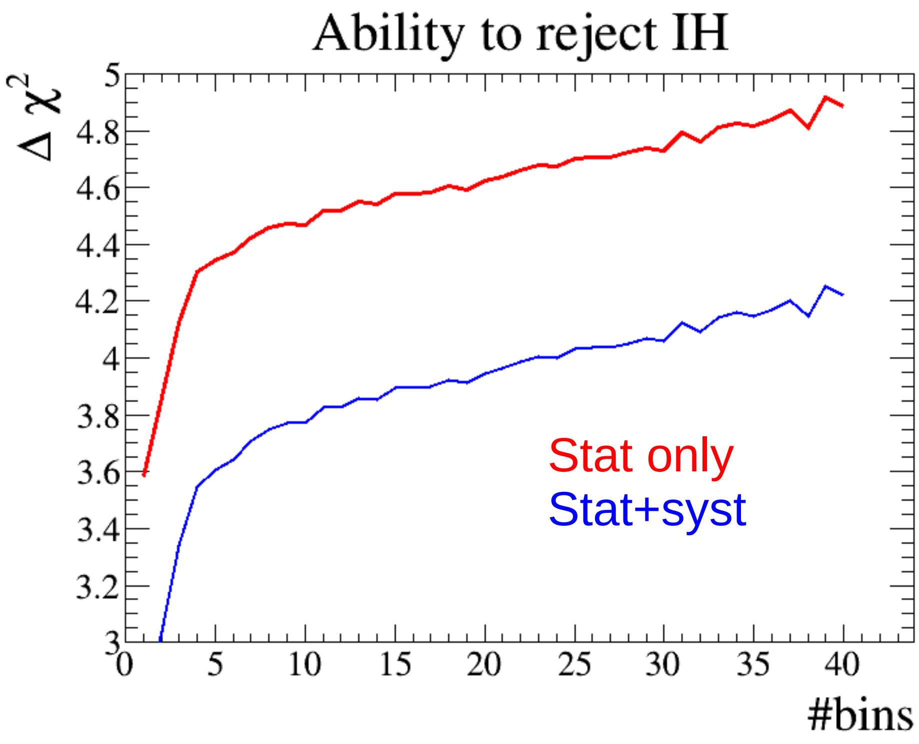
Studies on MH sensitivity as a function of number of bins lead to surprising results:

- Sensitivity keeps increasing linearly with number of bins
- Adding systematic uncertainties or MC stat. error did not change the pattern

20% overall normalization error

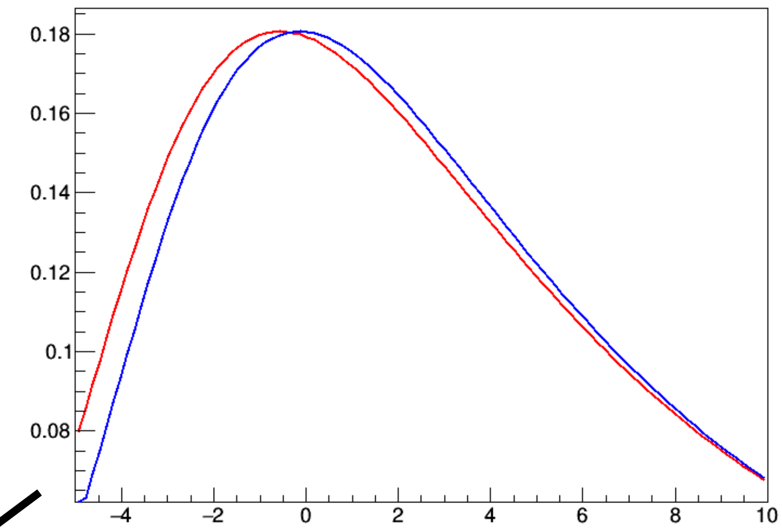


20% overall normalization error
+5% uncorrelated error in each bin

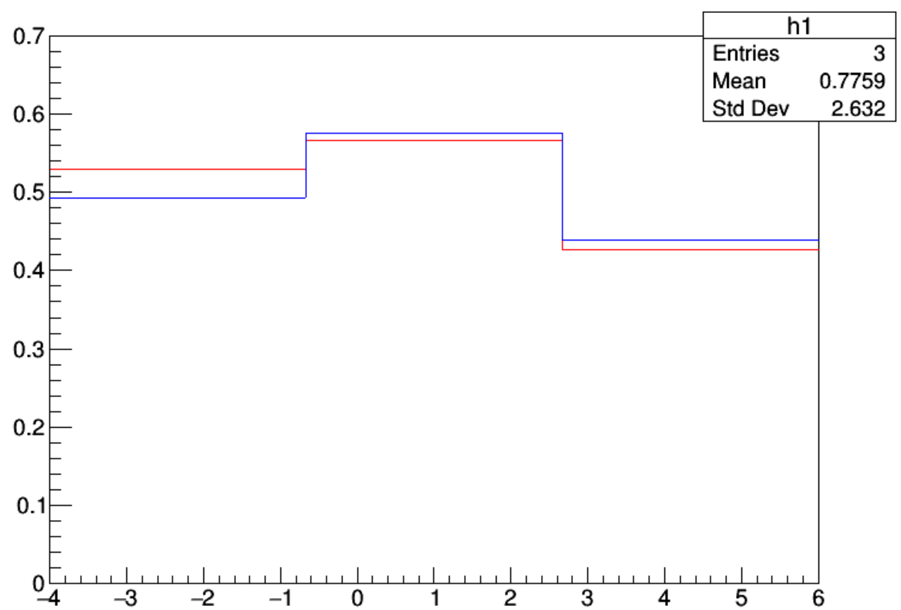


MC statistics and resolution

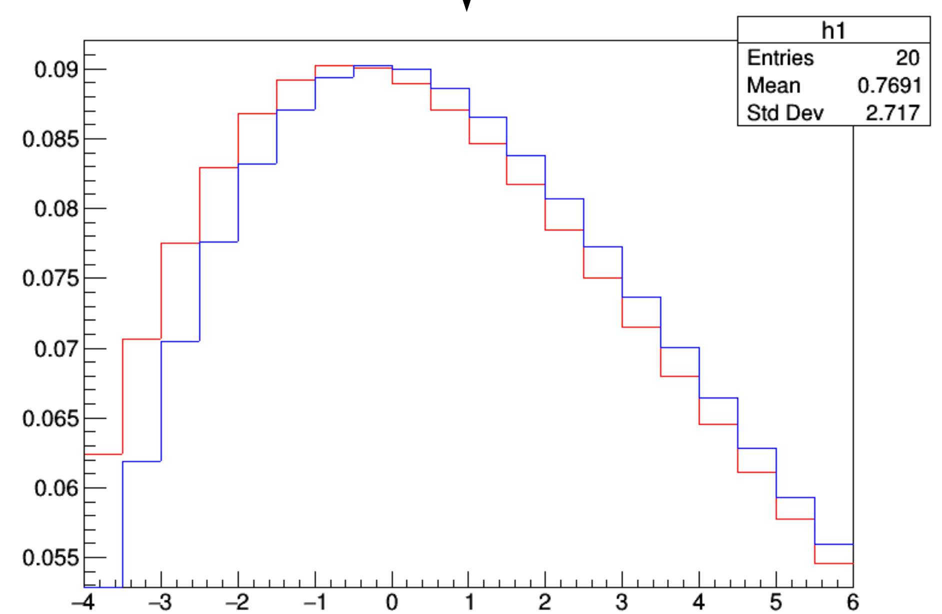
Toy study:
- Try to separate 2 distributions using similar method as for NH/IH
- Fill bins with average value of each distributions
- Calculate log likelihood ratio to estimate “sensitivity”



3 bins



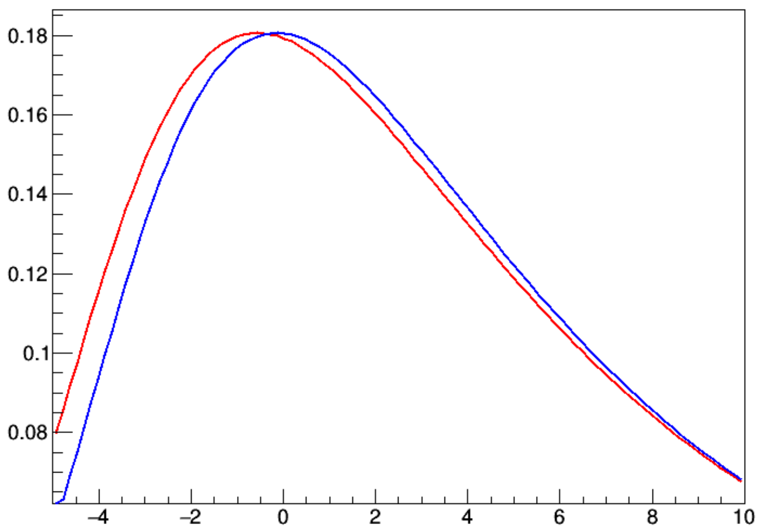
20 bins

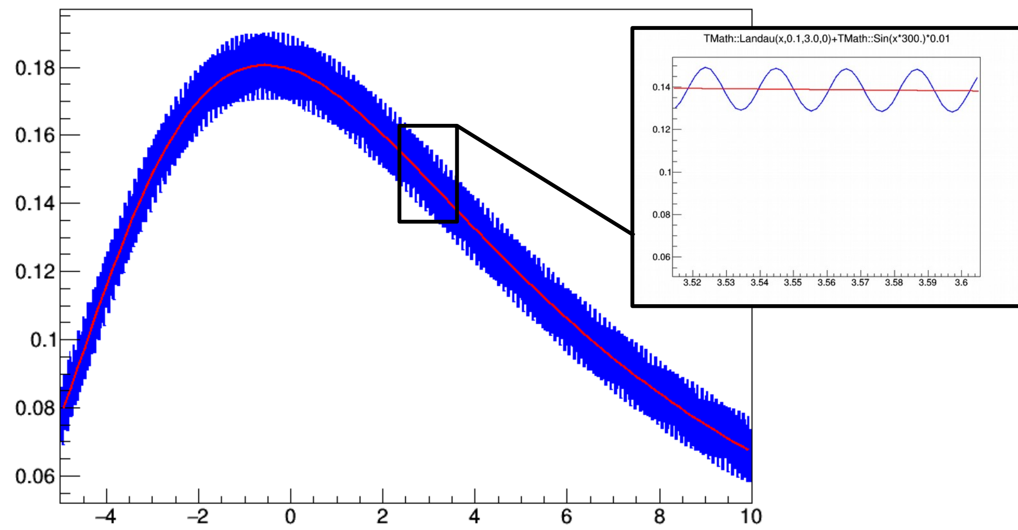


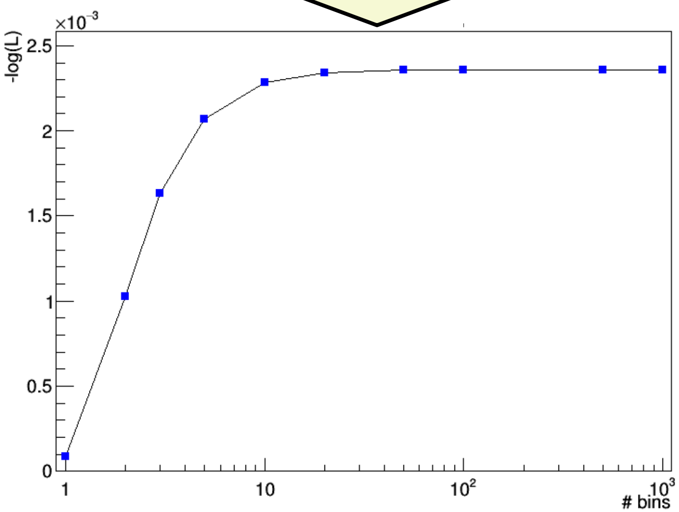
MC statistics and resolution

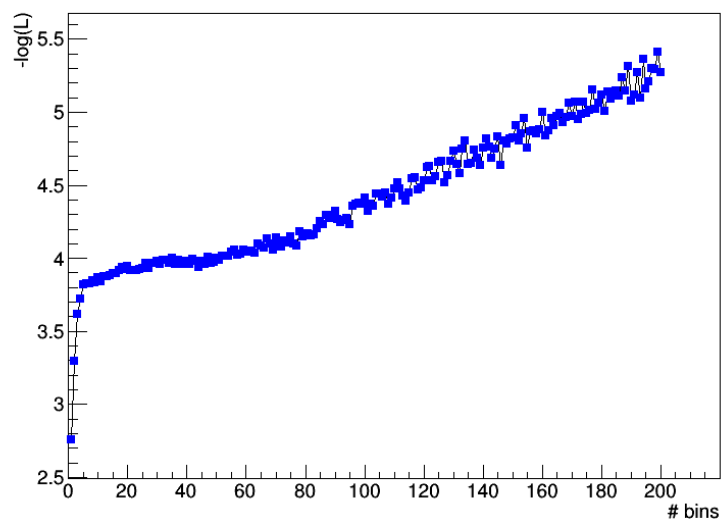
Can obtain similar pattern if:

- The 2 distributions differ on a shorter scale as well (fast oscillations)
- Bins are added regularly spaced in log scale



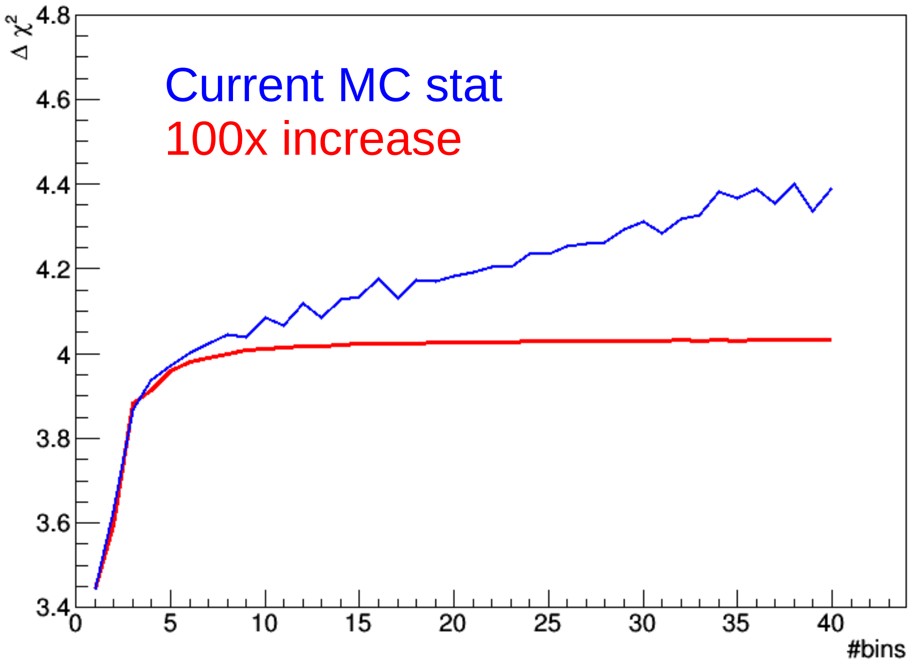






MC statistics and resolution

- Real detector should not be sensitive to those short scales differences due to limited E and L resolutions
- Smearing true → reconstructed quantities done by MC
- With insufficient MC statistics, small scale differences seem to survive in the reconstructed quantities



<u>Test:</u>

- Assume gaussian smearing from true to reconstructed energy
- For each MC event, randomly generate Erec from this gaussian smearing
- Increase MC stat. by re-using MC events

Is there a known way to determine necessary amount of MC or build a systematic error?

Background prediction

- IceCube DeepCore final samples contain ~5% atmospheric muons
- Background rejection cut very efficient: reduce by a factor 10^8
  → could not generate enough μ MC to properly estimate this background
- Use a data driven method instead

PRL 120, 071801 (2018)

$$\chi^2 = \sum_{i\in\{\text{bins}\}} \frac{(n_i^{\nu+\mu_{\text{atm}}} - n_i^{\text{data}})^2}{(\sigma_i^{\text{data}})^2 + (\sigma_{\nu+\mu_{\text{atm}},i}^{\text{uncor}})^2} + \sum_{j\in\{\text{syst}\}} \frac{(s_j - \hat{s}_j)^2}{\hat{\sigma}_{s_j}^2}$$

Additional uncertainty for each bin

New approach in preparation:

- Based on the Barlow method (Computer Physics Communications 77 (1993) 219—228)
- “This note shows how to incorporate the fact that the Monte Carlo statistics used are finite and thus subject to statistical fluctuations”

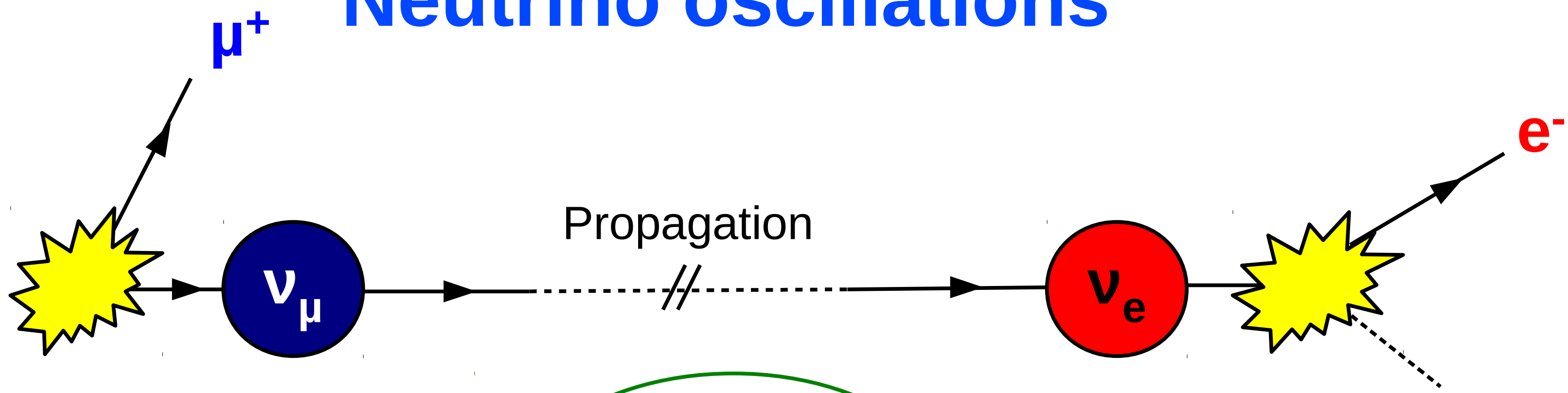
Summary

- Next generation of experiments studying atmospheric neutrinos will try to determine the neutrino mass hierarchy
- Currently running experiments already performed analysis with limited sensitivity, and started facing issues that will need to be addressed by next generation of experiments
- Studies using pseudo data samples can be used to determine the significance of an observation, but might become prohibitive in terms of computation for larger significance
- Other challenge is to predict precisely what should be observed from simulation:
  - how to determine how much MC is needed?
  - how to properly do analysis if enough MC cannot be produced?

Many thanks to J. Hignight for explanations on IceCube DeepCore approach and issues faced

Additional slides

Neutrino oscillations



Flavor eigenstates (interaction)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \times \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigenstates (propagation)

Mixing (or Pontecorvo-Maki-Nagawa-Sakata) matrix
link between the two sets of eigenstates

$P(\nu_\alpha \rightarrow \nu_\beta)$ oscillates as a function of distance L traveled by the neutrino

- Amplitude of oscillations depends on the mixing matrix U
- Phase of the oscillation depends on energy and difference of mass squared: $\mathbf{\Delta m^2_{ij} L/E}$

$(\Delta m^2_{ij} = m^2_i - m^2_j)$

Neutrino oscillations
Parameters

In practice, for neutrino oscillations:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

"Atmospheric" "Reactor" "Solar"

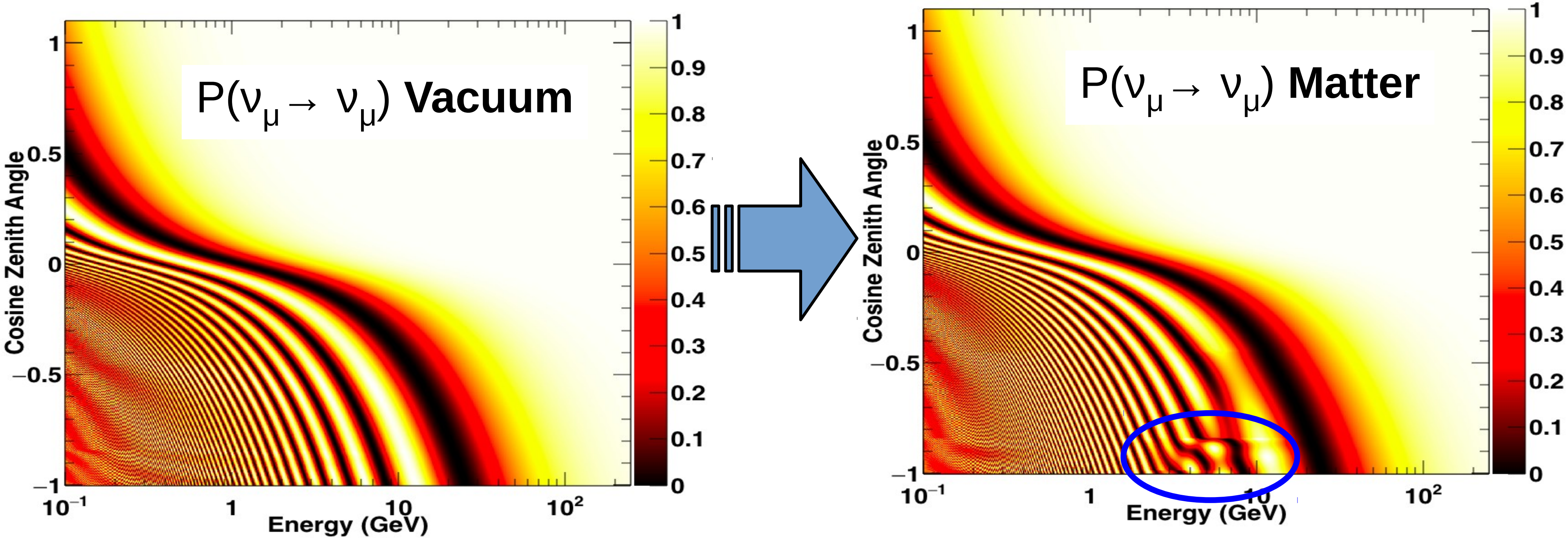
($c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$)

$P(\nu_\alpha \rightarrow \nu_\beta)$ depends on **6 parameters**:

- 3 mixing angles $\boldsymbol{\theta_{12}, \theta_{23}, \theta_{13}}$
- 2 independent mass splittings $\boldsymbol{\Delta m^2_{ij}}$
- 1 complex phase, the CP phase $\boldsymbol{\delta}$

- Observed both disappearance and appearance of neutrino flavors
- All mass splittings (Δm^2_{ij}) and mixing angles (θ_{ij}) measured to be non-zero
- Only δ still unknown (not well constrained by data)
- Sign of $\Delta m^2_{32/31}$ unknown

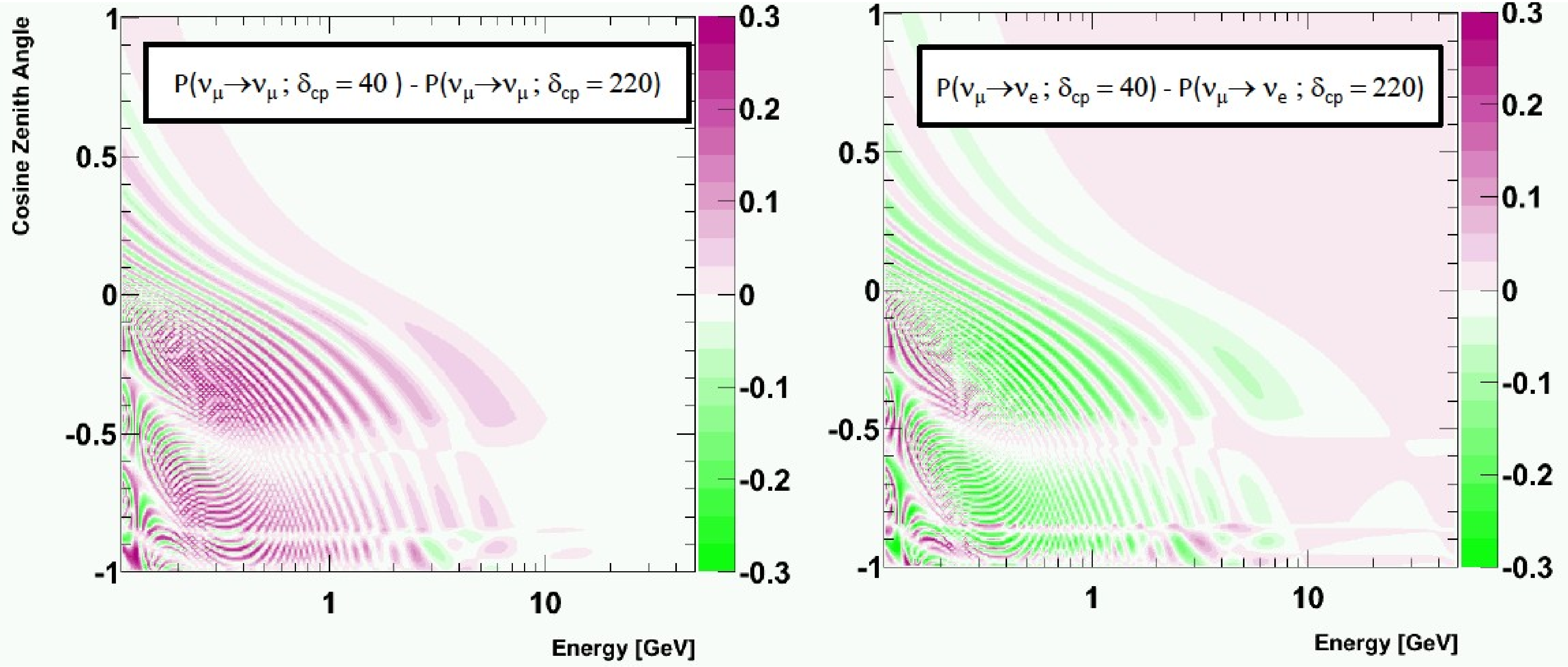
Atmospheric neutrino oscillations
Matter effects – muon neutrinos



Slightly more muon disappearance for neutrinos passing through the Earth's core

Atmospheric neutrino oscillations
Delta CP (Super-K case)

Value of δ_{CP} modifies the oscillation patterns in a complicated way



- Given neutrino flux and detector energy and angular resolution, sensitivity mainly comes from number of sub-GeV e-like events
- More ν_e appearance events for δ~220-240º, and less for δ~40-45º

Super-Kamiokande
Samples contributing to the mass hierarchy