Statistical Issues in Reactor Neutrino Experiments

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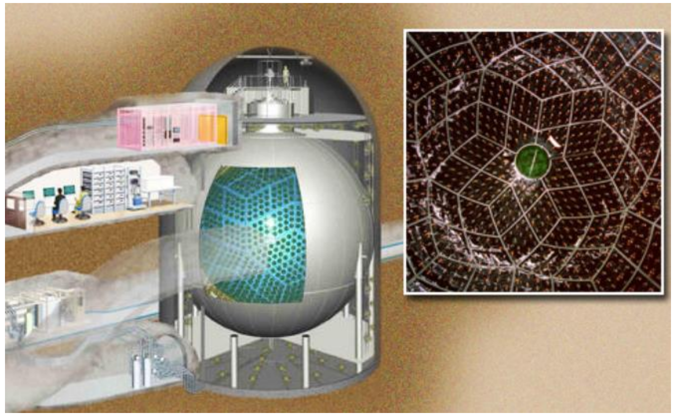
Outline

- Introduction to reactor neutrinos and oscillation experiments
- Statistical Issues in reactor experiments
  - Parameter estimation
    - θ_{12} and Δm^2_{21} (KamLAND)
    - θ_{13} and $|\Delta m^2_{31}|$ (Daya Bay, RENO, Double Chooz)
    - θ_{14} and Δm^2_{41} (PROSPECT and many other SBL reactor experiments)
  - Spectrum Unfolding
  - Combining experimental results
  - Hypothesis Testing
    - Neutrino Mass Ordering (JUNO)

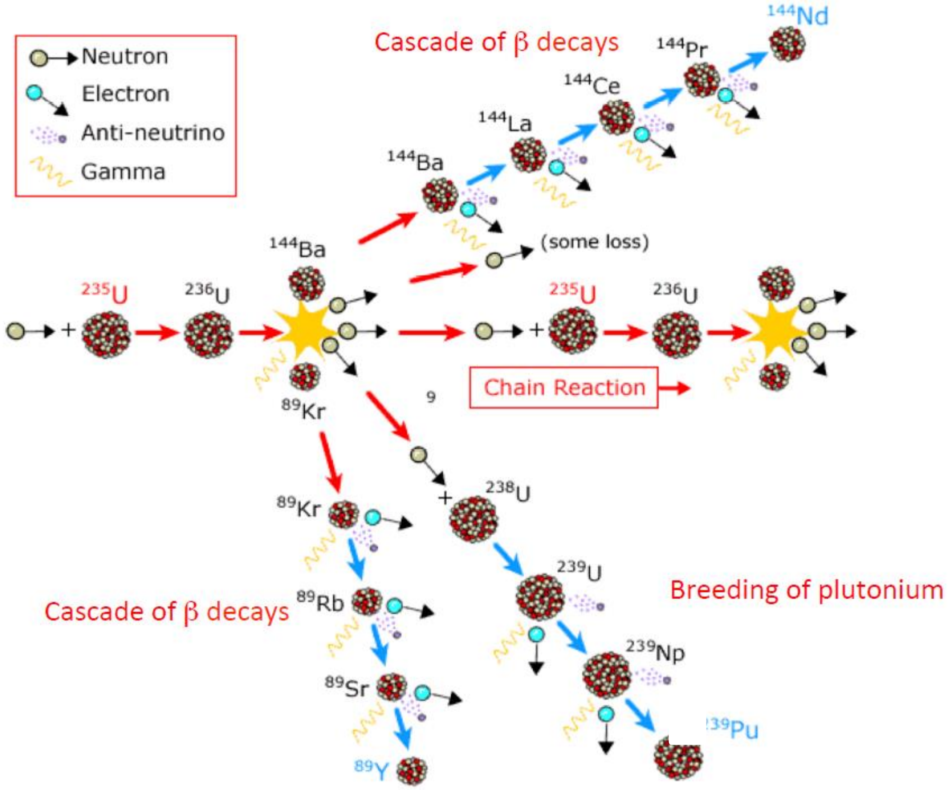
1950s
Reines & Cowan
Discovery of ν

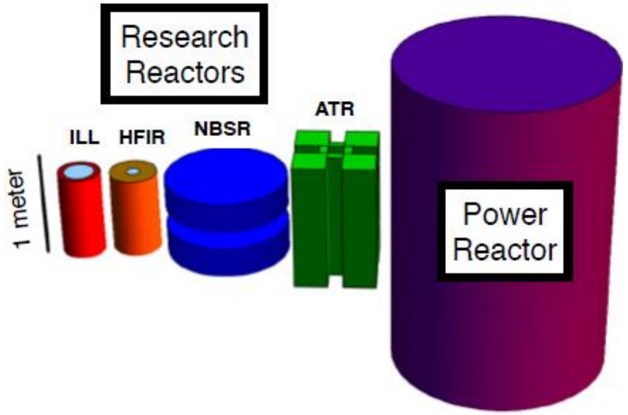


2000s
KamLAND
Solving solar ν problem on Earth

2010s
Daya Bay
Discovery of smallest osc. angle θ_{13}

Nuclear Reactor as Antineutrino Source



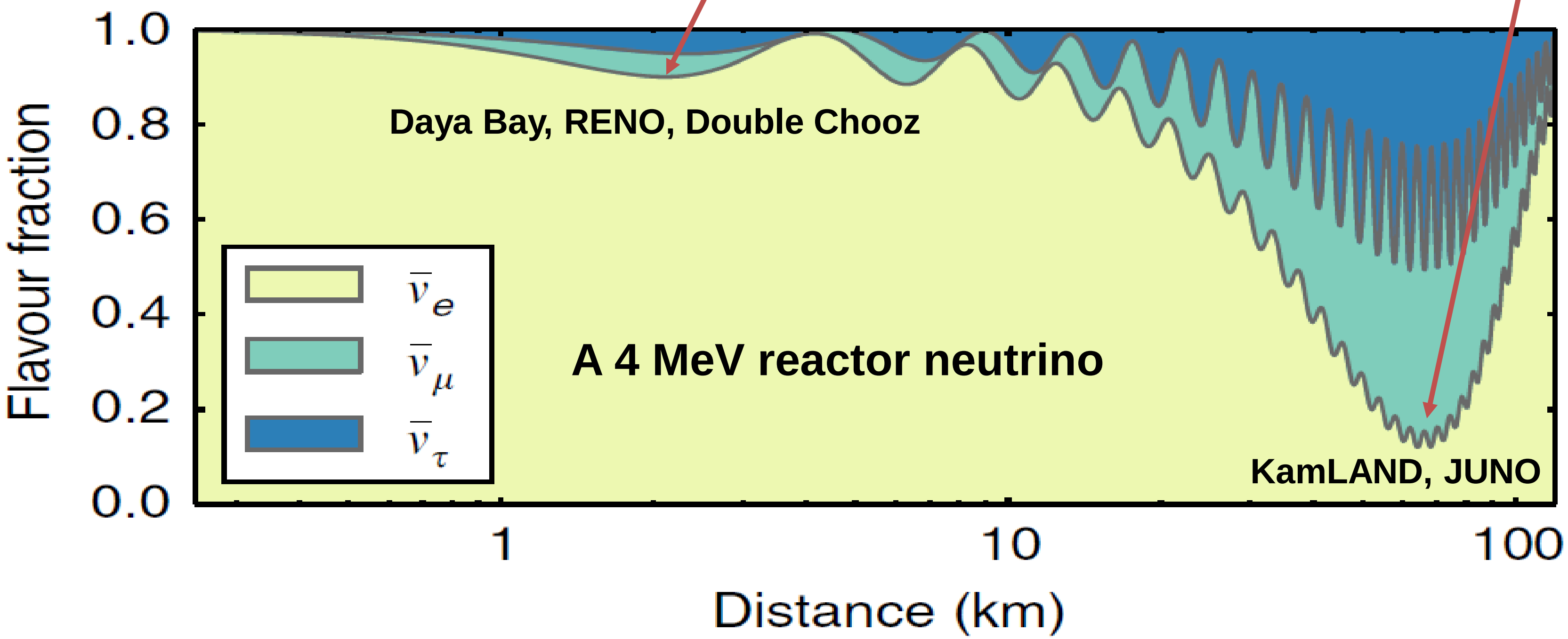


- Nuclear reactors produce pure $\overline{\nu}_e$ from beta decays of fission daughters
  - Low energy: < 10 MeV
- 6 $\overline{\nu}_e$ / fission
- 2 x 10^{20} ν_e / sec per GW_{th} (free for physicists)

- Commercial reactors in Nuclear Power Plants have low-enriched uranium (LEU) cores
  - Mixture of fissions: ^{235}U (~55%), ^{239}Pu (~30%), ^{238}U (~10%), ^{241}Pu (~5%)
  - Large power: ~3 GW_{th}
- Research reactors have highly-enriched uranium (HEU) cores
  - ^{235}U fission fraction ~99%
  - Lower power, few tens of MW_{th}
  - compact size

Reactor Neutrino Oscillations

$$P(\overline{\nu}_e \to \overline{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\Delta m^2_{ee} \cdot \frac{L}{E}\right) - \cos^4\theta_{13} \cdot \sin^2 2\theta_{12} \cdot \sin^2\left(\Delta m^2_{21} \cdot \frac{L}{E}\right)$$



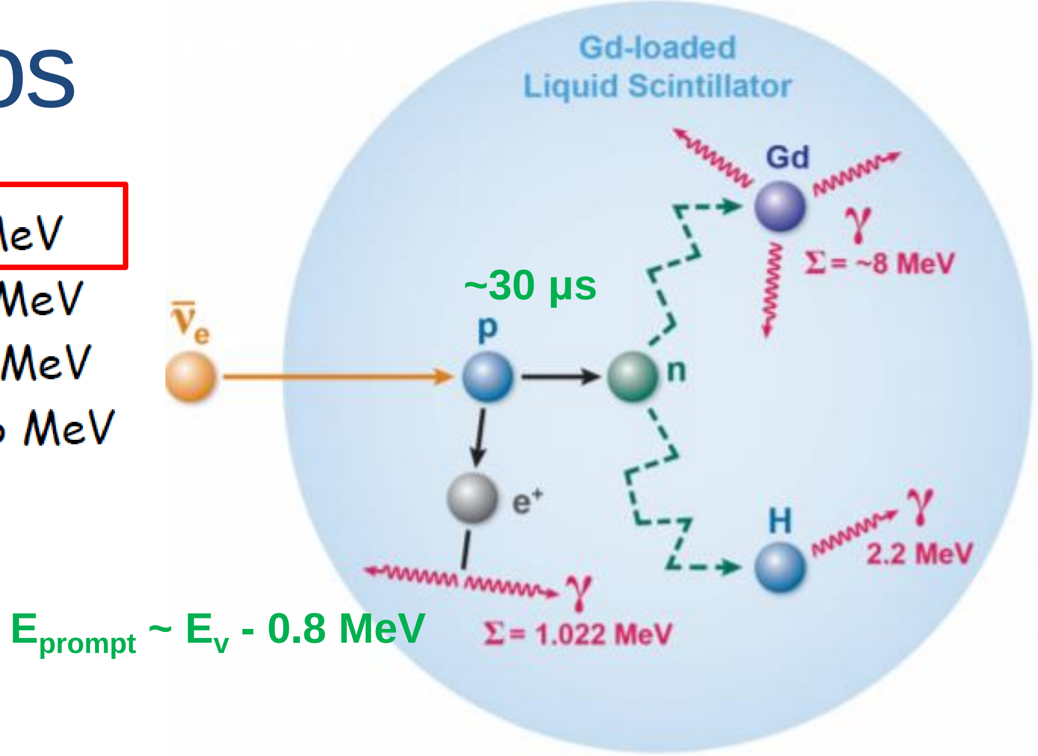
$$|\Delta m^2_{ee}| \sim |\Delta m^2_{3x}| \approx 2.4 \times 10^{-3} eV^2$$
$$>> |\Delta m^2_{21}| \approx 7.6 \times 10^{-5} eV^2$$

- Disappearance experiments, independent of CP violation
- Low energy: vacuum oscillation, negligible matter effects

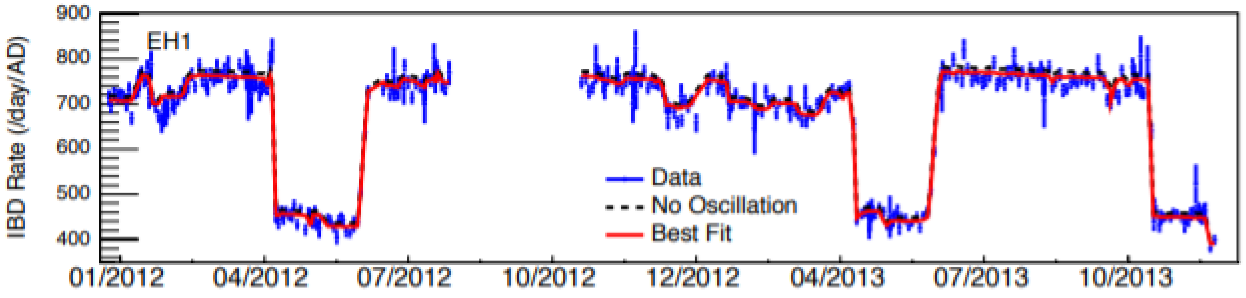
⇒ Clean measurement of oscillation parameters of interest

Detecting Reactor Antineutrinos

$\bar{\nu} + p \rightarrow e^+ + n$	ccp	$\sigma \simeq 63\times10^{-44}$ cm^2/fission	E_{th} = 1.8 MeV
$\bar{\nu} + d \rightarrow e^+ + n + n$	ccd	$\sigma \simeq 1.1\times10^{-44}$ cm^2/fission	E_{th} = 4.0 MeV
$\bar{\nu} + d \rightarrow \bar{\nu} + n + p$	ncd	$\sigma \simeq 3.1\times10^{-44}$ cm^2/fission	E_{th} = 2.2 MeV
$\bar{\nu} + e^- \rightarrow \bar{\nu} + e^-$	el. sc.	$\sigma \simeq 0.4\times10^{-44}$ cm^2/fission	E_{range} 1-6 MeV



- All have been used in the past, but the Inverse Beta Decay (IBD) process has clear advantages
  - Largest cross section (relatively speaking)
  - Low-cost material (liquid scintillator)
  - Distinctive coincidence signature



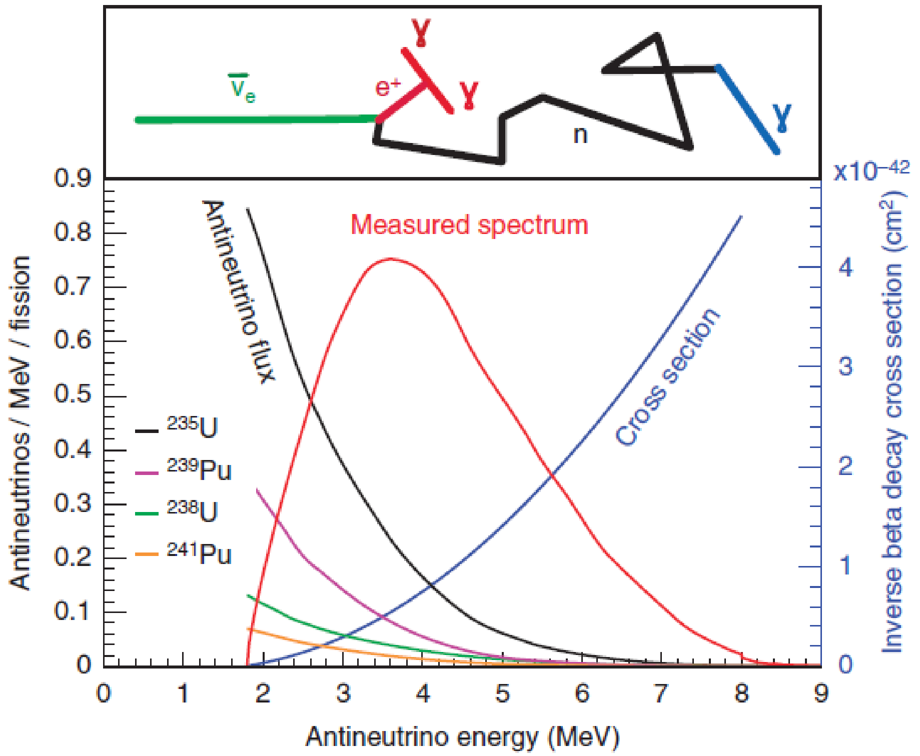
Daya Bay, Chin. Phys. C 41, 1 (2017)

Measured Reactor Spectrum

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

$$S(E_\nu) = c \cdot \sum_i f_i \cdot s_i(E_\nu) \cdot \sigma(E_\nu)$$

- c: reactor thermal power, energy released per fission, baseline, target protons, detection efficiency, oscillation, etc.
- f_i: isotope fission fraction
- $s_i(E_\nu)$: isotope neutrino spectra
- $\sigma(E_\nu)$: IBD cross section



P. Vogel, L. J. Wen, C. Zhang, Nature Commun. 6, 6935 (2015)

Reactor related uncertainties

Correlated		Uncorrelated	
Energy per fission	0.2%	Power	0.5%
IBD reaction per fission	3%	Fission fraction	0.6%
		Spent fuel	0.3%

Daya Bay, Phys Rev Lett 108, 171803

Predicting Reactor Neutrino Spectrum

Sonzogni et al, PRC 91, 011301

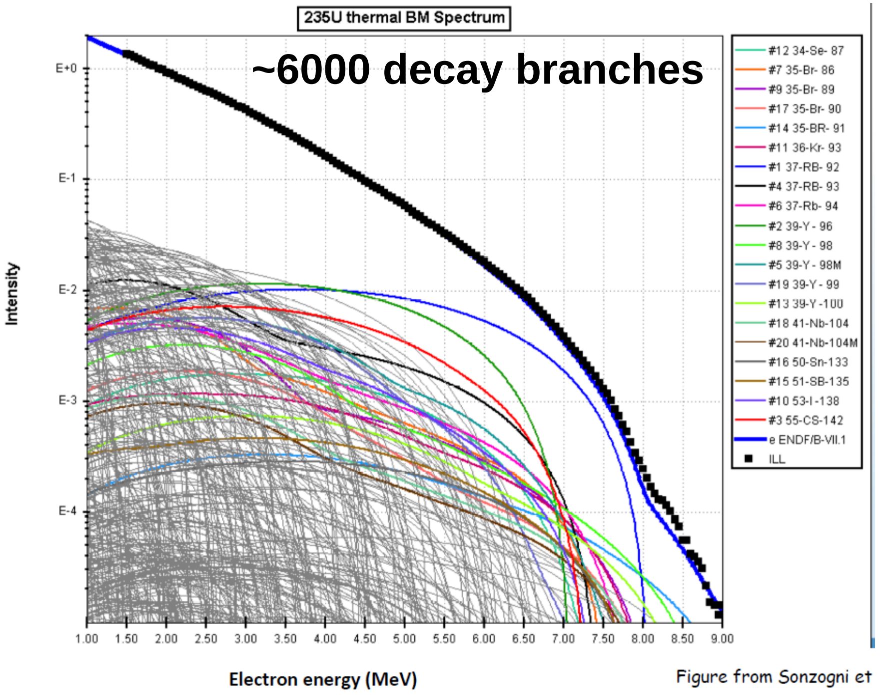
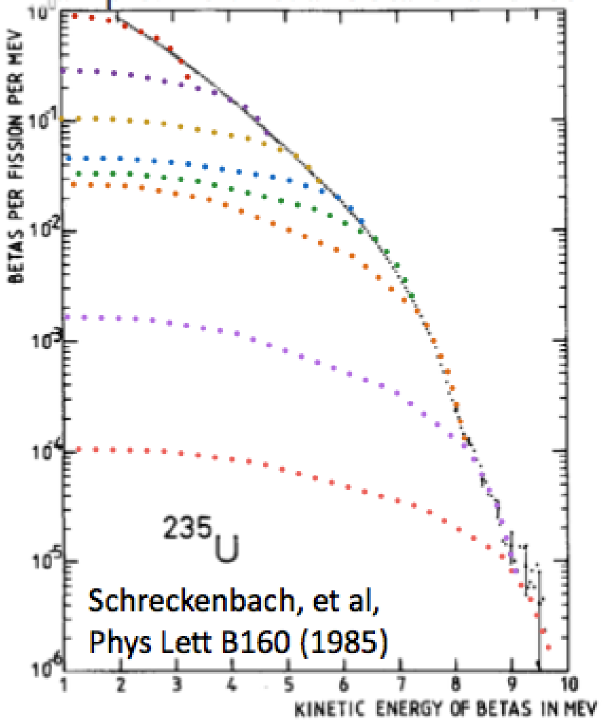


Figure from Sonzogni et

- **Summation (*ab initio*) method**
  - Calculate the spectrum of each branch using **nuclear databases**: <u>fission yields</u>, <u>decay schemes</u>
  - ~10% uncertainty

- **Conversion Method**
  - Measure total outgoing beta-decay electron energy spectra. (*Experiments done at ILL in the 1980s*)
  - Predict corresponding anti-neutrino spectra with >30 virtual branches
  - More precise: ~2.7% uncertainty. Default method in reactor neutrino experiments

Example: Fit virtual beta branches

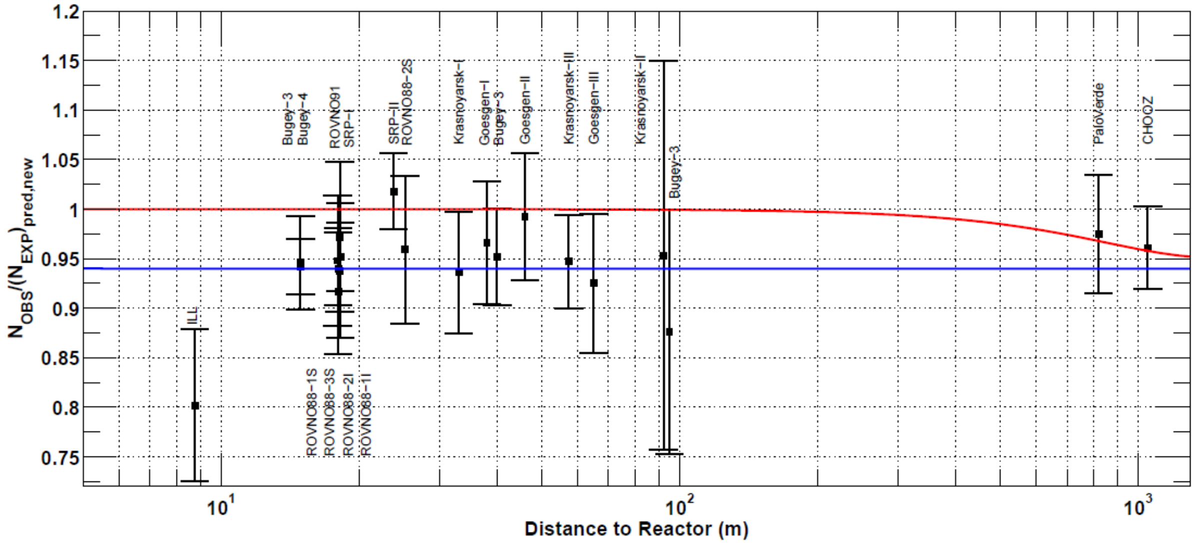


Converting electron spectrum -> neutrino spectrum is fundamentally an unfolding problem

Reactor Antineutrino Anomaly (2011):

- In 2011, two beta-conversion re-analyses increased predicted flux
  - conversion +3%, neutron lifetime +1%, non-equilibrium isotopes +1%
  - ~3σ tension with previous experiments
- Implications
  - Either reactor model uncertainty is underestimated (need to be > 5%)
  - Or is there a sterile neutrino?
- Practical Implications
  - θ_{13} experiments: should not rely on reactor models, need near/far detectors
  - Sterile neutrinos: should not rely on pure rate deficit, need look for L/E oscillation

PRD 83, 073006 (2011)



An interesting statistical mistake in the original paper enlarged the deficit by ~1.5%

$$\chi^2(R_g^{past}) = (R_g^{past} - R_i)\cdot V_{ij}^{-1}\left(R_g^{past} - R_j\right)$$

$$V = V^{exp} + V^{theory}$$

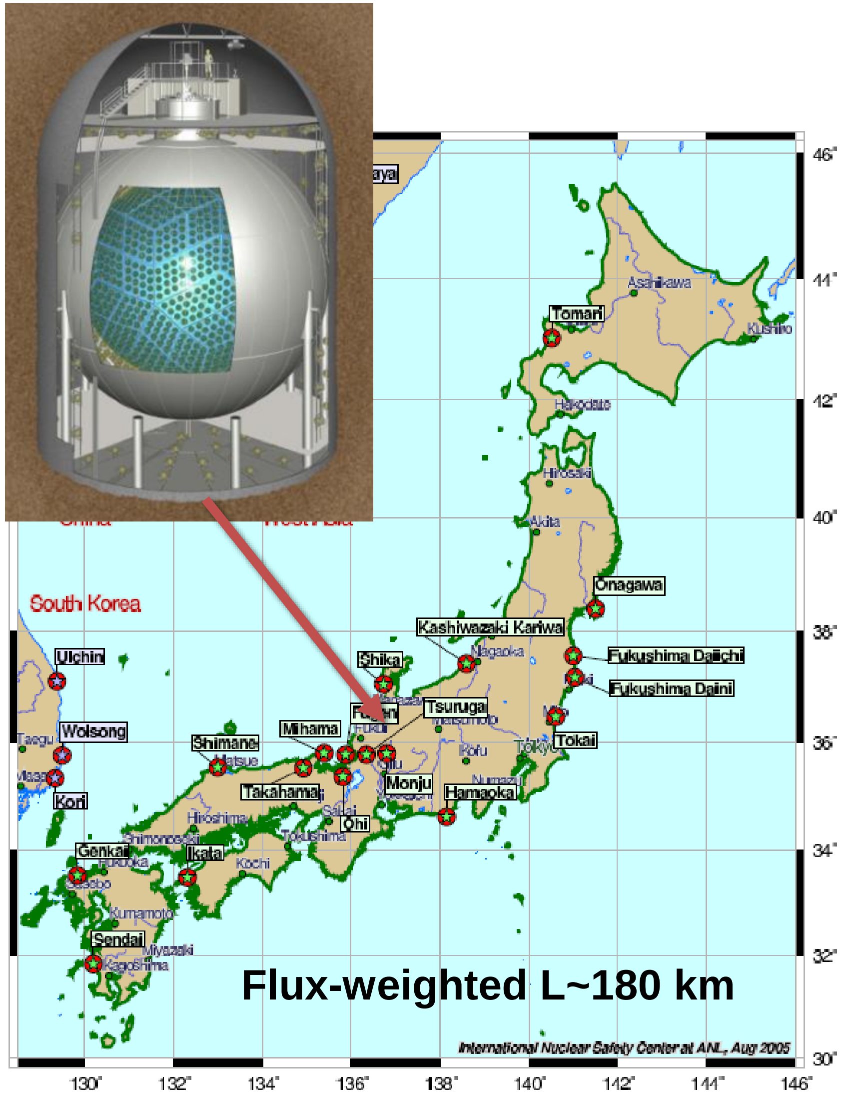
$$V^{theory} = R_i^{obs} R_j^{obs} (\sigma^{theory})^2$$

$$\text{should be } V^{theory} = R_i^{theory} R_j^{theory} (\sigma^{theory})^2$$

See also:

G. D'Agostini NIMA 346, 306 (1994),
V. Blobel, SLAC-R-0703, p101,
B. Roe arXiv:1506.09077

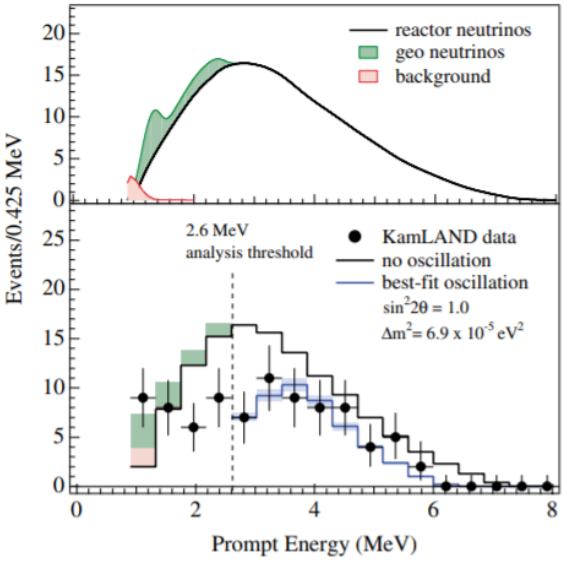
KamLAND (2001 - ~2011)



- Surrounded by 57 reactor cores
  - 35% of total electricity in Japan at the time
  - Contract with all power plants to obtain relevant reactor operation information (thermal power and fission fraction history)
- Relies on reactor model for prediction
  - Luckily, θ_{12} oscillation is large
- Low statistics: ~0.5 event /day (even with the 1 kt liquid scintillator detector)
- A three-stage physics analysis as statistics grow:
  - Rate-only
  - Rate plus shape
  - Precision measurement

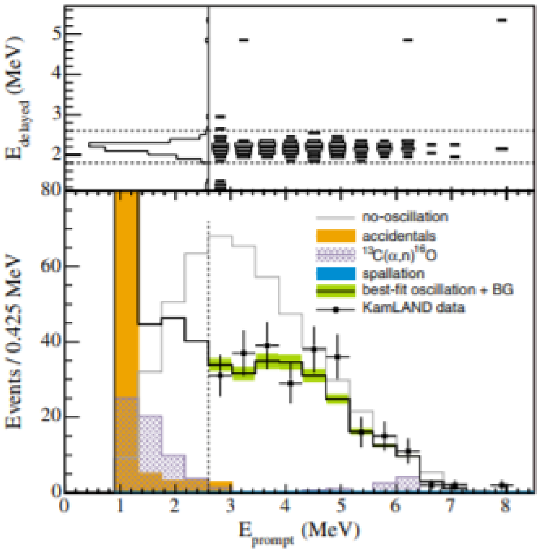
KamLAND Analysis

Phys. Rev. Lett. **90**, 021802



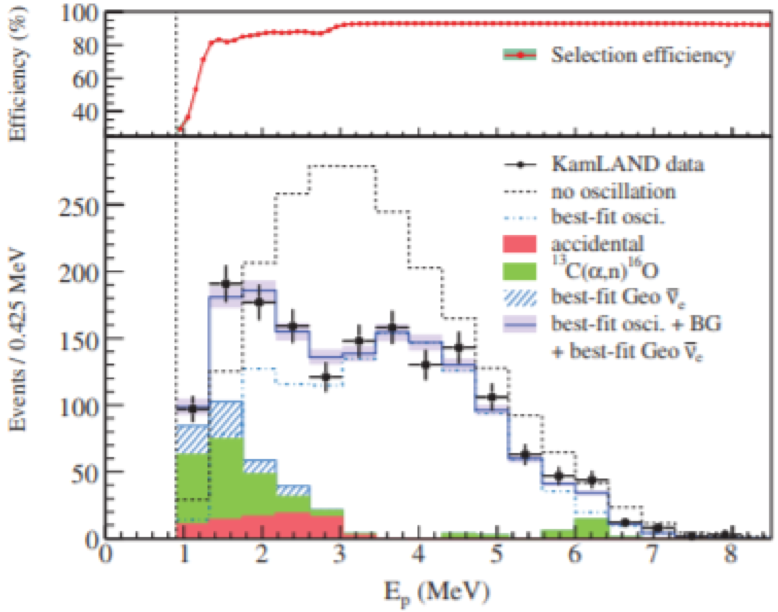
2004: 54 events
(3 +- 2 backgrounds)

Phys. Rev. Lett. **94**, 081801



2005: 258 events
(18 +- 7 backgrounds)

Phys. Rev. Lett. **100**, 221803

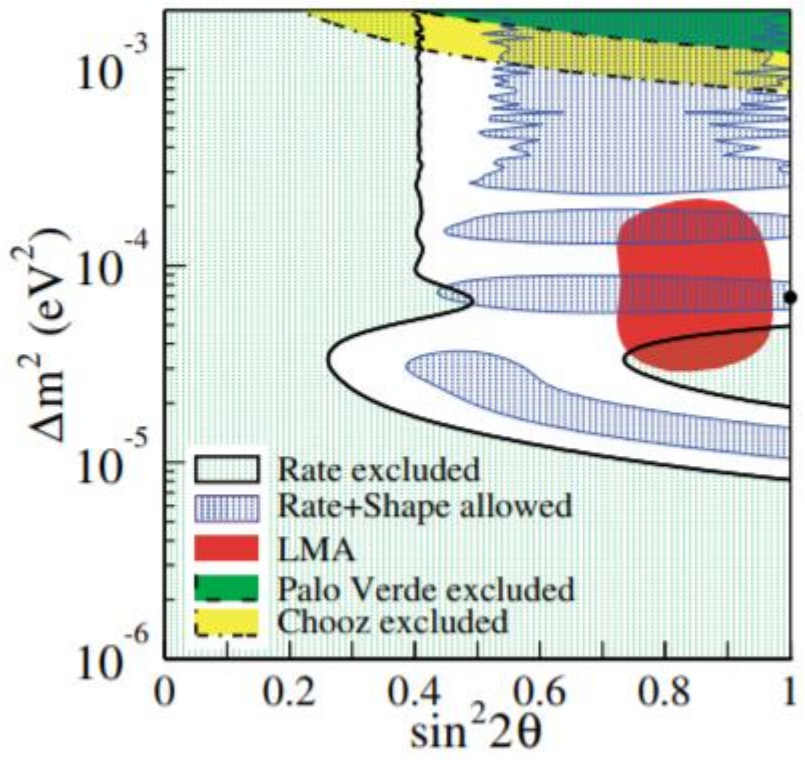


2008: 1609 events
(276 +- 23 backgrounds)

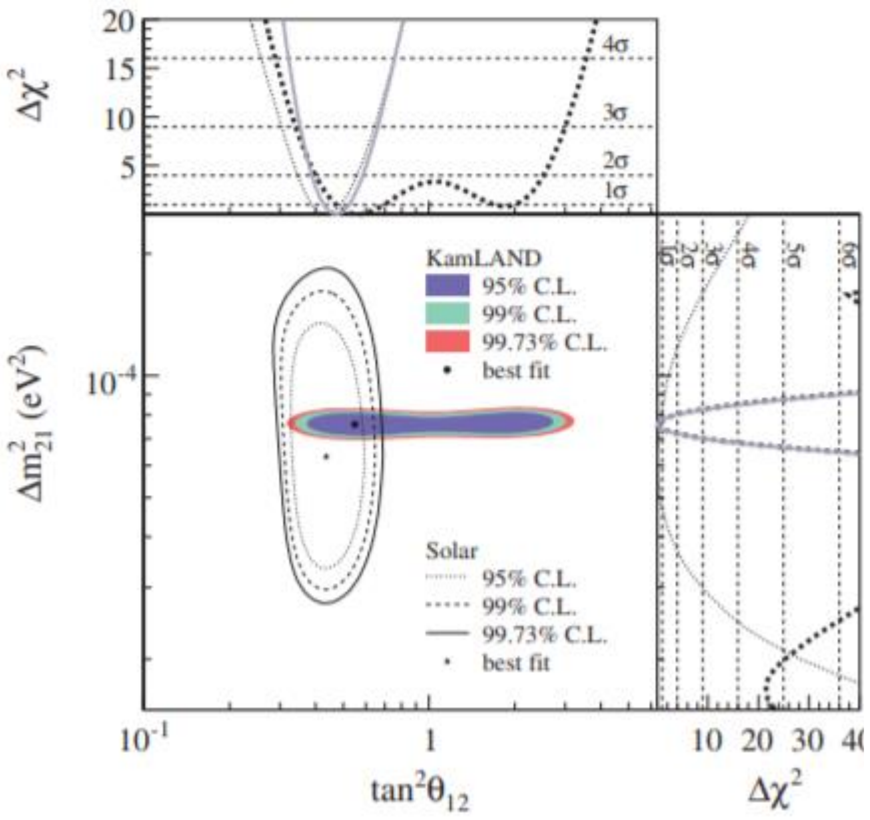
$$\chi^2 = \chi^2_{\rm rate}(\sin^2 2\theta, \Delta m^2, N_{\rm BG1\sim2}, \alpha_{1\sim4}) - 2\log L_{\rm shape}(\sin^2 2\theta, \Delta m^2, N_{\rm BG1\sim2}, \alpha_{1\sim4}) + \chi^2_{\rm BG}(N_{\rm BG1\sim2}) + \chi^2_{\rm distortion}(\alpha_{1\sim4}).$$

- Rate term: Poisson likelihood χ^2
- Shape term: multinomial likelihood χ^2 (Baker & Cousins, 1984)
- Pull terms: nuisance parameters (background, detector)
- Alternative analysis: unbinned likelihood ratio

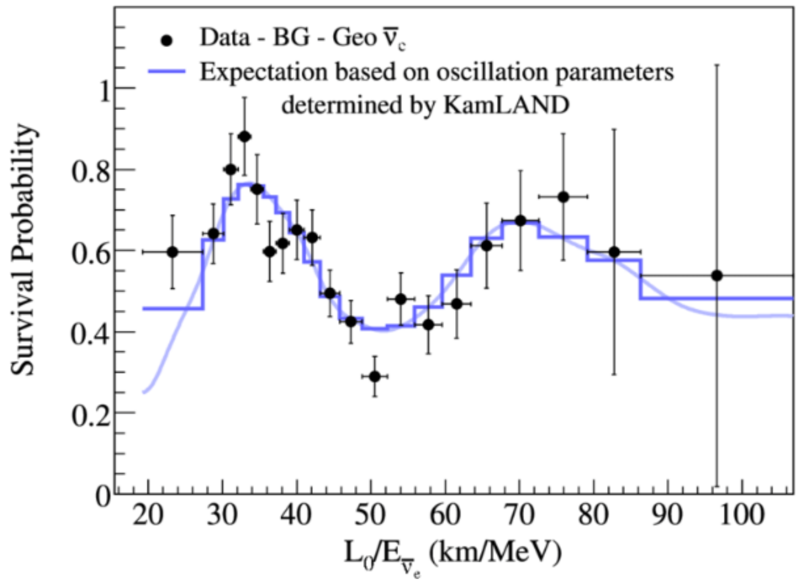
Confidence Intervals: θ_{12} and Δm^2_{21}



2004: 54 events
(3 +- 2 backgrounds)



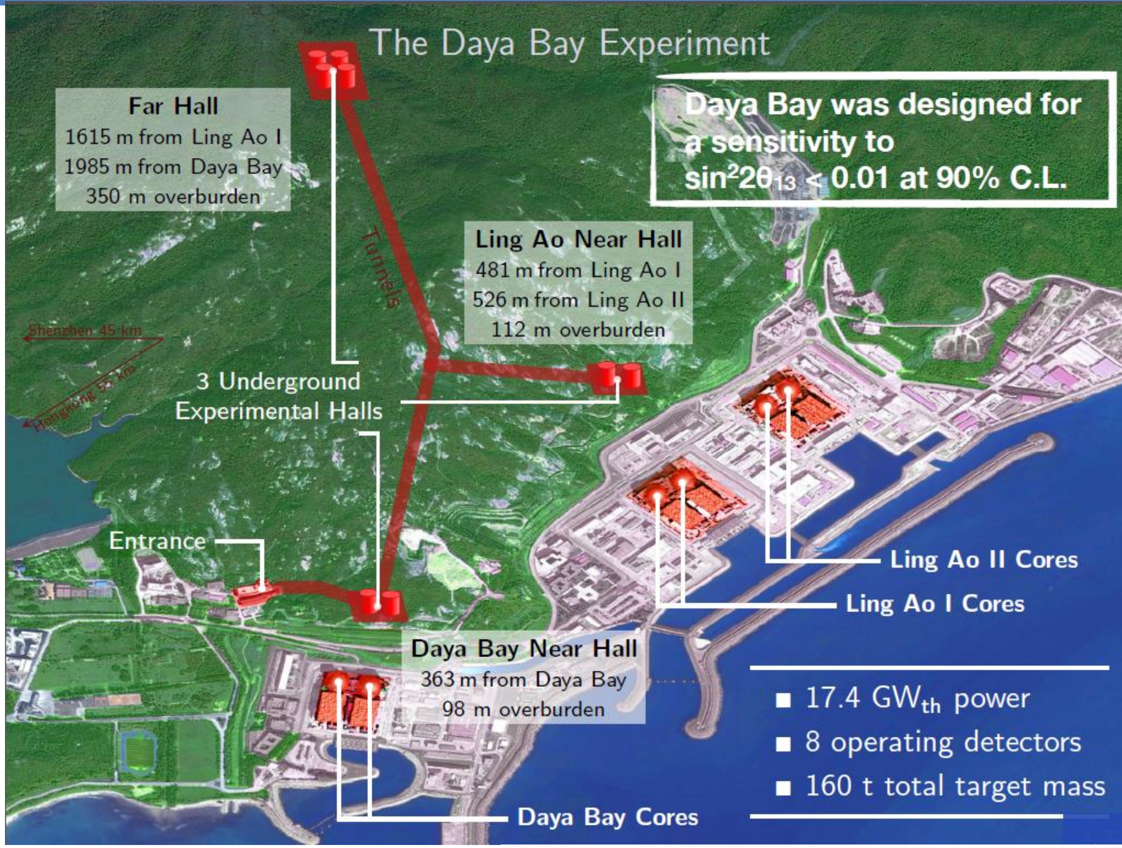
2008: 1609 events
(276 +- 23 backgrounds)

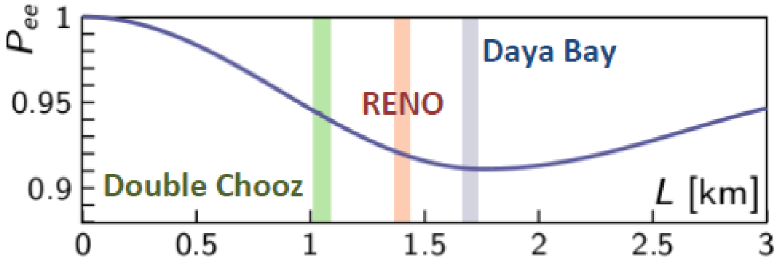


clear θ_{12}-driven L/E oscillation

θ_{13} Experiments (2011 - now): Daya Bay, RENO, Double Chooz

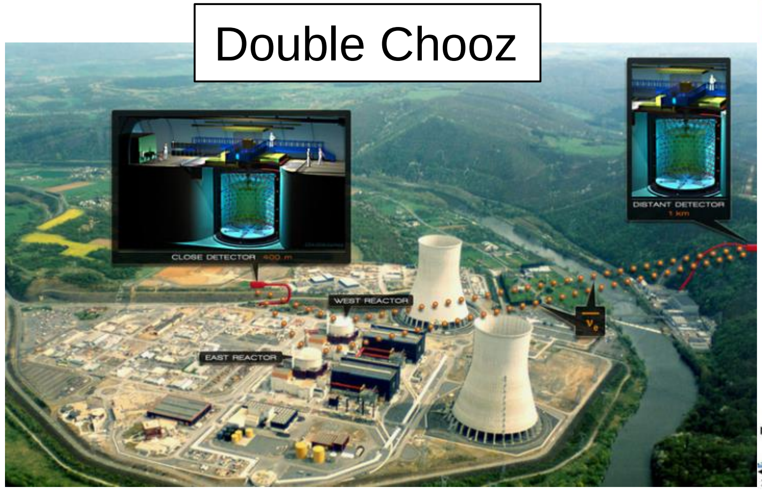
- Near/ Far measurement to cancel the reactor-related uncertainty
  - Suppression of reactor uncertainties: Daya Bay 95%, Double Chooz 88%, RENO 77%
- "Functionally identical detectors" to cancel detector-related uncertainties

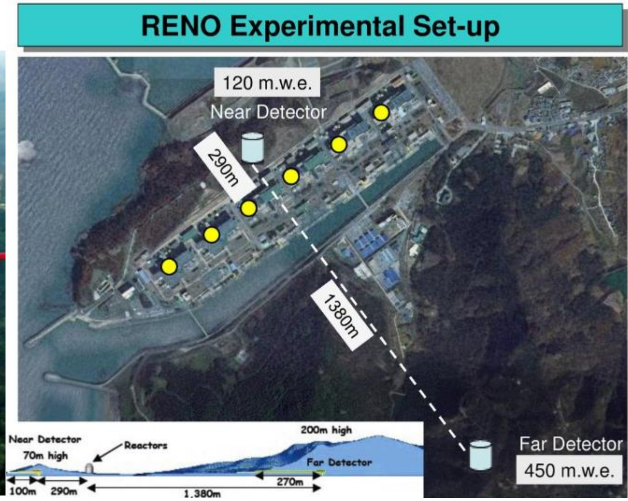




$$\frac{N_{far}}{N_{near}} = \left(\frac{N_{p,f}}{N_{p,n}}\right) \cdot \left(\frac{L_n}{L_f}\right)^2 \cdot \left(\frac{\varepsilon_f}{\varepsilon_n}\right) \cdot \left(\frac{P_{survival}(E,L_f)}{P_{survival}(E,L_n)}\right)$$

Far/Near Neutrino Ratio	Detector Target Mass	Distance from Reactor	Detector Efficiency	Survival Probability (θ_{13})





Daya Bay First Result (2012)

Rate-only, Neyman χ^2 with pull terms

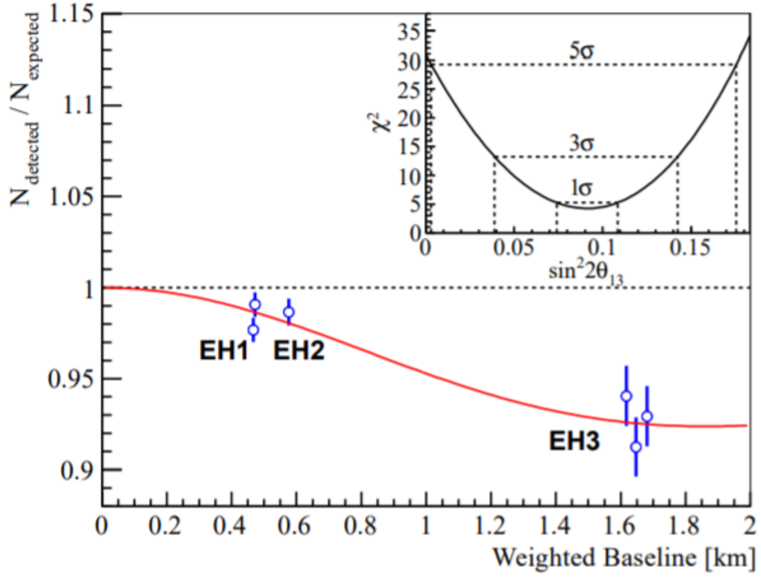
The value of $\sin^2 2\theta_{13}$ was determined with a χ^2 constructed with pull terms accounting for the correlation of the systematic errors [28],

$$\chi^2 = \sum_{d=1}^{6} \frac{\left[M_d - T_d\left(1+\varepsilon+\sum_r \omega_r^d \alpha_r + \varepsilon_d\right) + \eta_d\right]^2}{M_d + B_d} + \sum_r \frac{\alpha_r^2}{\sigma_r^2} + \sum_{d=1}^{6}\left(\frac{\varepsilon_d^2}{\sigma_d^2} + \frac{\eta_d^2}{\sigma_B^2}\right), \quad (2)$$

σ_d: uncorrelated detector uncertainty – largest systematics
σ_r: uncorrelated reactor uncertainty
σ_B: background uncertainty

ε: free parameter to absorb all correlated uncertainties

	Detector		
	Efficiency	Correlated	Uncorrelated
Target Protons		0.47%	0.03%
Flasher cut	99.98%	0.01%	0.01%
Delayed energy cut	90.9%	0.6%	0.12%
Prompt energy cut	99.88%	0.10%	0.01%
Multiplicity cut		0.02%	<0.01%
Capture time cut	98.6%	0.12%	0.01%
Gd capture ratio	83.8%	0.8%	<0.1%
Spill-in	105.0%	1.5%	0.02%
Livetime	100.0%	0.002%	<0.01%
Combined	78.8%	1.9%	0.2%



$\sin^2 2\theta_{13} = 0.092 \pm 0.016\ (\text{stat}) \pm 0.005\ (\text{syst})$

Rate + Shape Analyses

- Pure pull-term based analyses
  - Simple to construct and easy to check
  - Some systematic effects are difficult to model with nuisance parameters
  - Could be slow to minimize when number of nuisance parameters are large
- Pure covariance matrix based analysis
  - Fast minimization but need generate many MC samples varying systematics
  - Cov. matrix may depend on fitting parameters
- Hybrid approach
  - Pull terms for simpler systematics (detector, background)
  - Cov. matrix for more complicated systematics (reactor)

PRL, 108, 171803 (2012)

$$T = -2\mathrm{Log}\left(L_{stat}\right) - 2\mathrm{Log}\left(L_{sys}\right) + C$$

$$T_{stat} = 2\sum_{j}^{ADs,bin}\left(N_j^{pred} - N_j^{obs} + N_j^{obs}\cdot\mathrm{Log}\left(\frac{N_j^{obs}}{N_j^{pred}}\right)\right)$$

$$T_{sys} = T_{Detector} + T_{Background} + T_{Reactor} + T_{Oscillation}$$

$$\text{Example format } T_{sys}^{\eta} = \frac{\left(\eta - \overline{\eta}\right)^2}{\delta\eta^2} \text{ with } \mathrm{N}_j^{pred}\left(\eta\right)$$

PRL, 115, 11802 (2015)

$$\chi^2 = \sum_{i,j}(N_j^{\mathrm{f}} - w_j \cdot N_j^{\mathrm{n}})(V^{-1})_{ij}(N_i^{\mathrm{f}} - w_i \cdot N_i^{\mathrm{n}}),$$

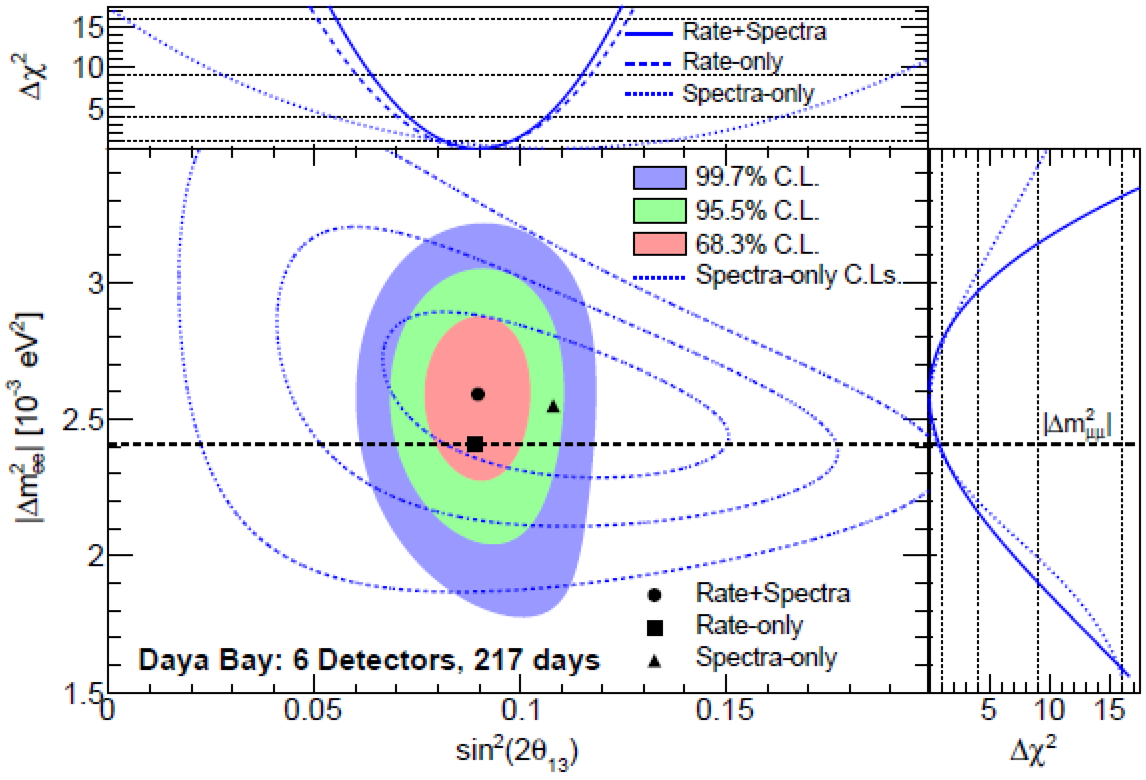
$$V_{ij} = \frac{1}{N}\sum^{N}\left(S_i^{\mathrm{f}} - w_i \cdot S_i^{\mathrm{n}}\right)\left(S_j^{\mathrm{f}} - w_j \cdot S_j^{\mathrm{n}}\right)$$

N is number of simulated experiments with varying systematics

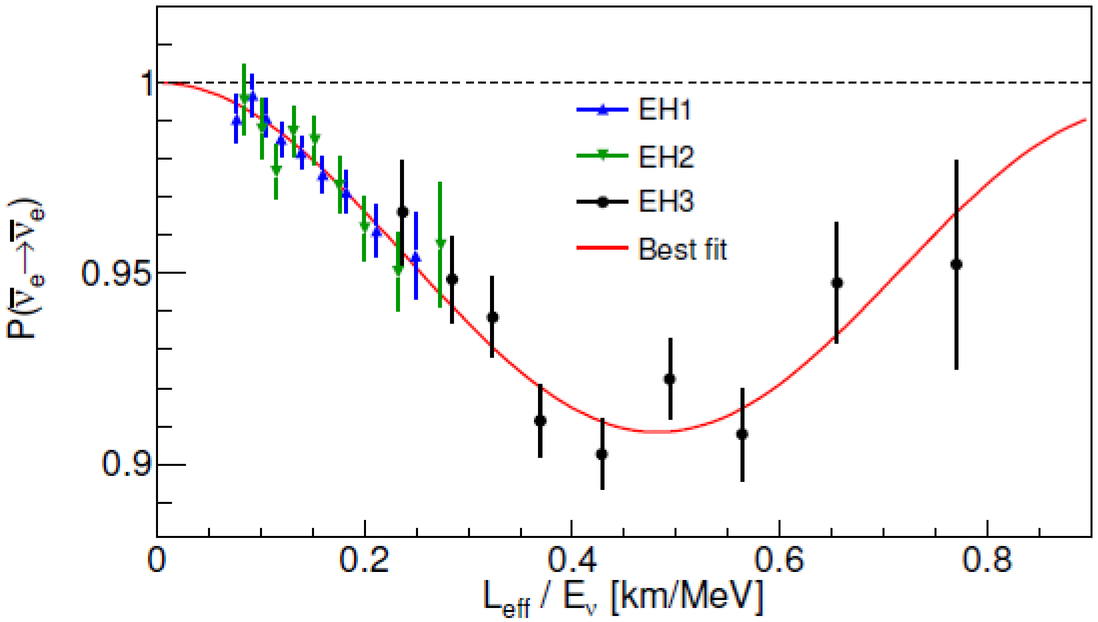
PRL,112, 061801 (2014)

$$T = -2\mathrm{Log}\left(L_{stat}\left(\eta\right)\right) - 2\mathrm{Log}\left(L_{other\ sys}\right) + \sum_{i,j}\left(\eta_i\right)\cdot\left(V_{reactor}\right)_{ij}^{-1}\cdot\left(\eta_j\right) + C$$

Confidence Intervals: θ_{13} and $|\Delta m^2_{ee}|$



PRL,112, 061801 (2014)



clear θ_{13}-driven L/E oscillation

Spectrum: “5 MeV” Bump (2014)

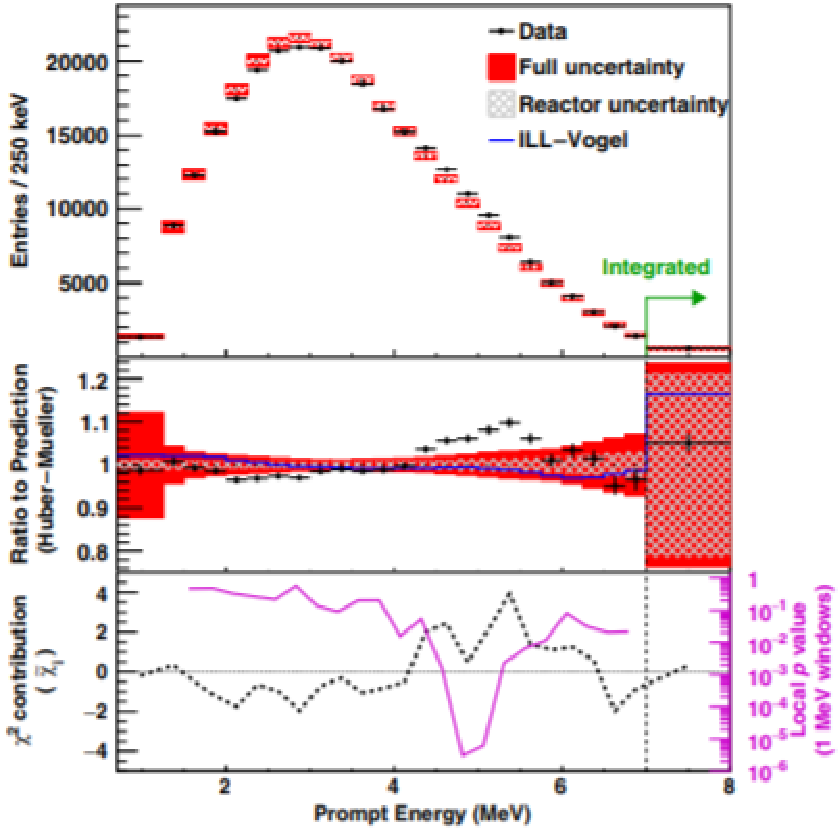
- Bump in (4 – 6 MeV) prompt energy region (when compared to the model prediction) observed in 2014 by the three θ_{13} experiments (Daya Bay, RENO, DC). Later also seen by NEOS (2016).
  - This is a shape-only comparison
- 2.6σ discrepancy for the full energy range
- 4.4σ local significance for 4~6 MeV

Nested-hypothesis test:

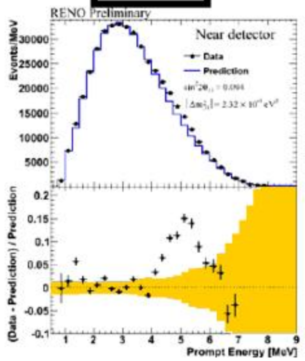
- H1: 8 additional nuisance parameters controlling the shape in 2-MeV window are added and allowed to freely vary
- H0: no additional nuisance parameters

$$\frac{\Delta\chi^2}{\Delta NDF} = \frac{37.4}{8}$$

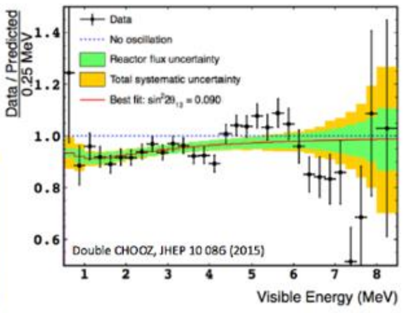
Daya Bay, Phys. Rev. Lett. **116**, 061801



RENO

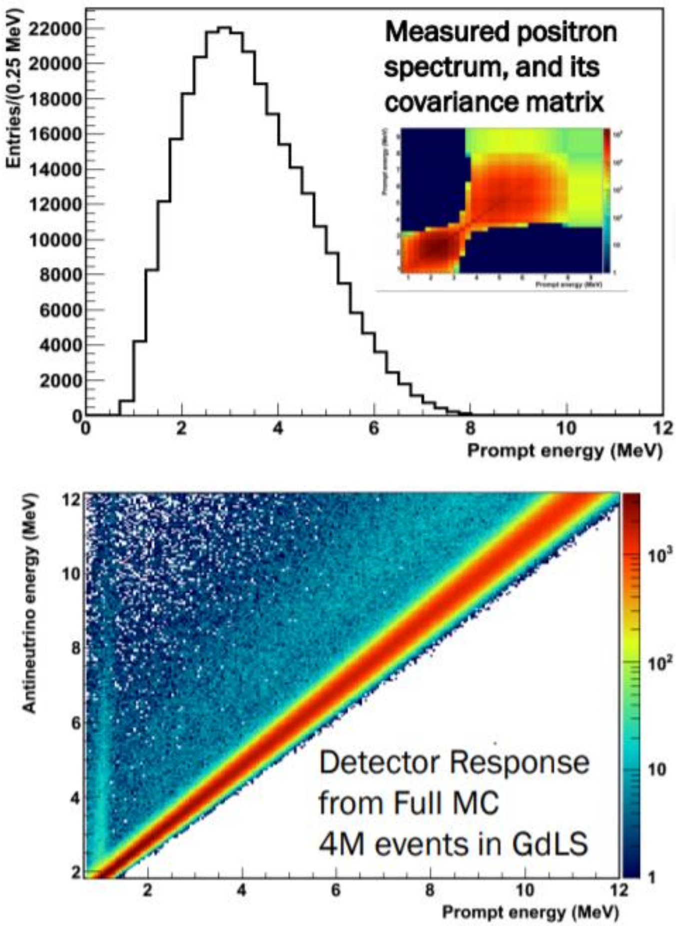


Double Chooz

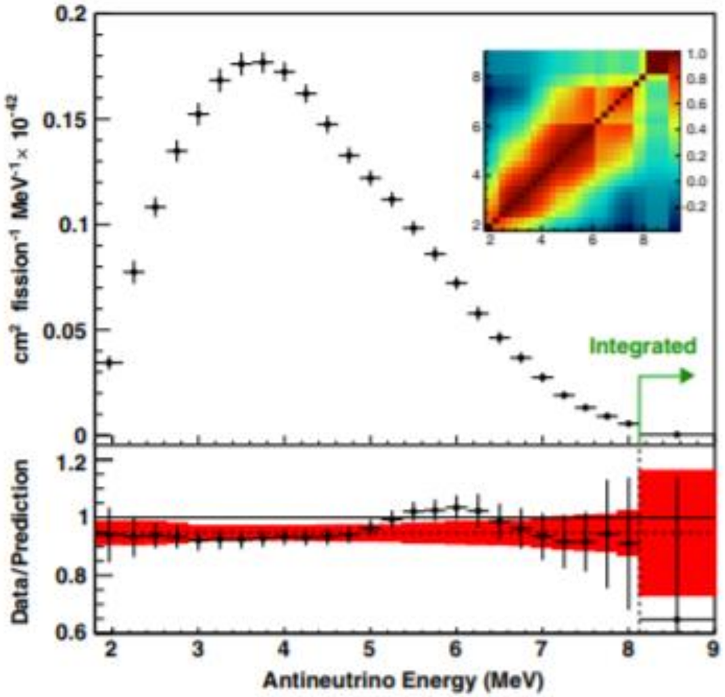


Spectrum Unfolding

Input



Output



Phys. Rev. Lett. **116**, 061801

- Unstable unfolding results caused by:
  - Fluctuation from statistical and systematic uncertainties
  - Smearing from finite energy resolution
- Methods:
  - SVD with regularizations
  - Bayesian iteration
- Bias and variance:
  - Systematic scan of regularization strength (or number of iterations for Bayesian method) with MC to balance between bias and variance

X. Li et al. JINST 12, P10002

Wiener-SVD Unfolding

- Inspired by the popular Wiener Filter used in digital signal processing to maximize signal-to-noise ratio
- Apply Wiener Filter to SVD spectrum ("effective frequency domain")
  - Construct WF based on expected signal (event counts) and noise (fluctuations)
- Advantage
  - Avoids the scanning of regularization strength
  - Naturally balances bias vs. variance, leading to a small MSE

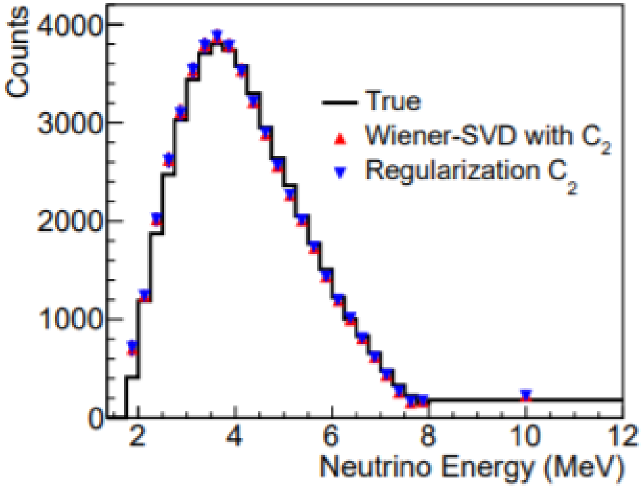
$$R = U \cdot D \cdot V^T \qquad W_i = \frac{d_i^2 \cdot s_i^2}{\overline{n_i^2} + d_i^2 \cdot s_i^2}$$

R: smearing matrix $\quad W$: Wiener filter

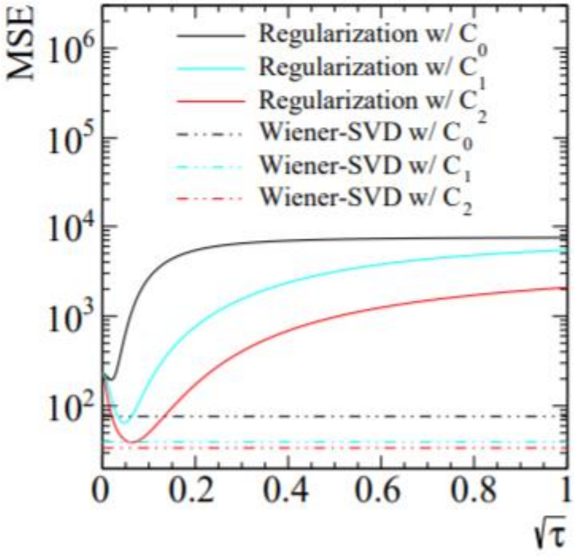
D: smearing in the effective frequency domain

s_i: expected signal in effective freq. domain

$\overline{n_i^2} \equiv 1$: (white) noise in effective freq. domain

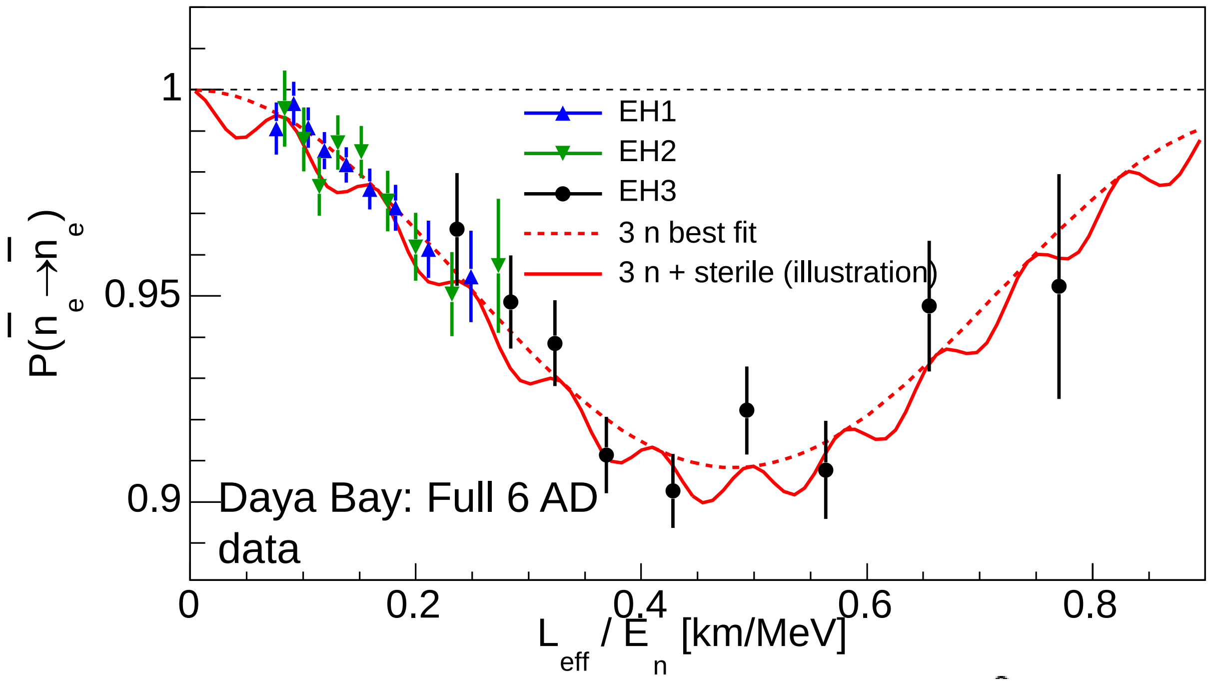


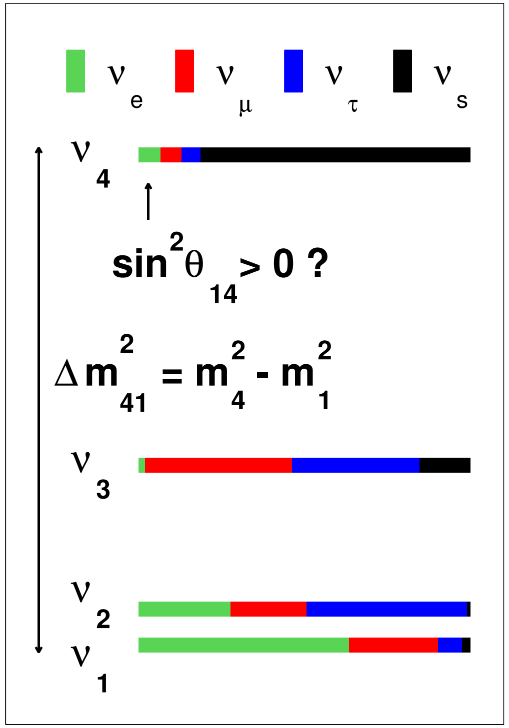
MSE = σ^2 + bias^2



Also see: X. Qian's talk on Thursday

Search for Light Sterile Neutrino Oscillation



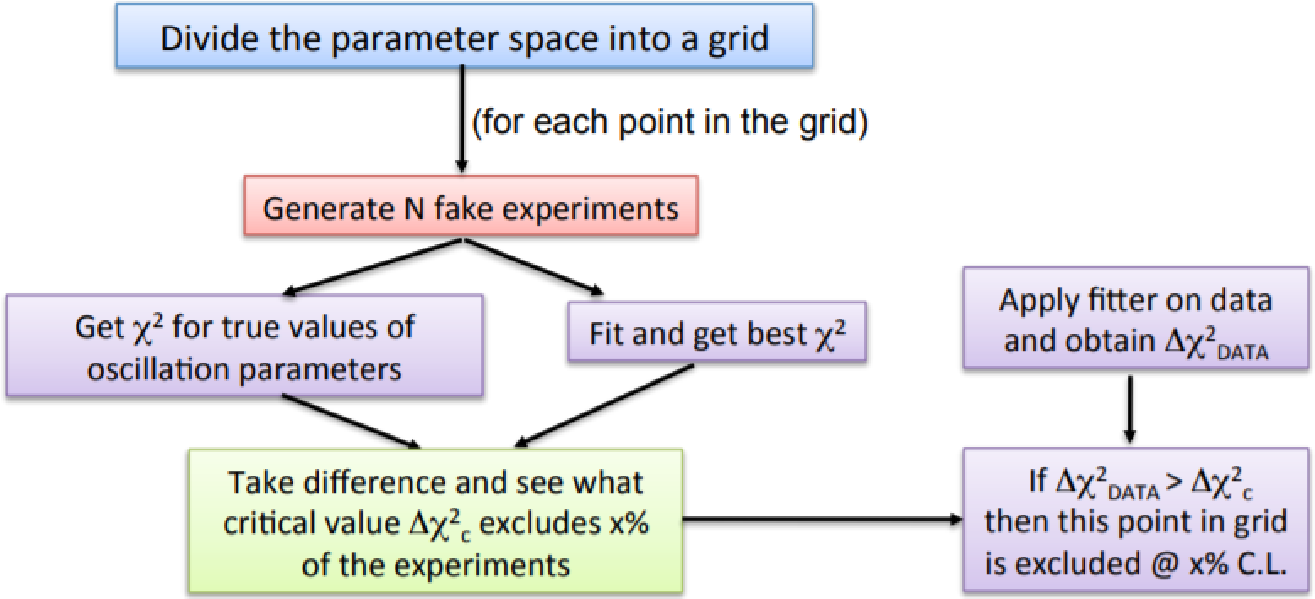


$$P(\bar{\nu}_e \to \bar{\nu}_e) \cong 1 - \cos^4\theta_{14}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{ee}L}{4E_\nu}\right) - \sin^2 2\theta_{14}\sin^2\left(\frac{\Delta m^2_{41}E}{4E_\nu}\right)$$

- A minimum extension of the 3-v model: 3(active) + 1(sterile)-v model
- Search for a higher frequency oscillation pattern besides $|\Delta m^2_{ee}|$

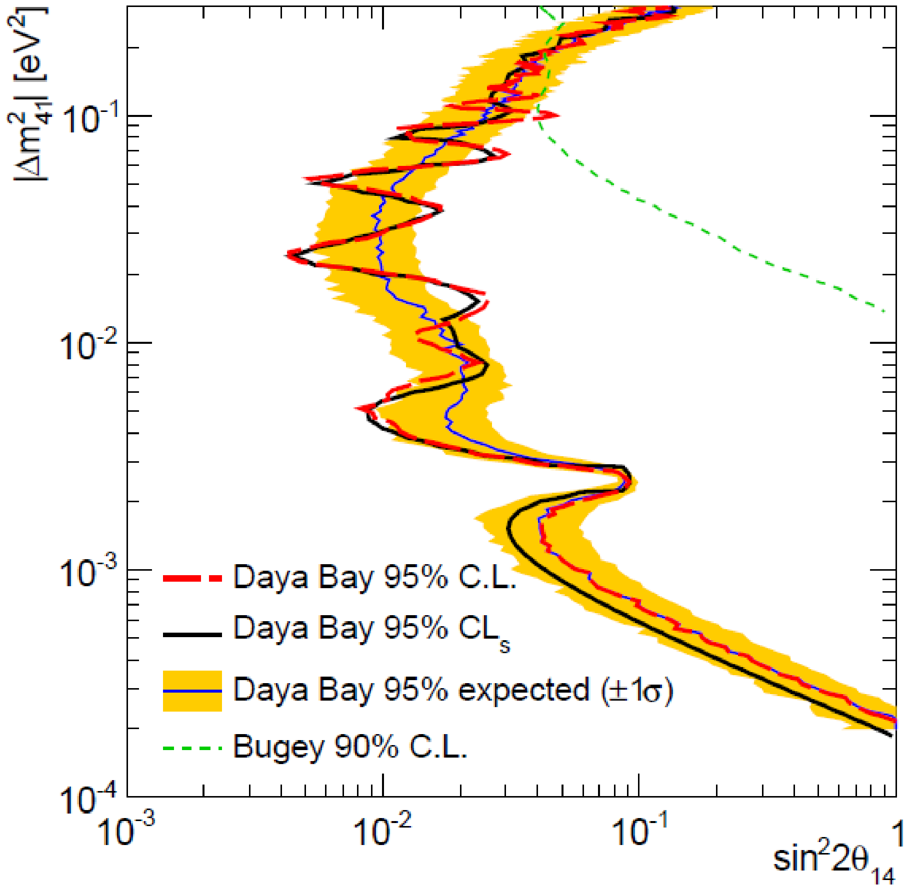
Confidence Intervals: Feldman-Cousins method

- Confidence Intervals are obtained using the Feldman-Cousins method (PRD 57, 3873 (1998))



$$\Delta\chi^2 = \chi^2(\theta_{14}, \Delta m^2_{41}) - \chi^2_{min}(\theta_{14}(min), \Delta m^2_{41}(min))$$

Could be computationally intensive for a fine grid



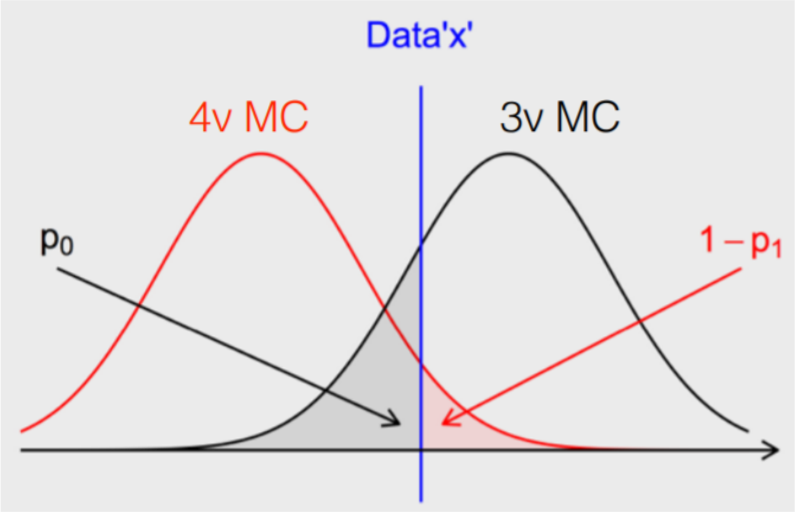
Daya Bay PRL 113, 141802, 2014

Alternative: Gaussian CL_s method to set limit

For each (θ_{14} , Δm^2_{41}) compare two hypotheses: 3ν and 4ν.

Define $\Delta\chi^2 = \chi^2_{4\nu} - \chi^2_{3\nu}$

then $CL_s = \frac{1-p_1}{1-p_0}$



For Gaussian CLs†, calculate

$\Delta\chi^2_{data}$ — data

$\Delta\chi^2_{3\nu}$ — 3ν Asimov data

$\Delta\chi^2_{4\nu}$ — 4ν Asimov data

$$CL_s = \frac{1 + Erf\left(\frac{\Delta\chi^2_{4\nu} - \Delta\chi^2_{data}}{\sqrt{8|\Delta\chi^2_{4\nu}|}}\right)}{1 + Erf\left(\frac{\Delta\chi^2_{3\nu} - \Delta\chi^2_{data}}{\sqrt{8|\Delta\chi^2_{3\nu}|}}\right)}$$

* A.L. Read J. Phys. G28, 2693
* T. Junk NIMA 434, 435
† X. Qian et al. NIMA 827, 63 (2016)

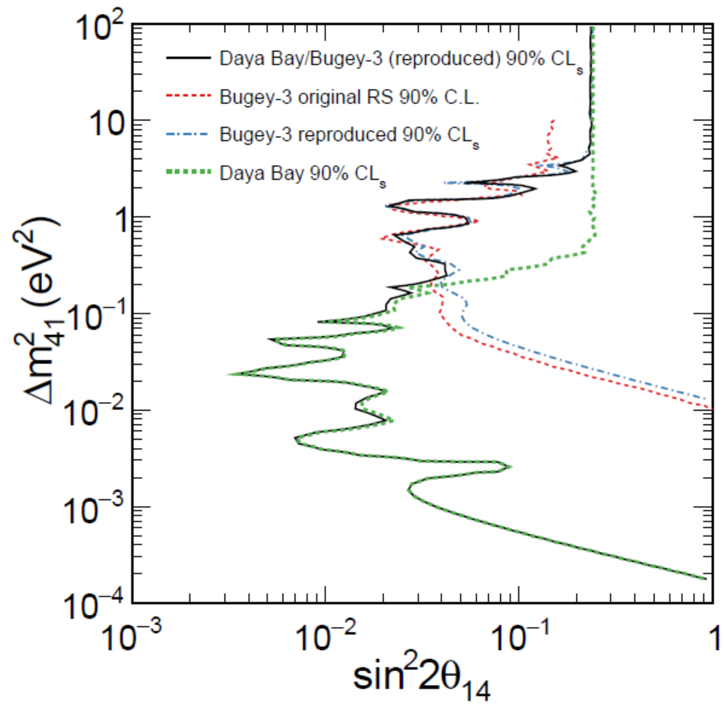
Advantage

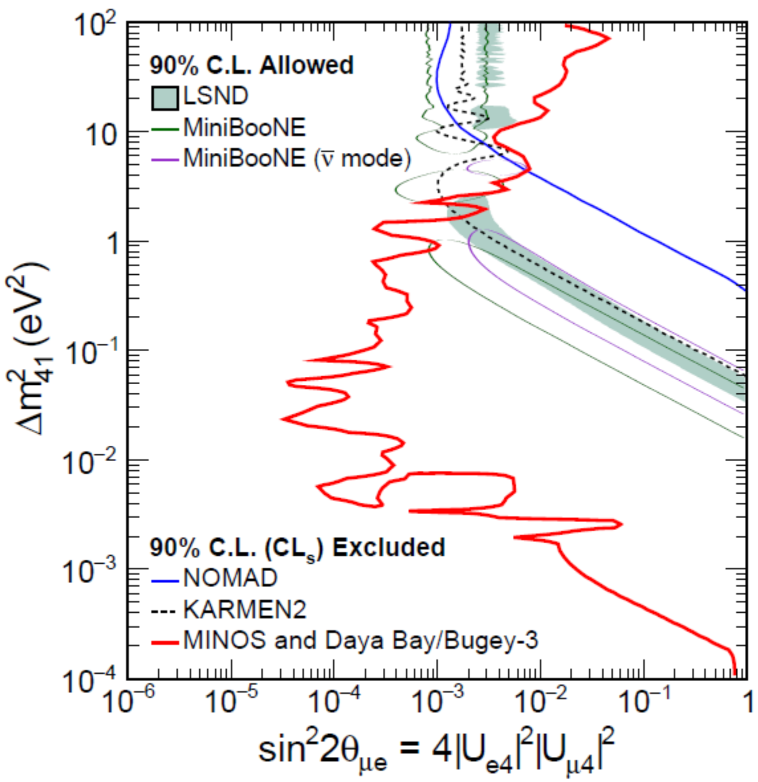
- **Computation**: for each (θ_{14}, Δm^2_{41}) only need:
  - Generate 1 Asimov MC sample
  - Perform 3 χ^2 minimizations
- **Combining experiments**
  - For proper F-C, need a combined fitter to find the global minimum of all experiments, which may be difficult
  - For CL_s, since all experiments are comparing with the same 3ν model, their $\Delta\chi^2$ values can be directly added to calculate CLs

Disadvantage: Can only be used to set exclusion regions

Example of Combining Sterile Search with CL_s

Phys. Rev. Lett. 117, 209901 (2016)





$$\Delta\chi^2_{data} = \Delta\chi^2_{data}|_{DayaBay} + \Delta\chi^2_{data}|_{Bugey}$$

$$\Delta\chi^2_{3\nu} = \Delta\chi^2_{3\nu}|_{DayaBay} + \Delta\chi^2_{3\nu}|_{Bugey}$$

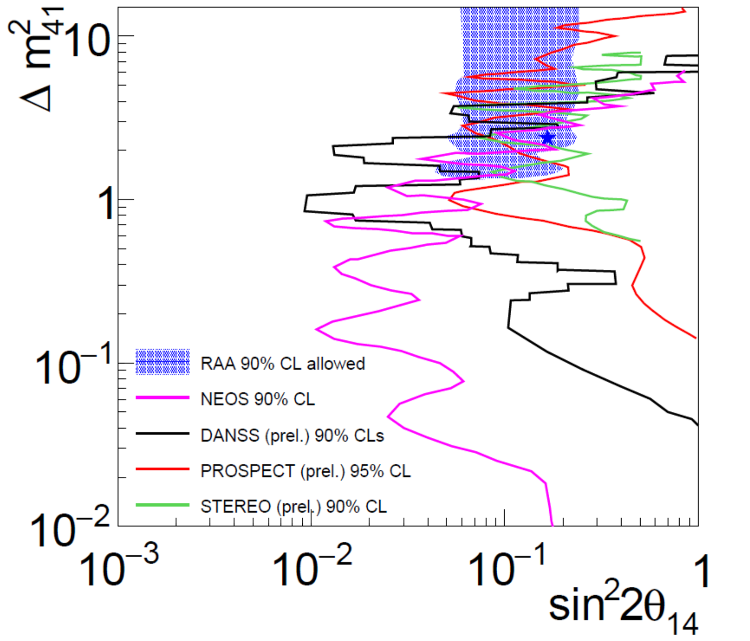
$$\Delta\chi^2_{4\nu} = \Delta\chi^2_{4\nu}|_{DayaBay} + \Delta\chi^2_{4\nu}|_{Bugey}$$

- Daya Bay/Bugey-3 and MINOS

$$\Delta\chi^2_{com} = \Delta\chi^2_{DB} + \Delta\chi^2_{M} \qquad \sin^2 2\theta_{\mu e} = \sin^2 2\theta_{14}\sin^2\theta_{24}$$

- Then calculate the CL_s value for each (Δm^2_{41}, $\sin^2 2\theta_{14}$, $\sin^2\theta_{24}$)

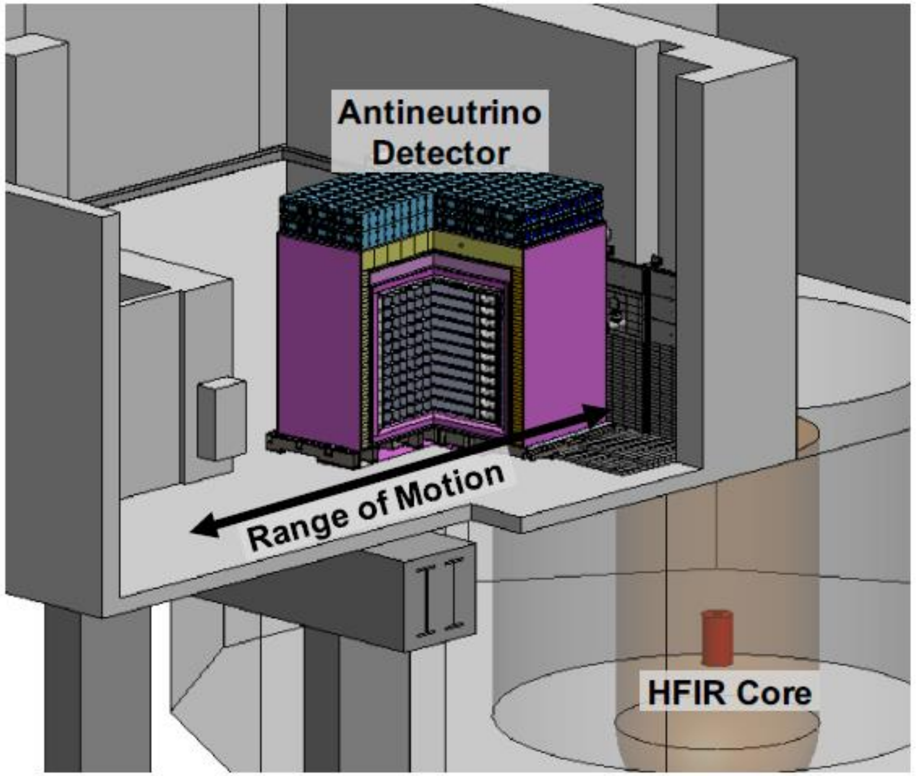
X. Qian and J-C Peng, arXiv:1801.05386, Reports on Progress in Physics

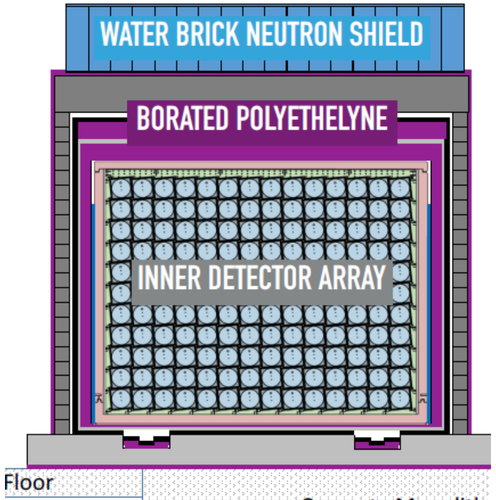


- ❑ Many future SBL reactors will have results soon.
- ❑ If no sterile neutrinos are observed, CL_s method provides an easy way to combine their results

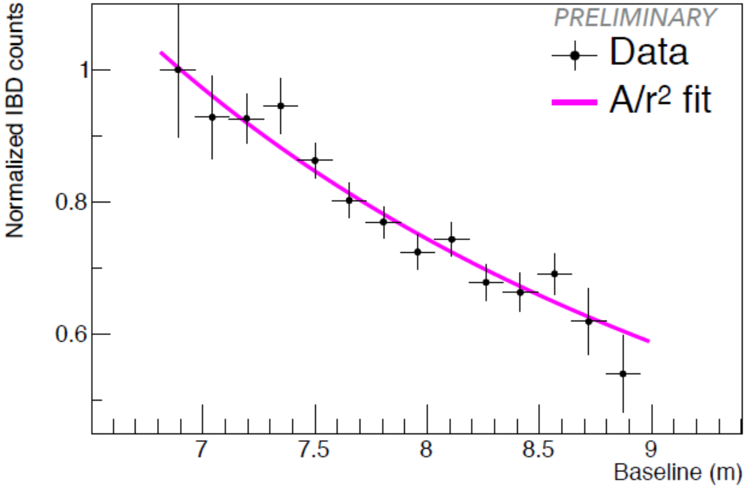
An Example Short Baseline (~m) Experiment: PROSPECT (2018 -)

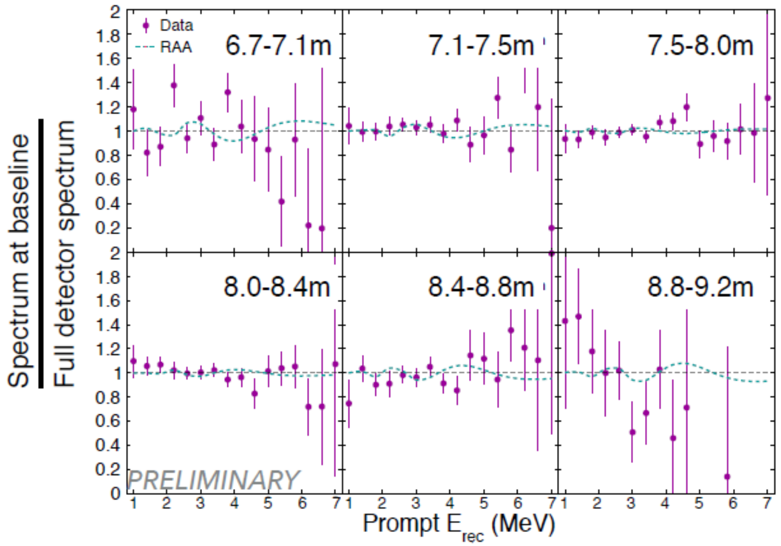
- Reactor
  - HIFR research reactor at ORNL, U.S.A
  - Baseline: 7-12 m
- Detector
  - 154 segmented cells ($15 \times 15 \times 119\ cm^3$) with 0.1% ^{6}Li-loaded LS: total 4 ton
- Unique statistical issues
  - <u>Low anti-v rate per segment</u>: ~ 6 /day
  - <u>Relatively high background</u>: S:B = 1.36
    - Mostly from cosmic neutrons (overburden < 1 m.w.e)
    - Use reactor-off period to measure
  - <u>Complicated detector systematics</u>





PROSPECT





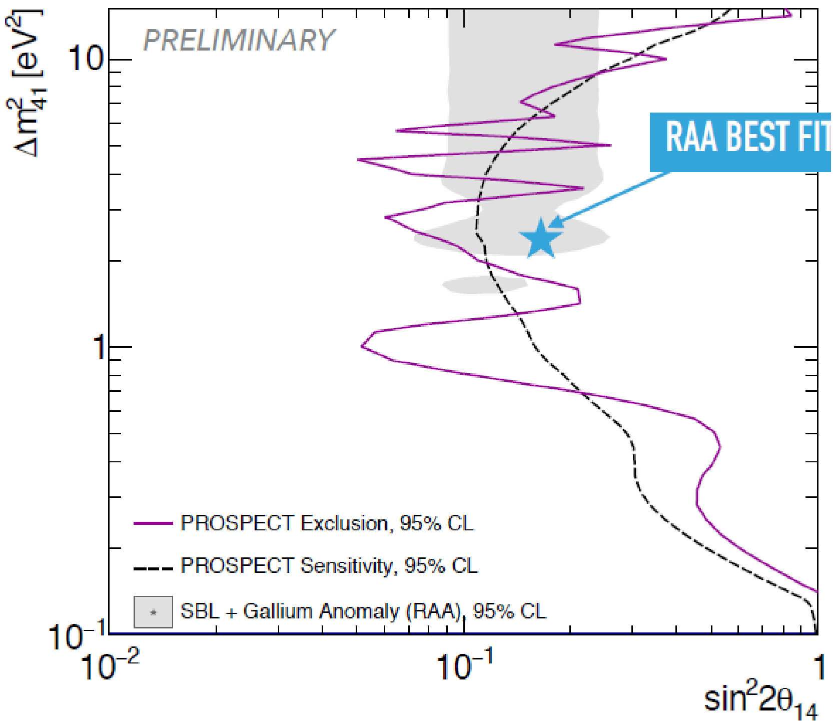
33 days reactor on (25k ν events)
28 days reactor off

$$\chi^2 = \Delta^T V_{tot}^{-1} \Delta.$$

$$\Delta_{l,e} = M_{l,e} - M_e \frac{P_{l,e}}{P_e}.$$

Δ is a 96-element vector representing the agreement between measurement and prediction in 6 position bins and 16 energy bins

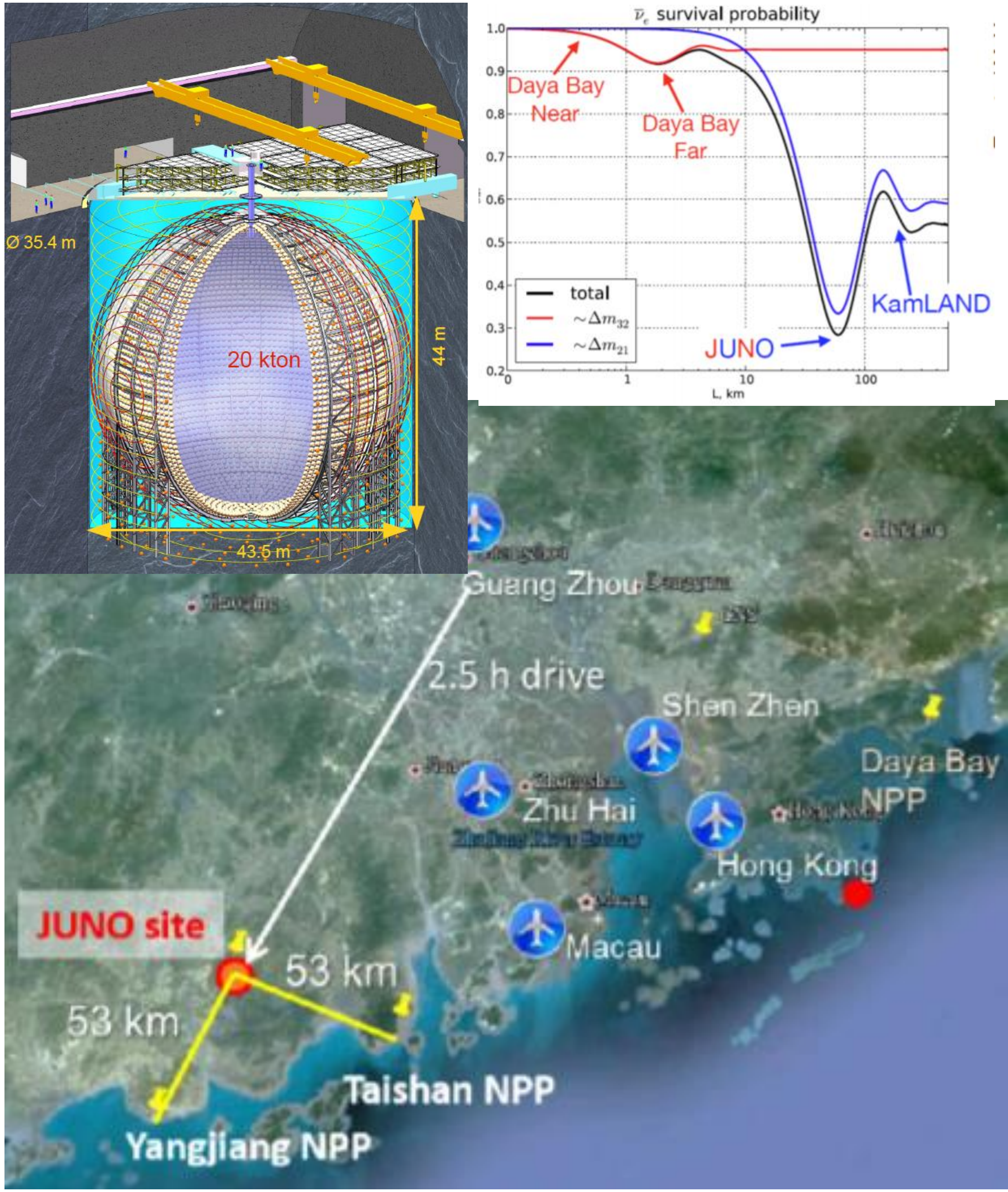
Phys. Rev. Lett. 121, 251802 (2018)



- Shape-only comparison
- Pure covariance matrix approach
  - Group the segments into 6 baseline groups to increase statistics of each group
- Alternative analysis: segment-by-segment fitter
  - Use combined Neyman + Pearson χ^2 to approximate Poisson χ^2 in low statistics (to reduce number of nuisance parameters from backgrounds)
- F-C and Gaussian CL_s methods were used to set exclusion regions

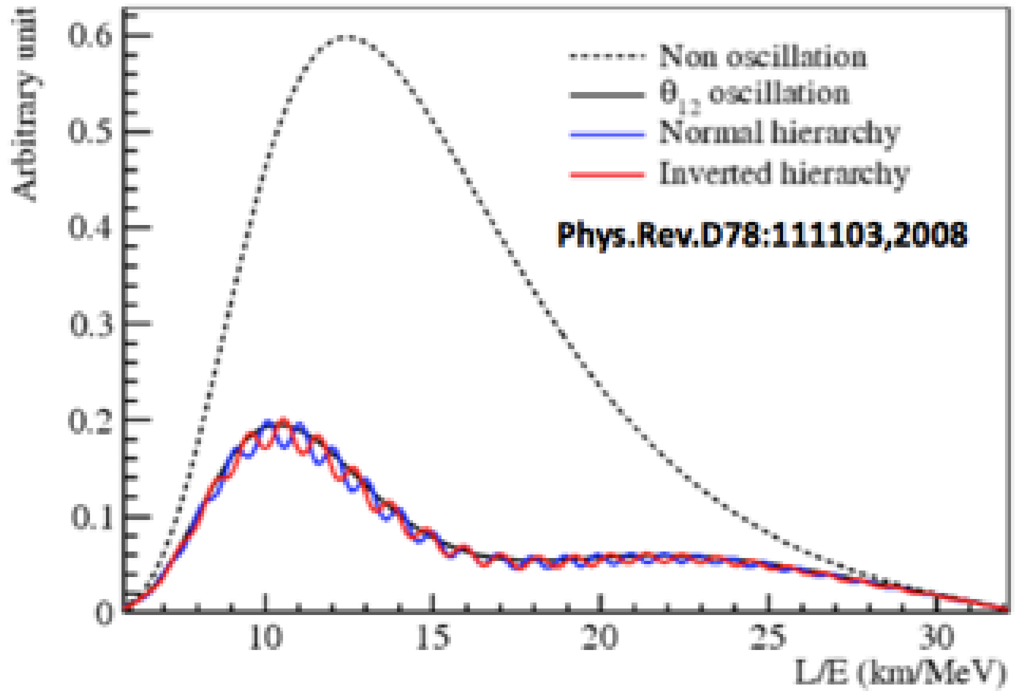
JUNO (2021 -)

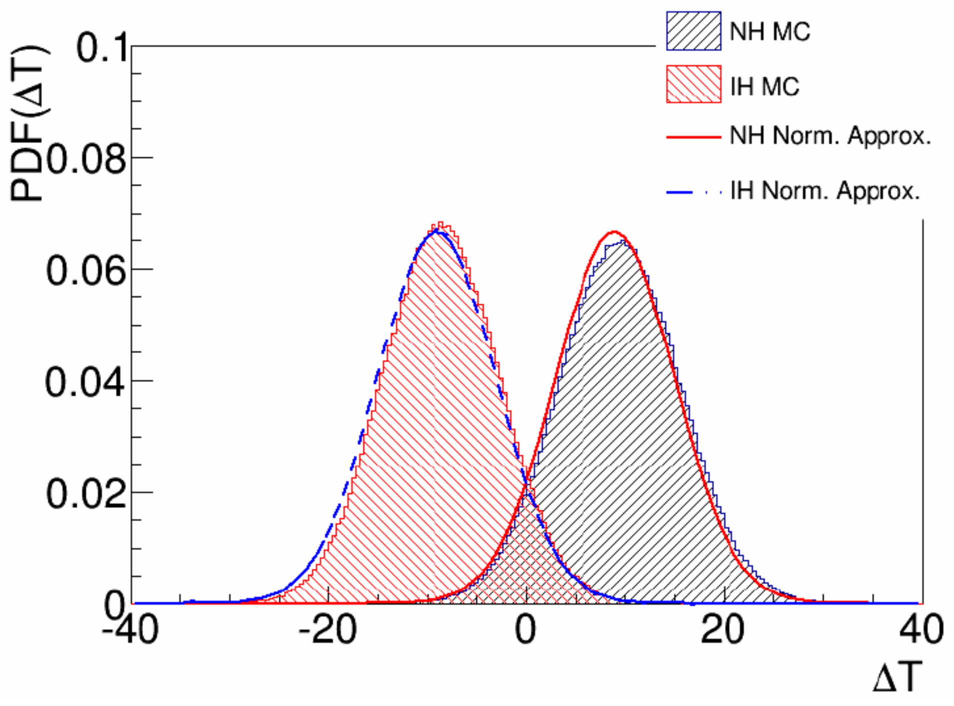
- Reactor neutrino experiment in China
  - Optimized baseline at 53 km from two large Nuclear Power Plants (36 GW_{th} total)
  - 20 kt liquid scintillator detector
- Expect ~60 reactor ν/day, ~4 bkg/day
- Key detector features
  - ~3% energy resolution (~80% photo-coverage)
  - <1% energy scale calibration
- Expect data taking 2021
- >3σ sensitivity to MH in 6 years.
  - Can reach >4σ with 1% constraint on $\Delta m^2_{\mu\mu}$ from future accelerator experiments



Mass Hierarchy Determination

MH determination (NH or IH) is a non-nested hypothesis test





- Typically define the test statistics as $\Delta\chi^2 = \chi^2_{IH} - \chi^2_{NH}$, which will then follow Gaussian distribution $N(\overline{\Delta\chi^2}, 2\sqrt{\overline{\Delta\chi^2}})$ under large statistics
- $\overline{\Delta\chi^2}$ (median sensitivity) is often quoted when comparing MH sensitivity between different experiments

See: X. Qian et al, PRD 86, 113011 (2012); M.Blennow et.al, JHEP 03, 028 (2014) among others

- MH sensitivity comes from the interference between the two slightly different Δm^2_{31} and Δm^2_{32}
- Largest systematics: energy resolution and energy scale uncertainty

See: Emilio & Fatma's talks on Thursday

Summary

- Reactor neutrino experiments play important roles in studying neutrino oscillations in the past and future
  - Long baseline: KamLAND
  - Kilometer baseline: Daya Bay, RENO, Double Chooz
  - Very short baseline: NEOS, DANSS, PROSPECT, STEREO, etc.
  - Medium baseline: JUNO
- Reactor neutrino analyses involve a broad range of statistical topics
  - parameter estimation, unfolding, hypothesis testing, etc.
  - Understanding of the statistical issues involved is very important.