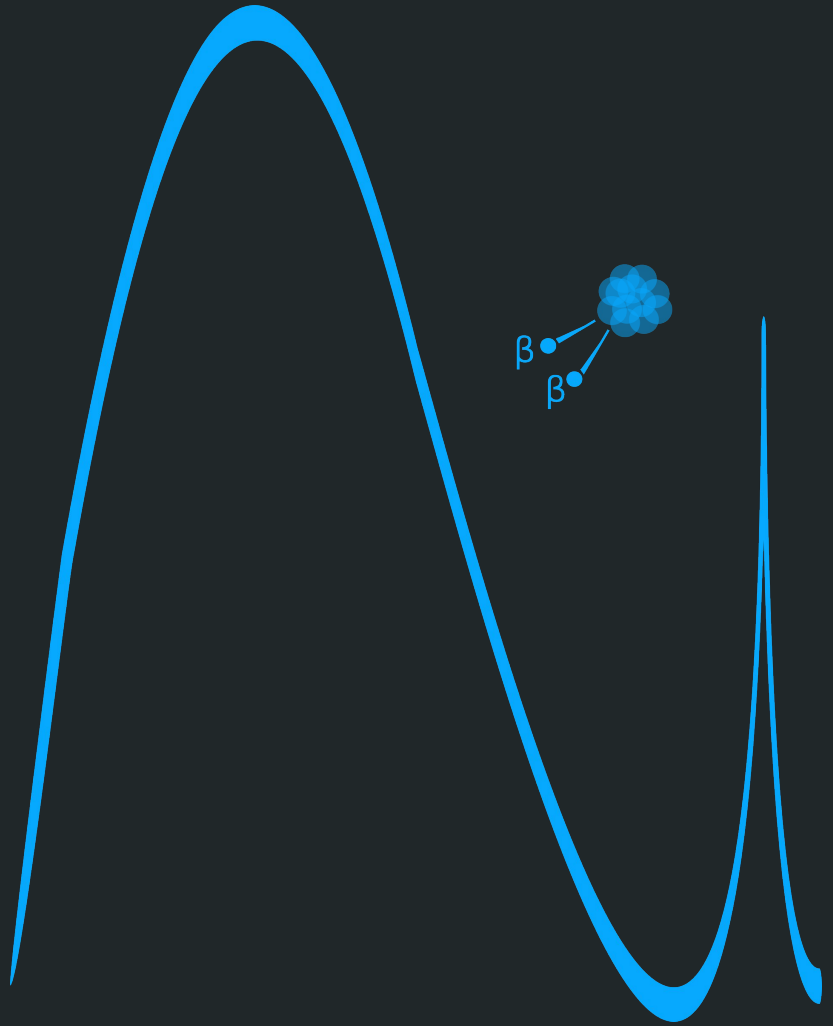



# Statistical Issues & Methods in Neutrinoless Double-β Decay Experiments

Matteo Agostini

Munich Technical University (TUM)

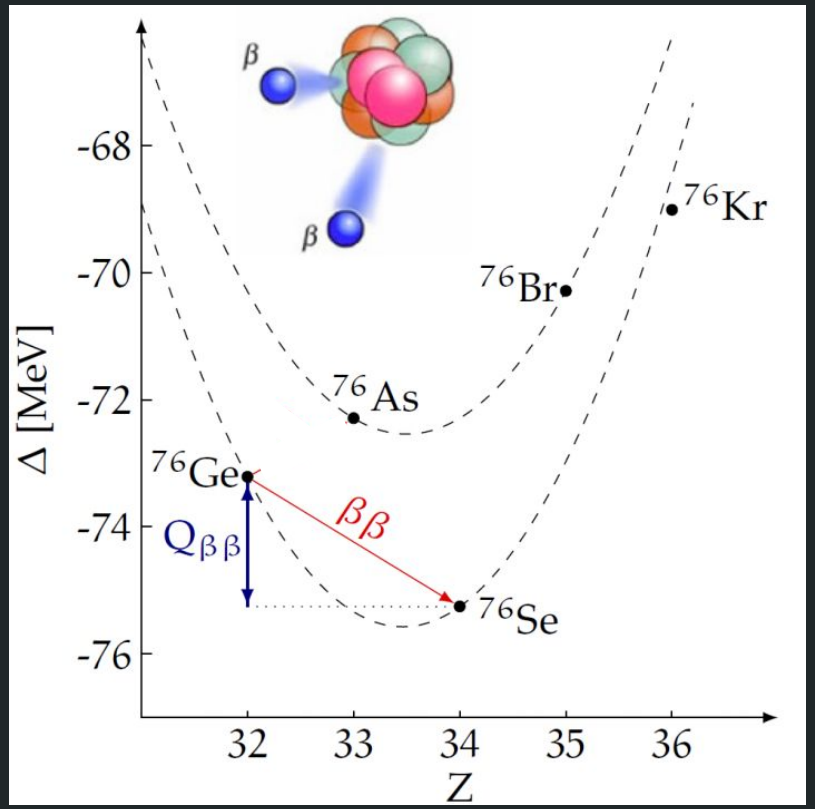
PhyStat-nu, CERN, Jan 22-25, 2019

# Neutrinoless Double-$\beta$ Decay ($0\nu\beta\beta$)

From the point of view of Nuclear Physics:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$

- ➢ second order nuclear transition
- ➢ atomic number Z increased by two units




[Courtesy of G. Benato]

# Neutrinoless Double-$\beta$ Decay ($0\nu\beta\beta$)

From the point of view of Nuclear Physics:

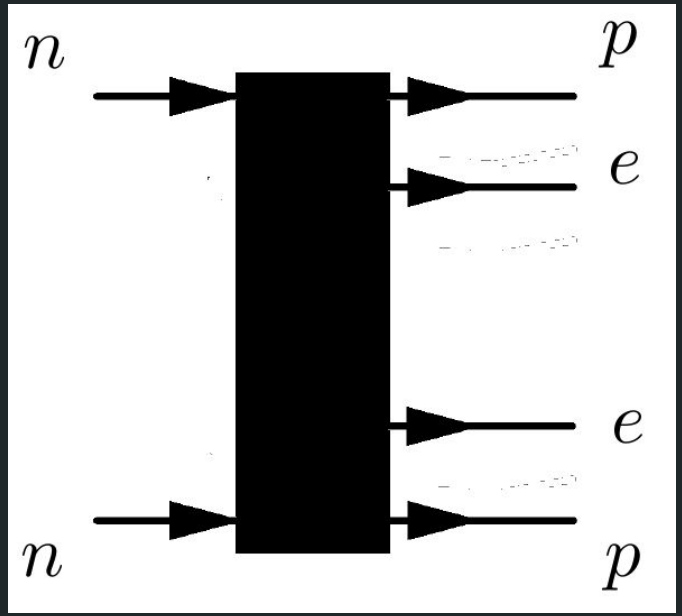
$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$

- ➢ second order nuclear transition
- ➢ atomic number Z increased by two units

From the point of view of Particle Physics:

$$2n \rightarrow 2p + 2e^-$$

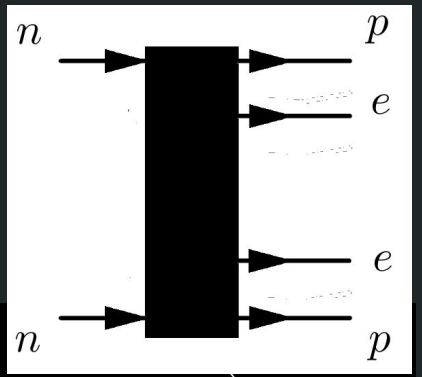
- ➢ channel depends on new physics
- ➢ 2 leptons produced w/o balancing anti-leptons

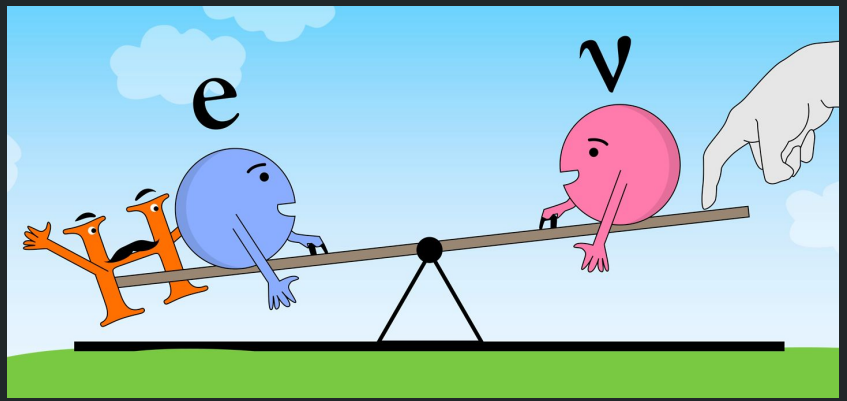


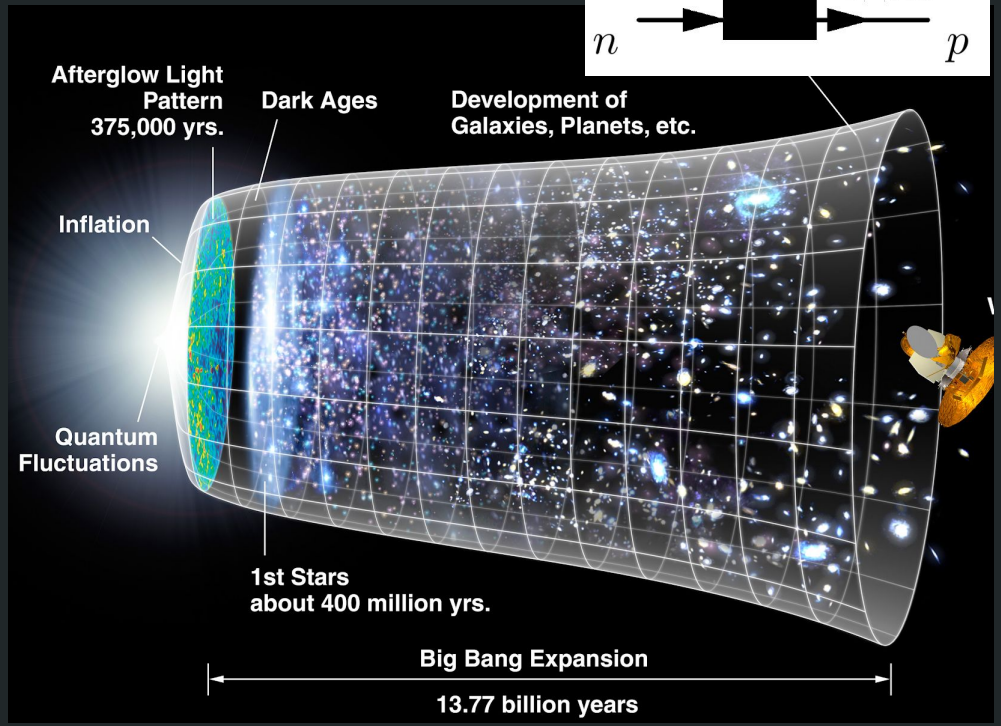
# Neutrinoless Double-$\beta$ Decay ($0\nu\beta\beta$)

**If $0\nu\beta\beta$ decay is discovered:**

- ➢ first observation of “matter creation"
  - ➢ matter-antimatter asymmetry
- ➢ neutrinos are their own antiparticle
  - ➢ origin neutrino masses



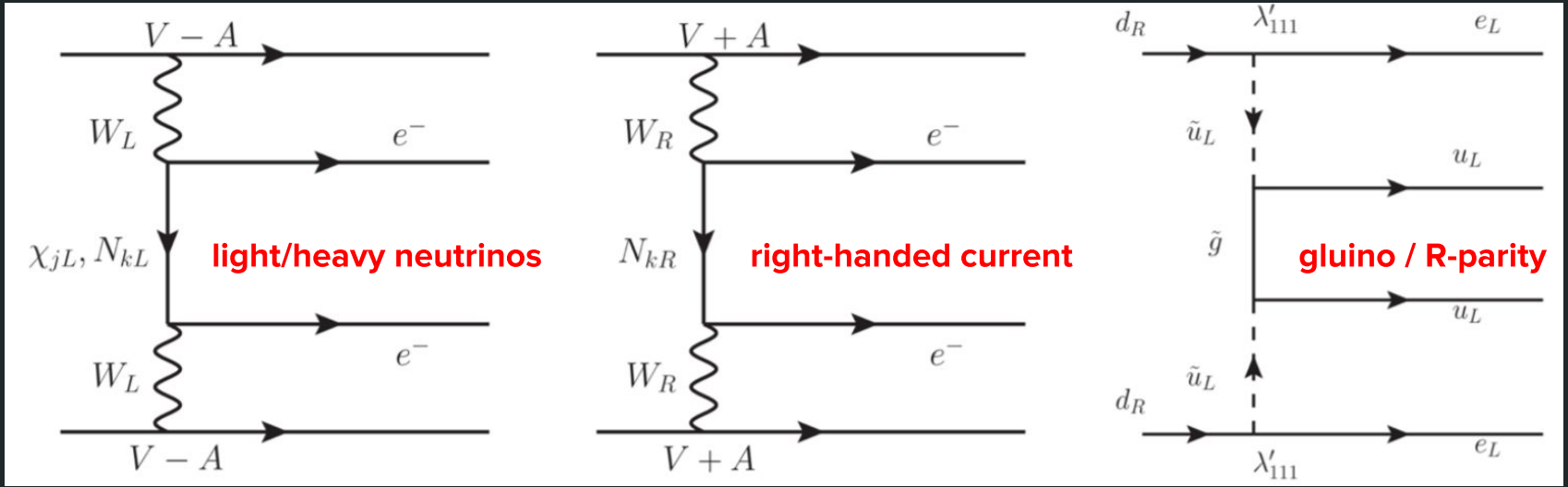

[www.symmetrymagazine.org]

# A portal to Physics beyond the Standard Model

Decay probability proportional to coherent sum of involved mechanisms:

$$\Gamma = \frac{1}{T_{1/2}} = G(Q, Z) \times \left| \sum_i \mathcal{M}_i \times \eta_i \right|^2$$

Phase Space Factor | Nuclear Physics | Propagator






[Faessler et al, PRD, 83, 11 (2011), 113003]

# A portal to Physics beyond the Standard Model

Decay probability proportional to coherent sum of involved mechanisms:

$$\Gamma = \frac{1}{T_{1/2}} = G(Q, Z) \times \left| \sum_i \mathcal{M}_i \times \eta_i \right|^2$$

Phase Space Factor | Nuclear Physics | Propagator

Propagator factor:

- Connects decay probability to energy scale of new physics
- $T_{1/2}$ is for $0\nu\beta\beta$ decay what the **collision energy** is for **LHC**
- Scale of new physics is unknown and a signal can be around the corner -> important to increase the sensitivity on $T_{1/2}$

Strongest constraints

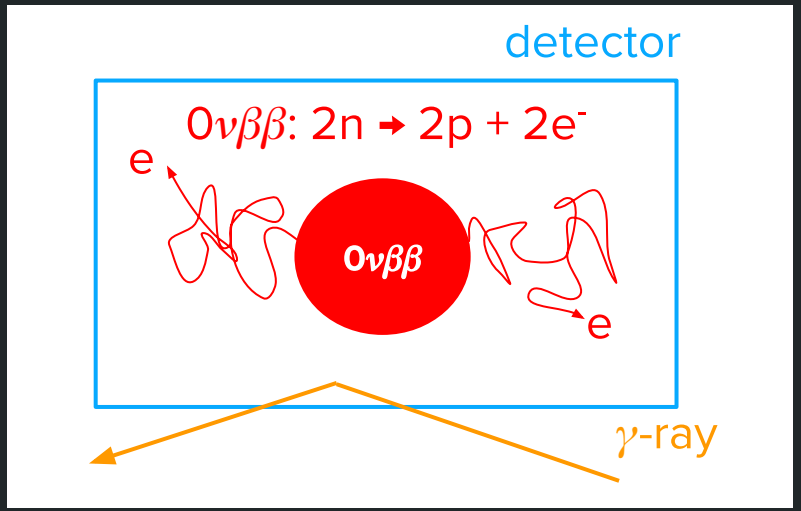
$T_{1/2} > 10^{26}$ yr

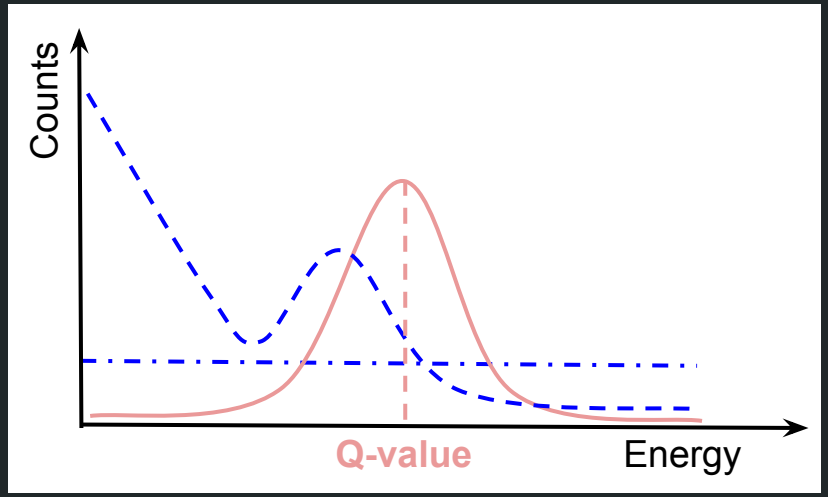
i.e. more than a **million trillion** times the age of the Universe!

# Experiment design

Detection approach:

- source in detector active material
- detector acts as a calorimeter
- additional observables used to constrain background and systematics




Signal:

- spread due to energy resolution
- peak at Q-value (often Gaussian)

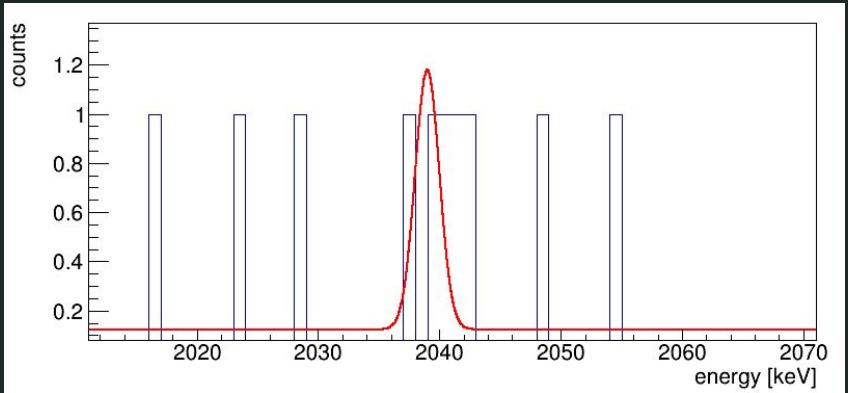
Background:

- typically flat if resolution is <0.1%
- not connected to physics mechanism generating the signal -> hard to model!

# Statistical analysis

Search for a peak at fixed position

- $\lambda_S$ signal expectation (typically very small)
- $\lambda_B$ background expectation
- **no look elsewhere**
- background control sample (on/off problem)




**1. Counting:**

$$\mathcal{L} = \mathrm{Poisson}(\mathcal{N}_{\mathrm{tot}}^{\mathrm{obs}} | \lambda_{\mathcal{S}} + \lambda_{\mathcal{B}}) \cdot \mathrm{Poisson}(\mathcal{N}_{\mathcal{B}}^{\mathrm{obs}} | \tau \lambda_{\mathcal{B}})$$

- $N_{tot}$ number of cts in signal window
- $N_B$ number of cts in control window
- $\tau$ ratio of windows size

- gives bulk of results
- window optimization is important

**2. Binned ML spectral fit:**

- full info on energy distribution
- goodness of background model

**3. Extended Unbinned ML spectral fit:**

- CPU convenient for sparse histogram
- not sensitive to background modeling

# Statistical issues

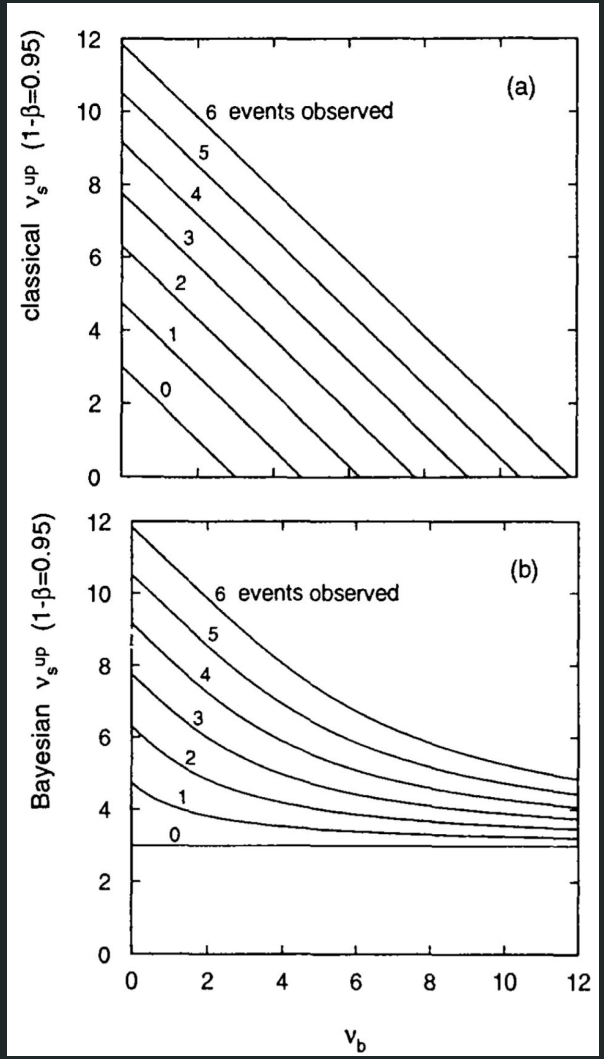
Frequentist framework:

- physical border ($\lambda_s \geq 0$)
- best fit in unphysical space when background under-fluctuates
- empty intervals emerge to preserve coverage
- stronger limits for higher background

Bayesian framework:

- low statistic -> strong impact of the prior

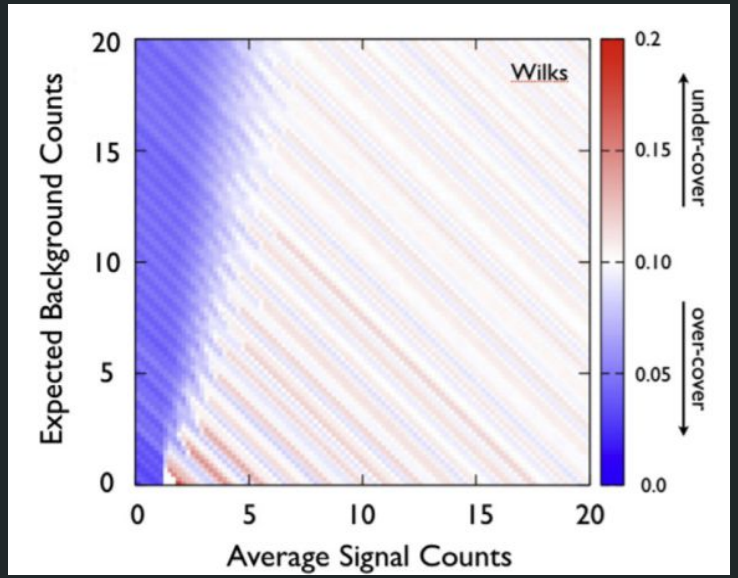
**Large deviations between different techniques!**

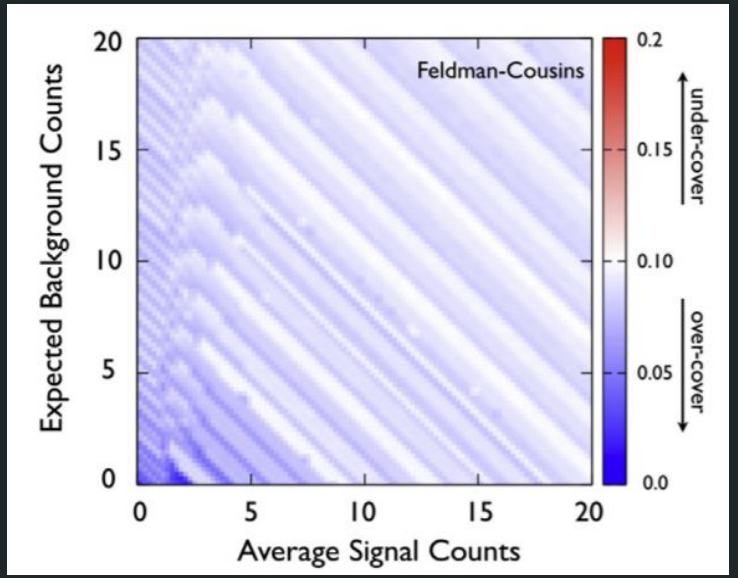



[Cowan, Statistical Data Analysis, Oxford Science Publication, 1998]

# Frequentist methods

| **Two-sided test statistic:** | **Confidence interval construction:** | **Coverage issues** |
|---|---|---|
| ➢ $\chi^2$<br>➢ profile likelihood | ➢ Wilks' or Feldman Cousins (FC)<br>➢ $\lambda_S$ bounded to positive values to avoid empty intervals | ➢ intrinsic in discrete problem<br>➢ Wilks' overcovers for $\lambda_S < \sqrt{\lambda_B}$<br>➢ FC overcovers for small $\lambda_S$ |

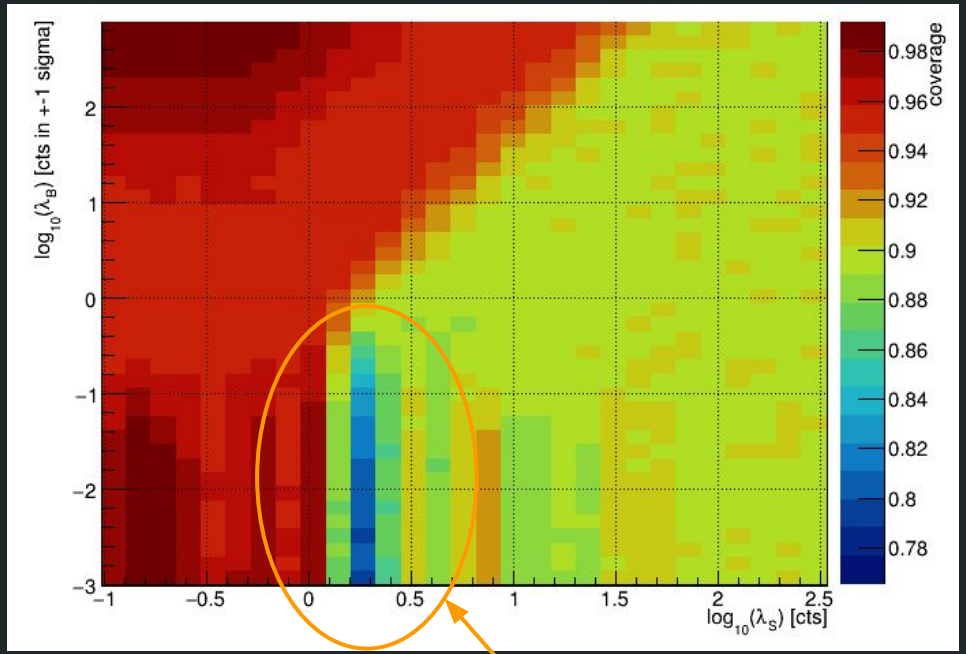



[Biller and Oser, Nucl.Instrum.Meth. A774 (2015) 103-119 ]

# “Deep Poisson” regime

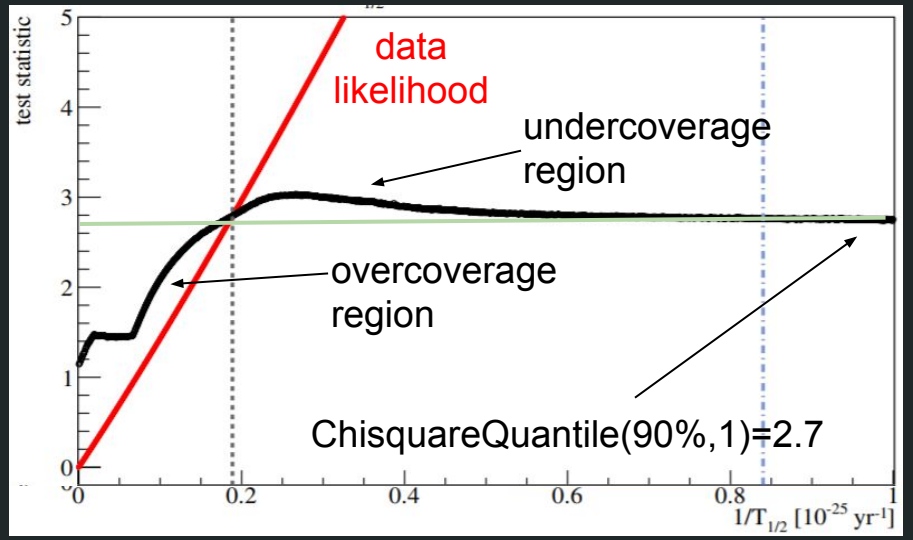
Spectral fit under Wilks’ approximation




Background-free condition

Target of many experiments

Threshold of test statistic for a 90% CL derived through a FC construction (GERDA Phase II)

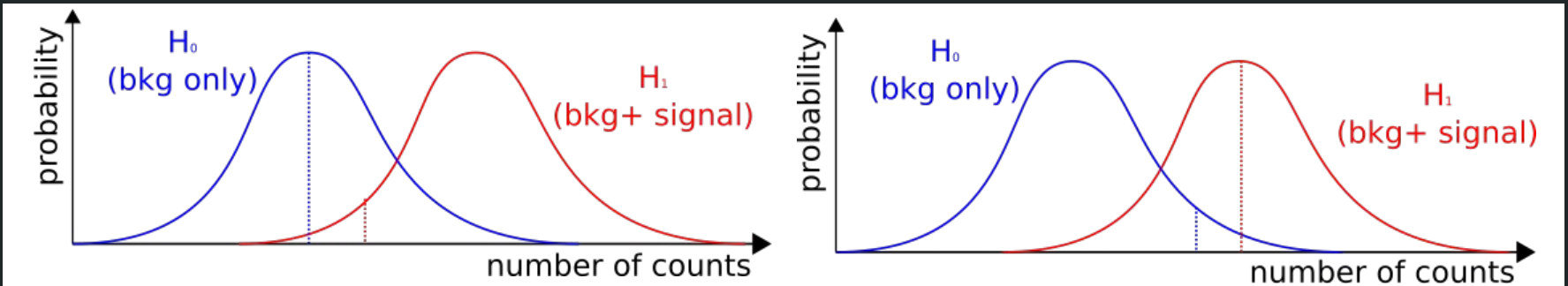



[ V. Wagner, MPIK, PhD thesis ]

# Sensitivity Definitions

| | CL | Concept | How to compute |
|---|---|---|---|
| limit setting | 90% | Assuming there is no signal, what is the expected upper limit on the signal expectation? | find signal expectations that would be rejected with a **median** significance of 90% CL assuming the null hypothesis |
| signal discovery | 99.7% (3σ) | Assuming there is a signal, how strong does it has to be to make a discovery? | find signal expectations for which the null hypothesis would be rejected with a **median** significance ≥3σ |

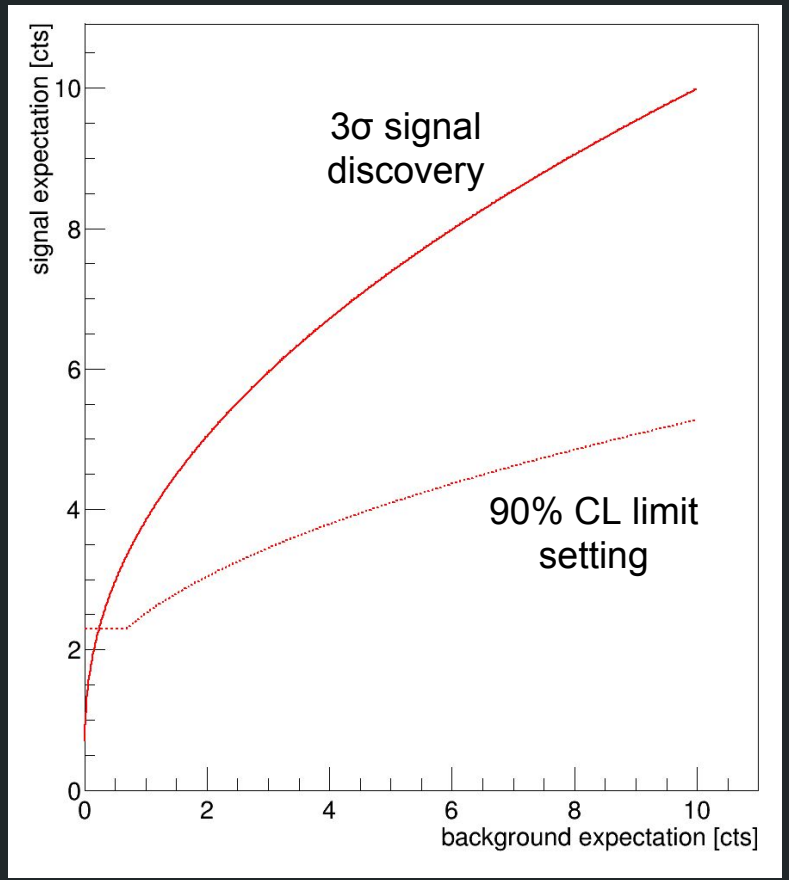
[Consistent with prescription from Cowan, Cranmer et al ]



# Counting Experiment Sensitivity

Simple calculation for signal + known background:

- based on distributions of expected frequency of observations
- Poisson CDF approximated with gamma functions




[**M.A.**, G Benato and J A Detwiler, PRD 96, 053001 (2017)]

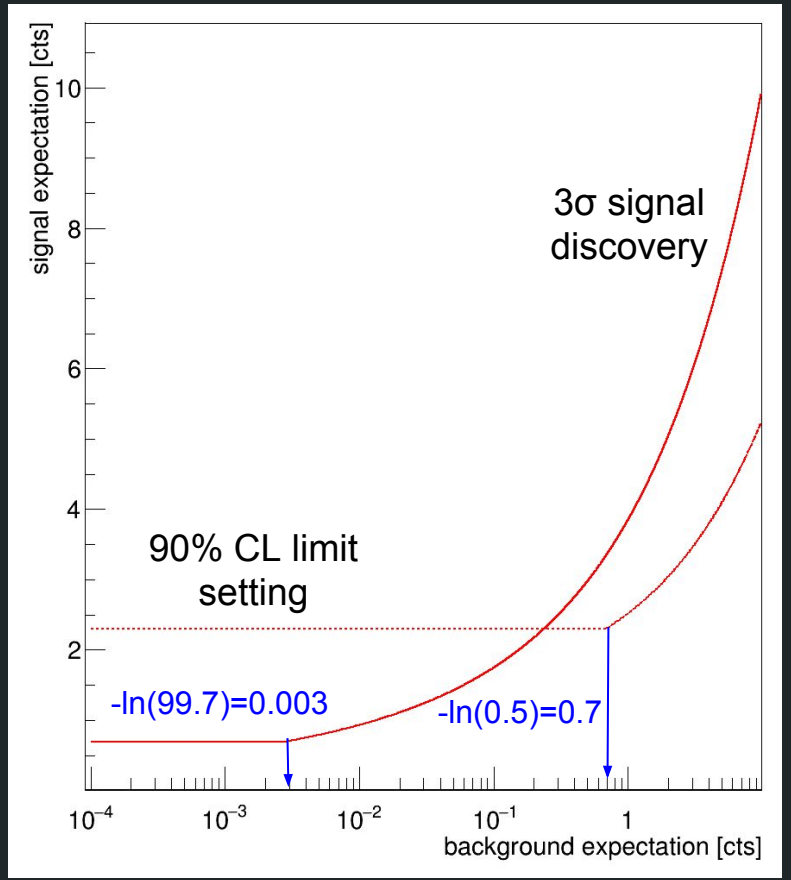
# Counting Experiment Sensitivity

Simple calculation for signal + known background:

- based on distributions of expected frequency of observations
- Poisson CDF approximated with gamma functions

Background free regime:

- **limit setting sensitivity:** flattens for a background expectation of .7 cts (median of $H_0$ is at 0)
- **signal discovery sensitivity:** flattens for a background expectation of 0.003 (99.7% quantile of $H_0$ is at 0)
- 1 count is a discovery for low enough background




[**M.A.**, G Benato and J A Detwiler, PRD 96, 053001 (2017)]

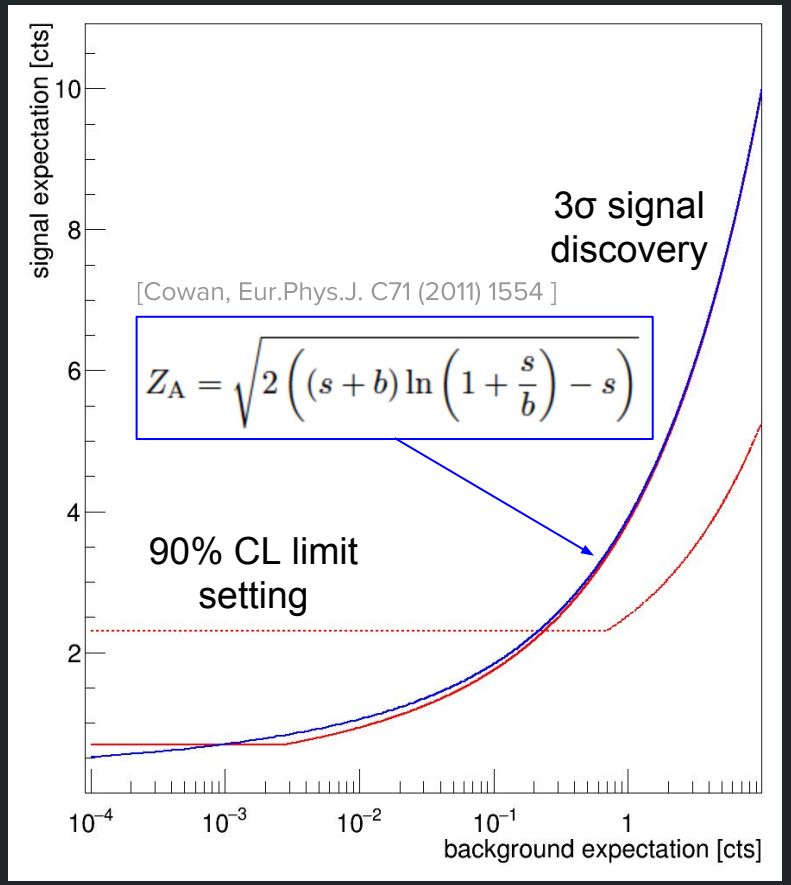
# Counting Experiment Sensitivity

Simple calculation for signal + known background:

- based on distributions of expected frequency of observations
- Poisson CDF approximated with gamma functions

Background free regime:

- **limit setting sensitivity:** flattens for a background expectation of .7 cts (median of $H_0$ is at 0)
- **signal discovery sensitivity:** flattens for a background expectation of 0.003 (99.7% quantile of $H_0$ is at 0)
- 1 count is a discovery for low enough background




[**M.A.**, G Benato and J A Detwiler, PRD 96, 053001 (2017)]

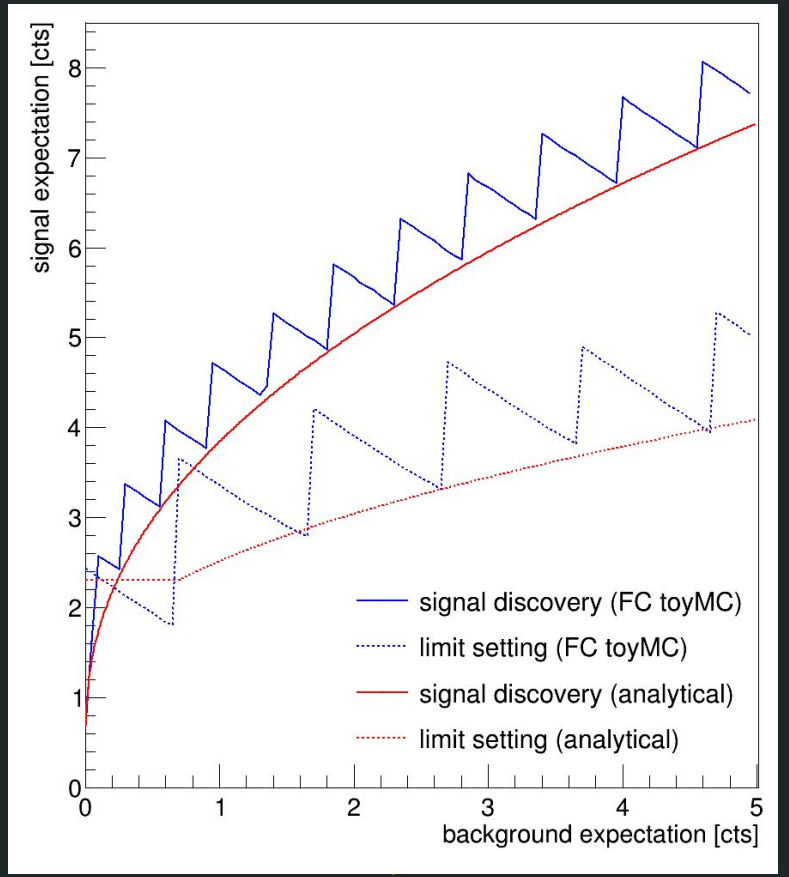
# Counting Experiment Sensitivity

Comparison with a FC construction:

- likelihood ratio
- test statistic distributions from toy MC
- median significance (not mean!)

Features:

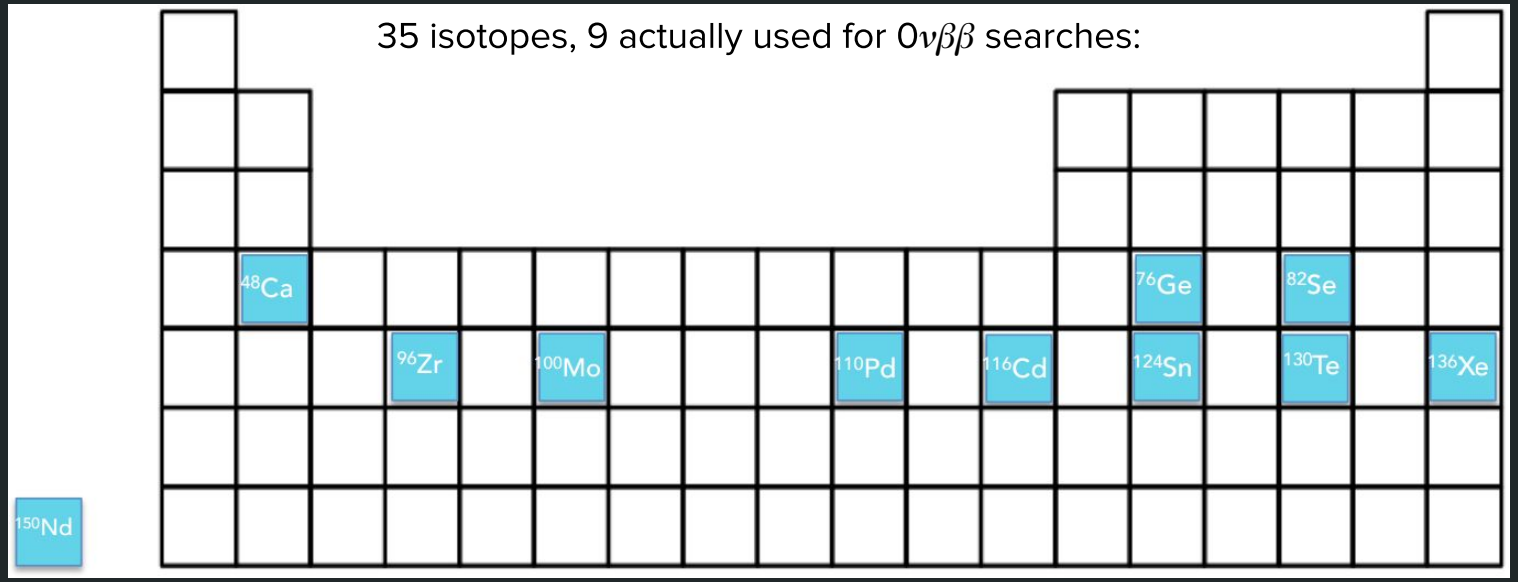
- jumps in coverage due to discrete nature of problem
- not monotonic functions -> apparent sensitivity improvement when increasing background
- “better than background free” regime






[**M.A.**, G Benato and J A Detwiler, PRD 96, 053001 (2017)]

# Target materials



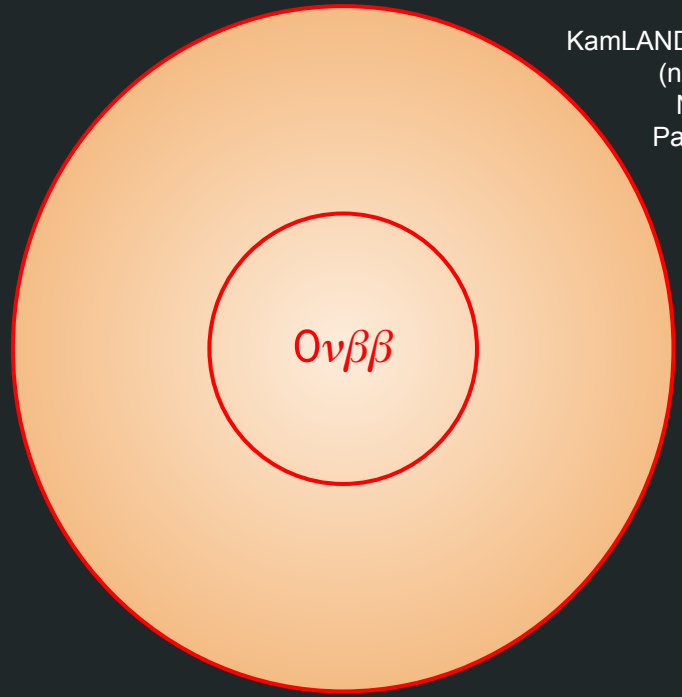

[Courtesy of K. Shaefner]

Different detection techniques available for different isotopes

Variety of the field is important for convincing discovery

# Liquid/Gas vs Solid Detectors

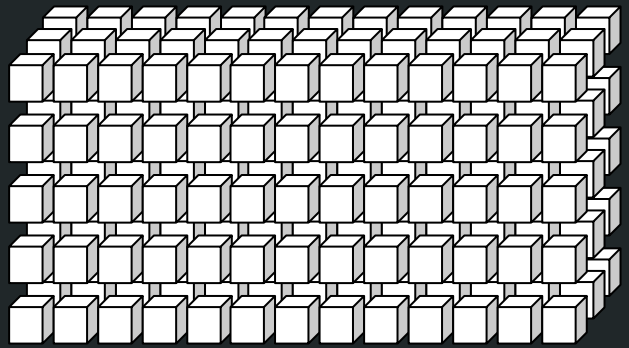
KamLAND-Zen
(n)EXO
NEXT
PandaX




**Loaded scintillator detectors or Xe Time Projection Chambers**

- $0\nu\beta\beta$ isotope mixed in the liquid/gas material
- self-shielding from external background
- volume fiducialization

CUORE
CUPID
AMORE
Majorana
GERDA

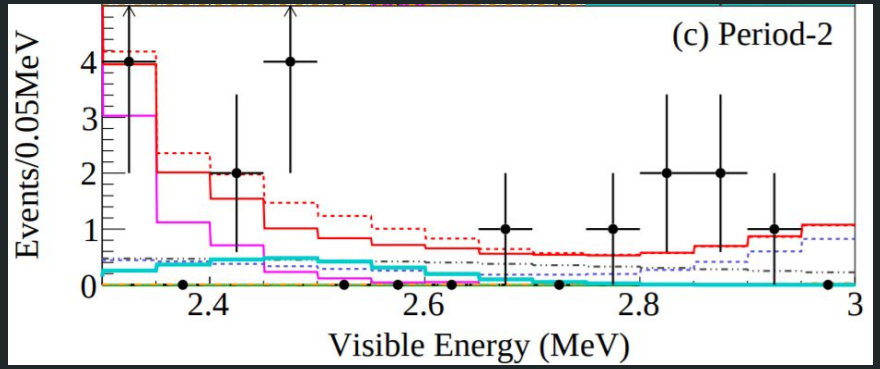


**Cryogenic Bolometers or Semiconductor detectors:**

- many crystals of isotopically enriched material
- detector granularity
- per mill energy resolution

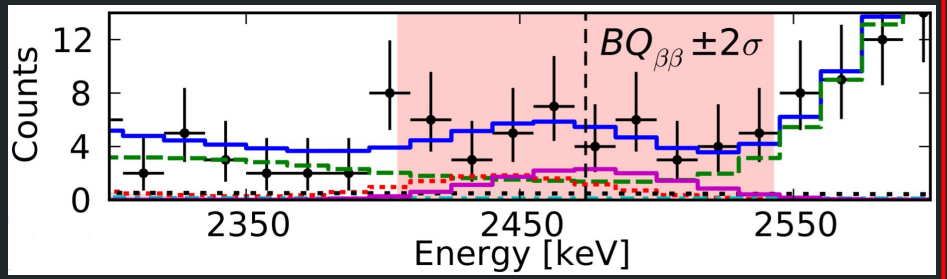
[Phys.Rev. Lett. 117 (2016) 109903]
[Phys.Rev. Lett. 120 (2018) 072701]
[Phys.Rev. Lett. 120 (2018) 132501]

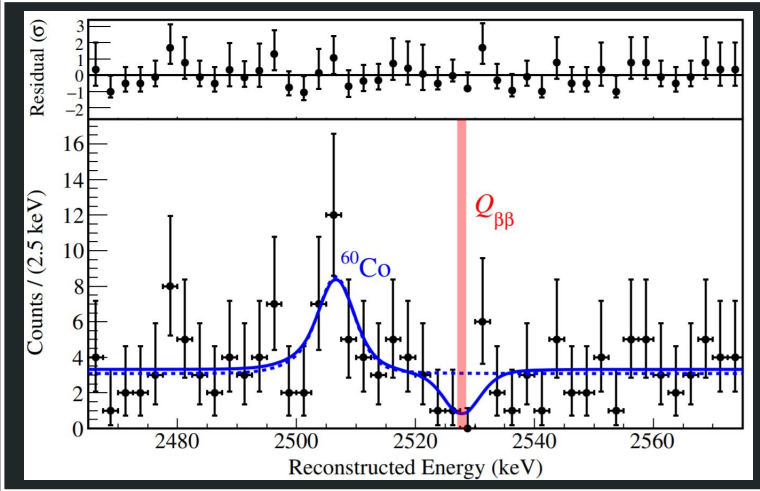
# Liquid/Gas vs Solid Detectors




KamLAND-Zen
B=*O*(10) cts

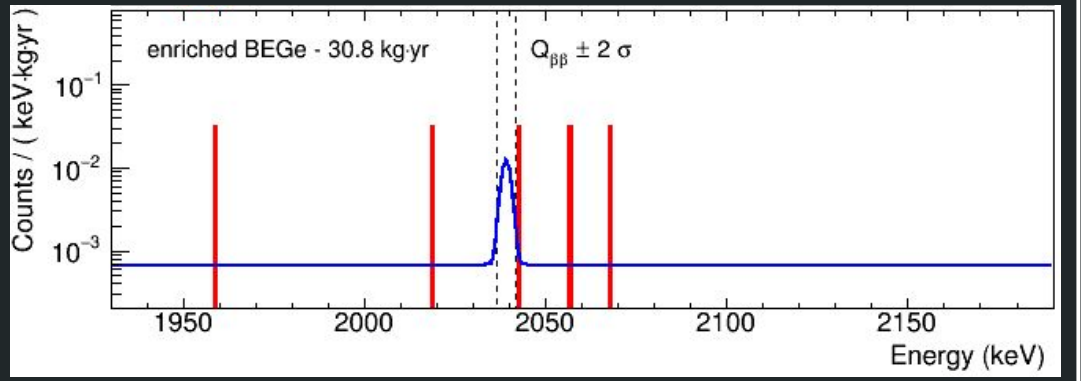
EXO *B*=*O*(10) cts



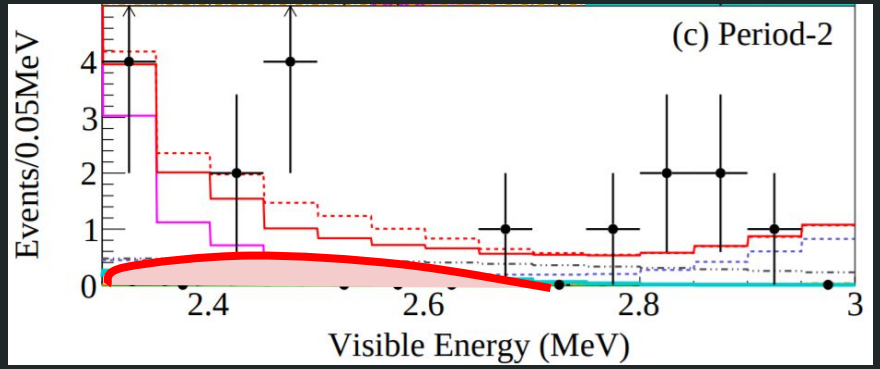

CUORE: *B*=*O*(10) cts

GERDA: *B*=*O*(0.1) cts



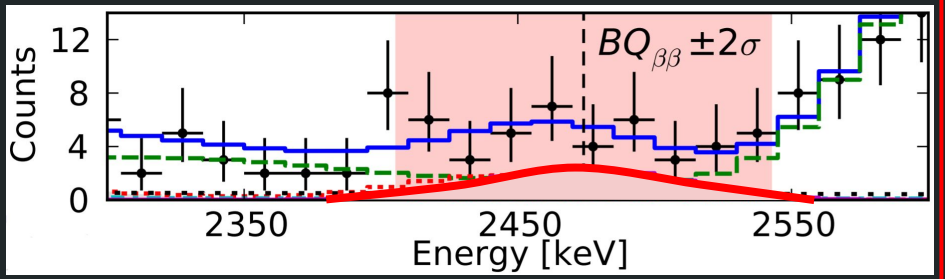
[Phys.Rev. Lett. 117 (2016) 109903]
[Phys.Rev. Lett. 120 (2018) 072701]
[Phys.Rev. Lett. 120 (2018) 132501]

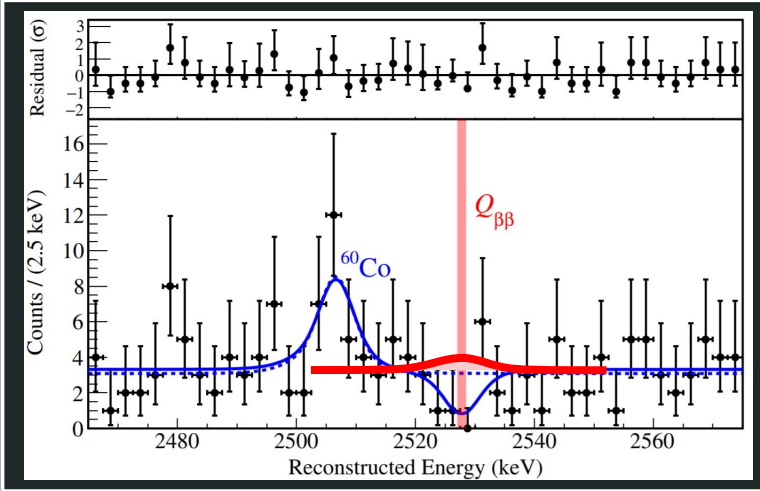
# Liquid/Gas vs Solid Detectors




KamLAND-Zen
B=*O*(10) cts

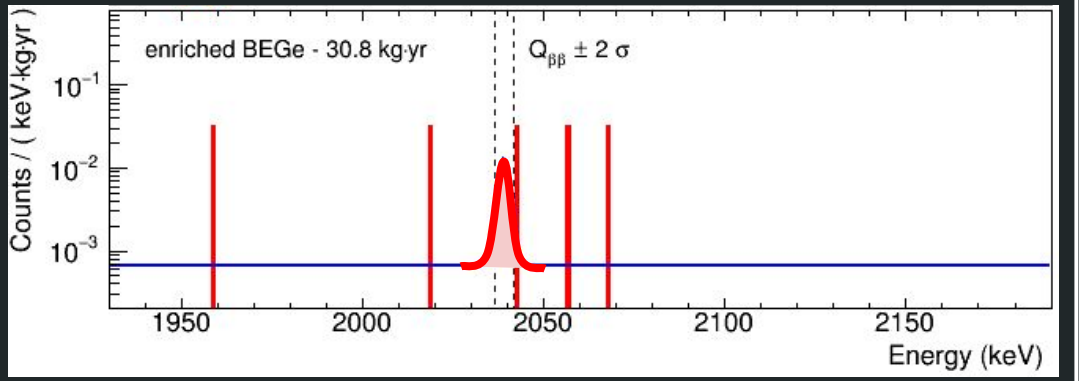
EXO *B*=*O*(10) cts



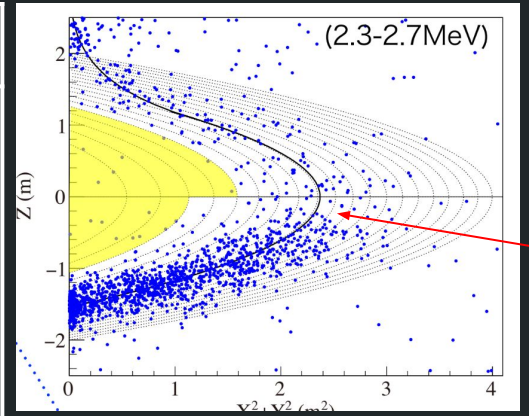

CUORE: *B*=*O*(10) cts

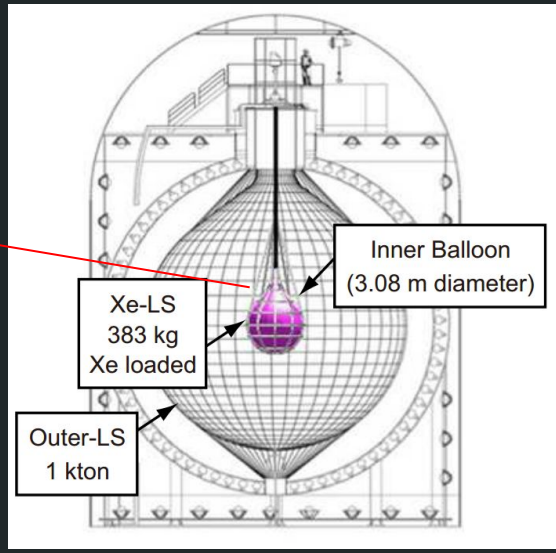
GERDA: *B*=*O*(0.1) cts



## KamLAND-Zen

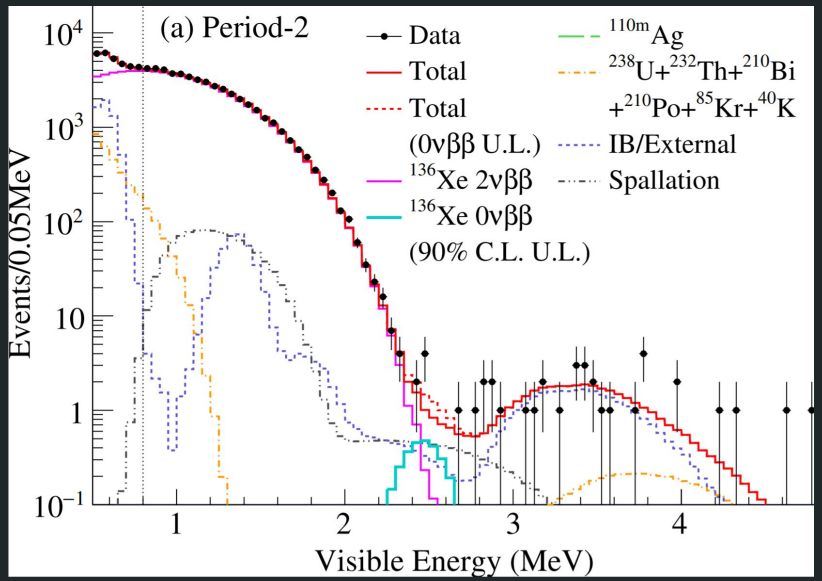
| | |
|---|---|
| Location | Kamioka, Japan |
| Isotope | $^{136}$Xe [$Q_{\beta\beta}$=2458 keV] |
| Technology | Xe-loaded liquid scintillator |
| Isotope Mass | 350 kg |
| $0\nu\beta\beta$ efficiency | 16% |
| Resolution [$\sigma$] | 100-120 keV |
| Latest results | $T_{1/2} > 1.1 \cdot 10^{26}$ yr (90% CL) |
| Sensitivity | $T_{1/2} > 5.6 \cdot 10^{25}$ yr (90% CL) |

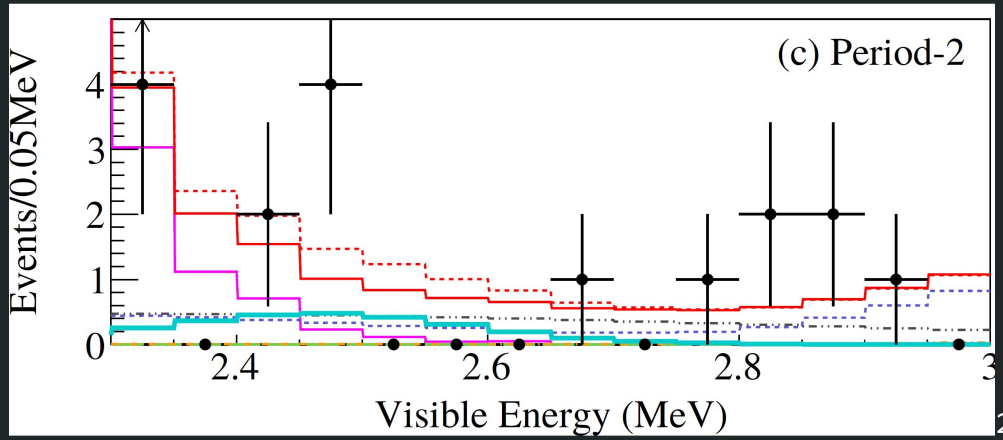



Frequentist likelihood fit

- Multivariate: E vs R
- Wilks' approximation tested with toy MC

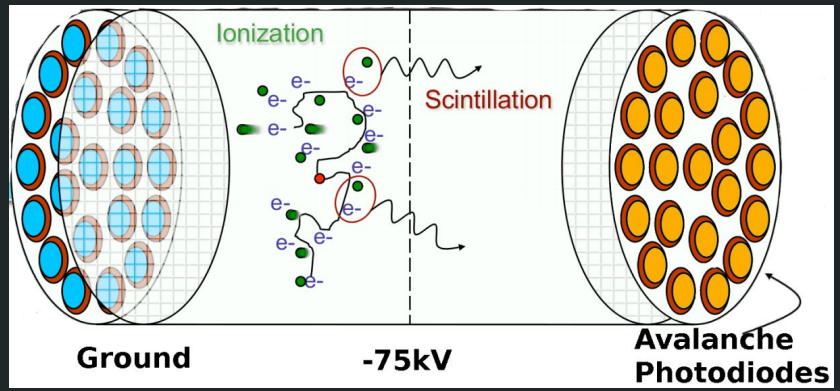


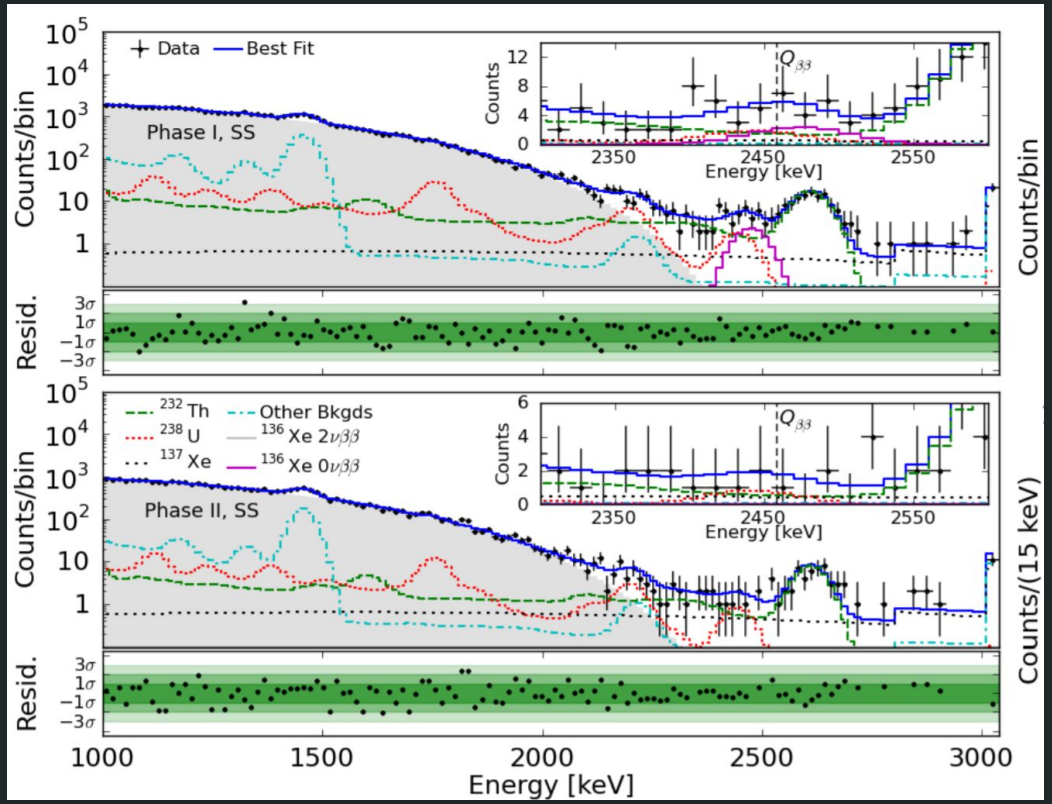

[Phys.Rev.Lett. 117 (2016) no.10, 109903]

## EXO-200

| | |
|---|---|
| Location | WIPP, New Mexico, USA |
| Isotope | $^{136}$Xe [$Q_{\beta\beta}$=2458 keV] |
| Technology | TPC with liquid Xe |
| Isotope Mass | 76 kg |
| $0\nu\beta\beta$ efficiency | 80% |
| Resolution [$\sigma$] | 34 keV |
| Latest results | $T_{1/2} > 1.8 \cdot 10^{25}$ yr (90% CL) |
| Sensitivity | $T_{1/2} > 3.7 \cdot 10^{25}$ yr (90% CL) |




[Phys.Rev.Lett. 120 (2018) no.7, 072701]

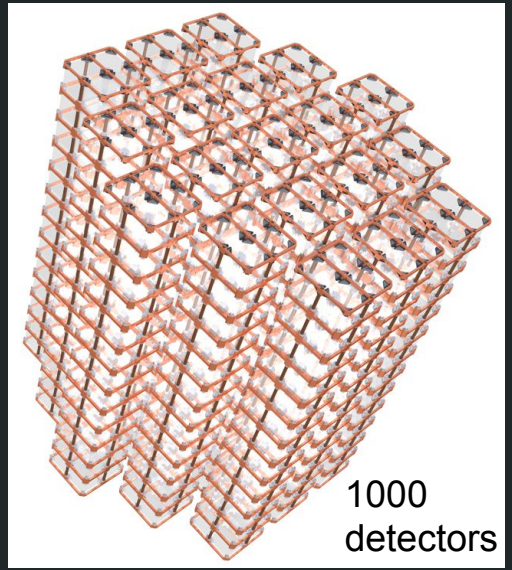



Frequentist binned likelihood fit:

- multivariate (energy, position, TMVA observables)
- Wilks' approximation valid (coverage tested)

## CUORE

| | |
|---|---|
| Location | LNGS, Italy |
| Isotope | $^{130}$Te [$Q_{\beta\beta}$=2527 keV] |
| Technology | Cryogenic calorimeters |
| Isotope Mass | 206 kg |
| $0\nu\beta\beta$ efficiency | 68% |
| Resolution [$\sigma$] | 3.3 keV |
| Latest results | $T_{1/2} > 1.5 \cdot 10^{25}$ yr (90% CL) |
| Sensitivity | $T_{1/2} > 0.7 \cdot 10^{25}$ yr (90% CL) |

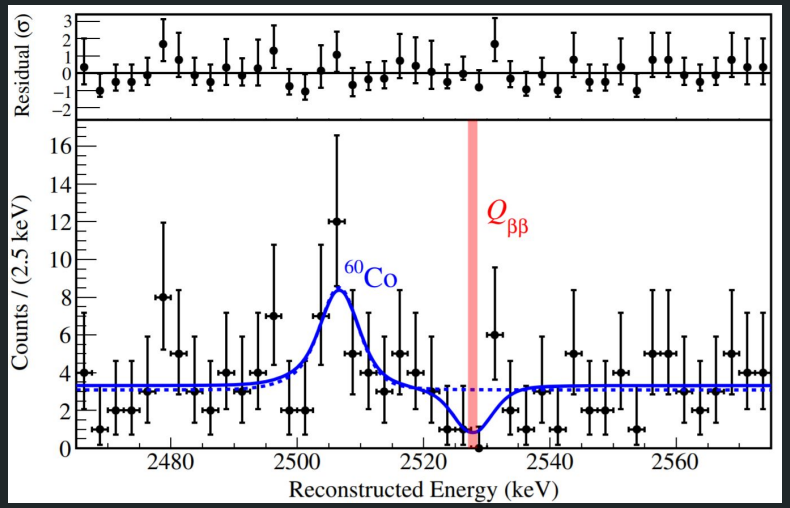



Bayesian:

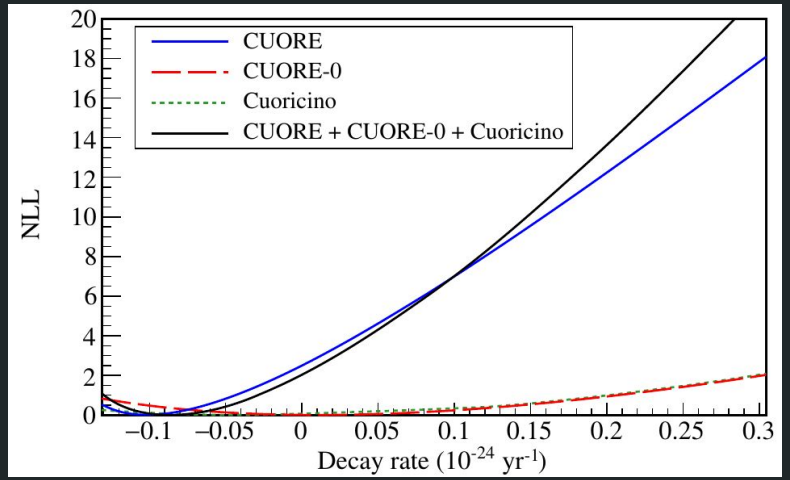
- flat prior
- profiling instead of marginalization

Frequentist:

- bounded profile likelihood
- Wilks approximation

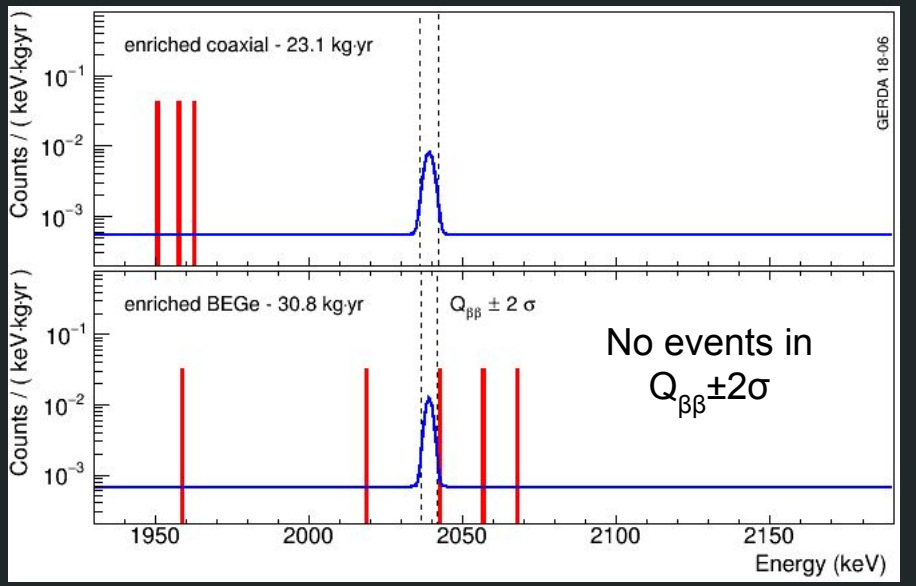



[Phys. Rev. Lett. 120, 132501 (2018)]



## GERDA

| | |
|---|---|
| Location | LNGS, Italy |
| Isotope | $^{76}$Ge [$Q_{\beta\beta}$=2039 keV] |
| Technology | Semiconductor Ge detectors |
| Isotope Mass | 35 kg |
| $0\nu\beta\beta$ efficiency | 65% |
| Resolution [$\sigma$] | 1.3 keV |
| Latest results | $T_{1/2} > 0.9 \cdot 10^{26}$ yr (90% CL) |
| Sensitivity | $T_{1/2} > 1.1 \cdot 10^{26}$ yr (90% CL) |



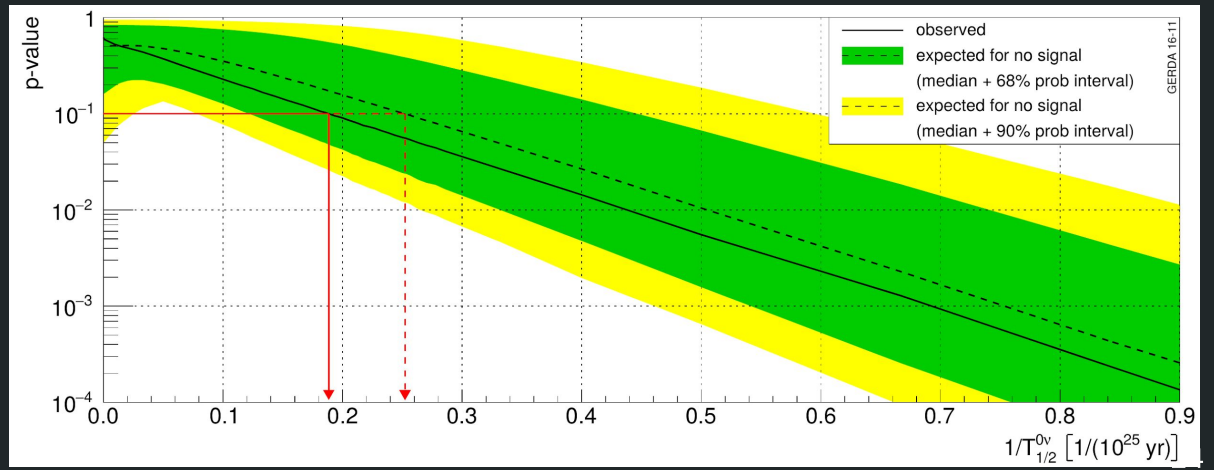

[Nature 544 (2017) 47]

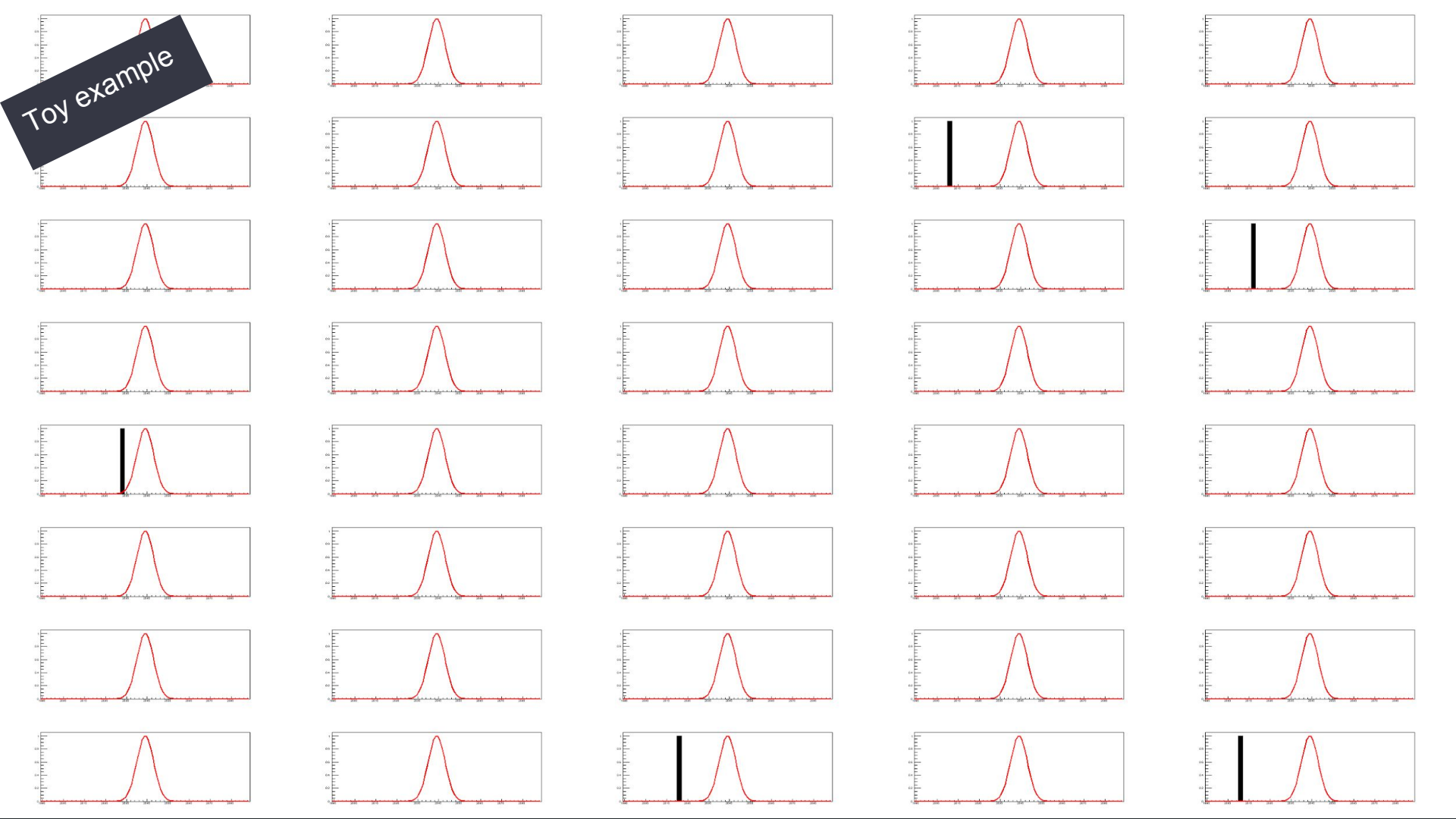
Frequentist:

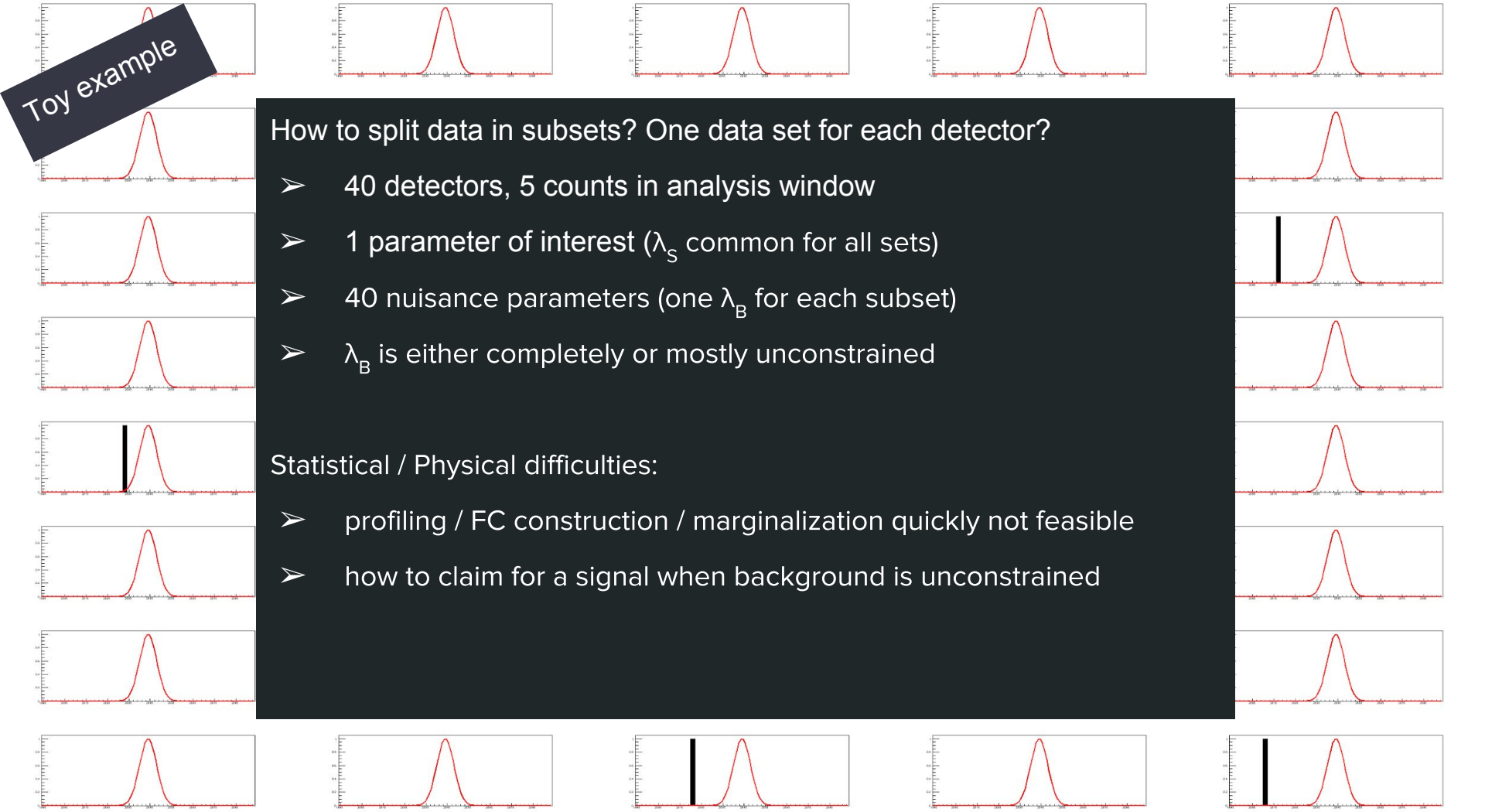
- ➢ extended unbinned likelihood
- ➢ profile likelihood
- ➢ FC construction (only for best fit value of nuisance parameters)

Bayesian:

- ➢ flat prior




Toy example



Toy example

How to split data in subsets? One data set for each detector?

- 40 detectors, 5 counts in analysis window
- 1 parameter of interest ($\lambda_S$ common for all sets)
- 40 nuisance parameters (one $\lambda_B$ for each subset)
- $\lambda_B$ is either completely or mostly unconstrained

Statistical / Physical difficulties:

- profiling / FC construction / marginalization quickly not feasible
- how to claim for a signal when background is unconstrained

# Discovery analysis with 1 candidate event

Conceptual problem:

- focus on a single signal-candidate event
- this event comes with a set of experimental parameters
- we feel like the analysis should use those parameters and not average values over a data set
- splitting data in more sets would reduce the difference between average parameters and those of a single event but leads to proliferation of nuisance parameters associated to empty data sets and larger uncertainties on their central values

Current approach:

- find a trade-off  between number of data sets and variance of the average parameters
- blinded analysis (including blinded data set definition) avoid biases in the process

Other Ideas?

# Systematics

Main systematics:

- background shape (liquid/gas experiments)
- energy scale and resolution
- signal detection efficiency
  - active volume
  - analysis cuts
  - ...

Approaches:

- nuisance parameters / pull terms
- fit with different models
- toy-MC generated under different models

Impact of systematic uncertainties

- not informative data -> impact determined by shape of priors or pull term
- 1% level for solid state detectors (no counts to pull nuisance parameters)
- 5-10% level for gas/liquid experiments
- different in case of signal

EXO-200 systematic uncertainties:

| Source | Phase I | Phase II |
|---|---|---|
| Signal detection efficiency | 3.0% | 2.9% |
| Background errors | | |
| Spectral shape agreement | 2.1% | 1.7% |
| Background model | 5.6% | 5.9% |
| Energy scale and resolution | 1.5% | 1.2% |
| Total | 6.2% | 6.2% |

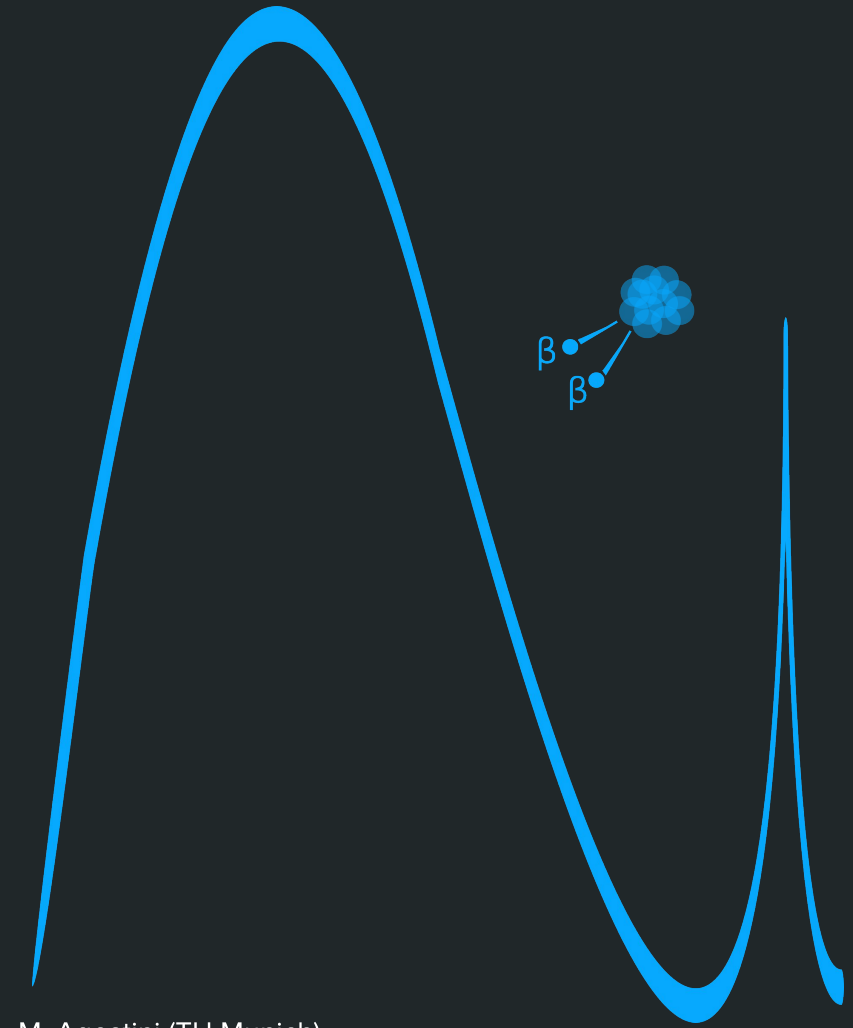
[Phys.Rev.Lett. 120 (2018) no.7, 072701]

# Statistics & Double-$\beta$ Decay Community

- most experiments quote results from multiple methods and give enough info to reproduce the analysis
- sensitivity always reported (sometimes also for Bayesian methods)
- blind analysis is almost the standard
- frequentist intervals still used as Bayesian intervals (even when Bayesian interval is available)
- sensitivity computed for the no signal hypothesis, more interesting to quote discovery power

[Phys. Rev. Lett. 120, 132502 (2018)]

| Statistical Method in the last PRL of the MAJORANA DEMONSTRATOR | $T_{1/2}$ lower limit 90% prob [$10^{25}$ yr] | $T_{1/2}$ lower limit sensitivity [$10^{25}$ yr] |
|---|---|---|
| Counting (FC) | 1.6 | |
| Unbinned likelihood fit (FC) | 1.9 | 2.1 |
| Unbinned likelihood fit & CLs | 1.5 | 1.4 |
| Bayesian flat prior | 1.6 | |
| Bayesian Jeffreys prior | 2.6 | |

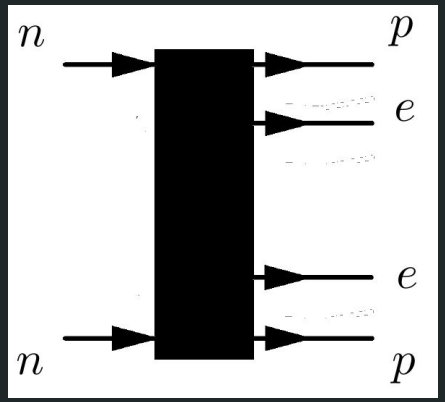



# Outlook

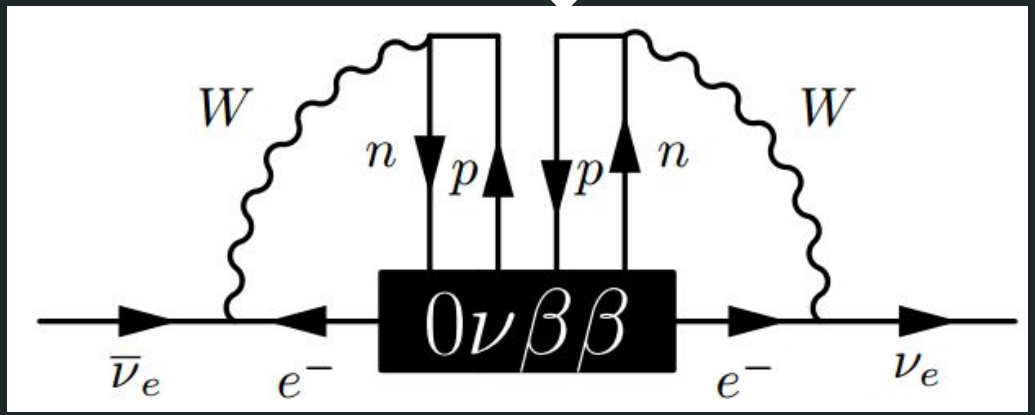
- 0νββ decay is a portal to new physics and experiments are interested in signals at the edge of their sensitivity
- The search for a peak with background keeps on posing new challenges: (too) many methods available, very different results
- Many experiments operate in the “Deep Poisson” regime where even a simple concept as sensitivity requires attention
- Important to shift focus towards a discovery analysis and define in advance how to dial with it a discovery based on a single count

# Backup slides

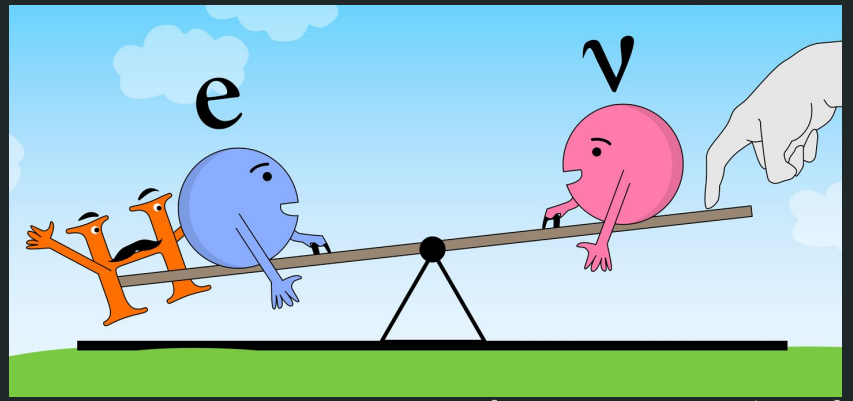
# $0\nu\beta\beta$ and $\nu$ Mass Origin




**Black Box theorem**:
$0\nu\beta\beta$ operator can be rearranged into a $\nu$-$\bar{\nu}$ oscillation
(i.e. a Majorana mass term)




[Schechter, Valle, PRD 25 (1982) 2951]




[www.symmetrymagazine.org]

**If $0\nu\beta\beta$ decay is discovered:**

- neutrinos are their own antiparticle
- neutrinos can have a Majorana mass
- neutrino small masses can be explained through see-saw models

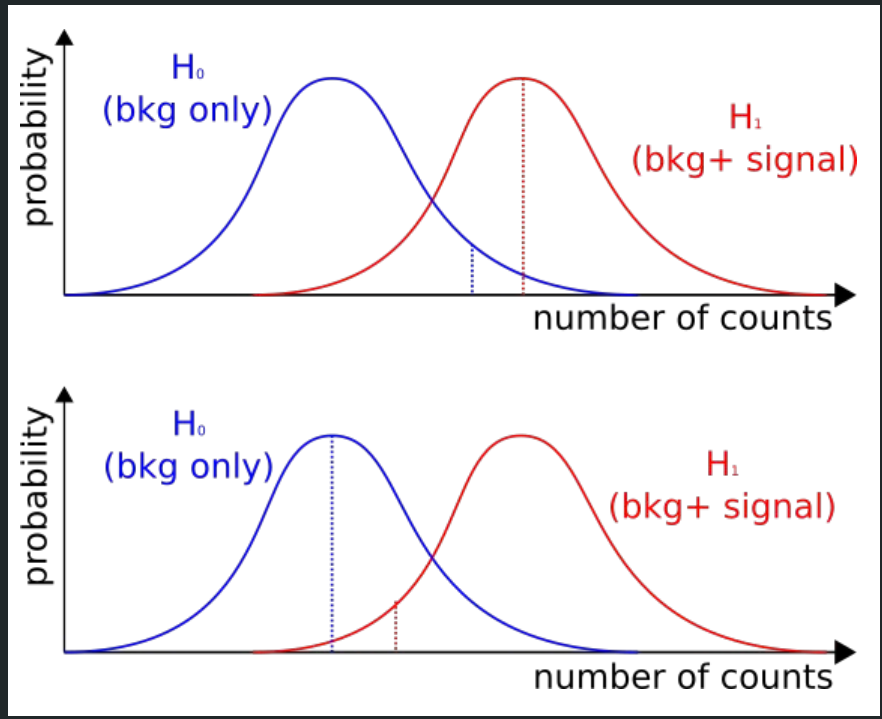
# Sensitivity of an experiment

- **3σ signal discovery sensitivity**

  assuming there is a signal, find $H_1$ such that $H_0$ is rejected at 3σ in 50% of the cases

- **90% CL limit setting sensitivity**

  assuming there is no signal, find $H_1$ that is rejected in at 90% CL in 50% of the cases



# Analytical computation of sensitivity

Signal discovery

- Find the number of counts $C_{3\sigma}$ such that: $CDF(C_{3\sigma}|B) = erf(3/\sqrt{2})$
- Solve: $CDF(C_{3\sigma} \mid S_{3\sigma} + B) = 50\%$
- $C_{3\sigma}$ is an integer: $S_{3\sigma}$ has discrete jumps ➜ Approximate the Poisson CDF with the upper incomplete gamma function so that the above equations can be inverted with standard numerical methods

Limit Setting:

- Find the median number of cts expected from bkg only $C_{med}$ : $CDF(C_{med}|B) = 50\%$
- Solve: $CDF(C_{med}| S_{90\%CL} + C_{med}) = 10\%$

[more in M.A. et al., Phys.Rev. D96 (2017) no.5)]

# The counting experiment with a profile likelihood

**if B is perfectly known:**

**B** := background expectation

**S** := signal expectation

**N** := number of cts in ROI

**L(s) = Pois(N|S+B)**

**t(s)** = -2 [Pois (N|S+B) - Pois(N|$S_{best}$+B)]

with $S_{best}$=max(0,N-B)

**if B is derived from a side band or control region:**

**τ** := side band width / ROI width

(for GERDA: **τ** = 220/6 ~40)

**M** := number of cts in side band

**L(s) = Pois(N|S+B) * Pois(M|τB)**

**t(s)** =-2[LogPois(N|S+$B_{cond}$)+LogPois(M|τ$B_{cond}$)

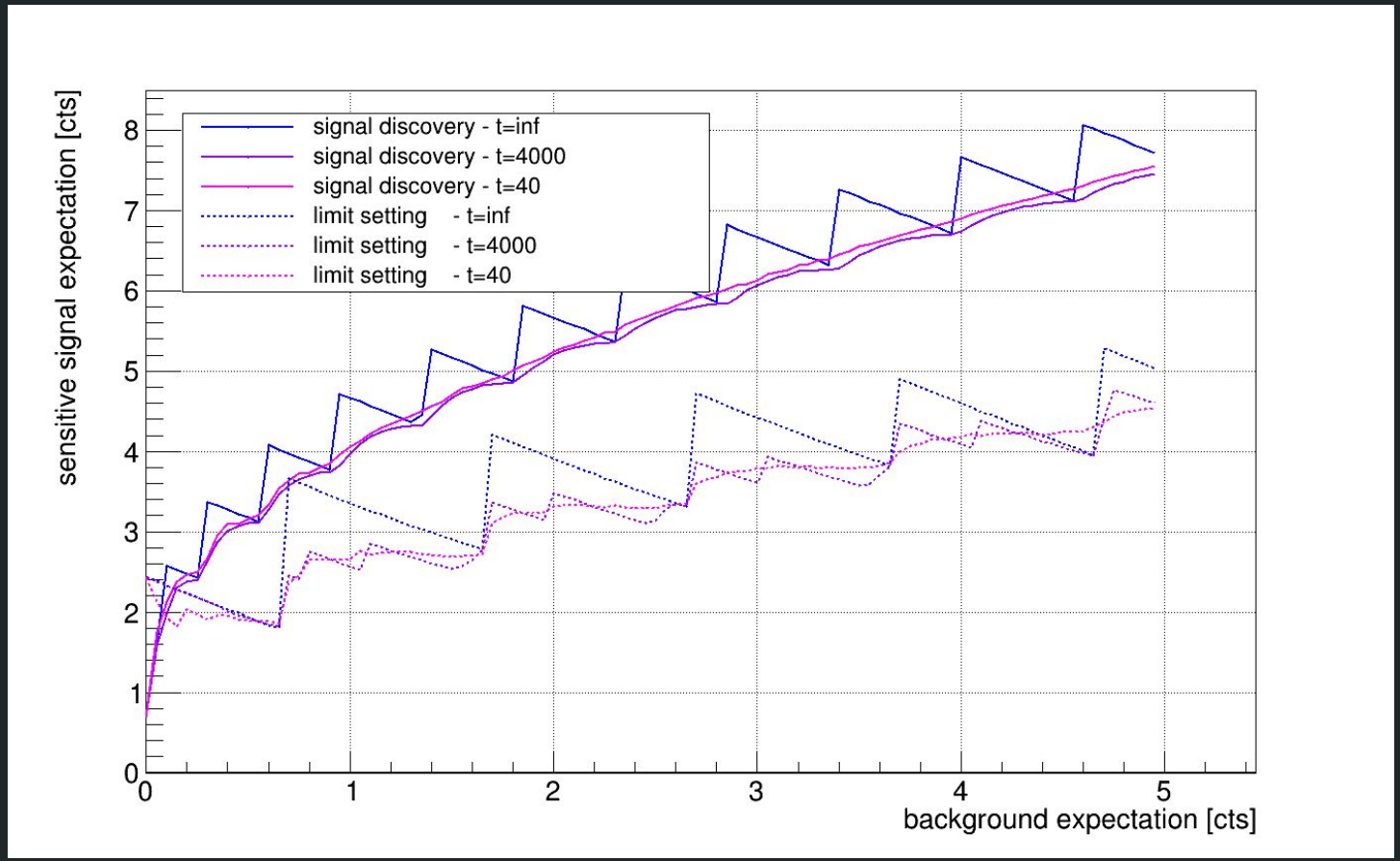
-LogPois(N|$S_{best}$+$B_{best}$) - LogPois(M|τ$B_{best}$)]

with: $S_{best}$=N-M/τ,

$B_{best}$= M/τ

$B_{cond}$ = N+M-(1+t)S+sqrt((N+M-(1+t)S)$^2$ + 4(1+t)SM)]/ [2(1+t)]

# Sensitivity with background uncertainty



[Adapted from A Giuliani, Neutrino2018]

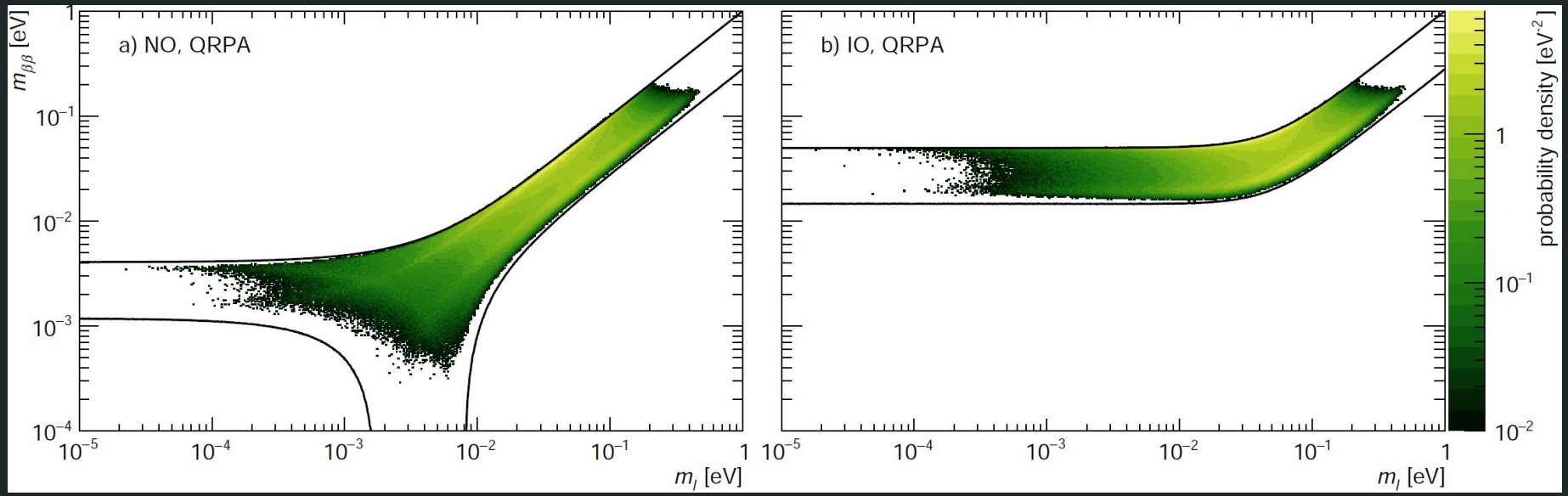
# Major $0\nu\beta\beta$ Projects

current gen → mid-term → long-term

| | | | | |
|---|---|---|---|---|
| Gas/Liquid detector | Liquid scintillator | KZ | KZ-800<br>$SNO^+$ phase I | KamLAND2-Zen<br>$SNO^+$ phase II |
| | Time Projection chambers | EXO<br>NEXT-10 | NEXT-100<br>PANDA-X-III | nEXO<br>NEXT-2.0<br>PANDAX-III 1t |
| Solid detectors | Cryogenic Calorimeters | CUORE<br>CUPID-0<br>AMORE | AMORE II | CUPID |
| | Ge semiconductor | GERDA<br>MJD | LEGEND-200 | LEGEND-1000 |
| External detectors | Magnetized tracking | NEMO | | SUPERNEMO |

# Probability Density from Global Fits

In absence of neutrino mass mechanisms or flavour symmetries that fix the value of the Majorana phases or drive $m_{light}$ to zero, the probability distribution for $m_{\beta\beta}$ is pushed to large values:



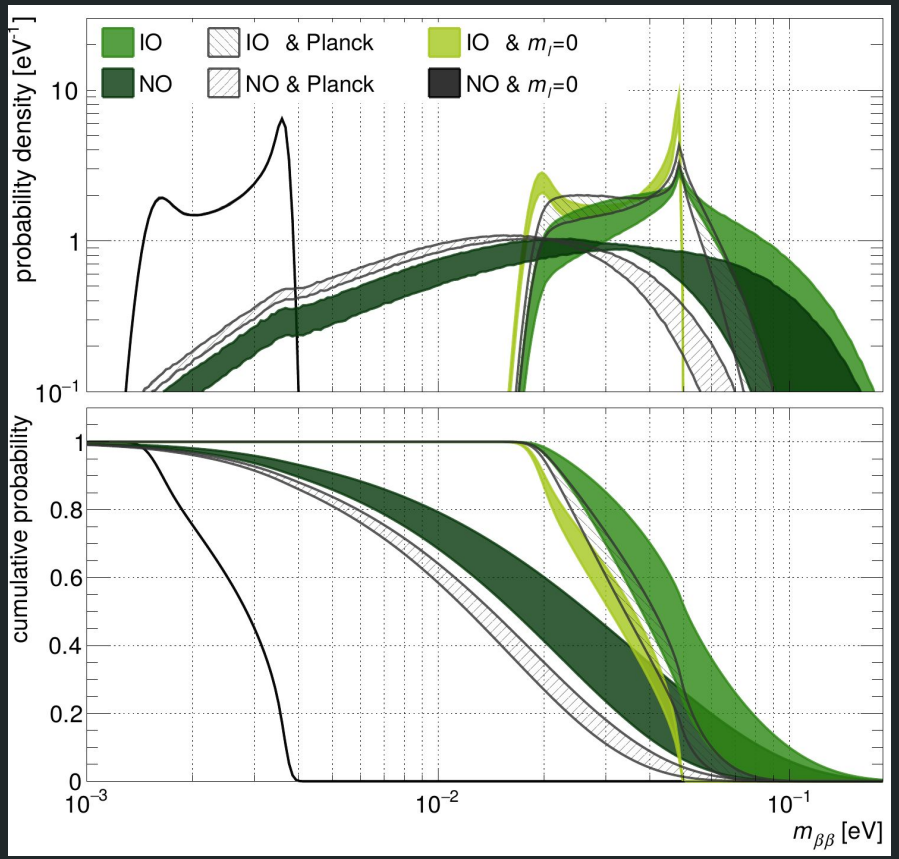

[**M.A.**, G Benato and J A Detwiler, PRD 96, 053001 (2017)]

Flat prior for the Majorana phases ➜ small $m_{\beta\beta}$ values require a fine tuning of the parameters

# Probability density from global fits

- data in the analysis: oscillations + $0\nu\beta\beta$ + (cosmology)
- bands shows deformation due to NME uncertainty
- $0\nu\beta\beta$ constraints on $m_{lightest}$ competitive with cosmology

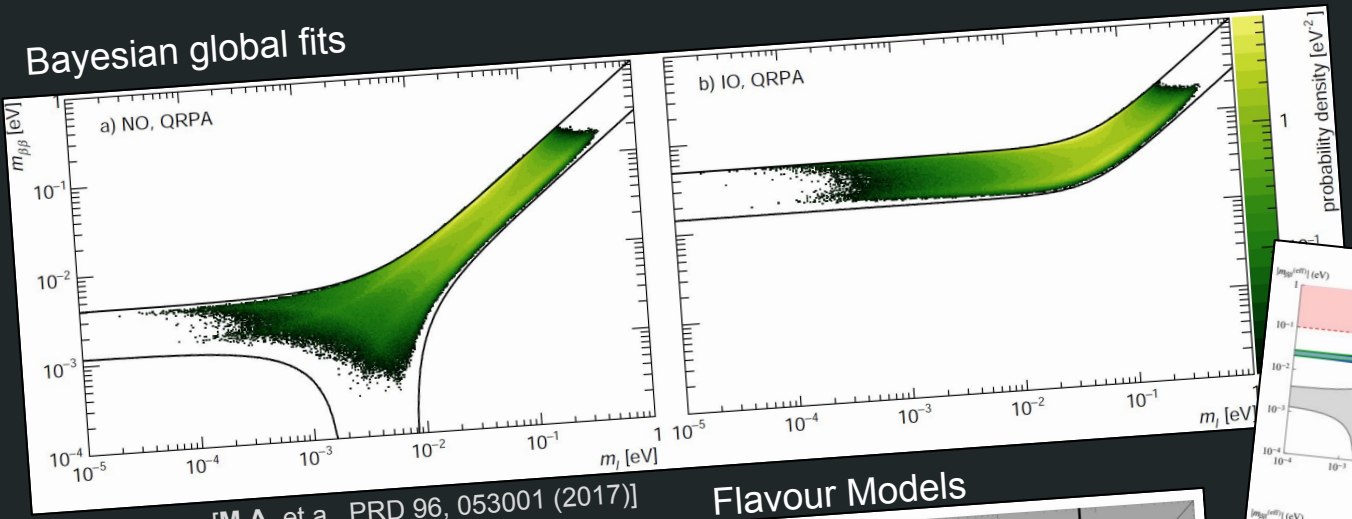
Bulk of probability at reach with next generation experiments




[**M.A.**, G Benato and J A Detwiler, Phys. Rev. D 96, 053001 (2017)]
see also [A Caldwell et al, Phys.Rev. D96 (2017) no.7, 073001]

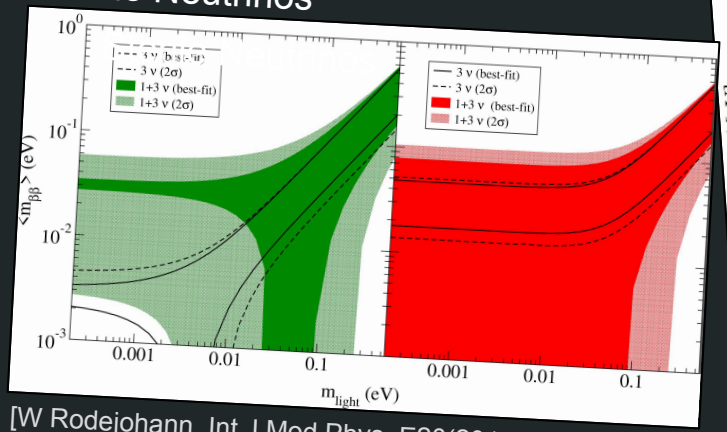
# Discovery Power

## Bayesian global fits



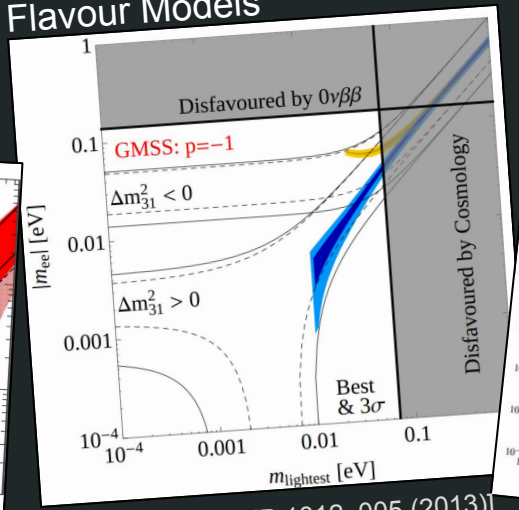

[**M.A.** et a., PRD 96, 053001 (2017)]

## Sterile Neutrinos



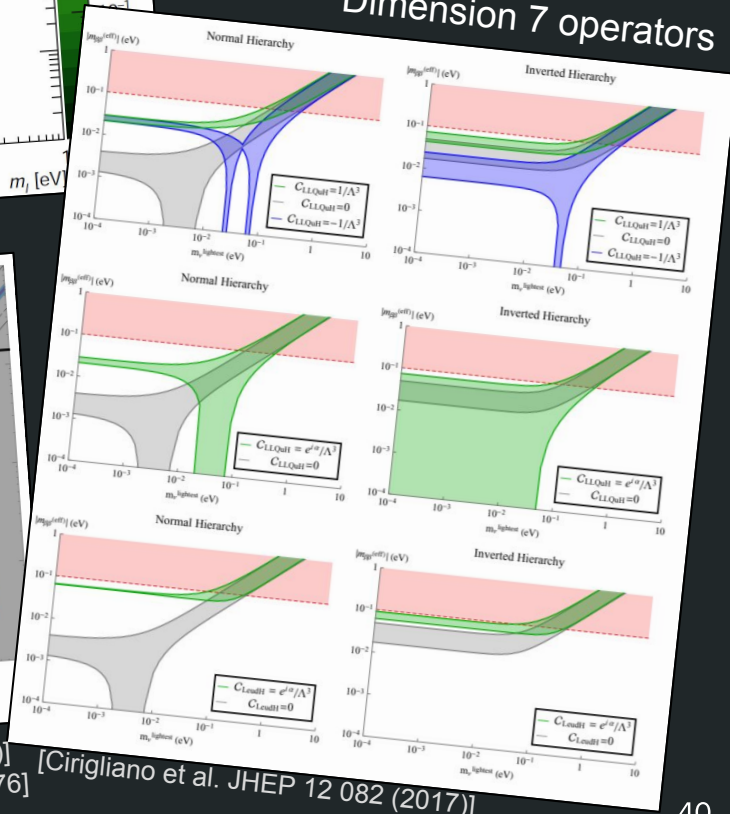

[W Rodejohann, Int.J.Mod.Phys. E20(2011)]

## Flavour Models




[King et al,, JHEP 1312, 005 (2013)]
[**M.A.** et al., EPJ C76 (2016) no.4, 176]

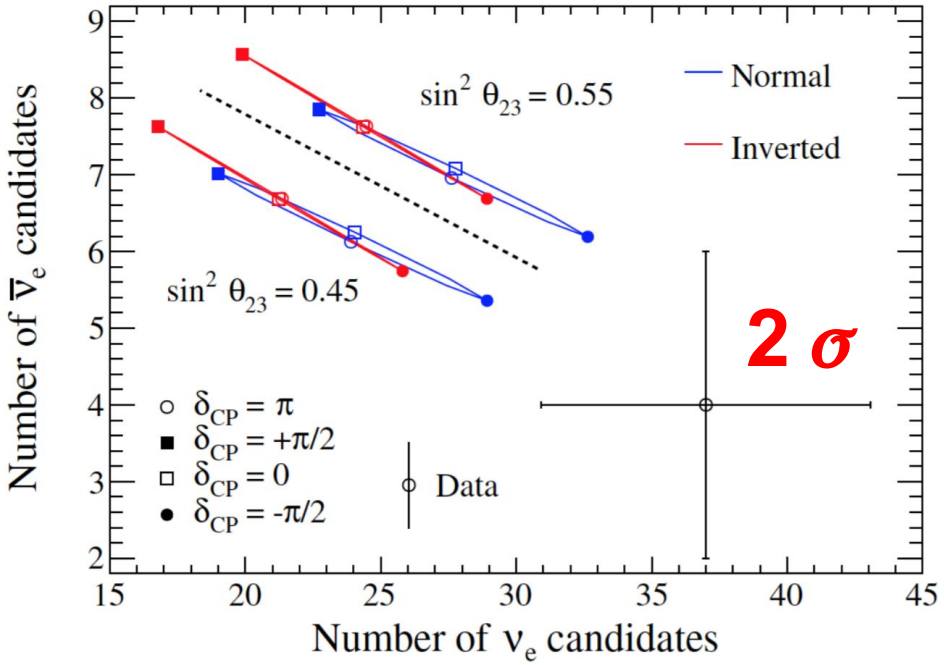
## Dimension 7 operators



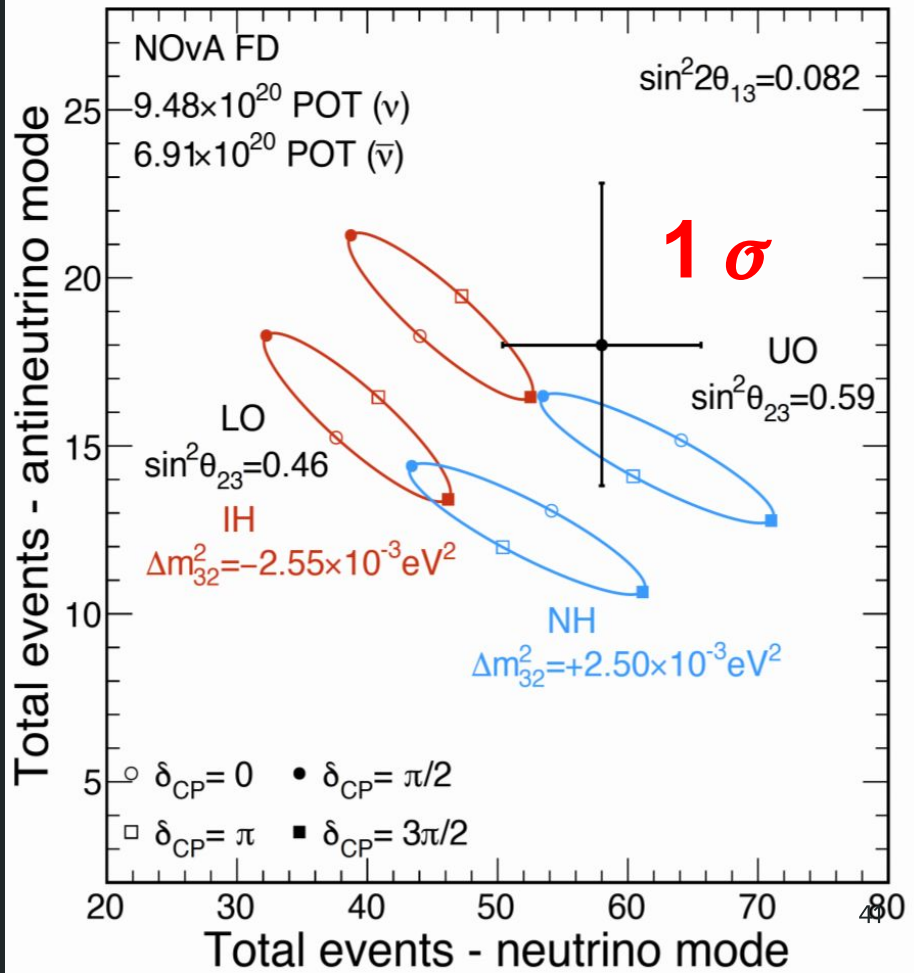

[Cirigliano et al. JHEP 12 082 (2017)]

# Normal vs Inverted ordering

T2K, PRD **96**, 092006 (2017)

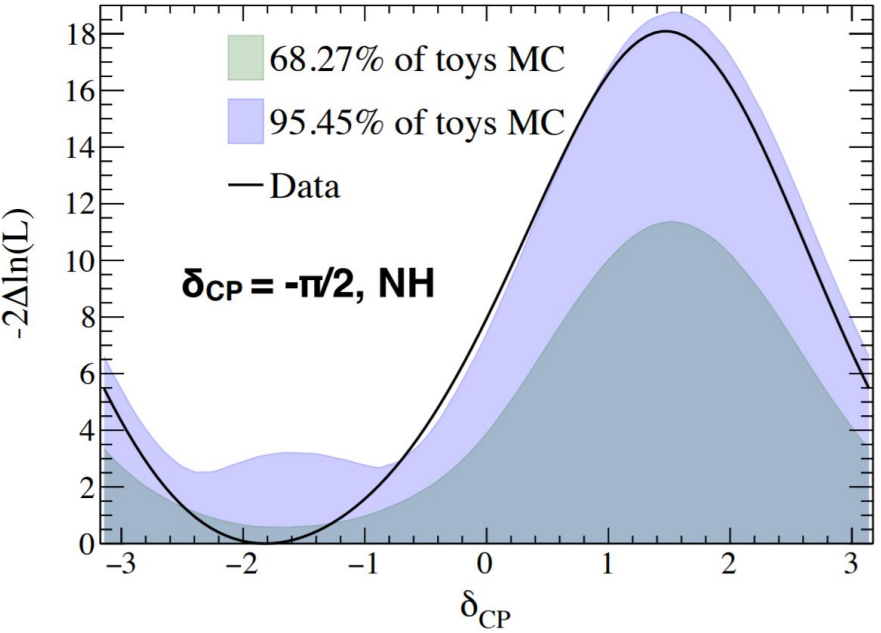


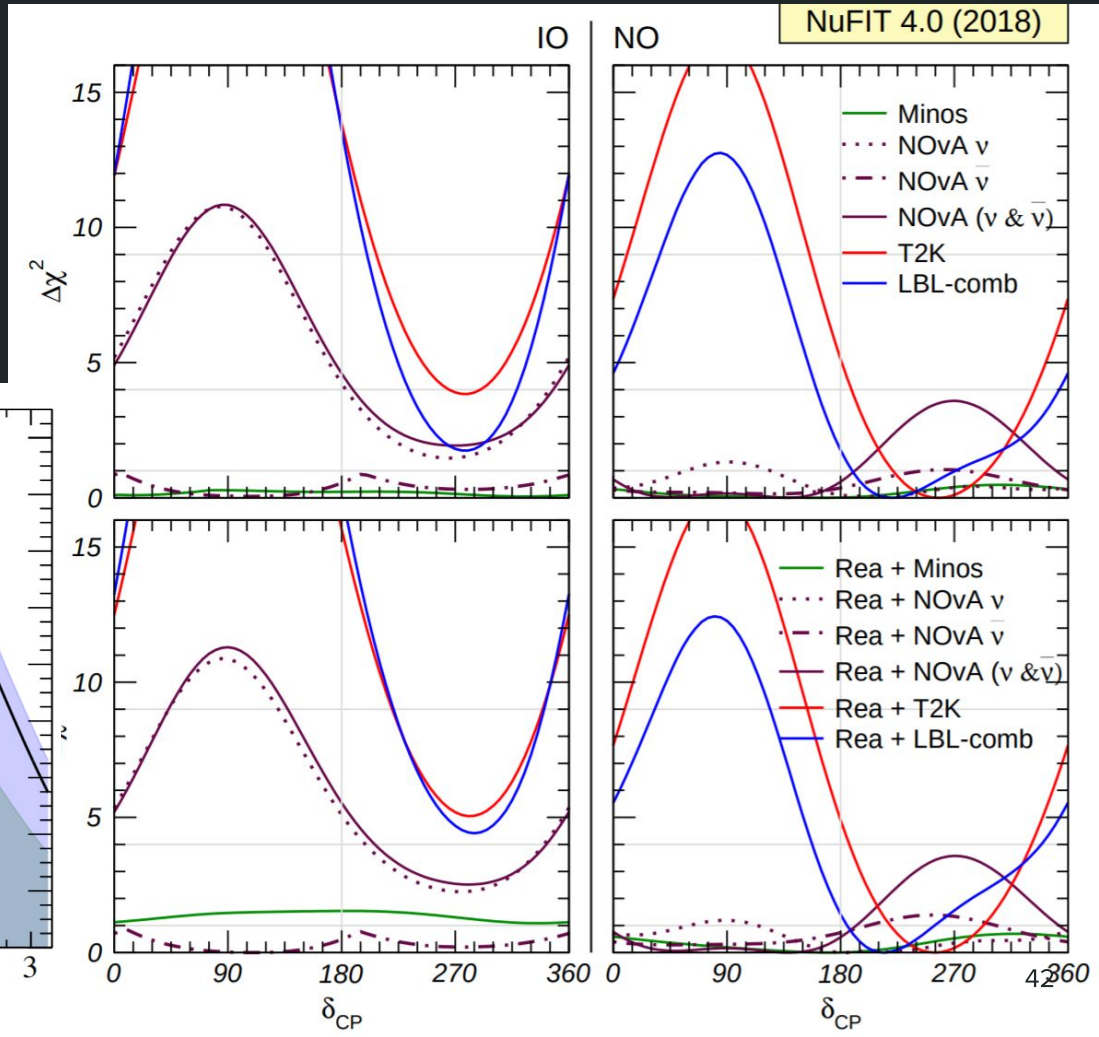

NOvA, Nu2018



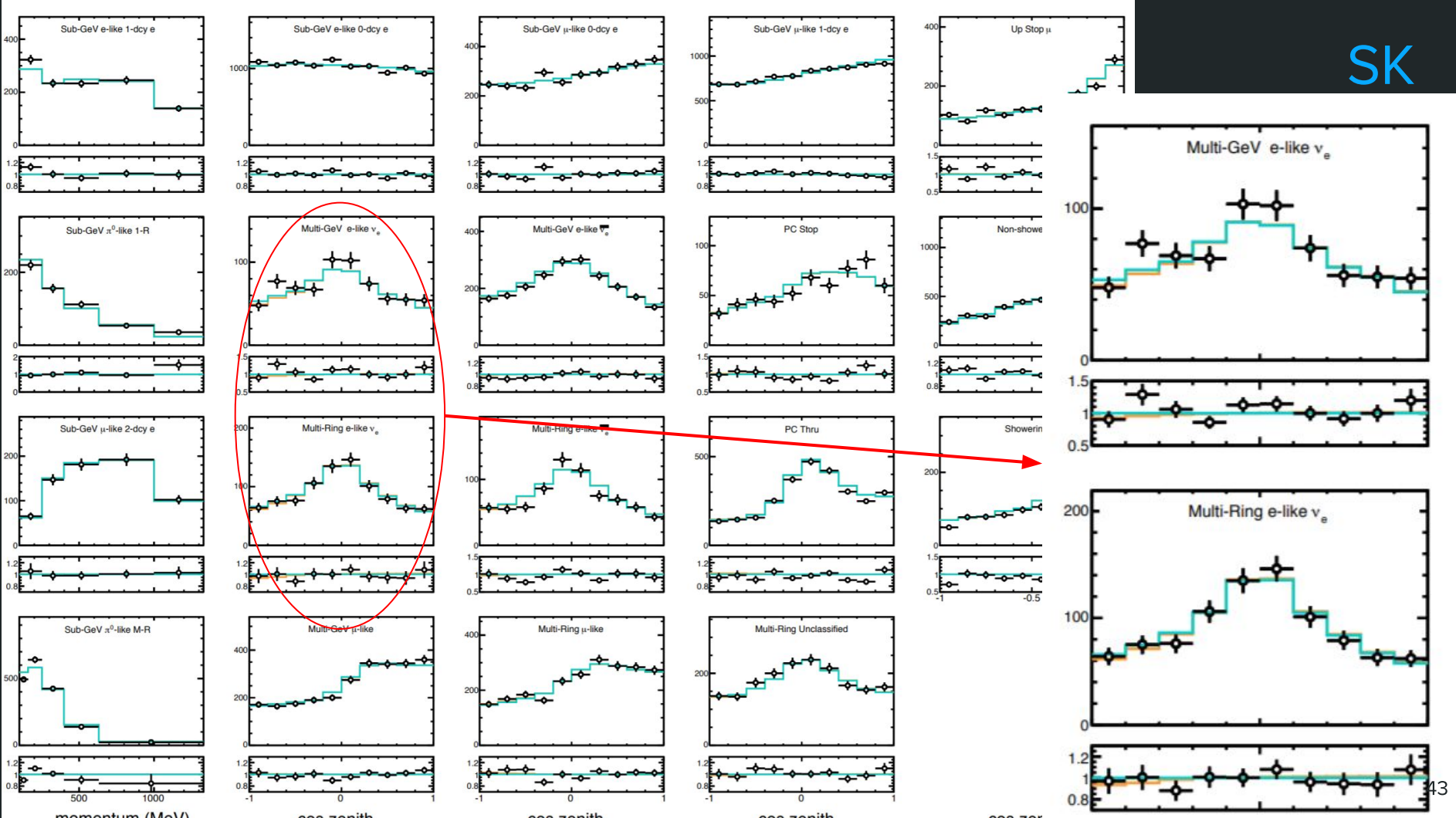
# Normal vs inverted ordering

- NOVA (no sensitive)
- T2K (no sensitive)



# SK


Sub-GeV e-like 1-dcy e
Sub-GeV e-like 0-dcy e
Sub-GeV μ-like 0-dcy e
Sub-GeV μ-like 1-dcy e
Up Stop μ
Sub-GeV π0-like 1-R
Multi-GeV e-like νe
Multi-GeV e-like ν̄e
PC Stop
Non-showe
Sub-GeV μ-like 2-dcy e
Multi-Ring e-like νe
Multi-Ring e-like ν̄e
PC Thru
Showerin
Sub-GeV π0-like M-R
Multi-GeV μ-like
Multi-Ring μ-like
Multi-Ring Unclassified
momentum (MeV)
cos zenith
cos zenith
cos zenith
cos zer
Multi-GeV e-like νe
Multi-Ring e-like νe