Uncertainty in the Reactor Neutrino Spectrum and Mass Hierarchy Determination

Emilio Ciuffoli - Institute of Modern Physics, CAS

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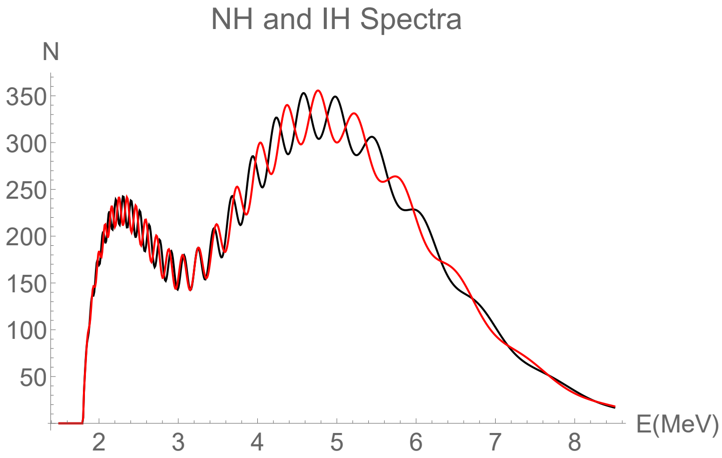
Neutrino Oscillations

In neutrino experiments, the expected number of events is given by

$$\frac{dn}{dE} = \phi(E) \times P(E, L)$$

$\phi(E)$ = **unoscillated** spectrum (including cross section, etc...),
$P(E, L)$ = oscillation probability
An uncertainty on the unoscillated spectrum could affect the measurement of the oscillation probability.
In the case of the mass hierarchy, this could be a serious problem because, due to degeneracy with Δm^2_{32}, the signal we are looking for is very small

Why MH determination is difficult?



Expected spectra for normal and inverted hierarchy at 53 km

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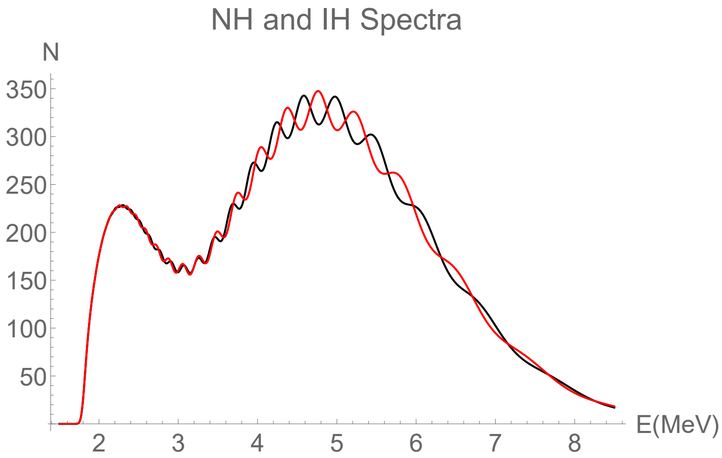
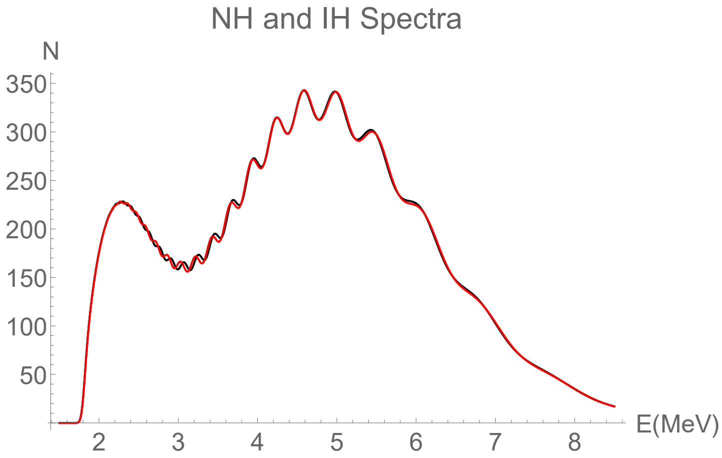


Expected spectra for normal and inverted hierarchy at 53km, finite energy resolution

Why MH determination is difficult?



Expected spectra for normal and inverted hierarchy at 53km, finite energy resolution. Inverted hierarchy: Δm^2_{23} shifted (by $\simeq 1\sigma$'s)

Reactor Neutrino Spectrum

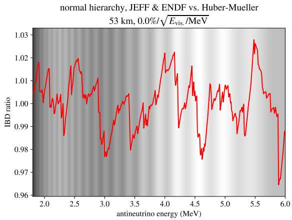
Theoretical model for reactor neutrino fluxes:

- *Ab initio* approach: calculate spectrum branch-by-branch (however, problematic for the large number of isotopes, $\simeq 10^3$, and branching ratio, $\simeq 10^4$)
- Conversion method: measure the beta spectrum directly and then convert to $\bar{\nu}_e$

In the last years with the measurement of the "5 MeV bump" it became clear that the current models for reactor neutrinos are not very reliable. Moreover, only available measurements of reactor neutrino spectrum with low energy resolution ($\simeq 6\%/\sqrt{E}$; for MH $3\%/\sqrt{E}$ needed).

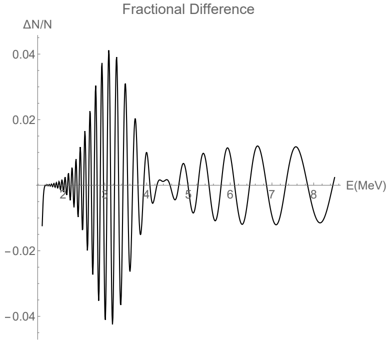
A fine structure ("sawtooth-like features') is most likely present (showed also in "ab initio" calculations) and currently undetected; it could be a problem for the MH determination

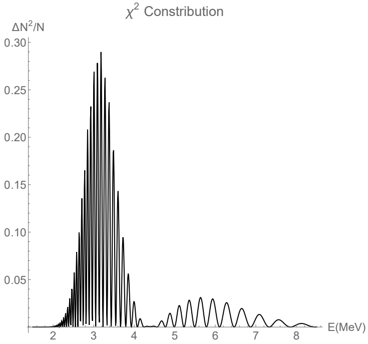
See also D. Forero, R. Hawkins, P. Huber, arXiv:1710.07378, and Danielson *et al*, arXiv:1808:03276



Danielson *et al*, arXiv:1808:03276

Difference between the MH's



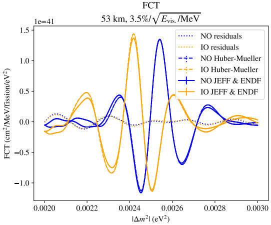


Fourier Transform

It is possible to determine the MH from the shape of the Fourier Sine (FST) and Cosine (FCT) Transform of the spectrum

- FCT: L/R= depth of the valleys on the left and right of the main peak, respectively; $(L-R)/(L+R) > (<) 0$ if NH (IH)
- FST: P/V= value of the peak (valley); $(P-V)/(P+V) > (<) 0$ if NH (IH)

It was shown that using the Fourier transform the fine structure has little if any effect on the MH determination. However this does not mean that the same is true if other approaches are used (ex: χ^2 test)



Danielson *et al*, arXiv:1808:03276

Problems with FT

However, several problems related to this approach:

- Sensitive to the high-energy part of the spectrum, which is not know very well (EC, J. Evslin and X.M. Zhang, Phys.Lett. B728)
- More difficult to control systematic errors
- Much more sensitive to non-linearity (Danielson *et al.* claim uncertainty below 0.5% is required)

For these reasons, very few papers in the last 4-5 years discuss this approach

χ^2 Test

The method most commonly used for the MH determination is the χ^2 test. We define

$$\Delta\chi^2 = \chi^2_{IH} - \chi^2_{NH} \qquad \Delta\chi^2 > (<) 0 \Rightarrow \text{NH (IH)}$$

The nuisance parameters are minimized. In this presentation

- Interference and degeneracy with Δm^2_{32} taken into account
- Background, non-linearity not considered

It was recently stated that the JUNO experiment will include a near detector: JUNO-TAU, Taishan Antineutrino Observatory; (See B. Wonsak, Neutrino2018, and Y.P. Cheng, NuPhys2018):
Data from the near detector + nuisance parameters (one for every energy bin, totally uncorrelated) $\Rightarrow$ no dependence on theoretical model. Near Detector:

- 35 m from one of the Taishan cores
- 1-3 ton Gd+LS

Many Nuisance Parameters

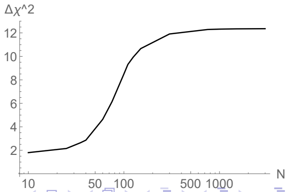
Discrete Approximation:

$$\int \mathrm{d}E\mathrm{d}E' e^{-(E-E')^2/2\sigma(E)^2}\phi(E)P(L,E) \to GP\vec{N}' = G_{ij}P_jN'_j$$

Where G_{ij} is a matrix represents the Gaussian convolution, $P_{ij} = P_i\delta_{ij}$ the oscillation probability and N_i the expected event (with no oscillation) in the energy bin (for simplicity, same energy resolution assumed at near and far)

	Asimov Data Set	Fit	σ^2
Far Detector	$GP_{NH}\vec{N}'$	$GP_{IH}(\vec{N}'+\vec{\beta}')$	$GP_{NH}\vec{N}'$
Near Detector	$1/\mathcal{R}G\vec{N}'$	$1/\mathcal{R}G(\vec{N}'+\vec{\beta}')$	$1/\mathcal{R}G\vec{N}'$

However N additional parameters must be minimized; moreover, since we are looking for small oscillations, a large number of energy bins is required (here used 700 bins in the 1.5-8.5 MeV range)



Gaussian Convolution

With perfect energy resolution the minimization of $\vec{\beta}$ is trivial (each β_i can be minimized separately), however the Gaussian convolution mixes the nuisance parameters of different energy bins; since in the far detector we have $GP\vec{\beta'}$ while in the near detector we have $G\vec{\beta'}$, any analytical solution involve the computation of G^{-1}. Approximation:

$$G(P'\vec{N'}) \to (GP')(G\vec{N'})$$

Physical meaning:

- The spectrum at near is assumed to be the "real" one (nuisance parameters take into account statistical fluctuations)
- Oscillated spectrum given by the product of the (Gaussian convoluted) oscillation probability and unoscillated spectrum
- Neglects that the Gaussian convolution of P should be weighted by the number of events in the bin: small error, because of good energy resolution

Gaussian Convolution

Is this a good approximation? Not exactly.... The error is less than 1%, however the difference between the normal and inverted hierarchy (*i.e.* $G\Delta P\vec{N}$) is very small: χ^2 can change significantly. However if this approximation is used for the Asimov data set as well, the 1% error is not on $GP\vec{N}$ anymore, but on $G\Delta P\vec{N}$

Asimov/Fit	$GP'\vec{N'}$	$(GP')(G\vec{N'})$
$GP'\vec{N'}$	12.17	11.10
$(GP')(G\vec{N'})$	-	12.11

$$GP' \to P \qquad G(\vec{N'} + \vec{\beta'}) \to \vec{N} + \vec{\beta}$$

The minimization over β is now trivial

$$\Delta\chi^2 = \sum_i \frac{(N_i \Delta P_i)^2}{N_i P_{NH,i} + \mathcal{R} P_{IH,i}^2 N_i}$$

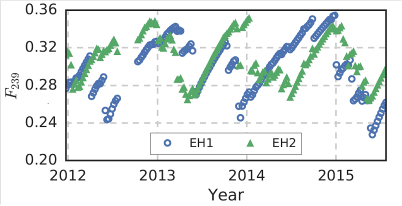
Chemical Composition

The reactor neutrino spectrum depends on the chemical composition of the fuel. For JUNO:

Near Detector $\rightarrow$ Taishan (one core)
Far Detector $\rightarrow$ Taishan + Yangjiang

Moreover, Taishan and Yangjiang are nuclear power plants of different model and generations (EPR, Gen. III and CPR-1000, Gen. II) $\Rightarrow$ The spectrum at near and far will be different.

However, studying the time evolution of the spectrum at near, it is possible to reconstruct the spectrum at far.



Daya Bay, PRL 118 (2017)

Chemical Composition

Considered only two isotopes: ^{239}P and ^{235}U. Spectrum at near divided into n time bins

$$\begin{aligned} \text{Near Detector} \quad & \frac{1}{\mathcal{R}} N_{j,i} = \frac{1}{\mathcal{R}}(\rho_j N_{P,i} + (1-\rho_j) N_{U,i}) \\ \text{Far Detector} \quad & N_i = \rho N_{P,i} + (1-\rho) N_{U,i} = \sum \alpha_j N_{j,i} \end{aligned}$$

$$\sum \alpha_j \rho_j = \rho \qquad \sum \alpha_j = 1$$

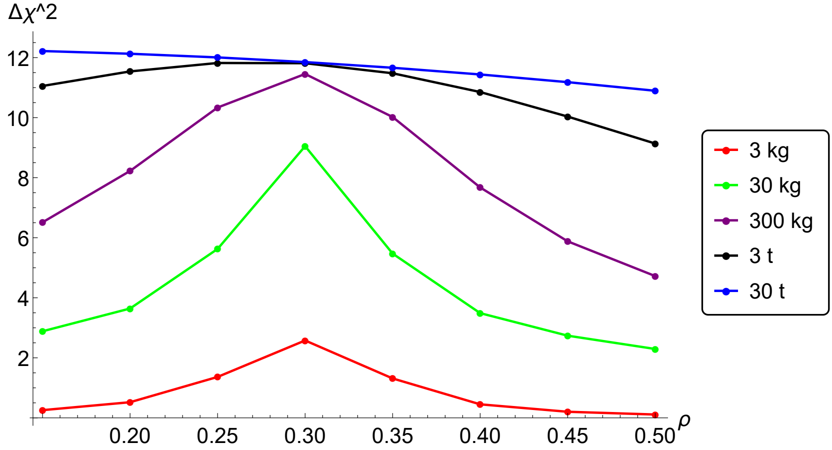
$$\chi^2 = \sum_i \frac{(N_i \Delta P_i)^2 (\sigma_{U,i}\sigma_{P,i} - \sigma_{M,i}^2)}{(\sigma_{U,i}\sigma_{P,i} - \sigma_{M,i}^2) P_{NH,i} N_i + ((1-\rho)^2 \sigma_{P,i} + \rho^2 \sigma_{U,i} - 2\rho(1-\rho)\sigma_{M,i}) \mathcal{R}^2 P_{IH,i}^2}$$

Where

$$\sigma_{U,i} = \sum_j \frac{\mathcal{R}(1-\rho_j)^2}{N_{j,i}} \qquad \sigma_{P,i} = \sum_j \frac{\mathcal{R}\rho_j^2}{N_{j,i}} \qquad \sigma_{M,i} = \sum_j \frac{\mathcal{R}(1-\rho_j)\rho_j}{N_{j,i}}$$

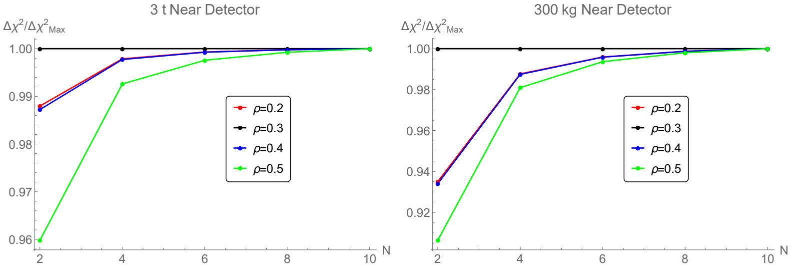
There is no dependence on α_j!

Chemical Composition



$\Delta\chi^2$ for different chemical compositions of the spectrum seen at the far detector. Two time bin assumed ($\rho_1 = 0.275$, $\rho_2 = 0.325$).

Number of Time Bins



$\Delta\chi^2$ as a function of the number of time bins (assuming ρ from 0.25 to 0.35, $\langle\rho\rangle = 0.3$).

Non-linearity

The same approach can be used to have a model-independent treatment of non-linearity. Let's call $E_{r(v)}$ = real (visible) energy,

$$E_r(E_v) = E_v(1 + \epsilon(E_v)) \qquad \int \mathrm{d}E \frac{\mathrm{d}n}{\mathrm{d}E} P(E, L) = F(E)$$

PN_i is the number of events between E_{i+1} and E_i (visible energy):

$$PN_i = F(E_{i+1}(1 + \epsilon_i)) - F(E_i(1 + \epsilon_i)) \simeq F(E_{i+1}) - F(E_i) + \epsilon_i \delta_i$$

$$\delta_i = EF'(E) = E \frac{\mathrm{d}n}{\mathrm{d}E} P(E, L)\Big|_{E_i}^{E_{i+1}}$$

Where ϵ_i are uncorrelated nuisance parameters.
However, define the penalty term for ϵ_i could be problematic (depends on the calibration, could lead to an unwanted dependence on the number of energy bins)

Conclusions

- Fine structure could affect the determination of the hierarchy: theoretical models are not reliable and no data available with sufficient energy resolution
- Fourier transform is not affected by this systematic error, however there are other problems (for example: very strong constrain on non-linearity needed)
- A near detector (which is now in program for the JUNO experiment) in the ton range will reduce almost completely the impact of fine structure
- Even if the near detector will see a different spectrum, from the time evolution it is possible to reconstruct a generic chemical composition (possible to use the data for other experiments as well)

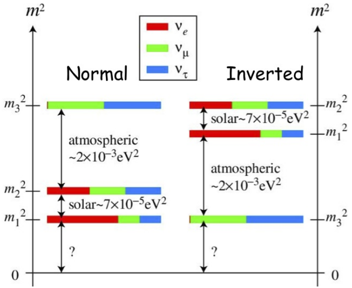
Backup Slides

The neutrino mass hierarchy

There are three light, mostly-active neutrino mass eigenstates called ν_1, ν_2 and ν_3 with masses are m_1, m_2 and m_3. Define

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

- Vacuum oscillations of ultrarelativistic $\nu \Rightarrow |\Delta m^2_{ij}|$
- Coherent interactions of solar neutrinos with the Sun $\Rightarrow \Delta m^2_{21} > 0$



The neutrino mass hierarchy (MH) is Sign(Δm^2_{31}).

From vacuum oscillations, we know $|\Delta m^2_{ij}|$.

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| \pm |\Delta m^2_{21}|$$

Precise measurement of $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$ (or two linear combinations of them) $\Rightarrow$ MH determination

Reactor Neutrino Experiments

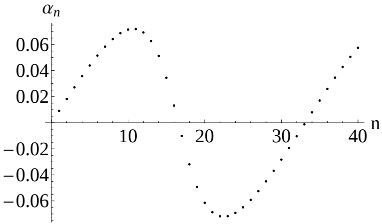
It is possible to determine the neutrino mass hierarchy studying the oscillation probability of **reactor neutrinos** at **intermediate baselines** (S. Choubey, S. T. Petcov, and M. Piai, Phys.Lett. B533 (2002) 94-106), where L/E is large enough that the effect of Δm^2_{21} is not negligible, but small enough that the 2-3 oscillations are not smeared out completely by the finite energy resolution ($|\Delta m^2_{31}| \sim |\Delta m^2_{31}| \sim 30|\Delta m^2_{21}|$).

$$
\begin{aligned}
P_{ee} &= 1 - \sin^2(2\theta_{12})\cos^4(\theta_{13})\sin^2\left(\frac{1.27\Delta m^2_{21}L}{E}\right) \\
&- \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2\left(\frac{1.27\Delta m^2_{13}L}{E}\right) \\
&- \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2\left(\frac{1.27\Delta m^2_{23}L}{E}\right)
\end{aligned}
$$

The MH can be determined studying the peaks of 1-3 and 2-3 oscillations: indeed locations of the peaks is given by

$$\frac{L}{E} = \frac{\pi}{1.27\Delta m^2_{23}}(n \pm \alpha_n)$$

- $\alpha_n > 0$ $(< 0) \Rightarrow$ NH (IH)
- However, at small n α_n is almost linear $\Rightarrow$ MH degenerate with shift in Δm^2_{23} (oscillations are dominated by an effective mass)
- It is necessary to measure the peaks at small and large n $\Rightarrow$ Intermediate baselines



(EC, J. Evslin and X.M. Zhang, JHEP 2013)

Challenges

Challenges in mass hierarchy reactor neutrino experiments:

- Degeneracy between a shift of Δm^2_{32} and a change of hierarchy
- It is necessary to detect a large range of oscillations $\Rightarrow$ excellent energy resolution needed ($\leq 3\%/\sqrt{E}$)
- Interference effect due to the different baselines of reactor cores decreases the precision (EC, J. Evslin and X.M. Zhang, JHEP 1212)
- Systematic error in the energy reconstruction (non-linearity) can be a problem $\Rightarrow$ great precision required (See, for example, X. Qian *et al.*, Phys. Rev. D 87; Y.-F. Li *et al.* Phys. Rev. D 88; EC *et al.*, Phys.Rev. D89; F. Capozzi, E. Lisi, and A. Marrone, Phys. Rev. D 92)
- Theoretical models for reactor neutrinos are not reliable ("5 MeV bump", reactor anomaly)

Lot of literature on these topics, references above are just a (very) partial list