DOUBLE PARTON SCATTERING AT SMALL RELATIVE TRANSVERSE DISTANCES

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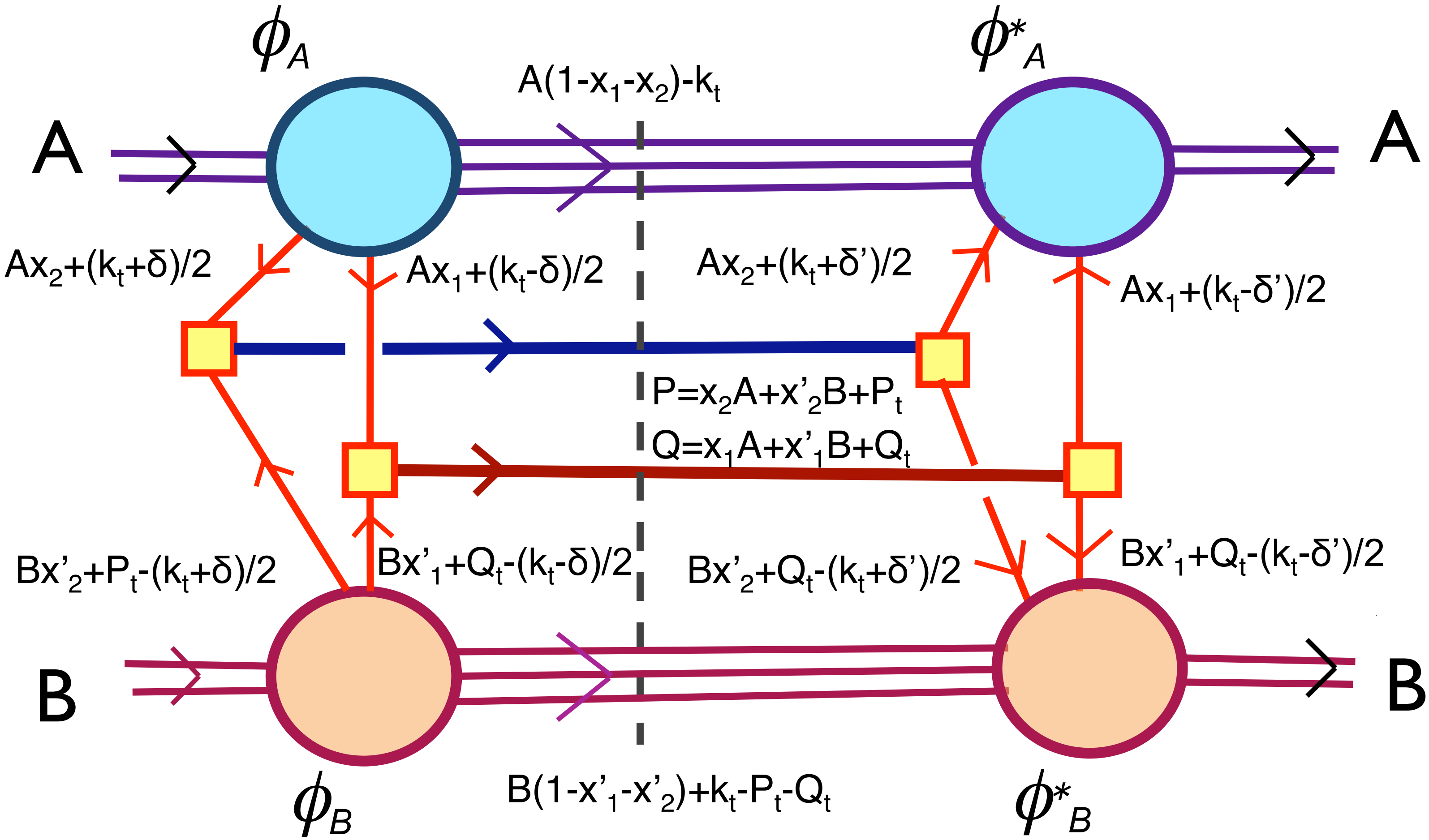
OUTLINE

- DPS at leading order, transverse integrations

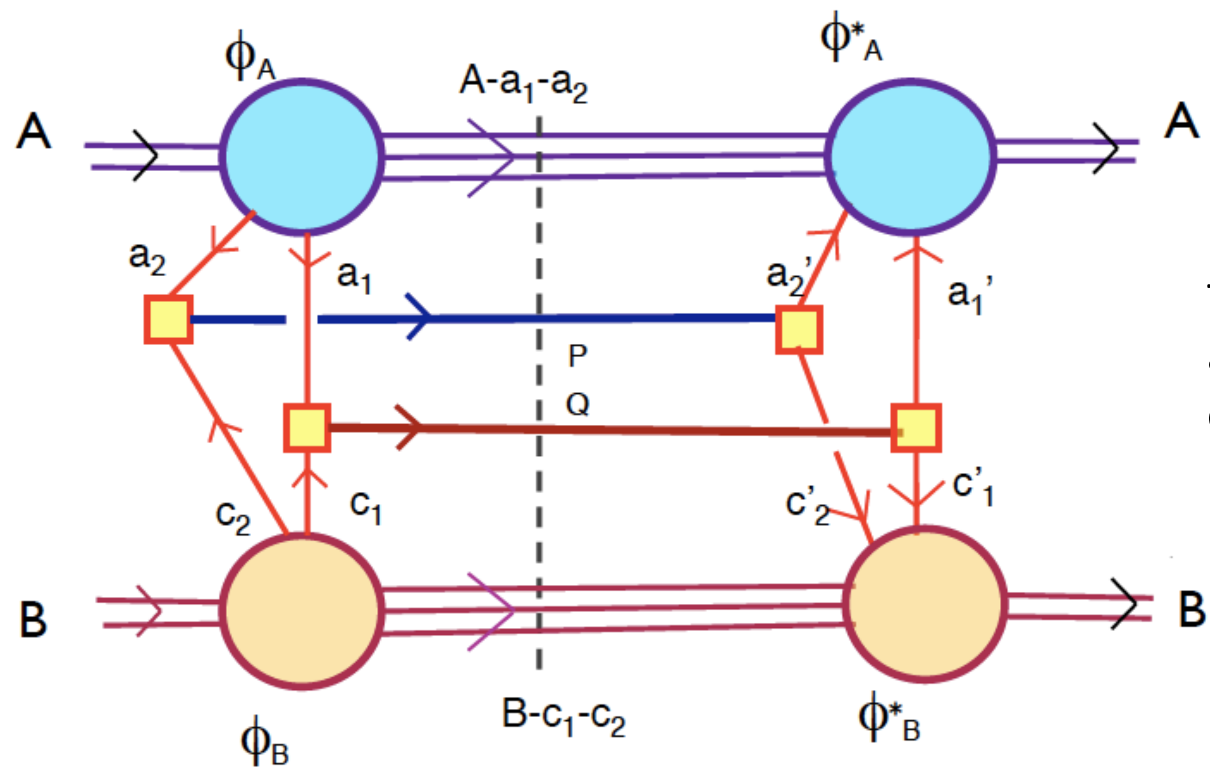
- Dominant contribution at small relative transverse distances

- Azimuthal correlations induced by parton splitting

The DPS contribution to the forward A+B scattering amplitude is a 5-loops diagram. P and Q are the c.m. momenta of the partonic states generated by the two hard collisions



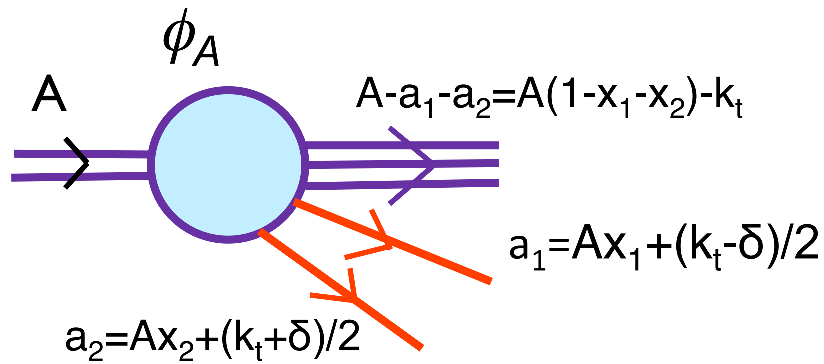
The light cone + components are large in the upper part of the diagram



The transverse components are of the same size all over the diagram

The light cone - components are large in the lower part of the diagram

To evaluate the leading contribution to the DPS cross section, only the longitudinal momentum components which grow as $s^{1/2}$ are to be taken into account. In this way the integrations on the light cone components which depend on δ_-, δ_-' and δ_+, δ_+' involve respectively only the upper and the lower parts of the diagram.

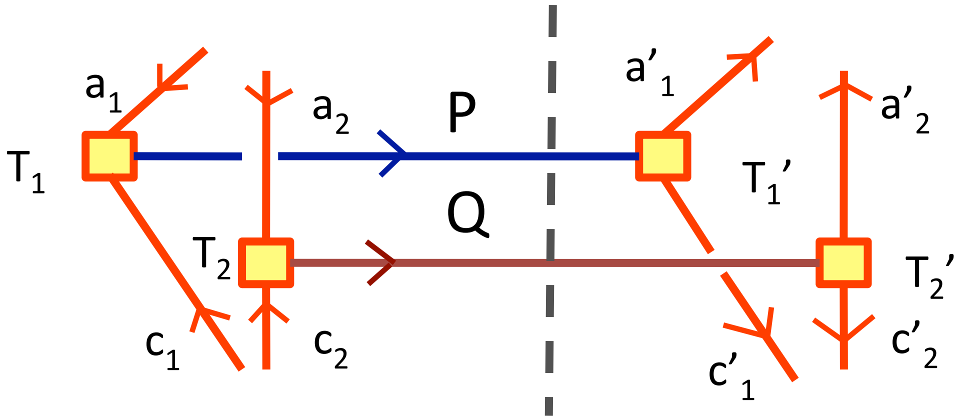


One may thus define the quantities:

$$\Psi_A(x_i, k_t, \delta_t) = \frac{1}{\sqrt{2}} \int \frac{\phi_A}{a_1^2 a_2^2} \frac{d\delta_-}{2\pi}, \qquad \Psi_B(x_i', k_t, \delta_t) = \frac{1}{\sqrt{2}} \int \frac{\phi_B}{c_1^2 c_2^2} \frac{d\delta_+}{2\pi}$$

$$\Psi_A^*(x_i, k_t, \delta_t') = \frac{1}{\sqrt{2}} \int \frac{\phi_A^*}{a_1'^2 a_2'^2} \frac{d\delta_-'}{2\pi}, \qquad \Psi_B^*(x_i', k_t, \delta_t') = \frac{1}{\sqrt{2}} \int \frac{\phi_B^*}{c_1'^2 c_2'^2} \frac{d\delta_+'}{2\pi}$$

At leading order in $S^{1/2}$, the central part of the diagram does not depend on the transverse coordinates.



$$\Rightarrow \quad |T_1(a_1 + c_1 \to P)|^2 |T_2(a_2 + c_2 \to Q)|^2$$

It's convenient to introduce the Fourier transforms

$$\tilde{\Psi}_A(x_i, b_1, b_2) = \frac{1}{(2\pi)^2}\int e^{i\mathbf{b}_1\cdot\mathbf{a}_{1,t}} e^{i\mathbf{b}_2\cdot\mathbf{a}_{2,t}} \Psi_A(x_i, a_{1,t}, a_{2,t}) d^2a_{1,t} d^2a_{2,t} \qquad \text{etc.}$$

By expressing the functions Ψ in terms of the transverse coordinates *b*, one can in fact perform all integrations on the transverse momenta

The explicit expression of all transvers integrations is

$$\frac{1}{(2\pi)^{10}}\int \tilde{\Psi}_A(b_1, b_2)\tilde{\Psi}_B(b_3, b_4)\tilde{\Psi}^*_A(b'_1, b'_2)\tilde{\Psi}^*_B(b'_3, b'_4)\ d^2b_1 d^2b_2 d^2b_3 d^2b_4 d^2b'_1 d^2b'_2 d^2b'_3 d^2b'_4 d^2\delta_t d^2\delta'_t$$
$$\times\exp\Big[i\Big\{\frac{1}{2}(\mathbf{k}_t+\boldsymbol{\delta}_t)\cdot\mathbf{b}_1 + \frac{1}{2}(\mathbf{k}_t-\boldsymbol{\delta}_t)\cdot\mathbf{b}_2 + [\frac{1}{2}(-\mathbf{k}_t-\boldsymbol{\delta}_t)+\mathbf{P}_t]\cdot\mathbf{b}_3 + [\frac{1}{2}(-\mathbf{k}_t+\boldsymbol{\delta}_t)+\mathbf{Q}_t]\cdot\mathbf{b}_4$$
$$-\frac{1}{2}(\mathbf{k}_t+\boldsymbol{\delta}'_t)\cdot\mathbf{b}'_1 - \frac{1}{2}(\mathbf{k}_t-\boldsymbol{\delta}'_t)\cdot\mathbf{b}'_2 - [\frac{1}{2}(-\mathbf{k}_t-\boldsymbol{\delta}'_t)+\mathbf{P}_t]\cdot\mathbf{b}'_3 - [\frac{1}{2}(-\mathbf{k}_t+\boldsymbol{\delta}'_t)+\mathbf{Q}_t]\cdot\mathbf{b}'_4\Big\}\Big]$$

The integrations on δ_t and δ_t' give

$$d^2\delta_t \to (2\pi)^2\delta(\mathbf{b}_1 - \mathbf{b}_2 - \mathbf{b}_3 + \mathbf{b}_4), \qquad d^2\delta_t' \to (2\pi)^2\delta(\mathbf{b}_1' - \mathbf{b}_2' - \mathbf{b}_3' + \mathbf{b}_4')$$

Which imply

$$\boxed{\mathbf{b}_1 - \mathbf{b}_2 = \mathbf{b}_3 - \mathbf{b}_4 \equiv \mathbf{b}, \qquad \mathbf{b}_1' - \mathbf{b}_2' = \mathbf{b}_3' - \mathbf{b}_4' \equiv \mathbf{b}'}$$

Only final state fragments with large k_t are observed. By integrating on k_t one obtains

$$d^2k_t \to (2\pi)^2\delta(\mathbf{b}_1 + \mathbf{b}_2 - \mathbf{b}_3 - \mathbf{b}_4 - \mathbf{b}_1' - \mathbf{b}_2' + \mathbf{b}_3' + \mathbf{b}_4')$$

By introducing the transverse coordinates of centers of mass

$$\frac{1}{2}(\mathbf{b}_1 + \mathbf{b}_2) \equiv \mathbf{B}_1, \qquad \frac{1}{2}(\mathbf{b}_3 + \mathbf{b}_4) \equiv \mathbf{B}_3, \qquad \frac{1}{2}(\mathbf{b}_1' + \mathbf{b}_2') \equiv \mathbf{B}_1', \qquad \frac{1}{2}(\mathbf{b}_3' + \mathbf{b}_4') \equiv \mathbf{B}_3'$$

The constraint above implies the following relations

$$\boxed{\mathbf{B}_1 - \mathbf{B}_3 = \mathbf{B}_1' - \mathbf{B}_3' \equiv \mathbf{\Delta} \quad \Rightarrow \quad \mathbf{B}_3 = \mathbf{B}_1 - \mathbf{\Delta},\ \mathbf{B}_3' = \mathbf{B}_1' - \mathbf{\Delta}}$$

and the expression, giving all transverse integrations, thus simplifies to

$$\frac{1}{(2\pi)^4}\int d^2\Delta\int\tilde{\Psi}_A(b,B_1)\tilde{\Psi}_B(b,B_1-\Delta)\times e^{i(\mathbf{P}_t+\mathbf{Q}_t)\cdot\mathbf{B}_1}e^{i(\mathbf{P}_t-\mathbf{Q}_t)\cdot\mathbf{b}/2}d^2B_1d^2b$$
$$\times\tilde{\Psi}^*_A(b',B'_1)\tilde{\Psi}^*_B(b',B'_1-\Delta)\times e^{-i(\mathbf{P}_t+\mathbf{Q}_t)\cdot\mathbf{B}'_1}e^{-i(\mathbf{P}_t-\mathbf{Q}_t)\cdot\mathbf{b}'/2}d^2B'_1d^2b'$$
$$=\int d^2\Delta\Big|\frac{1}{(2\pi)^2}\int\tilde{\Psi}_A(b,B_1)\tilde{\Psi}_B(b,B_1-\Delta)\times e^{i(\mathbf{P}_t+\mathbf{Q}_t)\cdot\mathbf{B}_1}e^{i(\mathbf{P}_t-\mathbf{Q}_t)\cdot\mathbf{b}/2}d^2B_1d^2b\Big|^2$$

When integrating on the transverse c.m. coordinates $\mathbf{P}_t$ and $\mathbf{Q}_t$ one obtains

$$\int|\tilde{\Psi}_A(x_i,b,B_1)|^2|\tilde{\Psi}_B(x'_i,b,B_1-\Delta)|^2d^2B_1d^2bd^2\Delta$$
$$=\int d^2b\int|\tilde{\Psi}_A(x_i,b,B_1)|^2d^2B_1\int|\tilde{\Psi}_B(x'_i,b,B'_1)|^2d^2B'_1$$

which, after multiplying by the flux factors of the elementary partonic collisions and by the factors deriving from the integration on the invariant mass of the residual hadron fragments, gives the product of the Double Parton Distributions of the two interacting hadrons integrated on the relative transverse distance **b** and one thus obtains the well known expression of the Double Parton scattering cross section

The transverse c.m. momenta of the two hard collisions can however be measured.

$\mathbf{P}_t$-$\mathbf{Q}_t$ is the conjugate variable of the relative transverse distance **b** between the two collisions and the dependence of the cross section on $\mathbf{P}_t$-$\mathbf{Q}_t$ is therefore of particular interest.

Going back to the expression giving the transverse integrations, by Fourier transforming the c.m. coordinates, one obtains

$$\frac{1}{(2\pi)^8}\int \bar{\Psi}_A(b,K_1)\bar{\Psi}_B(b,K_2)\bar{\Psi}^*_A(b',K_1')\bar{\Psi}^*_B(b',K_2')\ d^2B_1 d^2B_1' d^2b d^2b' d^2\Delta d^2K_1 d^2K_2 d^2K_1' d^2K_2'$$
$$\times\exp\Big\{i\big[(\mathbf{P}_t+\mathbf{Q}_t)\cdot\mathbf{B}_1+\mathbf{K}_1\cdot\mathbf{B}_1+\mathbf{K}_2\cdot(\mathbf{B}_1-\boldsymbol{\Delta})+(\mathbf{P}_t-\mathbf{Q}_t)\cdot\mathbf{b}/2$$
$$-(\mathbf{P}_t+\mathbf{Q}_t)\cdot\mathbf{B}_1'-\mathbf{K}_1'\cdot\mathbf{B}_1'-\mathbf{K}_2'\cdot(\mathbf{B}_1'-\boldsymbol{\Delta})-(\mathbf{P}_t-\mathbf{Q}_t)\cdot\mathbf{b}'/2\big]\Big\}$$

which, after integrating on Δ, B_1 and B_1', gives

$$\int d^2K_1\left|\frac{1}{2\pi}\int \bar{\Psi}_A(b,K_1)\bar{\Psi}_B(b,-K_1-P_t-Q_t)\,e^{i(\mathbf{P}_t-\mathbf{Q}_t)\cdot\mathbf{b}/2}\,d^2b\right|^2$$

Because *parton splitting*, when **b** goes to zero Ψ_A and Ψ_B are proportional to 1/b. To simplify the discussion, let us consider the following factorized expression:

$$\bar{\Psi}_{A,B}(x_i, b, K_1) = \Big[\frac{e^{-b^2/(4R^2)}}{b\sqrt{2\pi}}\Big]\varphi_{A,B}(x_i, K_1)$$

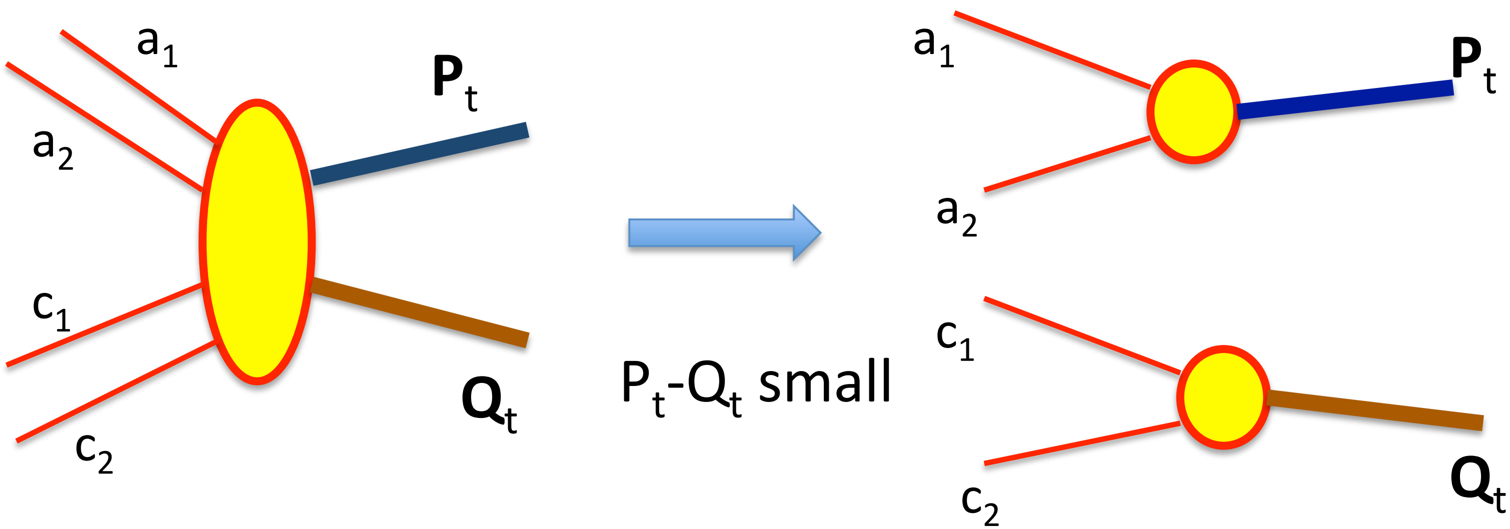
The singular contribution to the transverse integrations can thus be factorized as a product of two different integrals

$$\int d^2K_1 \Big|\varphi_A(x_i, K_1)\varphi_B(x_i', -K_1 - P_t - Q_t)\Big|^2 \times \Big|\frac{1}{(2\pi)^2}\int e^{i(\mathbf{P}_t - \mathbf{Q}_t)\cdot\mathbf{b}/2} e^{-b^2/(2R^2)}\,\frac{d^2b}{b^2}\Big|^2$$

The first integral gives the dependence of the non-perturbative component on the fractional momenta and on the total transverse c.m. hard process $\mathbf{P}_t$+$\mathbf{Q}_t$ through the convolution of the hadronic form factors of the two hadrons.

The second integral is singular when *b* goes to zero.

In a DPS the hard interaction factorizes in two almost uncorrelated components



One needs therefore to introduce a upper scale *S*, which represents the upper limit of $|\mathbf{P}_t\text{-}\mathbf{Q}_t|$ to consider the interaction a DPS process. The relative transverse distance *b* is conjugate to $|\mathbf{P}_t\text{-}\mathbf{Q}_t|$, so $b_{min}=1/S$

Notice that by choosing the value of *S*, one *defines* what one means by DPS cross section

At small relative transverse distances the process cannot be considered a DPS any more. The singular contribution therefore does not contribute to the DPS cross section and has to be subtracted:

$$\int_0^\infty \frac{e^{-b^2/(2R^2)}}{2\pi\, b}\left[1 - J_0(|\mathbf{P}_t - \mathbf{Q}_t|b/2)\right]db = \frac{1}{4\pi}\Big[\gamma - \mathrm{Ei}\Big(-\frac{|\mathbf{P}_t - \mathbf{Q}_t|^2 R^2}{2}\Big) + \log\Big(\frac{|\mathbf{P}_t - \mathbf{Q}_t|^2 R^2}{2}\Big)\Big]$$

After subtracting the singular contribution, the leading contribution at small relative transverse distances to the transverse integrations is given by

$$\int d^2K_1 \Big|\varphi_A(x_i, K_1)\varphi_B(x_i', -K_1 - P_t - Q_t)\Big|^2 \times \Big[\frac{1}{4\pi}\log\Big(\frac{|\mathbf{P}_t - \mathbf{Q}_t|^2 R^2}{2}\Big)\Big]^2$$

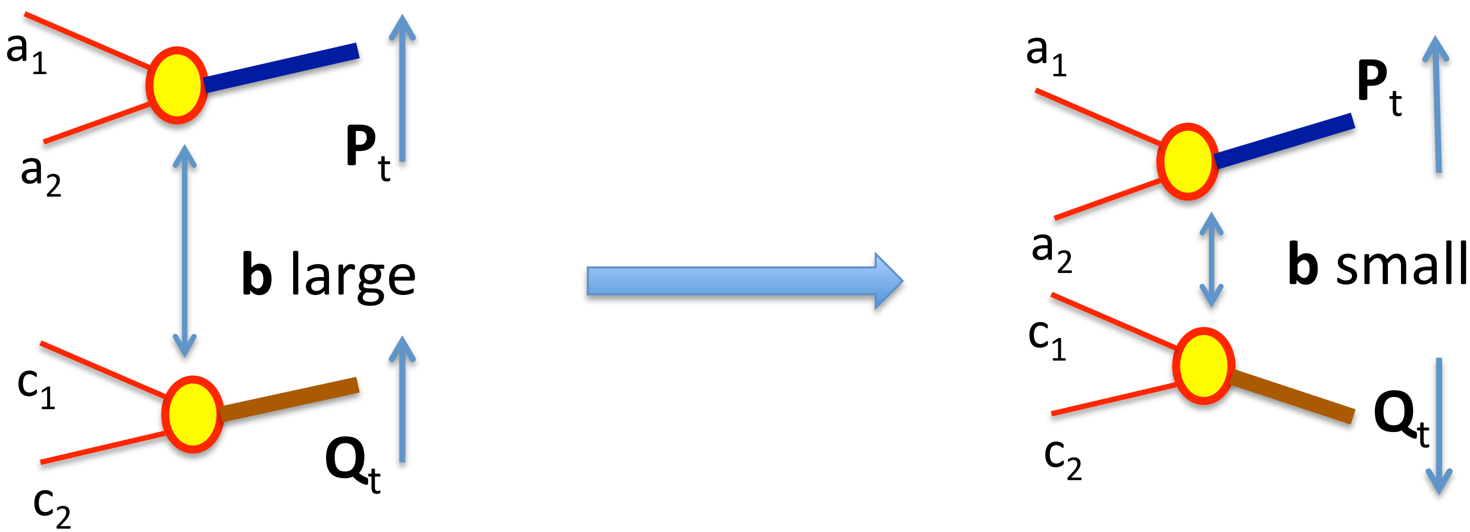
The resulting parton luminosity grows therefore logarithmically at small b.
The leading term being proportional to

$$\left[\log\left(|\mathbf{P}_t-\mathbf{Q}_t|\right)\right]^2, \qquad \text{where} \qquad |\mathbf{P}_t-\mathbf{Q}_t|<S=1/b_{min}$$

where S is the upper value of the relative transverse momentum between the two hard interactions, which allows considering the process a Double Parton Scattering

Because of parton splitting, the parton population grows as a $[\log|\mathbf{P}_t\text{-}\mathbf{Q}_t|]^2$ when $|\mathbf{P}_t\text{-}\mathbf{Q}_t|$ increases, namely when b decreases

Having selected the c.m. transverse momenta of the two hard interactions such that $|\mathbf{P}_t - \mathbf{Q}_t| < S$, the corresponding range of the relative transverse distance **b** runs from a minimum not smaller than 1/S, when $\mathbf{P}_t$ and $\mathbf{Q}_t$ are sizeably different in moduli and/or with a large relative angle, to a considerably larger maximum, which may reach also values of the hadron radius, when $\mathbf{P}_t$ and $\mathbf{Q}_t$ are close in moduli and with a small relative angle.



$|\mathbf{P}_t - \mathbf{Q}_t|$ small
→ lower parton luminosity

$|\mathbf{P}_t - \mathbf{Q}_t|$ large
→ higher parton luminosity

What happens with evolution ?

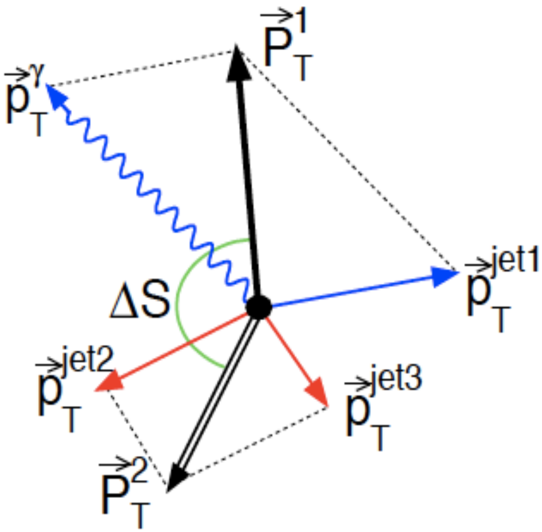
Independent evolution generates initial state configurations with much larger c.m. transverse momenta $\mathbf{P}_t$ and $\mathbf{Q}_t$ and with directions randomly distributed both in their relative angle and as a function of their relative transverse distance **b**.

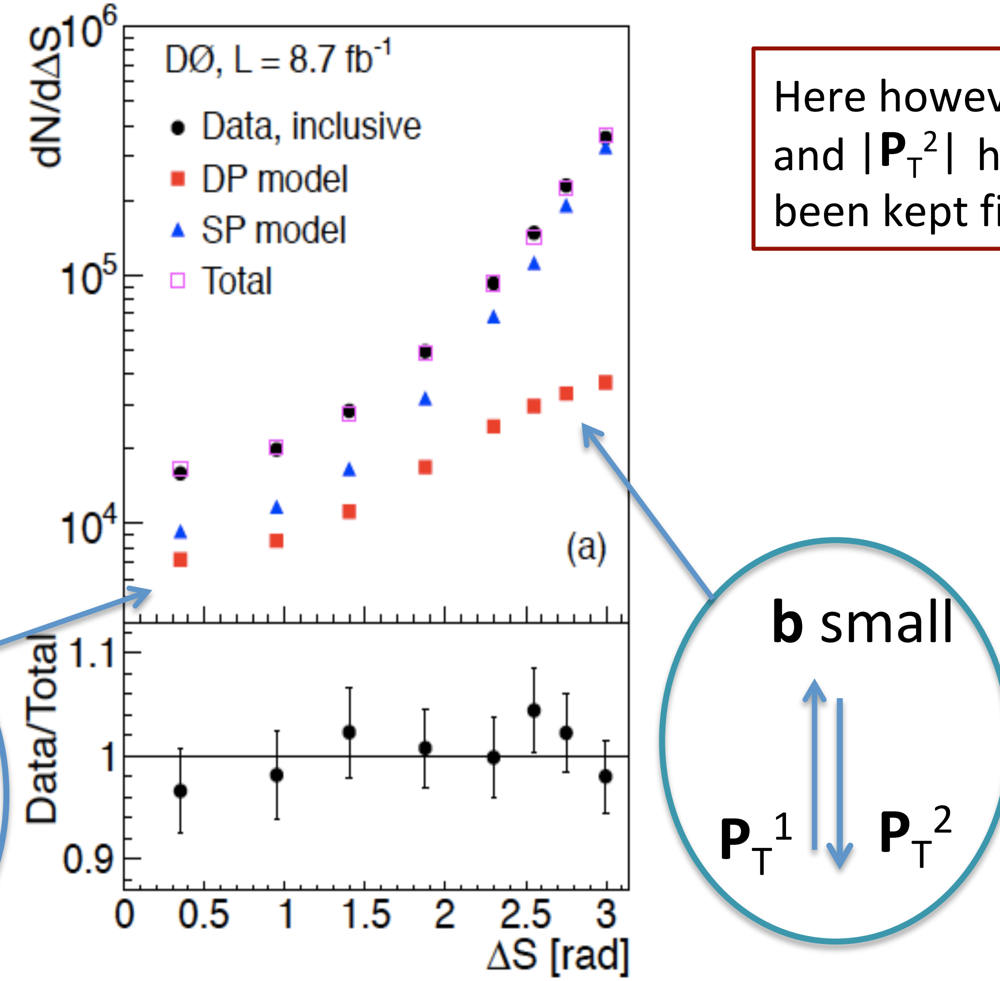
The initial flux of interacting parton pairs, generated by independent evolution depends therefore on the absolute values $|\mathbf{P}_t|$ *and* $|\mathbf{Q}_t|$.

When the source of initial state partons is short distance dynamics, the initial flux of interacting parton pairs grows, on the contrary, at small relative transverse distances, namely when the angle between $\mathbf{P}_t$ *and* $\mathbf{Q}_t$ *increases*. **Parton splitting therefore induces a azimuthal correlation between the transverse directions of $\mathbf{P}_t$ and $\mathbf{Q}_t$**

By keeping $|\mathbf{P}_t|$and $|\mathbf{Q}_t|$ fixed, One may thus have a direct indication on the relative importance of splitting versus independent evolution, by comparing the rate of DPS, in events where $\mathbf{P}_t$ and $\mathbf{Q}_t$ are parallel, with the rate of DPS in events where $\mathbf{P}_t$ and $\mathbf{Q}_t$ lie back to back.

D0 Collaboration, Phys Rev D 89, 072006 [arXiv:1402.1550]





Here however $|\mathbf{P}_T^1|$ and $|\mathbf{P}_T^2|$ have not been kept fixed

CONCLUDING SUMMARY

By working out the explicit dependence of DPS, as a function of the transverse momenta of the c.m. $\mathbf{P}_t$ and $\mathbf{Q}_t$ of the two hard partonic interactions, in the simplest instance of DPS at the lowest order in the coupling constant, we find that the relative transverse distance b between the two interactions is conjugate to $|\mathbf{P}_t - \mathbf{Q}_t|$.

Because of parton splitting, the Double Parton Distributions are proportional to $1/b^2$ at small b. As a consequence the parton population grows as the $\log(|\mathbf{P}_t - \mathbf{Q}_t|)$ at small b.

Small b values are hence correlated with large values of $|\mathbf{P}_t - \mathbf{Q}_t|$

Independent evolution does not induce any angular correlation between $\mathbf{P}_t$ and $\mathbf{Q}_t$.

One may thus have a direct indication on the relative importance of splitting versus independent evolution, by comparing the rate of DPS, in events where $\mathbf{P}_t$ and $\mathbf{Q}_t$ are parallel, with the rate of DPS in events where $\mathbf{P}_t$ and $\mathbf{Q}_t$ lie back to back.