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Istituto Nazionale di Fisica Nucleare

Mini-jet model estimate of the effective cross-section for double parton scattering

Giulia Pancheri

INFN Frascati National Laboratories

With A. Grau, S. Pacetti and Y.N. Srivastava

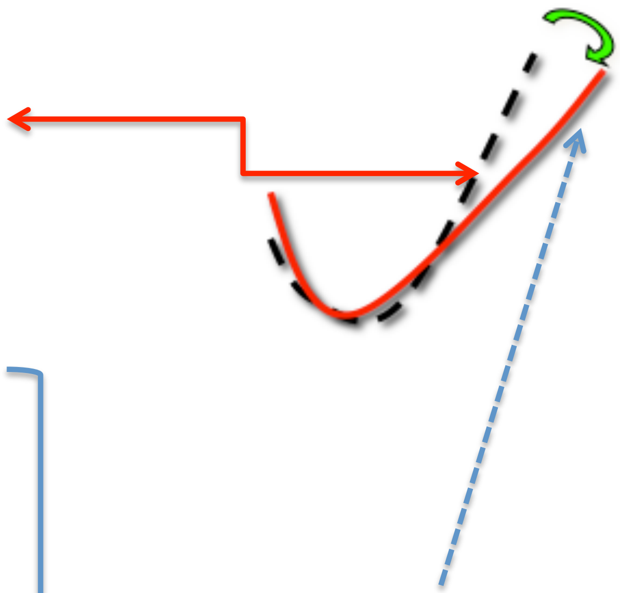
Outline

- The total cross-section: an overview
- The eikonal minijet model with soft gluon resummation (BN model)
  - Minijets
  - Parton impact parameter from soft gluon resummation => an ansatz for $k_t \sim 0$
- Application to calculation of the effective cross-section
- Comparison with existing estimates

The Bloch-Nordsieck (BN) inspired model for the total cross-section

with A. Grau, R.M. Godbole and Y.N.Srivastava (PRD 1999, 2004)

- Perturbative QCD
  → 1/x gluons → rise with energy



- Soft gluon resummation into the infrared

  +

- Eikonalize → unitarity
  - And resummation of multiple scattering

Tame the rise

Eikonal mini-jet model for the total cross-section

$$\bar{n}_{coll}(s) = 2\chi_I = A(b, s; p, PDF, p_{tmin})\sigma_{mini-jet}(s; p_{tmin}, PDF)$$

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi_I(b,s)}]$$

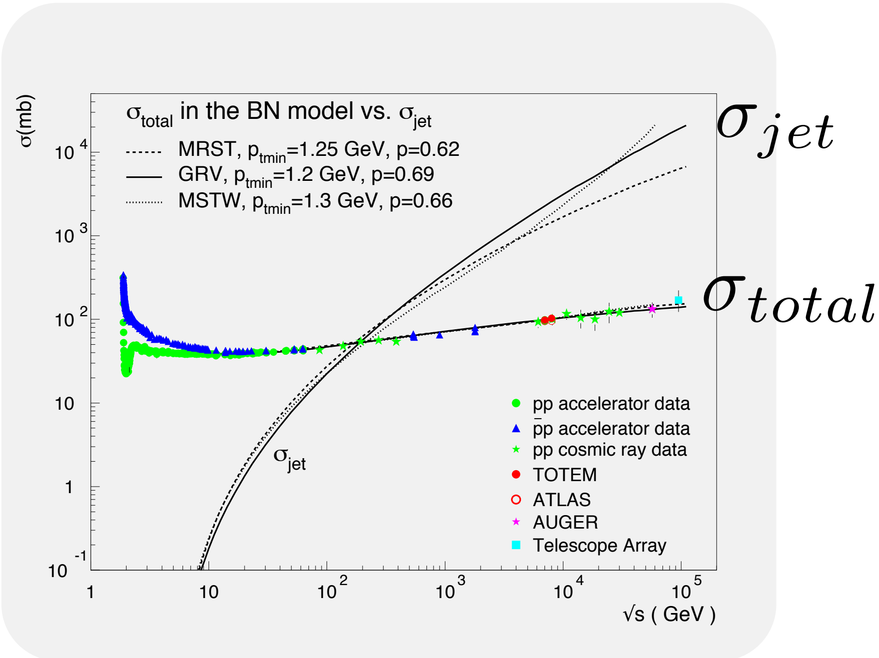
A.Grau,G.P.,Y.N.Srivastava
PRD 60 (1999)

- The eikonal function ~ real
- The rise is from pQCD → minijets with actual PDFs
- The taming (Froissart bound) of minijet rise is from all order resummation of soft gluons accompanying mini-jet producing collisions

PDF driven eikonal minijet model: Minijets vs total cross-section

$$\sigma^{AB}_{\rm jet}(s;p_{t_{min}}) = \int_{p_{t_{min}}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|A}(x_1,p_t^2) f_{j|B}(x_2,p_t^2) \frac{d\hat{\sigma}^{kl}_{ij}(\hat{s})}{dp_t}.$$

Negligible at low energies



But when

with $p_t \geq 1$ GeV → $x \leq 0.1$ $\alpha_{strong}(p_t) \lesssim 1 \rightarrow \alpha_{AF}$ → pQCD can be applied

$$\sigma^{AB}_{\rm jet}(s;p_{t_{min}} \simeq 1\ GeV) \sim 10\%\ \sigma^{AB}_{total} \qquad \sqrt{s} \simeq 20 \div 30\ GeV$$

The full eikonal in impact parameter space includes also a "soft" component

$$\bar{n}(b,s) = \bar{n}_{soft}(b,s) + \bar{n}_{mini-jets}(b,s) =$$
$$= A_{FF}(b,s)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)$$

F-transform of the proton form factor

$$A_{FF}(b) = \frac{\nu^2}{96\pi}(\nu b)^3 K_3(\nu b)$$

ν =0.71 GeV²

$$\sigma_{soft}(s) = constant \quad or \quad slowly \quad decreasing$$

$$A(b,s;p,PDF,p_{min}) \equiv A_{BN}(b,s)$$

BN from resummation of soft gluons → into $k_t \approx 0$ (Bloch & Nordsieck inspired)

We model the impact parameter distribution for partons →minijets as the Fourier-transform of ISR soft k_t distribution and thus obtain a cut-off at large distances →Froissart bound

$$A_{BN}(b,s) = N\int d^2\mathbf{K}_\perp \; e^{-i\mathbf{K}_\perp\cdot\mathbf{b}}\frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\; e^{-h(b,q_{max})}}$$

$$h(b,q_{max}) = \frac{16}{3\pi}\int_0^{q_{max}}\frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln\left(\frac{2q_{max}}{k_t}\right)[1-J_0(bk_t)]$$

1. $\alpha_{eff}(k_t\to 0)\sim k_t^{-2p}$ ← ½<p<1

2. $q_{max}(\sqrt{s};p_{tmin},PDF)$? **Calculated** from single gluon emission kinematics

A.Corsetti, A. Grau, G.P., Y.N. Srivastava PLB 1996

Semi-classical derivation (B. Touschek 1967)

$$d^2P(\mathbf{K}_t) = \sum_{n_\mathbf{k}} P(\{n_\mathbf{k}\}) d^2\mathbf{K}_t \delta^2(\mathbf{K}_t - \sum_\mathbf{k} \mathbf{k}_t n_\mathbf{k}) =$$

$$\sum_{n_k} \Pi_\mathbf{k} \frac{[\bar{n}_\mathbf{k}]^{n_\mathbf{k}}}{n_\mathbf{k}!} e^{-\bar{n}_\mathbf{k}} d^2\mathbf{K}_t \delta^2(\mathbf{K}_t - \sum_\mathbf{k} \mathbf{k}_t n_\mathbf{k})$$

Exchange Sum with Product →

$$d^2P(\mathbf{K}_t) = \frac{d^2\mathbf{K}_t}{(2\pi)^2} \int d^2\mathbf{b} e^{-i\mathbf{K}_t \cdot \mathbf{b}} exp\{-\sum_\mathbf{k} \bar{n}_\mathbf{k}[1 - e^{i\mathbf{k}_t \cdot \mathbf{b}}]\}$$

✪✃ Continuum limit → $\frac{d^2\mathbf{K}_t}{(2\pi)^2} \int d^2\mathbf{b} e^{-i\mathbf{K}_t \cdot \mathbf{b}} exp\{-\int d^3\bar{n}_\mathbf{k}[1 - e^{i\mathbf{k}_t \cdot \mathbf{b}}]\}$

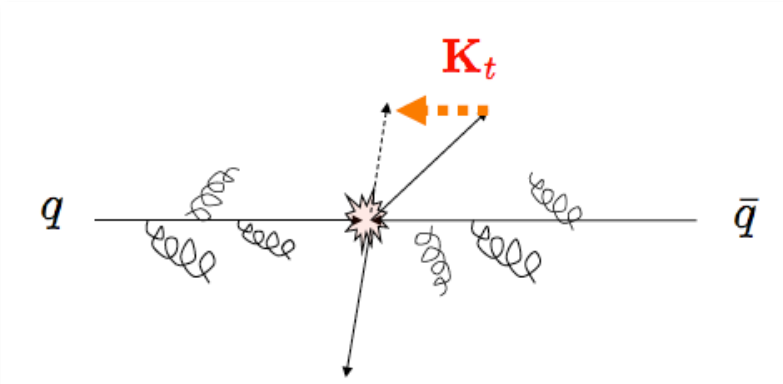
$$h(b, q_{max}) = \frac{16}{3\pi} \int_0^{q_{max}} \frac{dk_t}{k_t} \alpha_{eff}(k_t) \ln(\frac{2q_{max}}{k_t})[1 - J_0(bk_t)]$$

Regularized exponentiated soft gluon spectrum

→ Semiclassical Resummation procedure based on soft gluon Poisson distributions a' la Bloch and Nordsieck+ energy Momentum conservation

→ Needs integrable "effective" quark-gluon coupling constant

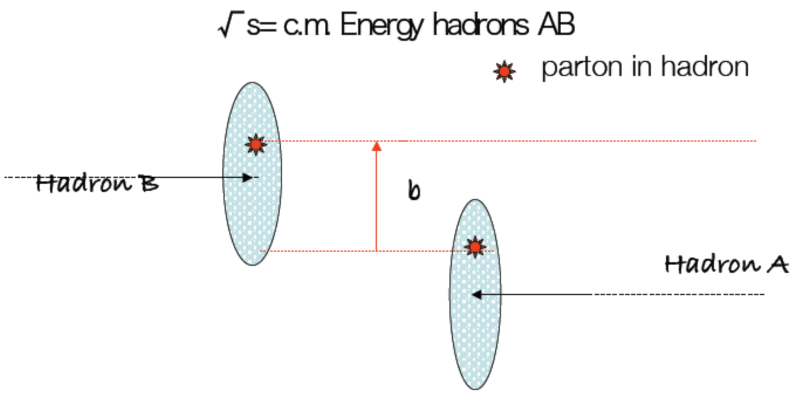
Nakamura, GP, Srivastava 1984



Implemented for impact parameter Distribution of partons

Corsetti, Grau,GP,Srivastava,PLB 1996

We model the impact parameter distribution for partons →minijets as the Fourier-transform of ISR soft k_t distribution and thus obtain a cut-off at large distances →Froissart bound



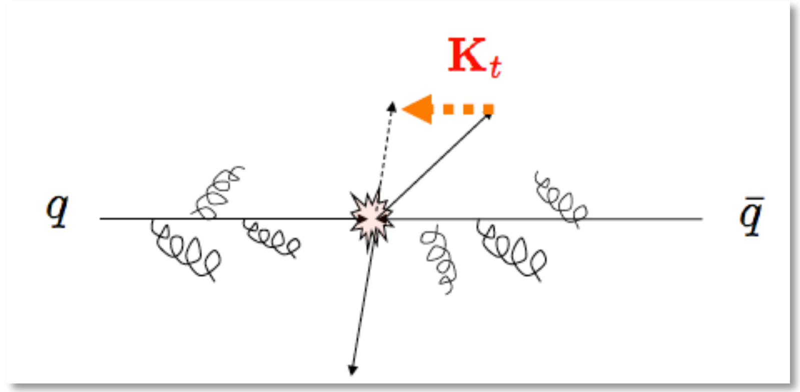
$$A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}$$

$$\bar{\Lambda} \equiv f(q_{max}, \Lambda_{QCD}, p)$$

$$q_{max}(\sqrt{s}; p_{tmin}, PDF) \quad \textbf{?}$$

Calculated from single gluon emission kinematics and averaged over densities (PRD 1999)

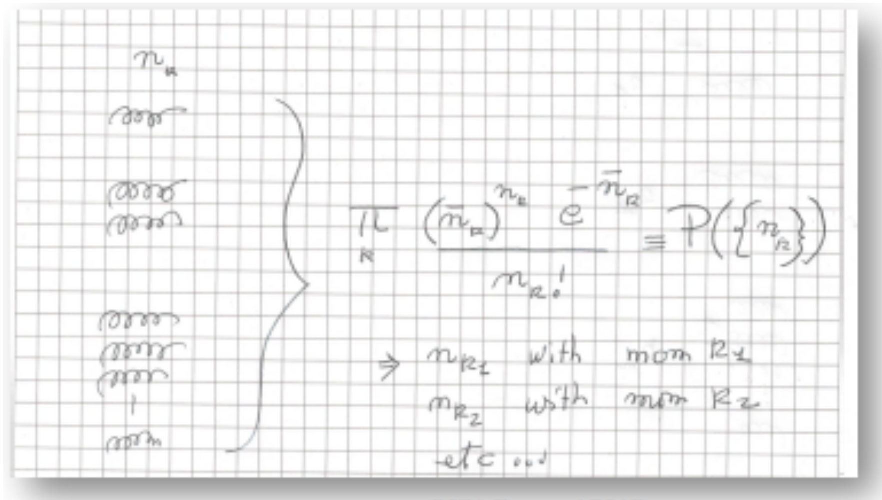
The BN inspired model for RESUMMING SOFT GLUONS



- Based on a democratic pathway to sum soft quanta – semiclassical approach with the ansatz :

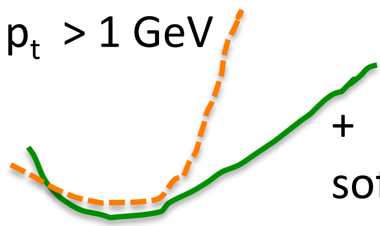
$$\alpha_{eff}(k_t \approx 0) \approx k_t^{-2p}$$

½<1<p



→ $$\sigma_{tot} \lesssim (\ln s)^{1/p}$$

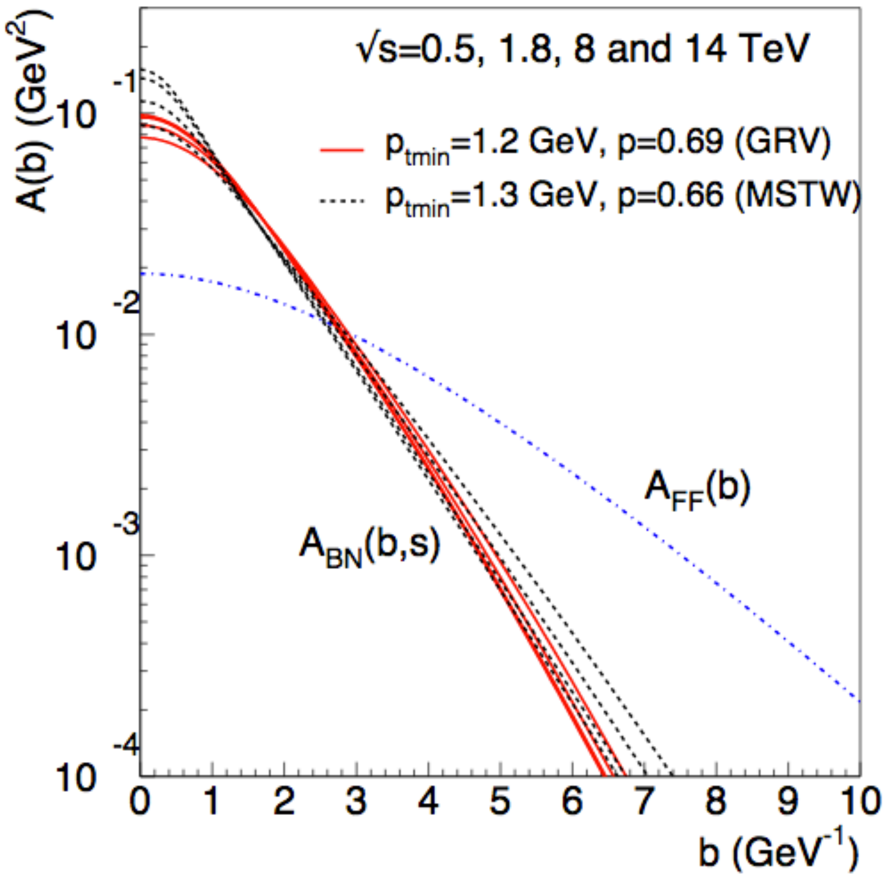
Only minijets p_t > 1 GeV



+ resummed soft gluons k_t <Λ

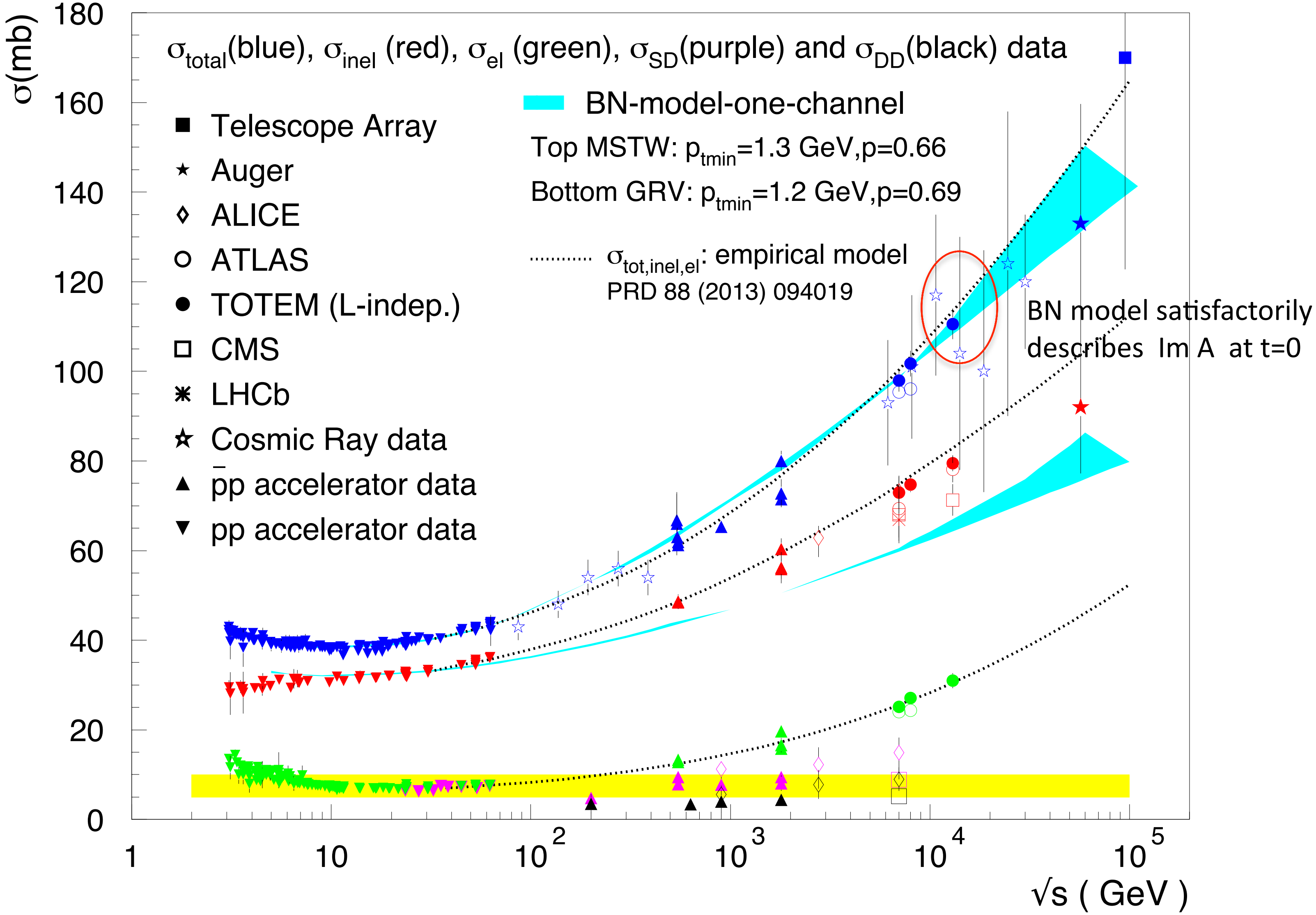
Grau, Godbole,GP,Srivastava, **Phys.Lett. B682 (2009)**

A(b,s):
Perturbative (resum), non perturbative (FF)



- At large √s perturbative partons have stronger b-fall-off for large b than FF
- Below and around ISR (not shown), similar between FF and perturbative

pp Total, elastic and inelastic cross-sections → 13 TeV updated



D. Fagundes, A.Grau, S. Pacetti, G.P.,Y.N.Srivastava, Phys.RevD (2013) empirical model
D.Fagundes ,A.Grau,G.P., O.Shekotsova, Y.N.Srivastava, Phys.RevD96(2017) BN model

NPS and $\sigma_{effective}$

$$\int (d^2\mathbf{b}) T(\mathbf{b}) = 1$$

$$\Sigma^{(n)} \equiv \int (d^2\mathbf{b}) T^n(\mathbf{b})$$

$$\sigma_{effective}^{DPS} = [\int d^2\vec{b}\; T^2(b)]^{-1}$$

$$\sigma_{eff}^{NPS} = [\Sigma^{(n)}]^{-1/(n-1)}$$

In eikonal minijet with resummation (BN) model

First attempt!

$$T(b) \rightarrow A_{resum}(b,s) = \frac{e^{-h(b,s)}}{\int d^2\vec{b}\; e^{-h(b,s)}}$$

$$\sigma_{eff}^{resum} = \frac{2\pi[\int bdb\; e^{-h(b,s)}]^2}{\int bdb\; e^{-2h(b,s)}}$$

Energy dependence of $\sigma_{effective}$

$$\sigma_{effective} \equiv \sigma_{eff}^{resum-BN}(s) = \sigma_{eff}^{resum-BN}(q_{max}(s))$$

$$q_{max}(s) \equiv < q_{max}(s, x_1, x_2) >_{densities} \quad \uparrow \sqrt{s}$$

$$h(b,s) \propto q_{max}(s) \quad \uparrow \sqrt{s}$$

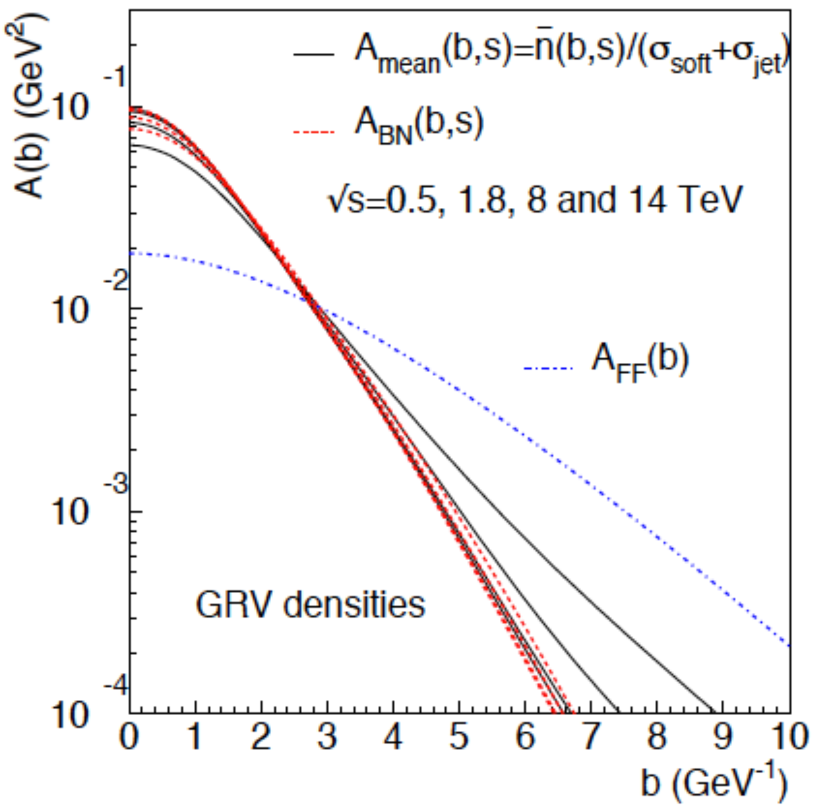
$$e^{-h(b,s)} \downarrow \quad \sqrt{s}$$

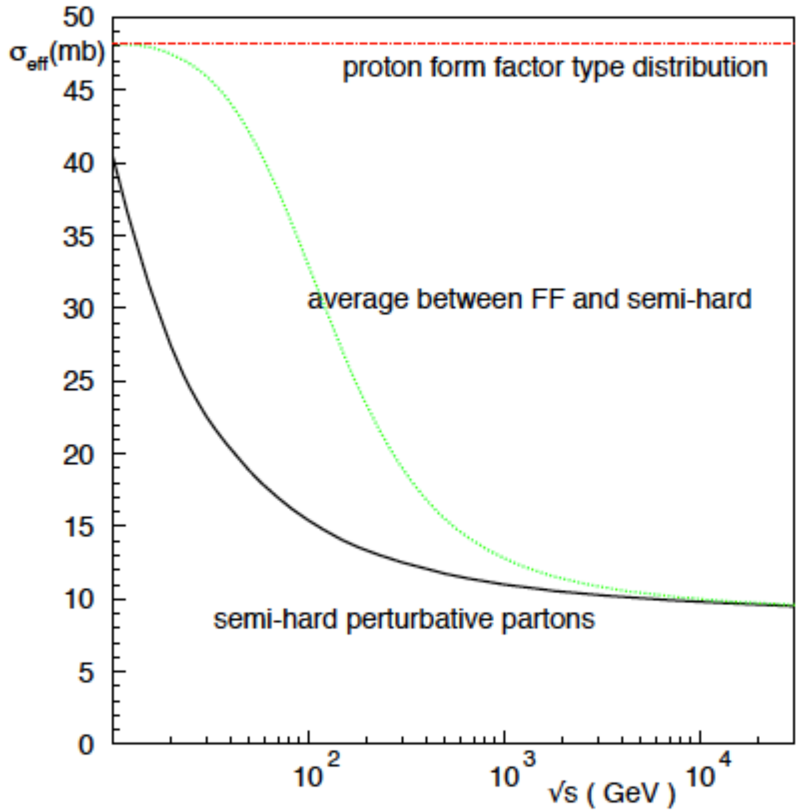
$$\sigma_{eff} \downarrow \quad \sqrt{s}$$

$$\sigma_{12} \propto [\sigma_{eff}]^{-1} \quad \uparrow \quad \sqrt{s}$$

A(b,s) → $\sigma_{eff}(s)$

$$A_{mean}(b,s) = \frac{A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)}{\sigma_{soft}(s) + \sigma_{mini-jets}(s)}$$



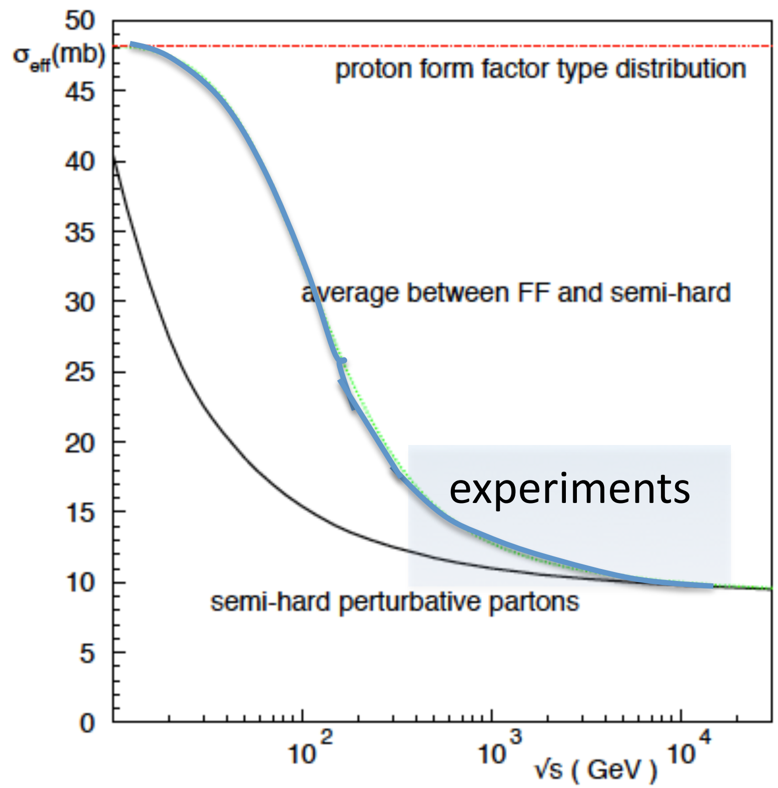


$\sigma_{eff}(s)$ Averaging over both semi-hard (from resummation) and non-perturbative partons

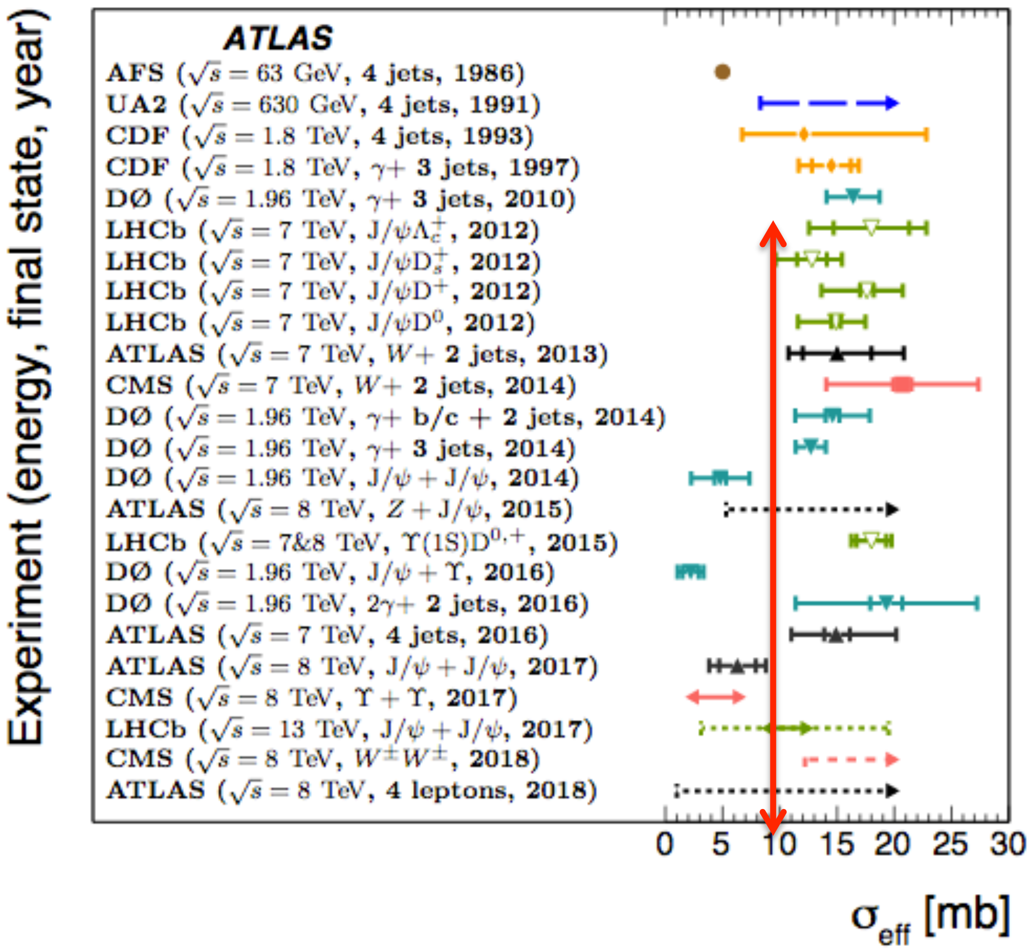
$$A_{mean}(b,s) = \frac{A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)}{\sigma_{soft}(s) + \sigma_{mini-jets}(s)}$$

- √s ≈ 10 GeV T(b) is dominated by Form Factor type partons
- √s ≈ 10 TeV T(b,s) is dominated by partons which engage with other partons from the other proton
- $\sigma_{eff}^{BN} \approx 10 mb$ at √s=13 TeV

≈ lower than Strickman,
≈ D'Enterria,
within experimental limit



ATLAS compilation 1811.11094



Our model first estimate

Where to now?

BN Model – toy (?) QCD inspired

- one-channel eikonal formalism
- includes two components:
  - Soft : no correlation between protons from Proton Form factor → exponential at large b → e^{-bM}
  - Minijets : correlation → s-dependence
    - s-dependence from mini-jets
    - b-dependence from soft gluon resummation

- With parameters inspired by total cross-section description

→ A(b,s) → $\sigma_{eff}^{DPS}(s)$

$\sigma_{eff}(s)$ Averaging over both semi-hard (from resummation) and non-perturbative partons

$$A_{mean}(b,s) = \frac{A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)}{\sigma_{soft}(s) + \sigma_{mini-jets}(s)}$$

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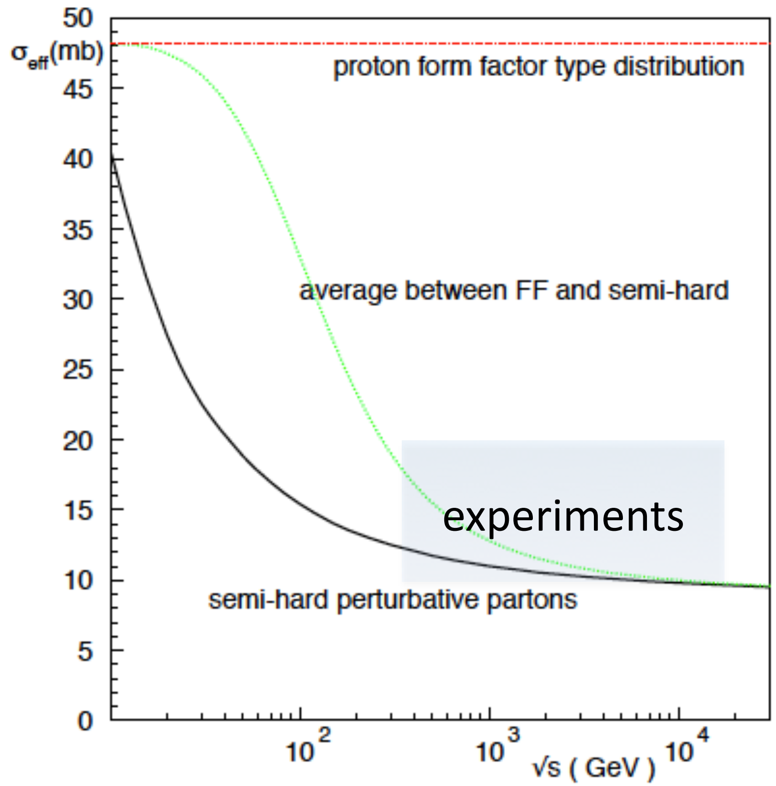


Spare or irrelevant

Similar procedure leads to K_t resummation in QCD but…

$$h^{(PP)}(b,s) = \frac{4}{3\pi^2}\int_{M^2}^{Q^2} d^2k_\perp [1 - e^{i\mathbf{k}_\perp \cdot \mathbf{b}}]\alpha_s(k_\perp^2)\frac{\ln(Q^2/k_\perp^2)}{k_\perp^2}$$

G.Parisi R.Petronzio 1979
With Asymptotic Freedom

Our Proposal (ZPC 1984)

$$M^2 \to 0$$

For integrability
And rising potential:

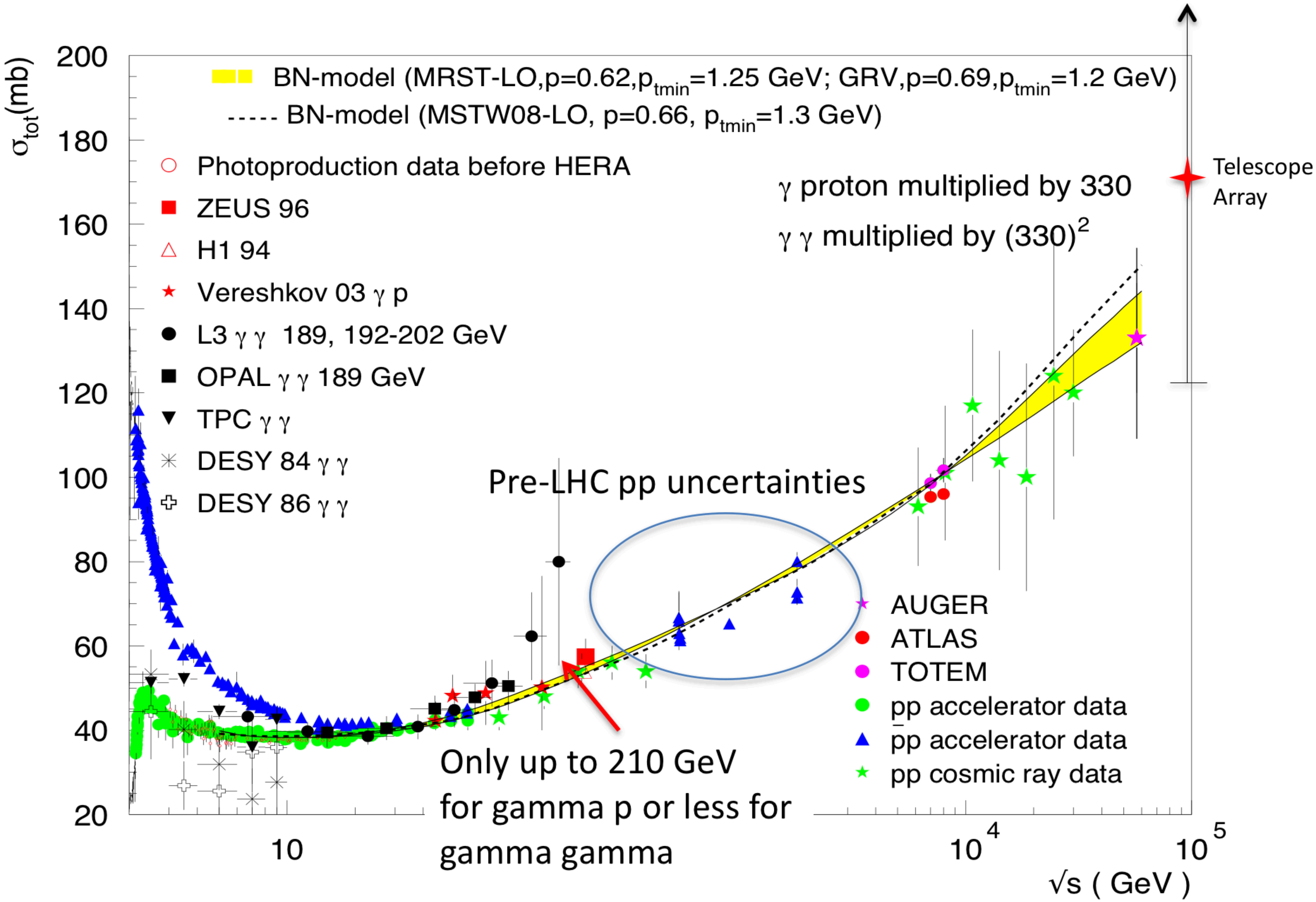
1/2<p < 1

$$\alpha_{strong}(k_t \leq \Lambda) \to \alpha_{eff}(k_t) \to [\frac{\Lambda}{k_t}]^{2p}$$

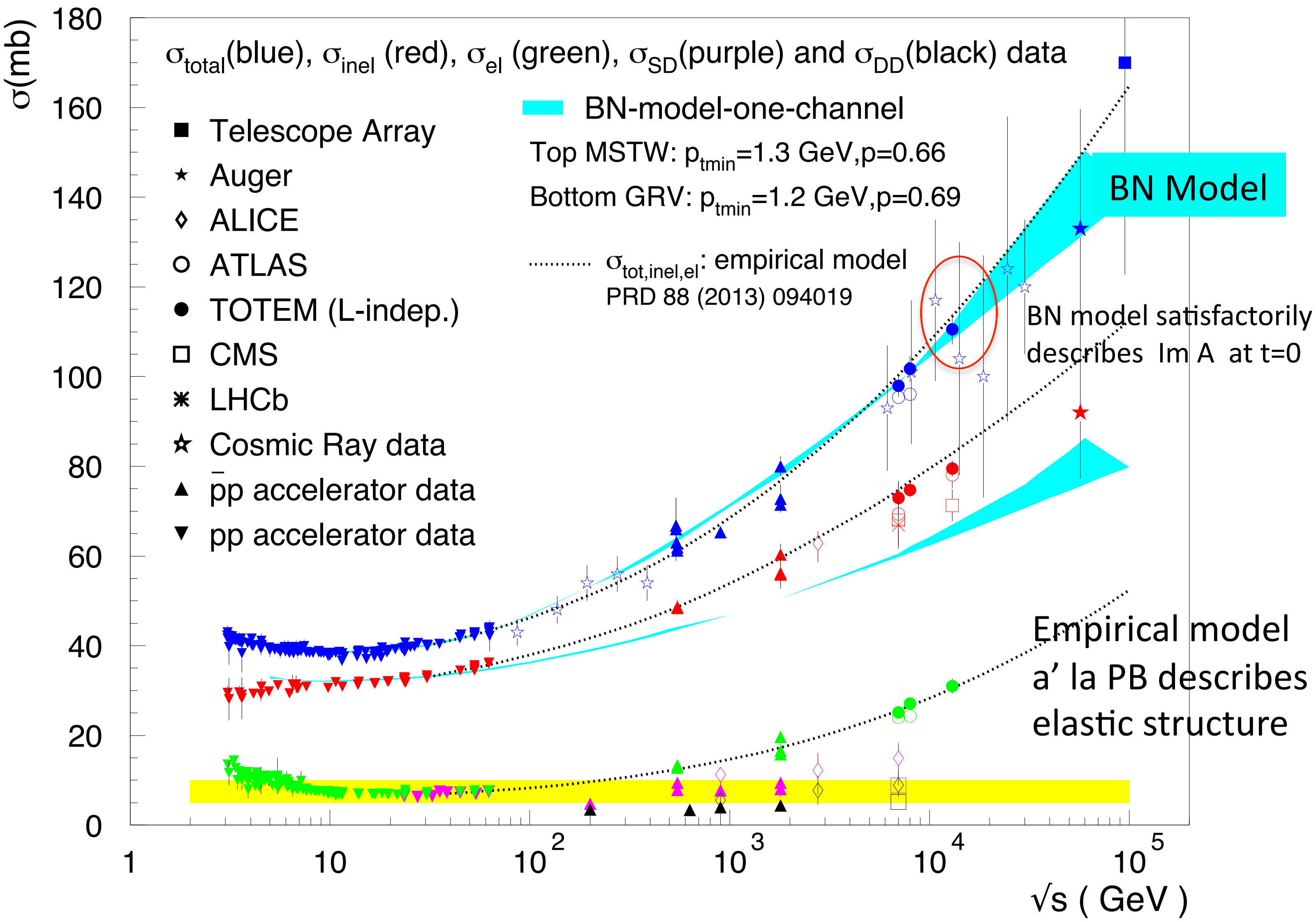
Total hadronic cross-sections

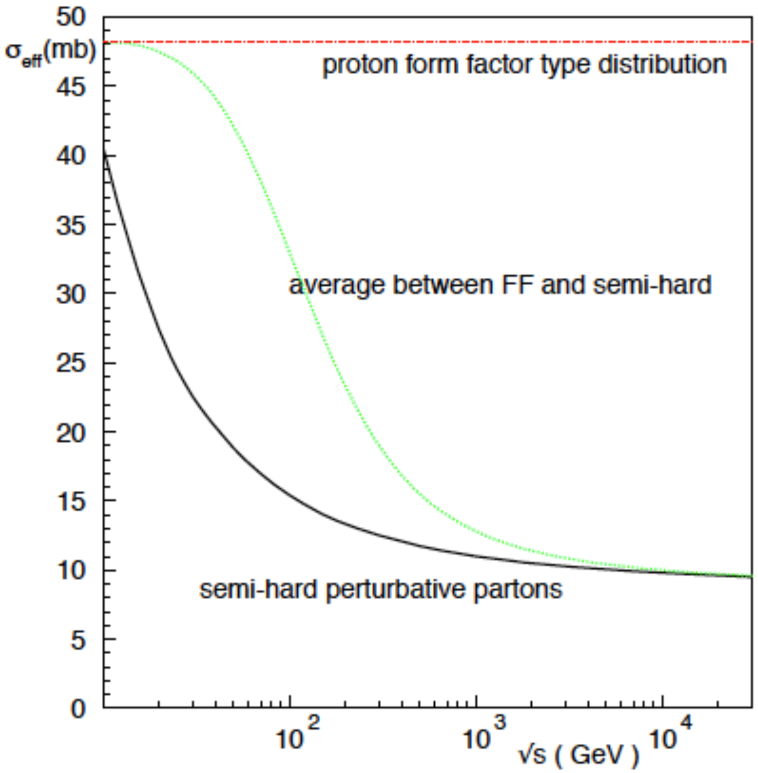
post-LHC update [before 13 TeV data]

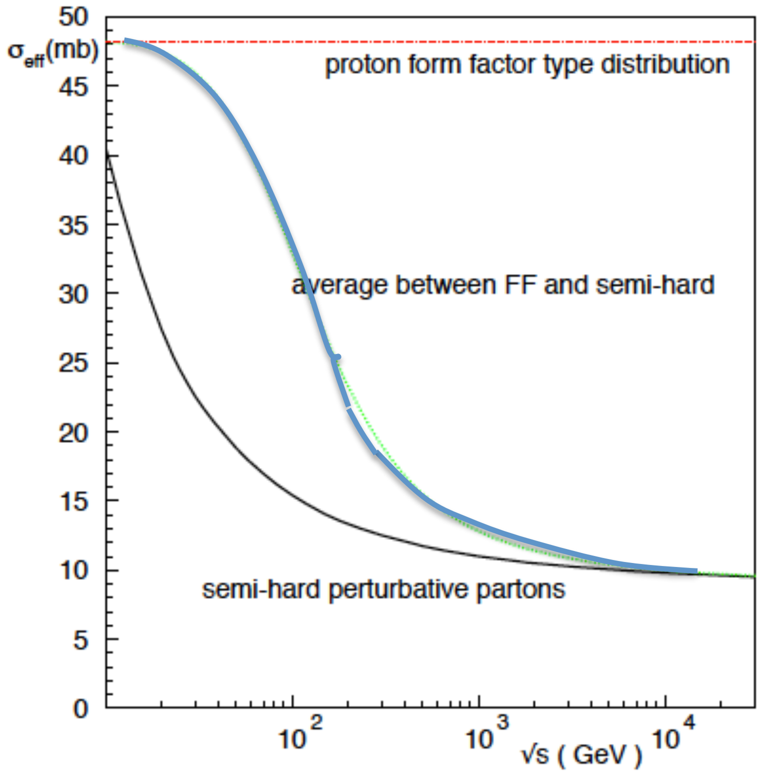
from R.M. Godbole, A. Grau, G. Pancheri, Y.N. Srivastava, Eur.Phys.J. C63 (2009) 69-85



pp Total, elastic and inelastic cross-sections → 13 TeV




50
σ_eff(mb)
45
40
35
30
25
20
15
10
5
0
proton form factor type distribution
average between FF and semi-hard
semi-hard perturbative partons
10^2
10^3
10^4
√s (GeV)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