Diffractive electroproduction of $\rho$-meson as discriminating testfield for the UGD in the proton

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Parton densities are relevant to the search for new Physics

They describe the internal structure of the nucleon in terms of its elementary components (quarks and gluons)

⇒ enter the expression for cross sections
⇒ nonperturbative objects
⇒ can be extracted from experiments through global fits

Several types of distributions...

- exhibit particular universality properties
- obey distinct evolution equations
- respect different types of factorization theorems
...A brief overview

Integrated parton densities:

- **PDF (or collinear) factorization**
  - inclusive processes
  - $\kappa_T \sim$ hardest scale

Unintegrated parton densities:

- **TMD factorization**
  - inclusive or seminclusive processes
  - $\kappa_T \ll$ hardest scale

- **GPD factorization**
  - exclusive processes
  - skewness effects

- **$\kappa_T$-factorization** (or small-$x$ factorization)
  - inclusive or exclusive processes
  - small-$x$, large $\kappa_T$
  - Unintegrated gluon distribution
Unintegrated Gluon Distribution (UGD)

- DIS: conventionally described in terms of PDFs
- less inclusive processes: need to use distributions unintegrated over the parton $\kappa_T$
  - example: virtual photoabsorption in $\kappa_T$-factorization

$$\sigma_{\text{tot}}(\gamma^* p \to X) = \text{Im}_s \{ \mathcal{A}(\gamma^* p \to \gamma^* p) \} \equiv \Phi_{\gamma^* \to \gamma^*} \otimes \mathcal{F}(x, \kappa^2)$$

- $\mathcal{F}(x, \kappa^2)$ is the unintegrated gluon distribution (UGD) in the proton
  - small-$x$ limit: UGD = [BFKL gluon ladder] $\otimes$ [proton impact factor]
Electroproduction of $\rho$ mesons at HERA

$e - p$ collisions provide

$$\gamma^* + \text{proton} \rightarrow \rho + \text{proton} \quad \ldots \text{exclusive process!}$$

- High-energy regime:
  $$s \equiv W^2 \gg Q^2 \gg \Lambda_{\text{QCD}}^2$$
  $$\Rightarrow \text{small } x = \frac{Q^2}{W^2}$$

- Photon virtuality $Q$ is the **hard scale** of the process

- **Process solved in helicity** $\Rightarrow$ so far **unexplored testfield** for UGD

  $\Rightarrow$ constrain $\kappa_T$-dependence of UGD in the HERA energy range

  $$2.5 \ \text{GeV}^2 < Q^2 < 60 \ \text{GeV}^2$$
  $$35 \ \text{GeV} < W < 180 \ \text{GeV}$$

- **Hierarchy of helicity amplitudes:**

  $$T_{00} \gg T_{11} \gg T_{10} \gg T_{01} \gg T_{1-1}$$


- **HERA data available for** $T_{11}/T_{00}$

  [H1 collaboration: F.D. Aaron et al., JHEP 05 032 (2010)]

- $\rho$-meson via **distribution amplitudes (DAs):**

  $$\varphi(y) = \varphi_{\text{WW}}(y) + \varphi_{\text{gen}}(y)$$
Helicity Amplitudes in $\kappa_T$-factorization

- Leading **helicity amplitudes** are known

**Assumption:**

- $\text{Im}_s \{ A(\gamma^* p \to \rho p) \}$
- Same $W$- and $t$-dependence for $T_{11}$ and $T_{00}$
  
  $\implies$ same physical mechanism, scattering of small transverse size of dipole on the proton target, at work $\implies \kappa_T$-factorization

\[
T_{\lambda_p \lambda_\gamma}(s; Q^2) = is \int \frac{d^2\kappa}{(\kappa^2)^2} \Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_p)}(\kappa^2, Q^2) F(x, \kappa^2), \quad x = \frac{Q^2}{s}
\]

**Interesting transitions:**

- $\gamma_L^* \to \rho_L \quad \text{encoded by} \quad \Phi^{\gamma_L^*\to\rho_L}$
- $\gamma_T^* \to \rho_T \quad \text{encoded by} \quad \Phi^{\gamma_T^*\to\rho_T}$

$\implies$ DAs enter $\Phi^{\gamma^*\to\rho} = [H_{LO}] \otimes [DA]$
Helicity Amplitudes in $\kappa_T$-factorization

$T_{11}$ and $T_{00}$

Assumption:

- **Wandzura-Wilczek (WW) approximation** $\rightarrow$ genuine terms neglected

\[
T_{11} = -is (\epsilon_\gamma \cdot \epsilon_\rho^*) 2 B C \frac{m_\rho}{Q^2} \int \frac{d^2 \kappa}{(\kappa^2)^2} \mathcal{F}(x, \kappa^2) \int_0^1 dy \ \varphi_{+\text{WW}}^W(y, \mu^2) \frac{\alpha (\alpha + 2y\bar{y})}{y\bar{y} (\alpha + y\bar{y})^2}
\]

\[
T_{00} = is \frac{4 B C}{Q} \int \frac{d^2 \kappa}{(\kappa^2)^2} \mathcal{F}(x, \kappa^2) \int_0^1 dy \ \varphi_{\text{as}}(y, \mu^2) \left( \frac{\alpha}{\alpha + y\bar{y}} \right)
\]

where $\alpha = \frac{\kappa^2}{Q^2}$, $B = 2\pi \alpha_s \frac{e}{\sqrt{2f_\rho}}$, $C = \frac{\delta_{ab}}{2N_c}$

$\Rightarrow$ **$\rho$-meson DAs** employed:

- **asymptotic** $\varphi_{1\text{as}}(y) \xrightarrow{\text{fixing}} a_2(\mu^2) = 0$

\[
\varphi_{+\text{WW}}^W(y, \mu^2) = (2y - 1) \varphi_{1T}^W(y, \mu^2) + \varphi_{AT}^W(y, \mu^2)
\]

$\Rightarrow \mathcal{F}(x, \kappa^2)$ has to be modeled!
Existence of several UGD models $\implies$ different behavior in $\kappa^2$-shape

- **ABIPSW**: $x$-independent model
  \[ F(x, \kappa^2) = \frac{A \delta_{ab}}{(2\pi)^2 M^2} \left( \frac{\kappa^2}{M^2 + \kappa^2} \right) \]
  [I. V. Anikin et al., *Phys. Rev. D* 84 (2011)]

- **Gluon mom. derivative**:
  \[ F(x, \kappa^2) = \frac{d x g(x, \kappa^2)}{d \ln \kappa^2} \]

- **IN**: soft and hard components to probe different regions of $\kappa$

- **HSS**: $\mathcal{G}_{BFKL} \otimes$ [proton IF]

- **WMR**: angular ordering of gluon emissions

- **GBW**: FT of dipole cross section
None of the models is able to reproduce data over the entire $Q^2$-range

$x$-independent ABIPSW and GBW $\rightarrow$ more suitable models
Numerical results

$T_{11}/T_{00}$ for GBW model - $W = 35$ GeV

- Genuine twist-3 effect included

![Graph showing $T_{11}/T_{00}$ as a function of $Q^2$ for different models.]

- Uncertainty band $\rightarrow$ variation of $a_2(\mu_0 = 1$ GeV)
\( \frac{T_{11}}{T_{00}} \) for GBW model - \( W = 180 \) GeV

- Genuine twist-3 effect included

GBW UGD model

\[ W = 180 \text{ GeV} \]

\[ 0 < a_2(\mu_0) < 0.6 \]

\[ \mu_0 = 1 \text{ GeV} \]

- Uncertainty band \( \rightarrow \) variation of \( a_2(\mu_0 = 1 \text{ GeV}) \)

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Diffractive electroproduction of \( \rho \)-meson

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Stability of $T_{11}/T_{00}$ on $k_{\text{min}}$ cutoff

- Uncertainty band $\rightarrow$ variation of $k_{\text{min}}$ between 0 and 1 GeV
- Small shift of $T_{11}/T_{00} \Rightarrow T_{11}$ and $T_{00}$ dominated by large $\kappa$ values

GBW UGD parametrization

$W = 100$ GeV

$0 < k_{\text{min}} < 1$ GeV

$a_2(\mu_0) = 0$
Conclusions...

Exclusive electroproduction of polarized $\rho$-meson as testing ground for UGD:

- **Exclusive** final state + small-$x$ limit $\Rightarrow \kappa_T$-factorization allowed
- **Process solved in helicity** $\Rightarrow T_{11}/T_{00}$ to constrain the $\kappa_T$-dependence of the UGD in the HERA energy range

  ✓ Importance of the region of small $\kappa_T$’s checked via predictions on the lower cutoff

  $\Rightarrow T_{11}$ and $T_{00}$ sensitive to large $\kappa_T$ values

...Outlook

- NLO impact factor in $\rho$-meson electroproduction $\Rightarrow \Phi^{\gamma^* \rightarrow \rho} = [H_{NLO}] \odot [DA]$
- Study and test of further UGDs
- Proposal of new UGD models and UGD extraction from different channels
- Consider other processes as testfield for UGD:
  - Heavy-quark and heavy-meson production
  - Forward Drell–Yan production

Thanks for your attention!!
BACKUP slides
Ivanov and Nikolaev’ (IN) UGD: a soft-hard model

\[ \mathcal{F}(x, \kappa^2) = \mathcal{F}^{(B)}_{\text{soft}}(x, \kappa^2) \frac{\kappa_s^2}{\kappa^2 + \kappa_s^2} + \mathcal{F}_{\text{hard}}(x, \kappa^2) \frac{\kappa^2}{\kappa^2 + \kappa_h^2}, \]

**The soft term:**

\[ \mathcal{F}^{(B)}_{\text{soft}}(x, \kappa^2) = a_{\text{soft}} C_F N_c \frac{\alpha_s(\kappa^2)}{\pi} \left( \frac{\kappa^2}{\kappa^2 + \mu_{\text{soft}}^2} \right)^2 V_N(\kappa) \]

- \( \mu_{\text{soft}}^2 \rightarrow \) soft parameter
- \( a_{\text{soft}} \rightarrow \) weight of soft term compared to the hard one

**The hard term:**

\[ \mathcal{F}_{\text{hard}}(x, \kappa^2) = \mathcal{F}^{(B)}_{\text{pt}}(\kappa^2) \frac{\mathcal{F}_{\text{pt}}(x, Q_c^2)}{\mathcal{F}^{(B)}_{\text{pt}}(Q_c^2)} \theta(Q_c^2 - \kappa^2) + \mathcal{F}_{\text{pt}}(x, \kappa^2) \theta(\kappa^2 - Q_c^2) \]

- \( \mathcal{F}_{\text{pt}}(x, \kappa^2) = \frac{\partial xg(x, \kappa^2)}{\partial \ln \kappa^2} \)
- \( \mathcal{F}^{(B)}_{\text{pt}}(x, \kappa^2) = C_F N_c \frac{\alpha_s(\kappa^2)}{\pi} \left( \frac{\kappa^2}{\kappa^2 + \mu_{\text{pt}}^2} \right)^2 V_N(\kappa) \)

**The coupling constant:**

\[ \alpha_s \leq 0.82 \ (\text{frozen}) \]

Hentschinski-Salas–Sabio Vera’ (HSS) model

\[ \mathcal{F}(x, \kappa^2, M_h) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2} \mathcal{C} \frac{\Gamma(\delta - i\nu - \frac{1}{2})}{\Gamma(\delta)} \left( \frac{1}{x} \right)^{\chi(\frac{1}{2} + i\nu)} \left( \frac{\kappa^2}{Q_0^2} \right)^{\frac{1}{2} + i\nu} \]

\[ \times \left\{ 1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0 \left( \frac{1}{2} + i\nu \right)}{8N_c} \log \left( \frac{1}{x} \right) \left[ -\psi \left( \delta - \frac{1}{2} - i\nu \right) - \log \frac{\kappa^2}{M_h^2} \right] \right\} \]

⋄ \( \chi_0 \left( \frac{1}{2} + i\nu \right) \equiv \chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma) \) LO eigenvalue of the BFKL kernel

⋄ \( \chi(\gamma) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) - \frac{1}{2} \bar{\alpha}_s^2 \chi'_0(\gamma) \chi_0(\gamma) + \chi_{RG}(\bar{\alpha}_s, \gamma) \) NLO eigenvalue of the BFKL kernel (collinearly improved and BLM optimized)

⋄ parametrization for the proton IF:

\[ \Phi_\rho(q, Q_0^2) = \frac{\mathcal{C}}{2\pi\Gamma(\delta)} \left( \frac{q^2}{Q_0^2} \right)^{\delta} e^{-\frac{q^2}{Q_0^2}} \]

⋄ parameters were fitted to the combined HERA data for the \( F_2(x) \) proton structure function → kinematically improved set chosen:

\[ Q_0 = 0.28 \text{ GeV}, \quad \delta = 6.5, \quad \mathcal{C} = 2.35 \]


Watt–Martin–Ryskin’ (WMR) model

\[ \mathcal{F}(x, \kappa^2, \mu^2) = T_g(\kappa^2, \mu^2) \frac{\alpha_s(\kappa^2)}{2\pi} \int_x^1 dz \left[ \sum_q P_{gq}(z) \frac{x}{z} q(\frac{x}{z}, \kappa^2) + \right. \\
\left. P_{gg}(z) \frac{x}{z} g(\frac{x}{z}, \kappa^2) \Theta \left( \frac{\mu}{\mu + \kappa} - z \right) \right] \]

- \( T_g(\kappa^2, \mu^2) = \exp \left( - \int_{\kappa^2}^{\mu^2} d\kappa_t^2 \frac{\alpha_s(\kappa_t^2)}{2\pi} \left( \int_{z_{\min}'}^{z_{\max}'} dz' z' P_{gg}(z') + N_f \int_0^1 dz' P_{qg}(z') \right) \right) \)

\( \rightarrow \) probability of evolving from the scale \( \kappa \) to the scale \( \mu \) without parton emission

- \( z_{\max}' \equiv 1 - z_{\min}' = \mu / (\mu + \kappa_t) \)

- \( \mu \) extra-scale \( \overset{\text{fixed at}}{\rightarrow} Q \)

Golec-Biernat–Wüsthoff’ (GBW) UGD

\[ \mathcal{F}(x, \kappa^2) = \kappa^4 \sigma_0 \frac{R_0^2(x)}{2\pi} e^{-\kappa^2 R_0^2(x)} \]

- derives from the effective dipole cross section \( \hat{\sigma}(x, r) \) for the scattering of a \( q\bar{q} \) pair off a nucleon through a reverse Fourier transform of

\[
\sigma_0 \left\{ 1 - \exp \left( -\frac{r^2}{4R_0^2(x)} \right) \right\} = \int \frac{d^2\kappa}{\kappa^4} \mathcal{F}(x, \kappa^2) (1 - \exp(i\vec{\kappa} \cdot \vec{r})) (1 - \exp(-i\vec{\kappa} \cdot \vec{r}))
\]

- \( R_0^2(x) = \frac{1}{\text{GeV}^2} \left( \frac{x}{x_0} \right)^{\lambda_p} \)

- The normalization \( \sigma_0 \) and the parameters \( x_0 \) and \( \lambda_p > 0 \) of \( R_0^2(x) \) have been determined by a global fit to \( F_2(x) \):

\[
\sigma_0 = 23.03 \text{ mb}, \quad \lambda_p = 0.288, \quad x_0 = 3.04 \cdot 10^{-4}.
\]