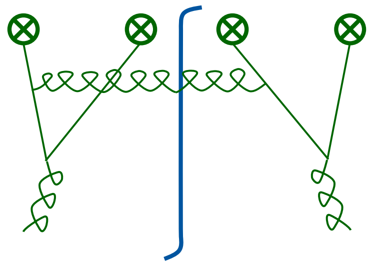
Two-loop splitting in double parton distributions



Jonathan Gaunt (CERN)

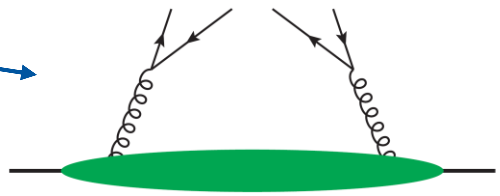
[arXiv:1812.xxxxx], …

In collaboration with Markus Diehl, Peter Plößl, Andreas Schäfer

10th International Workshop on Multiple Partonic Interactions at the LHC, Perugia, Italy, 11th December 2018

Parton splitting

In double parton scattering (DPS), two partons from a proton could have arisen as a result of one parton perturbatively splitting into (at least) two: 1 → 2 mechanism



This will be dominant contribution at small (perturbative) transverse separation between partons, ***y***

All-order form of double parton distribution F at small ***y*** is:

$$F_{a_1a_2}(x_1,x_2,y,\mu) = \frac{1}{\pi y^2}\sum_{a_0}\int_{x_1+x_2}^{1}\frac{dz}{z^2}\,V_{a_1a_2,a_0}\left(\frac{x_1}{z},\frac{x_2}{z},a_s(\mu),\log\frac{\mu^2y^2}{b_0^2}\right) f_{a_0}(z,\mu)$$

PDF

Overall $1/y^2$ dependence

Perturbative kernel

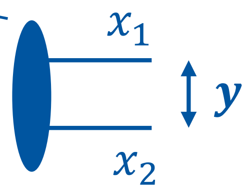
This $1/y^2$ dependence causes a power divergence when naïve formulation of DPS cross section is used: $\int d^2\boldsymbol{y} F(\boldsymbol{y})F(\boldsymbol{y})$. Related to leaking of DPS into SPS region.

Recap of DPS framework of Diehl, Gaunt, Schönwald (DGS)

[JHEP 1706 (2017) 083]

Use double parton distributions (DPDs) in **y** space, insert cut-off into **y** integration:

$$\sigma_{\mathrm{DPS}} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$



Cuts off integral for $\mathbf{y} \lesssim 1/\nu$, regulates power divergence

Use subtraction term in sum of SPS and DPS to avoid double counting:

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{DPS}} + \sigma_{\mathrm{SPS}} - \sigma_{\mathrm{sub}}$$

ν dependence cancelled order by order

DPS cross section with both DPDs replaced by fixed order splitting expression

F must reduce to perturbative expression at small **y**. When modelling F we used a sum of two terms: $F = F_{\mathrm{split}} + F_{\text{non-split}}$

$F_{\text{perturb.}}(y^*)\, e^{-y^2\Lambda^2}$ with $1/y^{*2} = 1/y^2 + 1/y_{\mathrm{max}}^2$

$f(x_1;\mu_0) f(x_2;\mu_0)\, \Lambda^2 e^{-y^2\Lambda^2}/\pi$

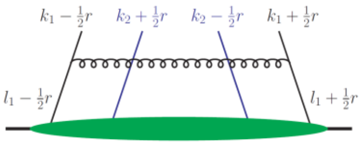
NLO corrections to DPS

Some key advantages of the DGS framework

- Can be formulated at all orders, with corrections that can be practicably computed. <u>Opens the way for NLO calculations of DPS!</u>
- Makes maximal use of existing SPS quantities.

What perturbative ingredients do we need for NLO DPS cross sections?

- NLO corrections to partonic cross sections: already known for many processes from SPS calculations ✓
- NLO ‘usual’ splitting functions - needed for evolution of $F(\boldsymbol{y})$: already known since the 80s ✓
- NLO corrections to the splitting (i.e. NLO V): not yet known ✗

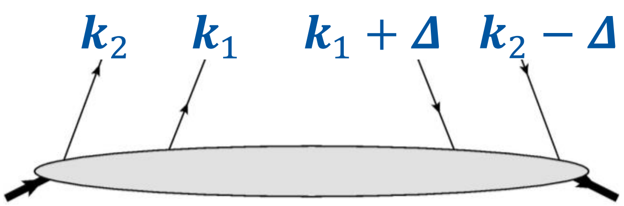


$$F_{a_1 a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} V_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2 y^2}{b_0^2}\right) f_{a_0}(z, \mu)$$

In this talk: computation of V at NLO.

DPDs in Δ-space

One can also consider Δ-space DPDs, where all divergences regularised using dimreg + $\overline{MS}$, and compute matching onto PDFs:



$$F_{a_1 a_2}(x_1, x_2, \Delta, \mu) = \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} W_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2}{\Delta^2}\right) f_{a_0}(z, \mu)$$

Evolution of Δ-space DPDs involves an inhomogeneous 1 → 2 splitting term:

$$\frac{d}{d\ln\mu^2} F_{a_1 a_2}(x_1, x_2, \Delta, \mu) = \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} P_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu)\right) f_{a_0}(z, \mu) + \{\text{homogeneous terms}\}\,,$$

We compute <u>also</u> W and P at NLO.

NLO DPDs

For our purpose: Ws are needed to link our y-space DPDs to $\overline{MS}$ Δ-space DPDs, latter of which satisfy momentum and number sum rules at $\Delta = 0$. Allows us to check to what extent our models for $F(\boldsymbol{y})$ satisfy the sum rules, and construct improved models.

[Gaunt, Stirling JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, arXiv:1811.00289]

DPDs used in DGS framework

$$F_{\Phi}^{a_1 a_2}(x_1, x_2, \boldsymbol{\Delta} = 0; \mu, \nu) = \int d^2\boldsymbol{y}\, \Phi(y\nu)\, F^{a_1 a_2}(x_1, x_2, \boldsymbol{y}; \mu)$$

$$F_{\overline{\text{MS}}}^{a_1 a_2}(x_1, x_2, \boldsymbol{\Delta} = 0; \mu) = F_{\Phi}^{a_1 a_2}(x_1, x_2, \boldsymbol{\Delta} = 0; \mu, \nu) + F_{\text{match}}^{a_1 a_2}(x_1, x_2; \mu, \nu)$$

Satisfy momentum and number sum rules

(see talk by Peter tomorrow)

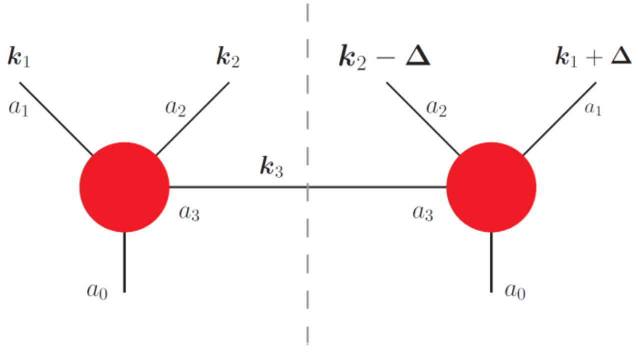
Need Ws here

In this talk, I'll focus on computation of matching coefficients and splitting functions for colour-singlet, unpolarised DPDs, for *all* parton channels. These will be made available shortly in [arXiv:1812.xxxxx].

[We are also computing the polarised + colour interference channels].

Computation

We initially compute bare Δ-space DPDs at $\mathcal{O}(\alpha_s^2)$ in a partonic state a_o: $F_B^{(1)}(\Delta)$



Renormalise, extract matching coefficient

$W^{(1)}(\Delta)$ (from ϵ^0 part of $F_B^{(1)}$)

$P^{(1)}$ (from ϵ^{-1} part of $F_B^{(1)}$)

Fourier transform in $2 - 2\epsilon$ dimensions

Bare y-space DPDs at $\mathcal{O}(\alpha_s^2)$: $F_B^{(1)}(y)$

Renormalise, extract matching coefficient

$V^{(1)}(y)$ (from ϵ^{-1} part of $F_B^{(1)}$)

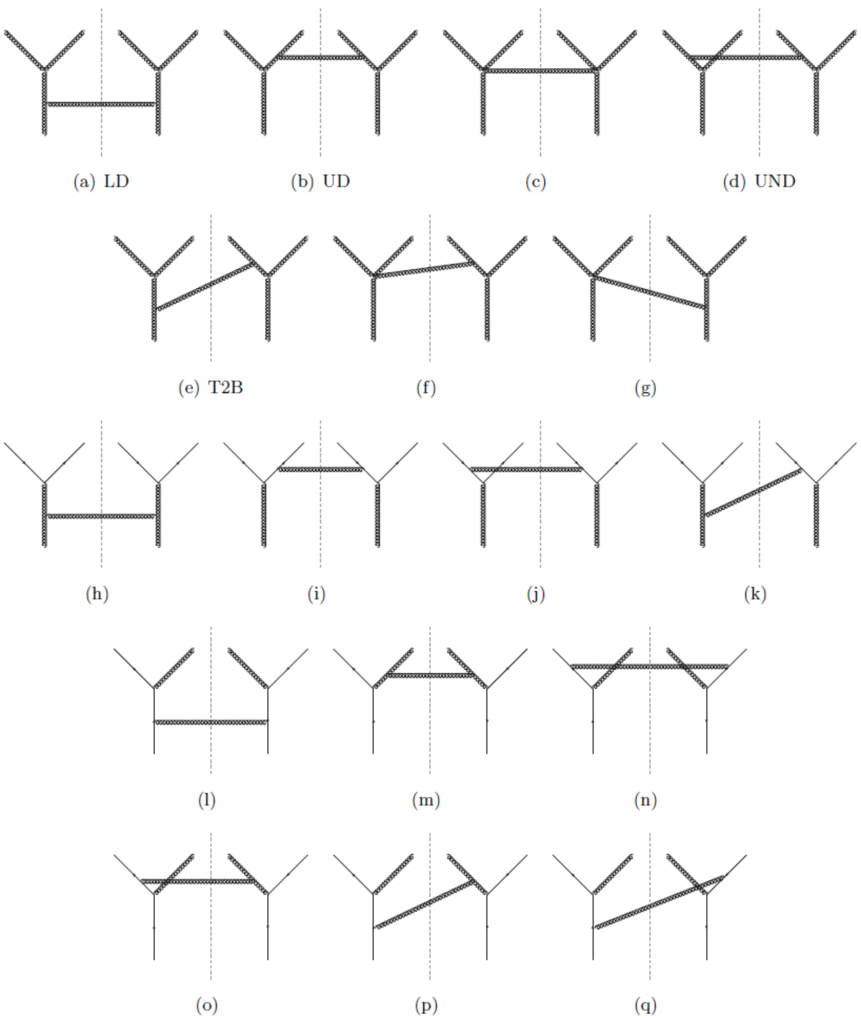
$$\int \frac{d^{2-2\epsilon}\boldsymbol{\Delta}}{(2\pi)^{2-2\epsilon}}\, e^{-i\boldsymbol{\Delta y}} \left(\frac{\mu}{\Delta}\right)^{2\epsilon n} = \frac{\Gamma(1-\epsilon)}{(\pi y^2)^{1-\epsilon}} \left(\frac{y\mu}{b_0}\right)^{2\epsilon n} n\epsilon\, T_{\epsilon,n}$$

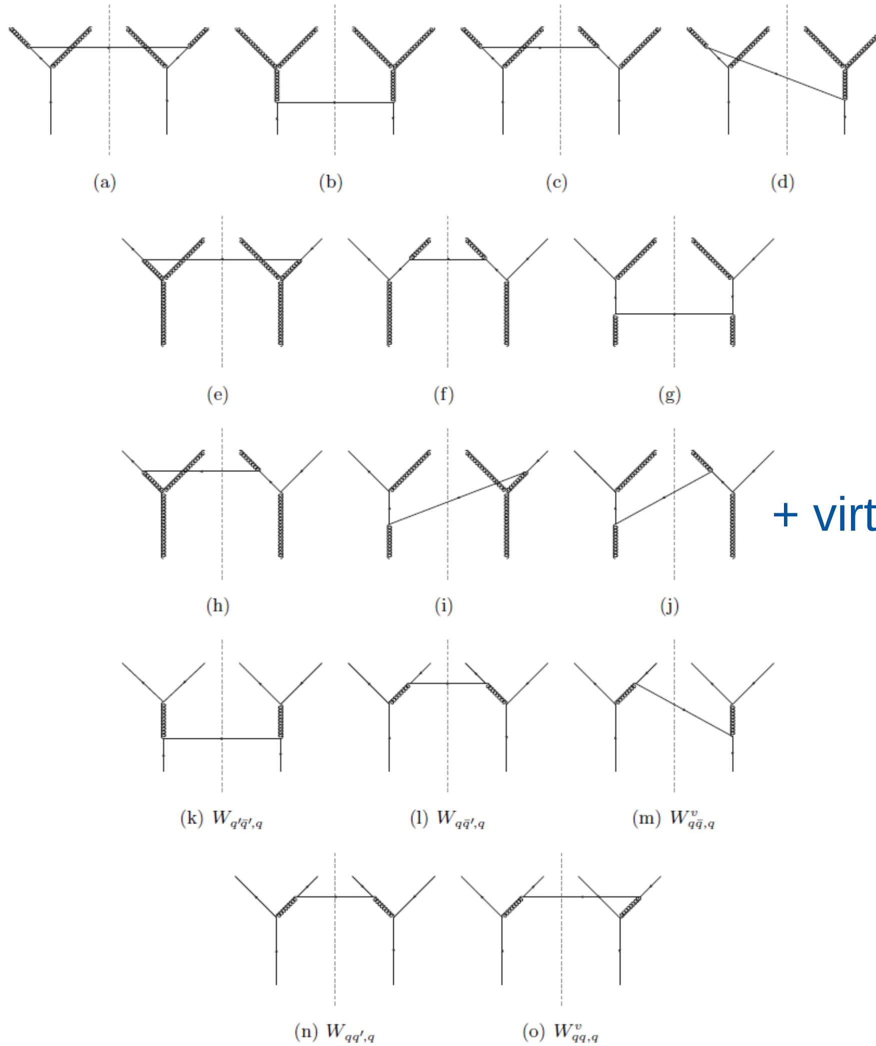
Dimensional analysis → $F_B^{(n)}$ depends on Δ like this

$T_{\epsilon,n} = 1 + \zeta_2 n\epsilon^2 + \cdots$

Graphs

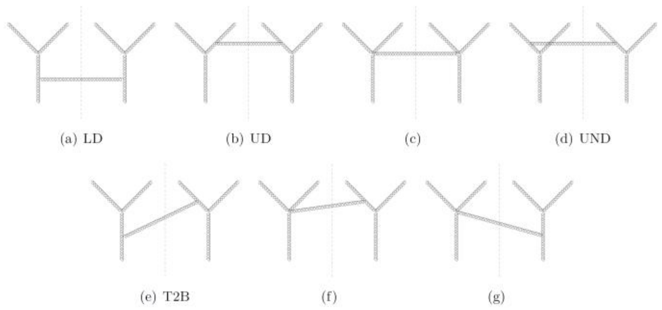
In light-cone gauge, graphs to compute:





+ virtuals

Method



Compute graph expressions (`FORM`, `FeynCalc`). Integrate over minus components using contours.

[Kuipers, Ueda, Vermaseren, Vollinga, Comput. Phys. Commun. 184 (2013) 1453-1467]
[Shtabovenko, Mertig, Orellana, Comput. Phys. Commun. 207 (2016) 432-444]

$$D_1 = \frac{(\boldsymbol{k}_1+\boldsymbol{\Delta})^2}{x_1} + \frac{(\boldsymbol{k}_2-\boldsymbol{\Delta})^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3} \qquad D_2 = \frac{\boldsymbol{k}_1^2}{x_1} + \frac{\boldsymbol{k}_2^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3}$$

$$D_3 = (\boldsymbol{k}_1+\boldsymbol{\Delta})^2 \qquad D_4 = \boldsymbol{k}_2^2 \qquad \tilde{D}_4 = \boldsymbol{k}_1^2 \qquad \tilde{D}_5 = (\boldsymbol{k}_1+\boldsymbol{k}_2)^2$$

$$I_1(a_1,a_2,a_3,a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}} \qquad I_2(a_1,a_2,a_3,a_4,a_5) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1\ldots3} D_i^{a_i} \prod_{i=4\ldots5} \tilde{D}_i^{a_i}}$$

$$I_1(1,1,0,0), I_1(0,1,1,0), I_1(1,1,1,0), I_1(1,0,1,1), I_1(1,1,1,1), I_1(2,1,1,1) \qquad I_2(0,1,1,0,1), I_2(1,1,1,1,0)$$

Integration-by-parts reduction to master integrals (`LiteRed`)

[Lee, J. Phys. Conf. Ser. 523 (2014)]

$$\begin{bmatrix} \frac{\partial I_1(1,1,0,0)}{\partial x_1} \\ \frac{\partial I_1(0,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,0,1,1)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,1)}{\partial x_1} \\ \frac{\partial I_1(2,1,1,1)}{\partial x_1} \end{bmatrix} = \begin{bmatrix} \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacksquare & 0 & 0 & 0 \\ 0 & \blacklozenge & 0 & \blacksquare & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \end{bmatrix} \begin{bmatrix} I_1(1,1,0,0) \\ I_1(0,1,1,0) \\ I_1(1,1,1,0) \\ I_1(1,0,1,1) \\ I_1(1,1,1,1) \\ I_1(2,1,1,1) \end{bmatrix}$$

Construct differential equations in x_1 and solve (`Fuchsia`)

[Gituliar, Magerya, Comput. Phys. Commun. 219 (2017) 329-338]

$$I_1(0,1,1,0) \to \pi^{3-2\epsilon} x_3^{1-\epsilon} (x_1 x_2)^{\epsilon} \frac{\Gamma[-\epsilon]}{\sin[2\pi\epsilon]\Gamma[1-3\epsilon]}$$

Computation of $x_3 \to 0$ limit of master integrals using method of regions (boundary conditions)

Results for bare graphs!

Cross-checks

- Full computation of bare graphs done using light-cone and covariant Feynman gauge ✓
- Master integrals satisfy differential equation in x_2 ✓
- Master integrals all checked numerically at 10 random points using FIESTA ✓
- Individual graphs have poles in ϵ up to ϵ^{-3}, as well as rapidity divergences. ϵ^{-3} pole + rapidity divergences cancel after summing over graphs, ϵ^{-2} pole is as predicted by renormalisation group equation ✓
- Splitting functions $P^{(1)}_{a_1 a_2, a_0}$ satisfy constraints related to number and momentum sum rules:

[Gaunt, Stirling JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, arXiv:1811.00289]

$$\int_0^{1-x_1} dx_2 \left[P_{a_1 q, a_0}(x_1, x_2) - P_{a_1 \bar{q}, a_0}(x_1, x_2) \right] = \left(\delta_{a_1 \bar{q}} - \delta_{a_1 q} - \delta_{a_0 \bar{q}} + \delta_{a_0 q} \right) P_{a_1 a_0}(x_1)\,,$$

$$\sum_{a_2} \int_0^{1-x_1} dx_2 \, x_2 \, P_{a_1 a_2, a_0}(x_1, x_2) = (1 - x_1)\, P_{a_1 a_0}(x_1) \qquad ✓$$

Small x_1, x_2 limit

Interesting processes/regions for studying DPS typically involve small x values (higher density of partons→greater chance of DPS, plus smaller Q such that power suppression is reduced).

→ Interesting to study matching coefficients and splitting functions in limits of small x_i. For example, small x_1, x_2 limit of $P^{(1)}_{gg,g}(x_1, x_2)$:

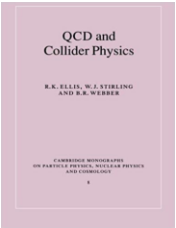
$$P^{(1)}_{gg,g}(x_1,x_2) \longrightarrow \frac{C_A^2\left(\left(1-6u+6u^2\right)+\left(8-\frac{2}{u}-4u+4u^2\right)\log[1-u]+\{u\leftrightarrow 1-u\}\right)}{x^2}$$

$x \equiv x_1 + x_2$
$u \equiv x_1/(x_1+x_2)$

Same $1/x^2$ behaviour for other splitting functions, and V kernels

$V^{(1)}(x_1,x_2) \sim 1/x^2 \Rightarrow F(x_1,x_2,\boldsymbol{y}) \sim \alpha_s^{n+2}\log^{n+1}(x)/x$ (for NLO splitting)
i.e. NLL in small x logarithms!

[$V^{(1)}(x_1,x_2) \sim \log(x)/x^2 \Rightarrow F(x_1,x_2,\boldsymbol{y}) \sim \alpha_s^{n+2}\log^{n+2}(x)/x$, i.e. LL]



Similar cf usual splitting functions, where $P^{(1)}(x) \sim 1/x$ and not $\log(x)/x$.

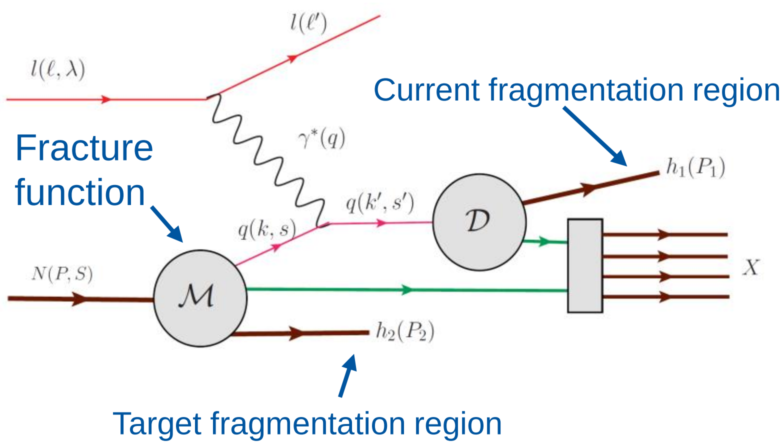
Comparison to other results in the literature

Various 1 → 2 splitting functions have been computed in the literature. Are they the same as our $P^{(1)}_{a_1 a_2, a_0}$ functions?

Fracture functions

$$\frac{\partial M^r_{i,h/P}(\xi,\zeta,M^2)}{\partial \log M^2} = \frac{\alpha_s(M^2)}{2\pi}\int_{\frac{\xi}{1-\zeta}}^{1}\frac{du}{u}\left[P^{(0)}_{i\leftarrow j}(u) + \frac{\alpha_s(M^2)}{2\pi}P^{(1)}_{i\leftarrow j}(u)\right] M^r_{j,h/P}\left(\frac{\xi}{u},\zeta,M^2\right)$$
$$+\frac{\alpha_s(M^2)}{2\pi}\frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\left[\tilde{P}^{(0)}_{ki\leftarrow j}(u,v) + \frac{\alpha_s(M^2)}{2\pi}P^{(1)}_{ki\leftarrow j}(u,v)\right] f^r_{j/P}\left(\frac{\xi}{u},M^2\right) D^r_{h/k}\left(\frac{\zeta}{\xi\, v},M^2\right)$$

Some of these NLO functions computed in Daleo, Sassot [Nucl. Phys. B673 (2003) 357-384] + Garcia Canal [Nucl. Phys. B662 (2003) 334-358]



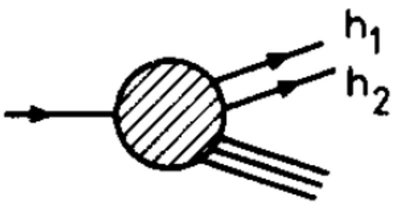
[From Kotzinian et al., Nuovo Cim. C035N2 (2012) 85-91]

Suggested by Ceccopieri [Phys. Lett. B697 (2011) 482-487] that after a simple transformation, these functions are equal to DPS $P^{(1)}_{a_1 a_2, a_0}$.

We find that this is not the case – in fact we observe that above $P^{(1)}$s are not symmetric under $x_1 \leftrightarrow x_2$

Comparison to other results in the literature

Di-hadron fragmentation functions



$$D_{a_1 a_2,i}(x_1, x_2, Q^2) = \sum_{b_1,b_2,j} \int_0^Y dy \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} \frac{1}{z_1 + z_2}$$

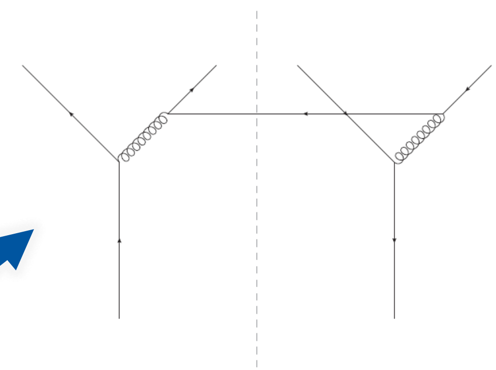
$$\times D_{a_1 b_1}(x_1/z_1, y) D_{a_2 b_2}(x_2/z_2, y)$$

$$\times \hat{P}_{j\to b_1 b_2}(z_1/(z_1+z_2)) D_{ji}(z_1 + z_2, Y - y),$$

[Konishi, Ukawa, Veneziano, Nucl.Phys. B157 (1979) 45-107]

NLO ‘two-body decay probabilities’ computed by Kalinowski, Konishi, Scharbach, Taylor, Nucl. Phys. B181 (1981) 253-276, Gunion, Kalinowski, Szymanowski, Phys. Rev. D32 (1985) 2303-2321

Generally different from our $P^{(1)}_{a_1 a_2, a_0}$ - an exception is the non-singlet contribution to $P^{(1)}_{qq,q}$. Likely because this is a very simple process: one Feynman diagram, and no subdivergences



Summary

- NLO matching of DPDs onto PDFs, and NLO $1 \to 2$ splitting functions, computed in unpolarised colour-singlet case ✓
- Corresponding matching coefficients to come for polarised + colour-nonsinglet channels.
- Then numerics!
  - Look at effect of NLO corrections on DPD y-profiles, parton luminosities, cross sections, etc.
  - Investigate perturbative convergence of DPS cross sections
  - Look for observables where we might be able to detect differences between LO and NLO predictions.

CERN