Hadronic uncertainties...

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G2G (Summer Edition)

Primosten, september 2018

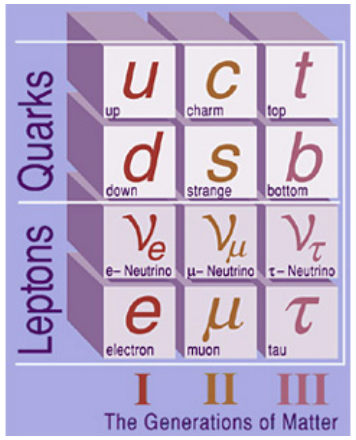
In the Standard Model

✗ Gauge sector entirely fixed by symmetry

$$i\bar{\psi}\not{D}\psi \qquad D_\mu = \partial_\mu - ig_s t_a A^a_\mu - ig\mathbf{T}\cdot\mathbf{W}_\mu - ig'\frac{Y}{2}B_\mu$$

✗ Flavor sector loose (a bunch of parameters)

13 of 19 are fermion masses and q.mixing parameters

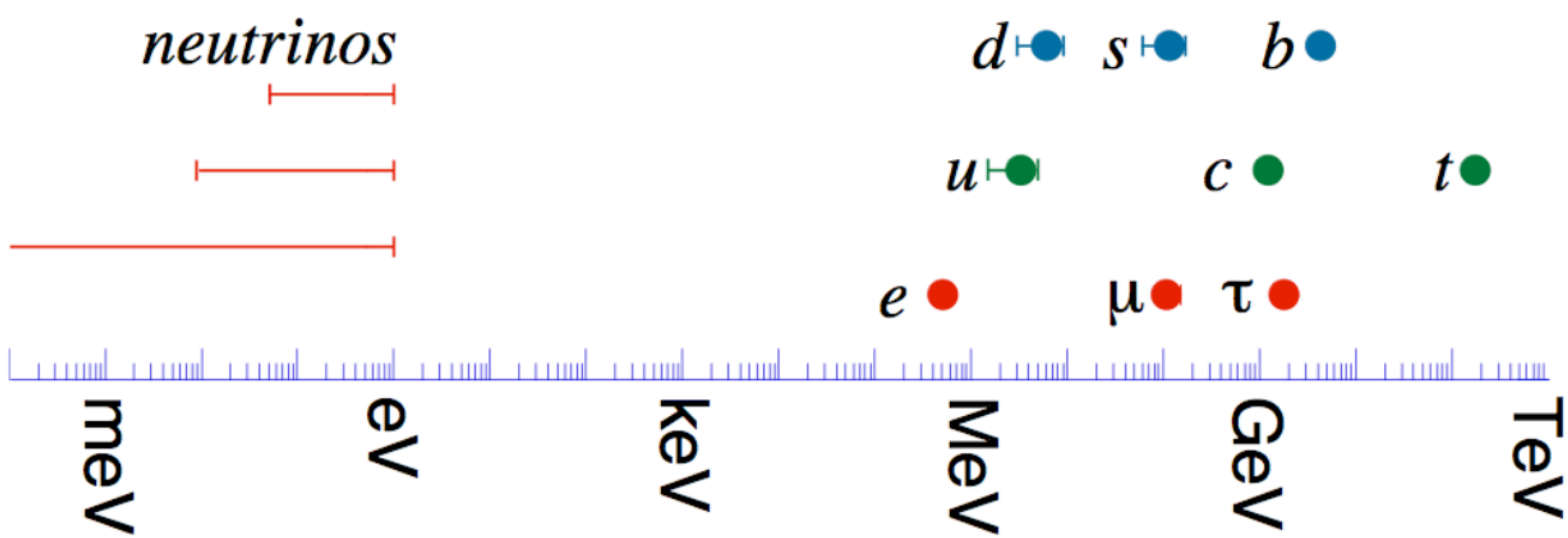


We know

- ✘ P and C broken by weak int. but CP is a symmetry (1 gen)
- ✘ Going from the gauge to mass basis

$$\mathcal{L}_Y^{\rm SM} = -Y_d^{ij}\overline{Q}_L^i \phi D_R^j - Y_u^{ij}\overline{Q}_L^i \widetilde{\phi} U_R^j + \text{h.c.}$$

$$\mathcal{L}_Y^{\rm SM} = -\left(1+\frac{h}{v}\right)\left[m_d \bar{d} d + m_u \bar{u} u + m_e \bar{e} e\right]$$



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- ✘ With 3 gen trickier - cannot simultaneously diagonalize *u* and *d* — mixing: CKM matrix
- ✘ V_{CKM} unitary ⇒ 3 real parameters + 1 phase (CPV!)

$$\lambda \qquad A \qquad \rho \qquad \eta$$

CKM-ology

$$\lambda \qquad A \qquad \rho \qquad \eta$$

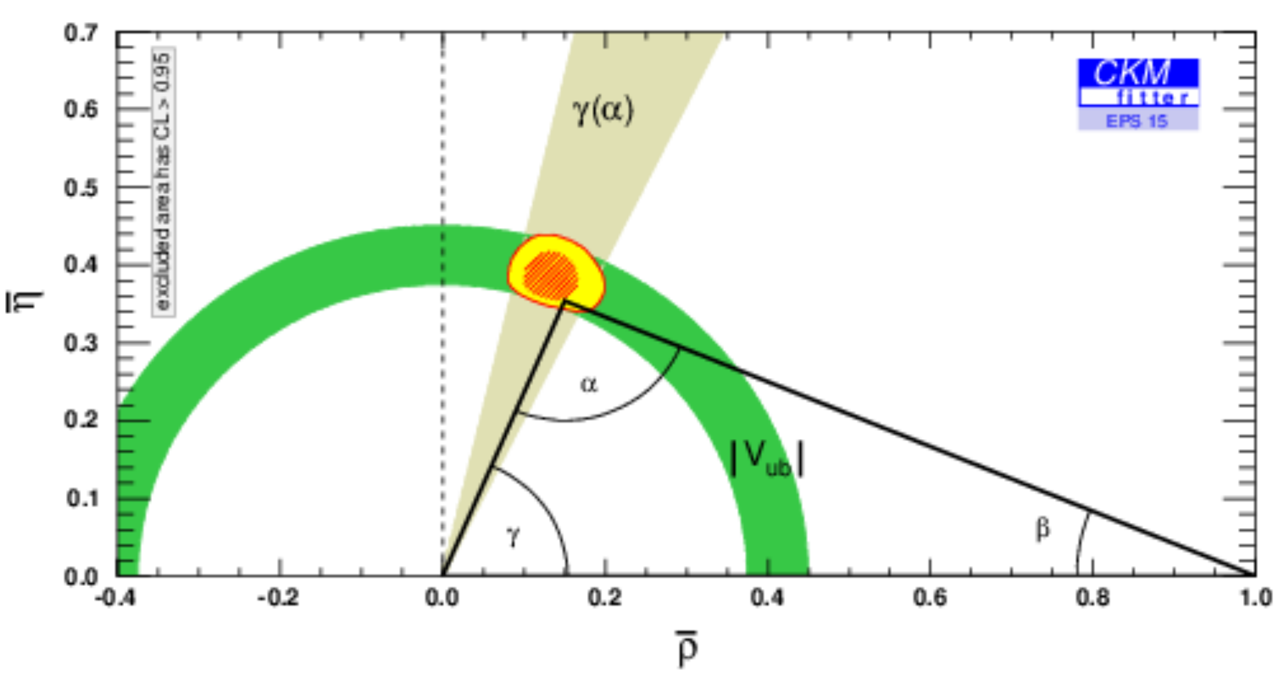
$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

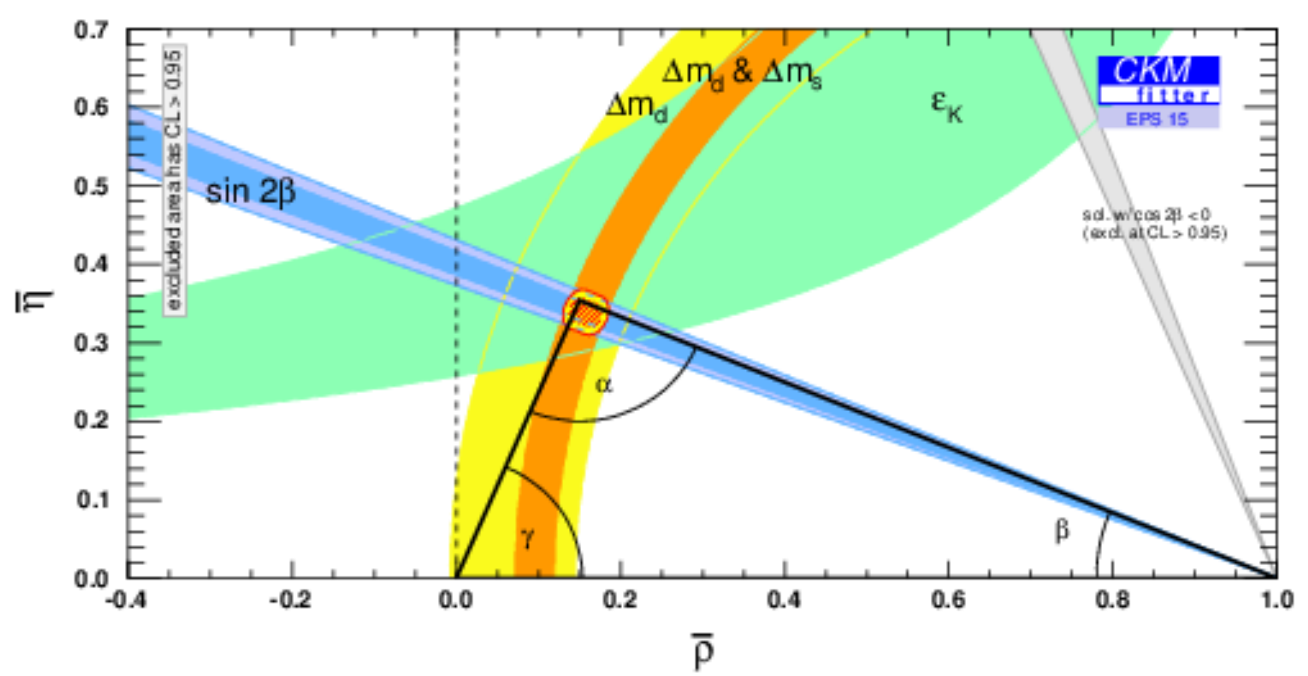
$$\lambda = \sin\theta_C \approx 0.224 \qquad A \simeq 0.82 \qquad \sqrt{\rho^2+\eta^2} \approx 0.45$$

- ✘ Fix CKM entries through tree level processes; over constrain by loop-induced ones
- ✘ V_{CKM} unitary ⇒ 3 real parameters + 1 phase (CPV!)

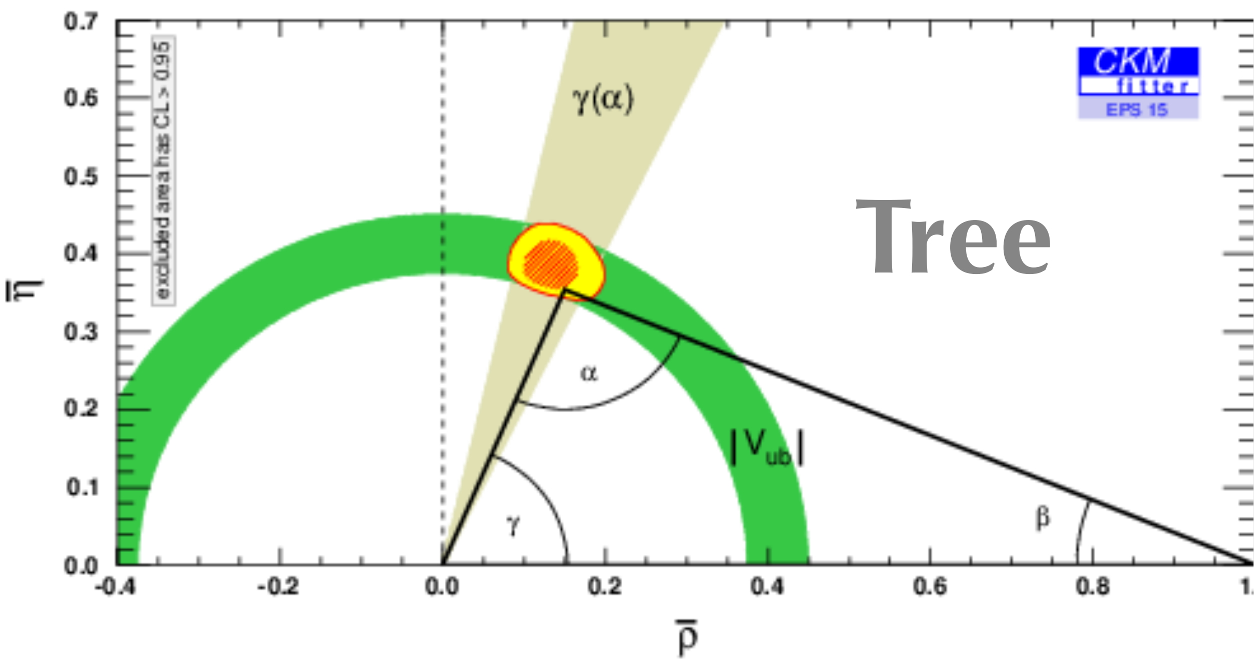
Experiments

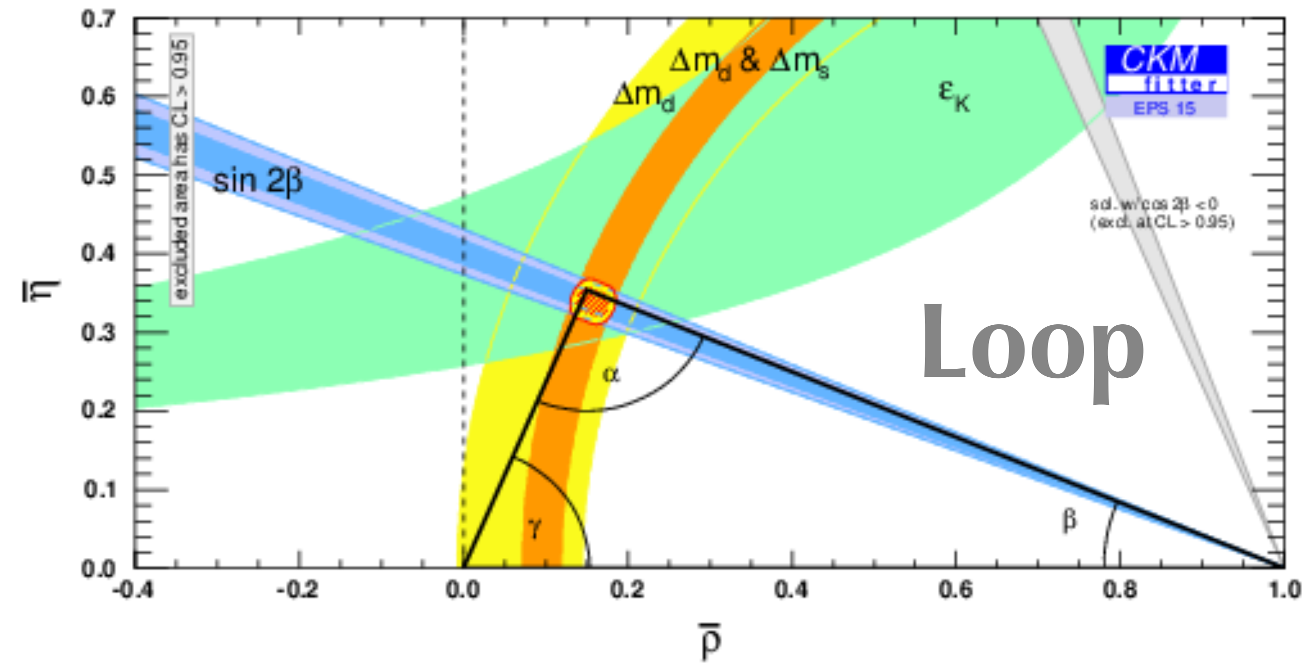
- K-factories u,d,s [NA62, KOTO]
- Tau-charm τ,c [BES III]
- B-factory b,c,τ [Belle II]
- LHC t,b,c
- LC t,…
- νF





CKM





Impressively — TL UT and LP UT agree to less than 10%
[Experiment will do better! Lattices will do better too!]
Only tensions in Vub and Vcb (inclusive Vs. exclusive) but all in all, CKM is very unitary!

2008, Nobel Prize

Example : Kaon physics

Tree level decays

hadronic uncertainty!

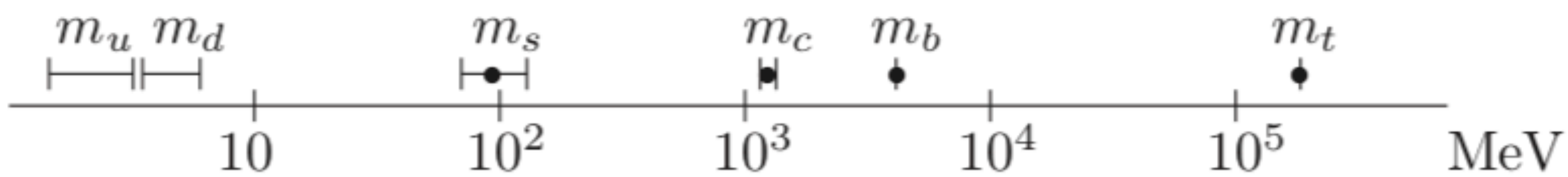
$K \to \pi \ell \nu$ ⟷ $\langle \pi | \bar{s} \gamma_\mu u | K \rangle \to f_{0,+}(q^2)$

$K \to \mu \nu$ ⟷ $\langle 0 | \bar{s} \gamma_\mu \gamma_5 u | K \rangle \to f_K$

f_K / f_π

Nonperturbative QCD - symmetries help (eg. Ademollo-Gatto) but ultimately needs LQCD

LQCD

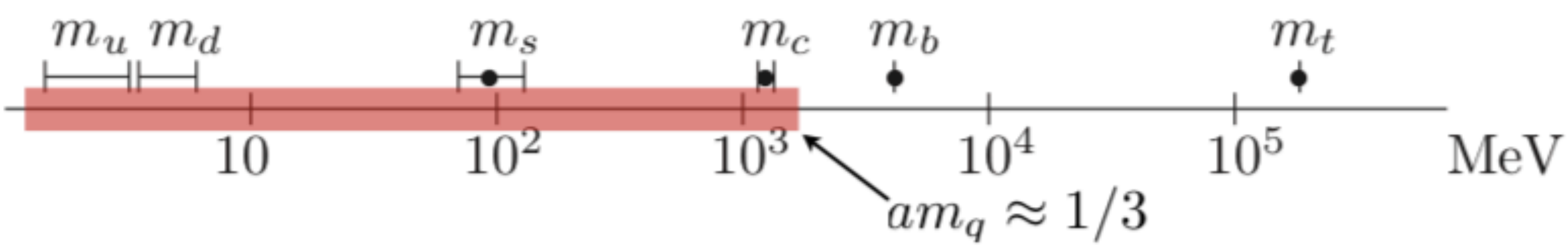


first-principles approach = control all systematic uncertainties

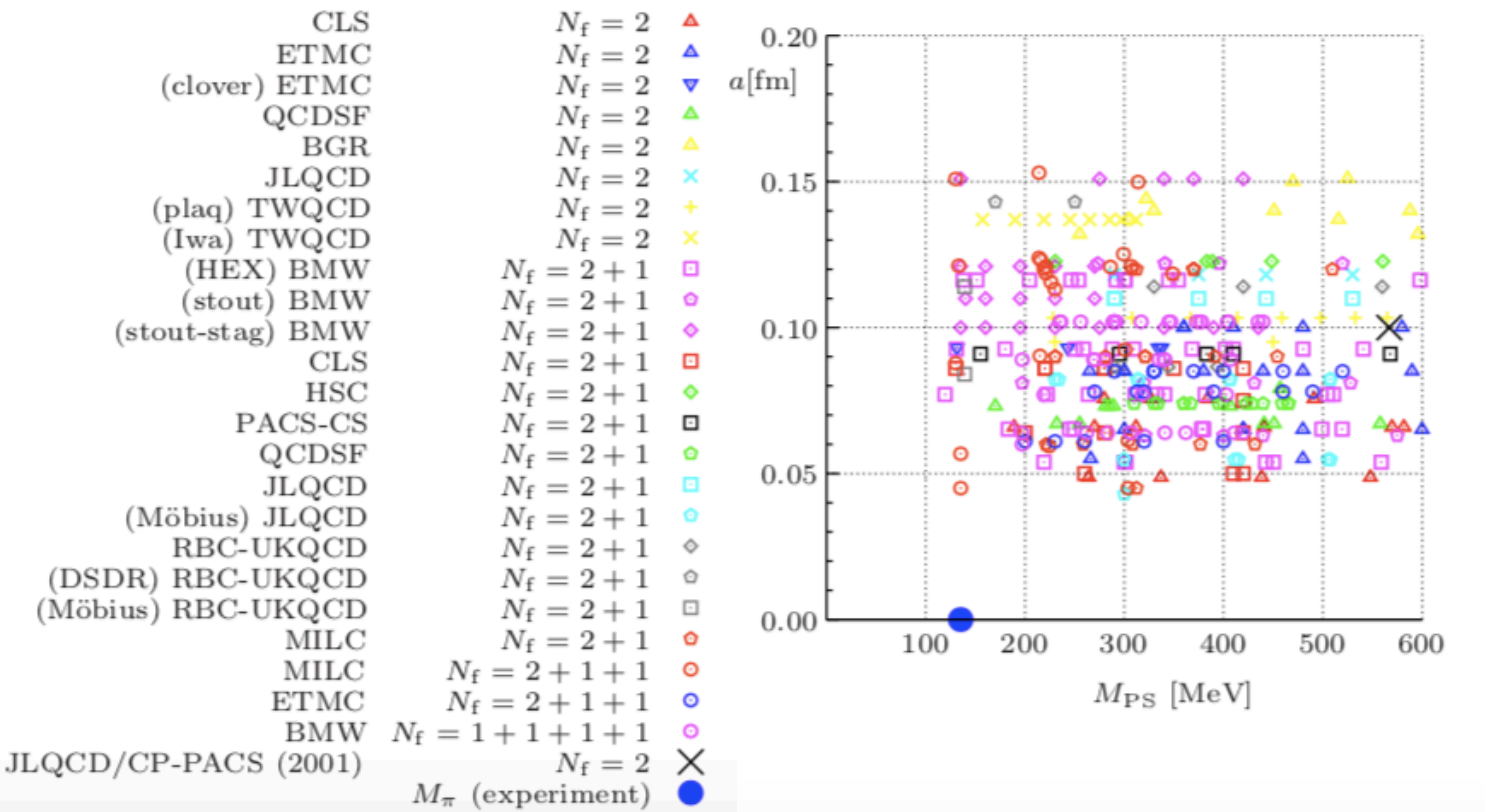
- cover all relevant scales: $L^{-1} \ll \mu \ll a^{-1}$
- control scaling (exploit universality!), renormalisation, ...
- ultimately: get rid of cutoffs at physical kinematics

complement with other first-principles/systematic approaches: dispersion relations, effective theories, ...

LQCD



m_u m_d
m_s
m_c
m_b
m_t
10
10^2
10^3
10^4
10^5
MeV
$am_q \approx 1/3$



CLS $N_f = 2$
ETMC $N_f = 2$
(clover) ETMC $N_f = 2$
QCDSF $N_f = 2$
BGR $N_f = 2$
JLQCD $N_f = 2$
(plaq) TWQCD $N_f = 2$
(Iwa) TWQCD $N_f = 2$
(HEX) BMW $N_f = 2+1$
(stout) BMW $N_f = 2+1$
(stout-stag) BMW $N_f = 2+1$
CLS $N_f = 2+1$
HSC $N_f = 2+1$
PACS-CS $N_f = 2+1$
QCDSF $N_f = 2+1$
JLQCD $N_f = 2+1$
(Möbius) JLQCD $N_f = 2+1$
RBC-UKQCD $N_f = 2+1$
(DSDR) RBC-UKQCD $N_f = 2+1$
(Möbius) RBC-UKQCD $N_f = 2+1$
MILC $N_f = 2+1$
MILC $N_f = 2+1+1$
ETMC $N_f = 2+1+1$
BMW $N_f = 1+1+1+1$
JLQCD/CP-PACS (2001) $N_f = 2$
M_π (experiment)
a[fm]
0.20
0.15
0.10
0.05
0.00
100
200
300
400
500
600
M_{PS} [MeV]

FLAG

what FLAG provides for each quantity:

- complete list of references
- summary of relevant formulae and notation
- quick-look summary tables
- quality assessment of computation setup: colour-coded tables
- averages/estimates (if sensible)
- a "lattice dictionary" for non-experts
- thorough appendix tables with details of all computations for experts
- between-editions updates at http://itpwiki.unibe.ch/flag

FLAG

tables:

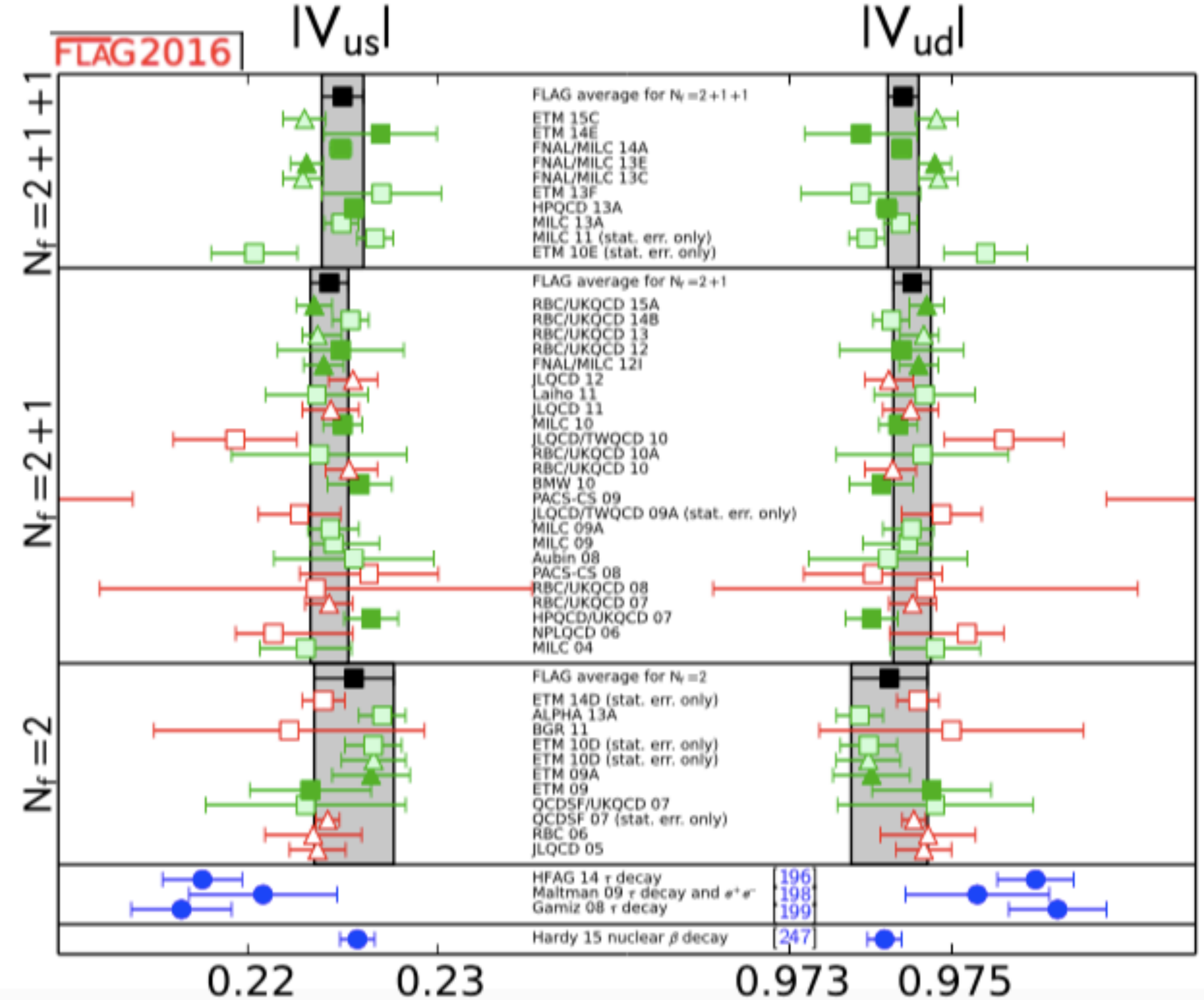
- ★/✓ allows for satisfactory control of systematics
- ○ allows for reasonable (but improvable) estimate of systematics
- ■ unlikely to allow for reasonable control of systematics

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization/ matching	Heavy-quark treatment	f_{B_s}/f_{B^+}	f_{B_s}/f_{B^0}	f_{B_s}/f_B
ETM 13E	[456]	2+1+1	C	★	○	○	○	✓	–	–	1.201(25)
HPQCD 13	[52]	2+1+1	A	★	★	★	○	✓	1.217(8)	1.194(7)	1.205(7)
RBC/UKQCD 14	[53]	2+1	A	○	○	○	○	✓	1.223(71)	1.197(50)	–
RBC/UKQCD 14A	[54]	2+1	A	○	○	○	○	✓	–	–	1.193(48)
RBC/UKQCD 13A	[457]	2+1	C	○	○	○	○	✓	–	–	$1.20(2)_{stat}$ [a]
HPQCD 12	[55]	2+1	A	○	○	○	○	✓	–	–	1.188(18)
FNAL/MILC 11	[48]	2+1	A	○	○	★	○	✓	1.229(26)	–	–
RBC/UKQCD 10C	[464]	2+1	A	■	■	■	○	✓	–	–	1.15(12)
HPQCD 09	[59]	2+1	A	○	○	○	○	✓	–	–	1.226(26)
ALPHA 14	[57]	2	A	★	★	★	★	✓	–	–	1.203(65)
ALPHA 13	[458]	2	C	★	★	★	★	✓	–	–	1.195(61)(20)
ETM 13B, 13C [b]	[20,58]	2	A	★	○	★	○	✓	–	–	1.206(24)
ALPHA 12A	[459]	2	C	★	★	★	★	✓	–	–	1.13(6)
ETM 12B	[460]	2	C	★	○	★	○	✓	–	–	1.19(5)
ETM 11A	[182]	2	A	○	○	★	○	✓	–	–	1.19(5)

cf. FLAG review in 1607.00299

New to appear soon — towards the end of 2018

LQCD



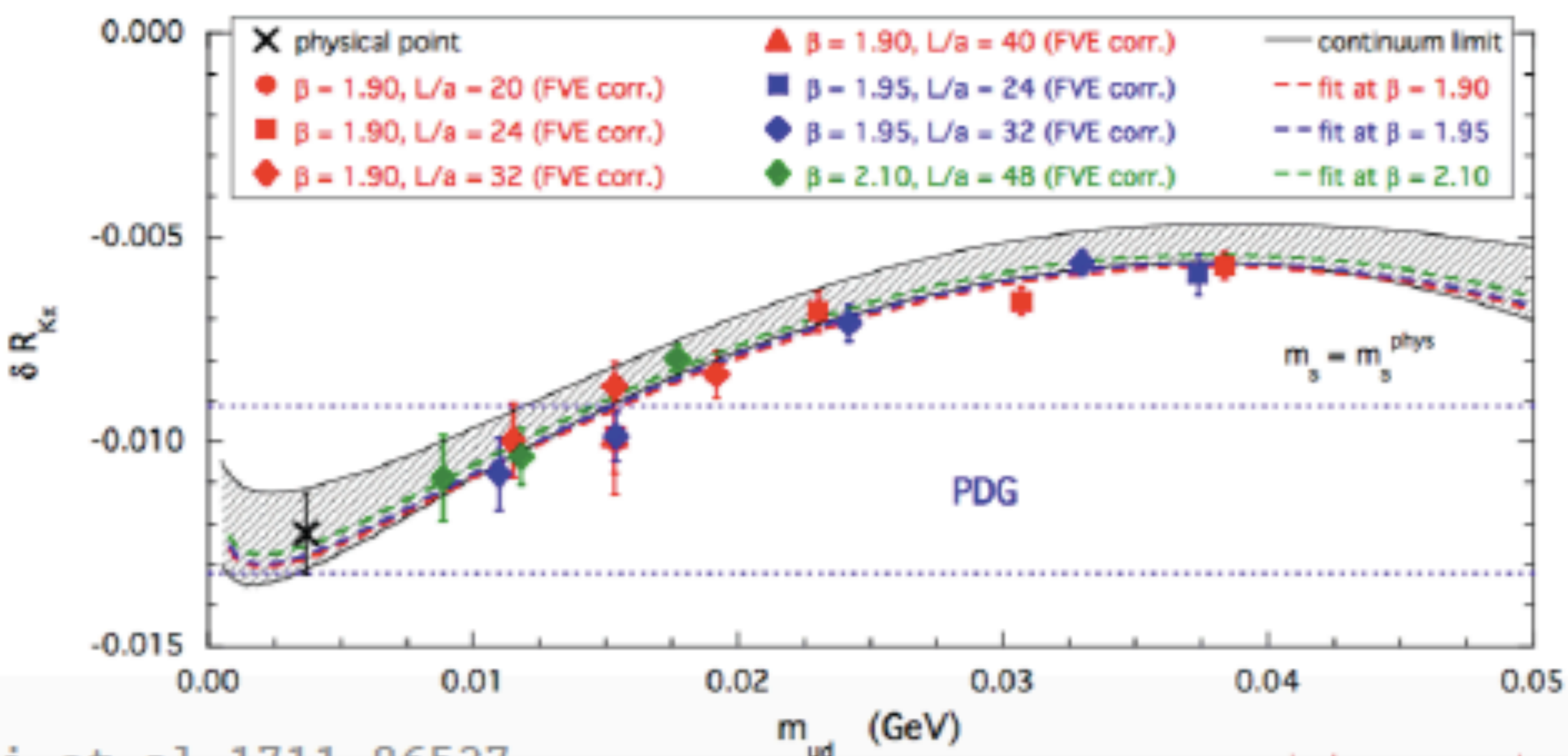
FLAG2016
$|V_{us}|$
$|V_{ud}|$
$N_f=2+1+1$
$N_f=2+1$
$N_f=2$
FLAG average for $N_f=2+1+1$
ETM 15C
ETM 14E
FNAL/MILC 14A
FNAL/MILC 13E
FNAL/MILC 13C
ETM 13F
HPQCD 13A
MILC 13A
MILC 11 (stat. err. only)
ETM 10E (stat. err. only)
FLAG average for $N_f=2+1$
RBC/UKQCD 15A
RBC/UKQCD 14B
RBC/UKQCD 13
RBC/UKQCD 12
FNAL/MILC 12I
JLQCD 12
Laiho 11
JLQCD 11
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
RBC/UKQCD 10
BMW 10
PACS-CS 09
JLQCD/TWQCD 09A (stat. err. only)
MILC 09A
MILC 09
Aubin 08
PACS-CS 08
RBC/UKQCD 08
RBC/UKQCD 07
HPQCD/UKQCD 07
NPLQCD 06
MILC 04
FLAG average for $N_f=2$
ETM 14D (stat. err. only)
ALPHA 13A
BGR 11
ETM 10D (stat. err. only)
ETM 10D (stat. err. only)
ETM 09A
ETM 09
QCDSF/UKQCD 07
QCDSF 07 (stat. err. only)
RBC 06
JLQCD 05
HFAG 14 τ decay
Maltman 09 τ decay and e^+e^-
Gamiz 08 τ decay
Hardy 15 nuclear β decay
196
198
199
247
0.22
0.23
0.973
0.975

QED corrections to leptonic decays

- Need $P \to \ell\nu + \ell\nu\gamma$ for KLN
- Real photon emission in pert.th up to a (tiny) ΔE_γ in P-rest frame
- IR divergences universal and cancel between virtual photon contribution (NP) and real photon emission (pert) - L acts as intermediate IR regulator Inclusive   Carrasco et al 1502.00257

$$\begin{aligned}\Gamma(P_{\ell 2}) &= \Gamma_0 + \Gamma_1^{pt}(\Delta E_\gamma) \\ &= \lim_{L\to\infty}\left[\Gamma_0(L) - \Gamma_0^{pt}(L)\right] + \lim_{\mu_\gamma\to 0}\left[\Gamma_0^{pt}(\mu_\gamma) + \Gamma_1^{pt}(\Delta E_\gamma, \mu_\gamma)\right]\end{aligned}$$

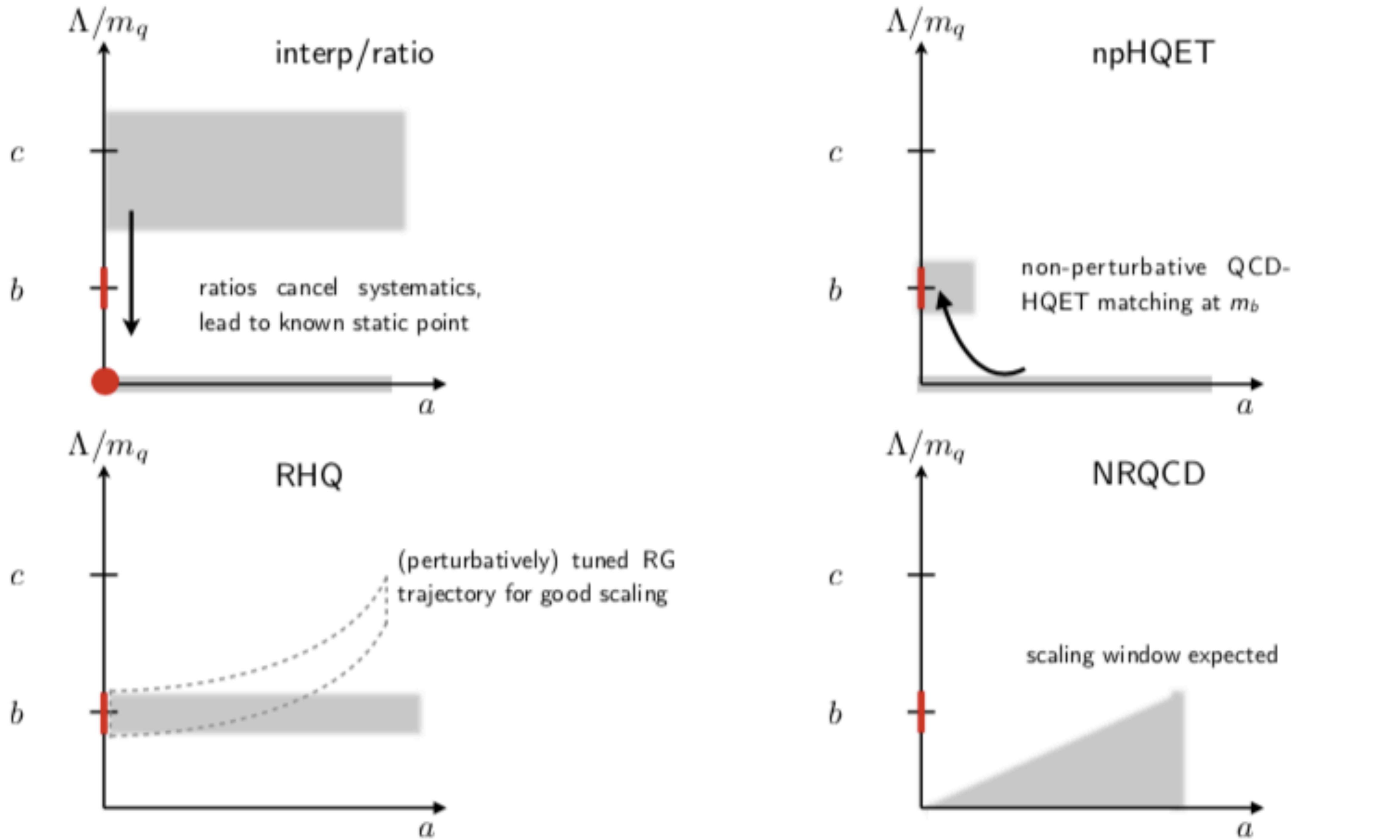
- Computed $\Gamma(P \to \ell\nu[\gamma]) = \Gamma_P^{tree} \times (1 + \delta R_P)$



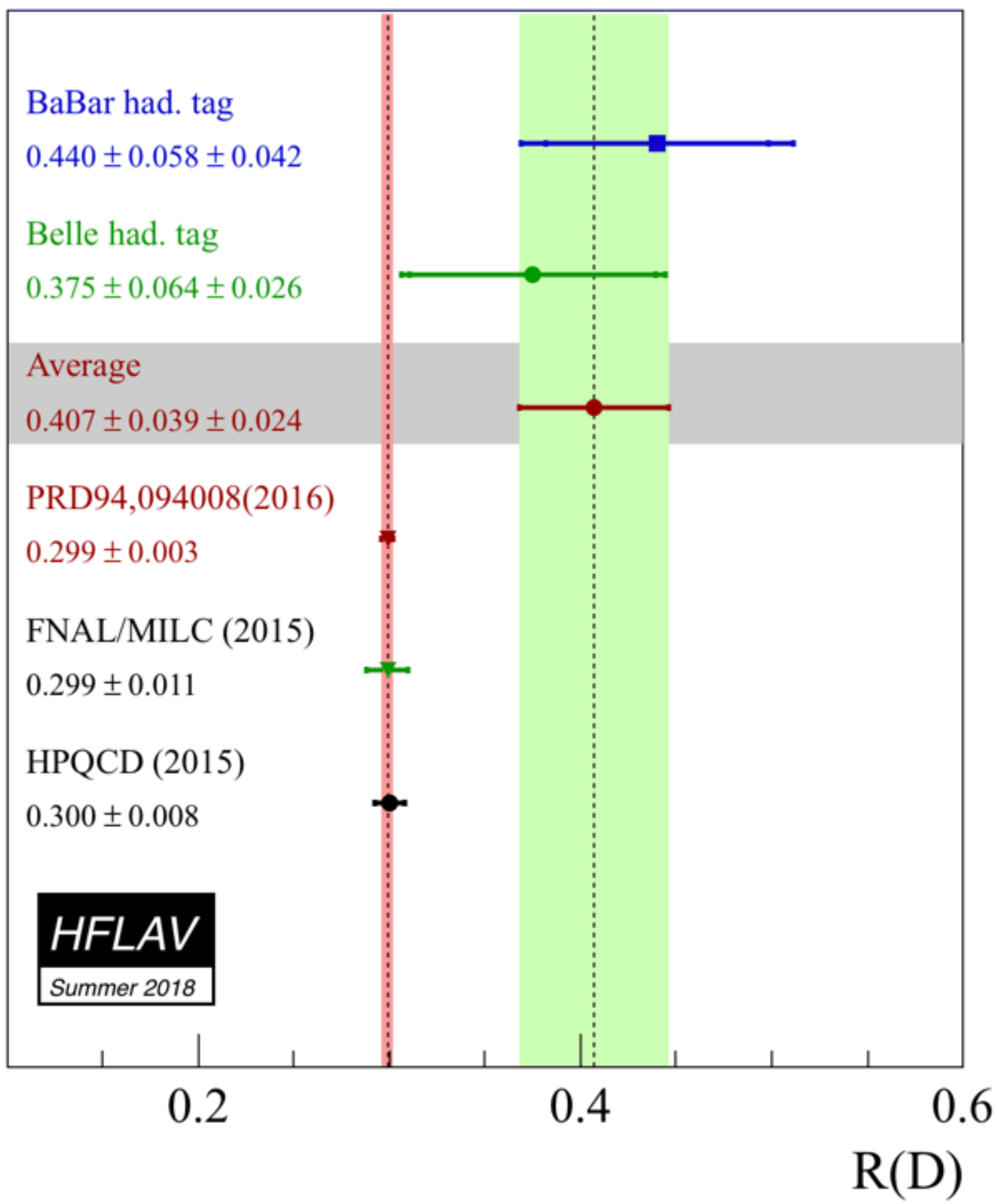
D. Giusti et al 1711.06537

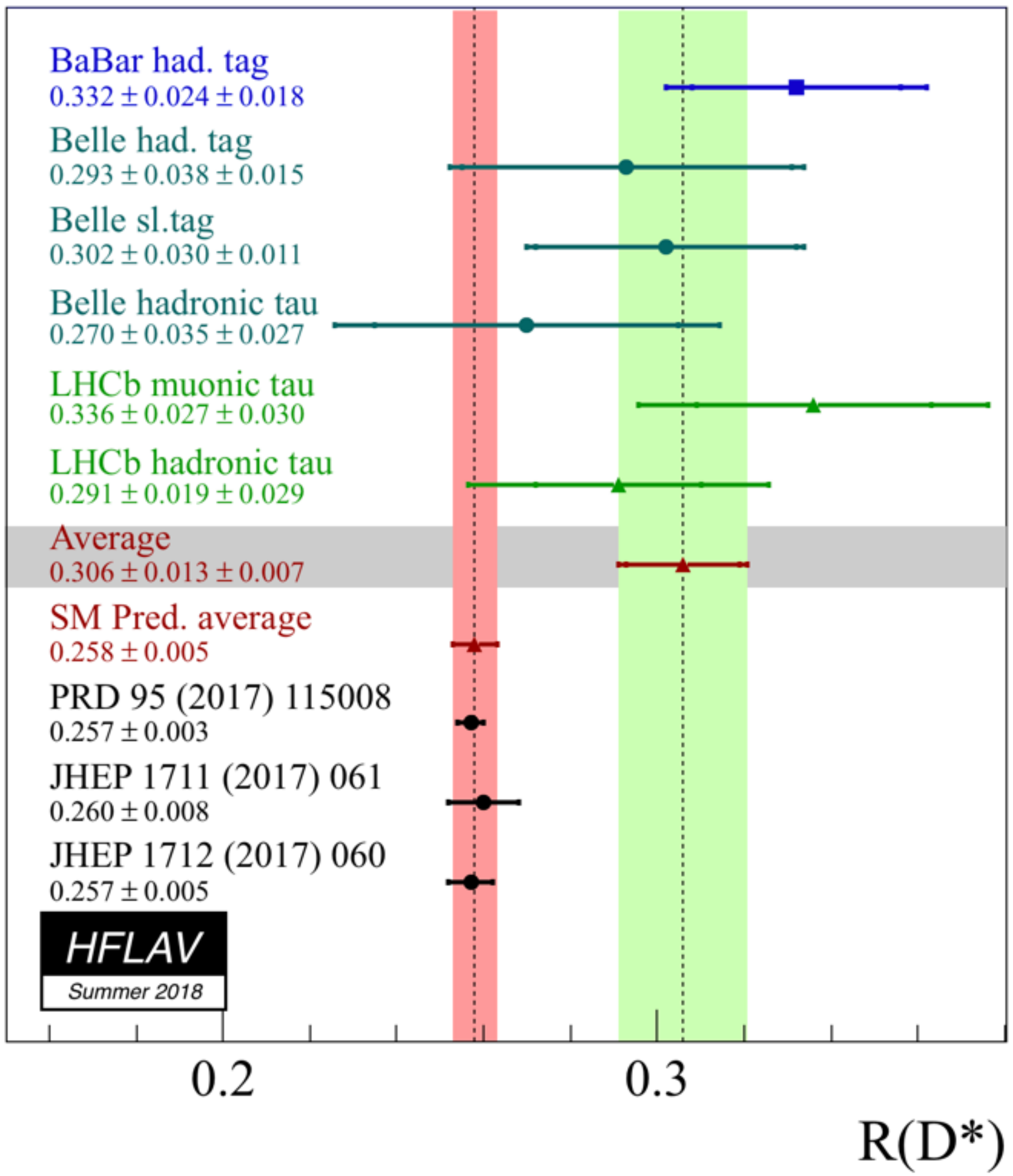
approaches to B physics

effective theory used differently, different pros/cons balance: **crosschecks crucial**

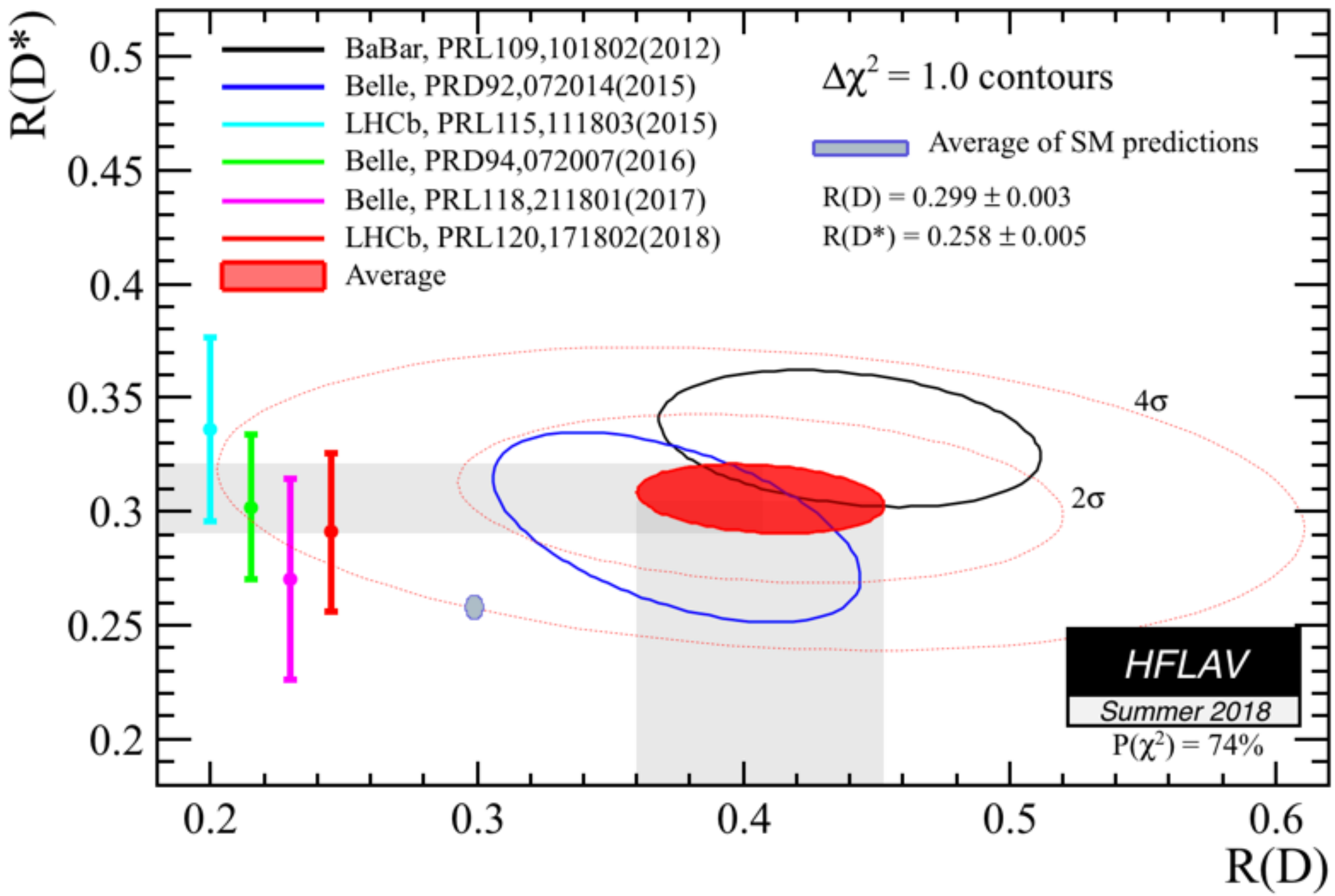


R_D R_{D^*}


BaBar had. tag
0.440 ± 0.058 ± 0.042
Belle had. tag
0.375 ± 0.064 ± 0.026
Average
0.407 ± 0.039 ± 0.024
PRD94,094008(2016)
0.299 ± 0.003
FNAL/MILC (2015)
0.299 ± 0.011
HPQCD (2015)
0.300 ± 0.008
HFLAV
Summer 2018
0.2
0.4
0.6
R(D)


BaBar had. tag
0.332 ± 0.024 ± 0.018
Belle had. tag
0.293 ± 0.038 ± 0.015
Belle sl.tag
0.302 ± 0.030 ± 0.011
Belle hadronic tau
0.270 ± 0.035 ± 0.027
LHCb muonic tau
0.336 ± 0.027 ± 0.030
LHCb hadronic tau
0.291 ± 0.019 ± 0.029
Average
0.306 ± 0.013 ± 0.007
SM Pred. average
0.258 ± 0.005
PRD 95 (2017) 115008
0.257 ± 0.003
JHEP 1711 (2017) 061
0.260 ± 0.008
JHEP 1712 (2017) 060
0.257 ± 0.005
HFLAV
Summer 2018
0.2
0.3
R(D*)

R_D R_{D^*}

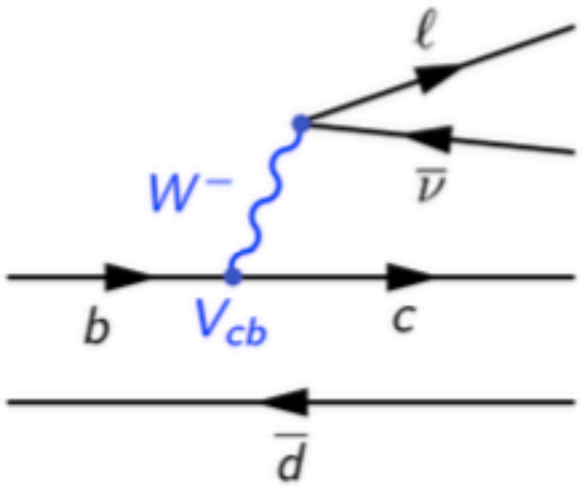


R(D*)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
BaBar, PRL109,101802(2012)
Belle, PRD92,072014(2015)
LHCb, PRL115,111803(2015)
Belle, PRD94,072007(2016)
Belle, PRL118,211801(2017)
LHCb, PRL120,171802(2018)
Average
Δχ² = 1.0 contours
Average of SM predictions
R(D) = 0.299 ± 0.003
R(D*) = 0.258 ± 0.005
4σ
2σ
HFLAV
Summer 2018
P(χ²) = 74%
0.2
0.3
0.4
0.5
0.6
R(D)

R_D R_{D^*}

- Tree-level process in the SM:

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}, \quad \ell = e, \mu.$$



- Non-perturbative QCD $\Longleftrightarrow$ form-factors (Lattice QCD)

  e.g. for $B \to D$, $\quad \langle D|\bar{c}\gamma_\mu b|B\rangle \propto f_{0,+}(q^2)$

- Situation less clear for $B \to D^*$ $\Rightarrow$ (more FFs, less LQCD results) [NP in τ – use angular distribution + HQET of Bernlochner et al 2017]

$B \to D(D^*)\ell\nu$ FF

$$\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_B m_D}\,[(v+v')_\mu h_+(w) + (v-v')_\mu h_-(w)]$$

$$\mathcal{G}(w) = h_+(w) + \frac{m_B - m_D}{m_B + m_D} h_-(w) \longrightarrow f_+(q^2)$$

$$\langle D^*(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} v^\alpha v'^\beta h_V(w)$$

$$\langle D^*(v')|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle = \frac{\sqrt{m_B m_{D^*}}}{2}\Big[\epsilon^*_\mu(1+w)h_{A_1}(w) + (v\cdot\epsilon^*)\big(v_\mu h_{A_2}(w) + v'_\mu h_{A_3}(w)\big)\Big]$$

$$w = v\cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}\,,$$

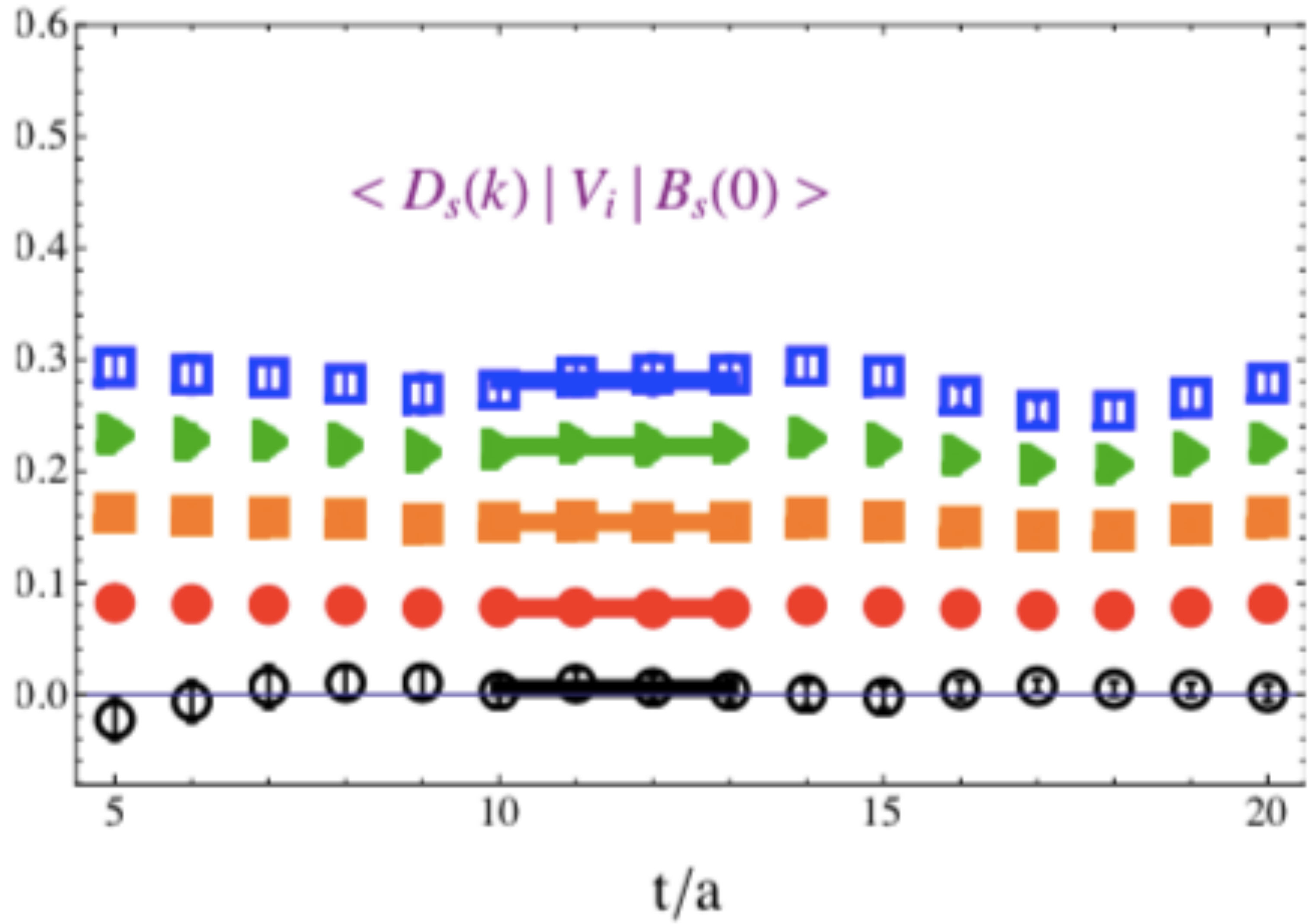
$B_{(s)} \to D_{(s)} \ell \nu$ eg. ETMC

$$C_\mu(\vec{q};t) = \sum_{\vec{x},\vec{y}} \langle P_{bs}(\vec{0},0) V_\mu(\vec{x},t) P^\dagger_{cs}(\vec{y},t_S) e^{-i\vec{q}(\vec{x}-\vec{y})} \rangle$$

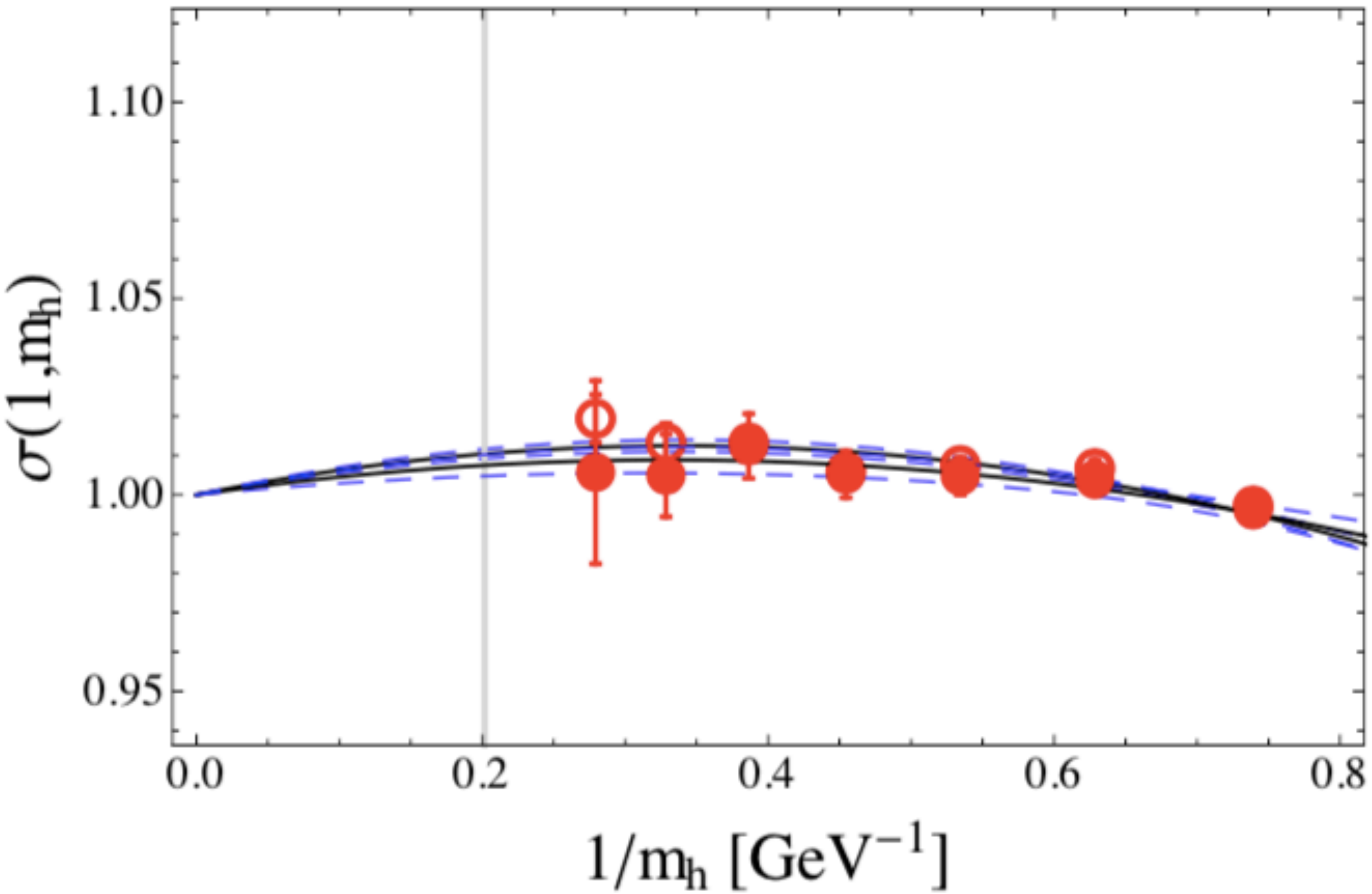
$$\to \quad \langle \sum_{\vec{x},\vec{y}} \mathrm{Tr}\left[\gamma_5 S_s(0,y) \gamma_5 S_c^{\vec{\theta}}(y,x;U) \gamma_\mu S_b(x,0;U) \right] \rangle$$

$B_{(s)} \rightarrow D_{(s)} \ell \nu$ eg. ETMC

$$C_\mu(\vec{q};t) = \sum_{\vec{x},\vec{y}} \langle P_{bs}(\vec{0},0) V_\mu(\vec{x},t) P^\dagger_{cs}(\vec{y},t_S) e^{-i\vec{q}(\vec{x}-\vec{y})} \rangle$$

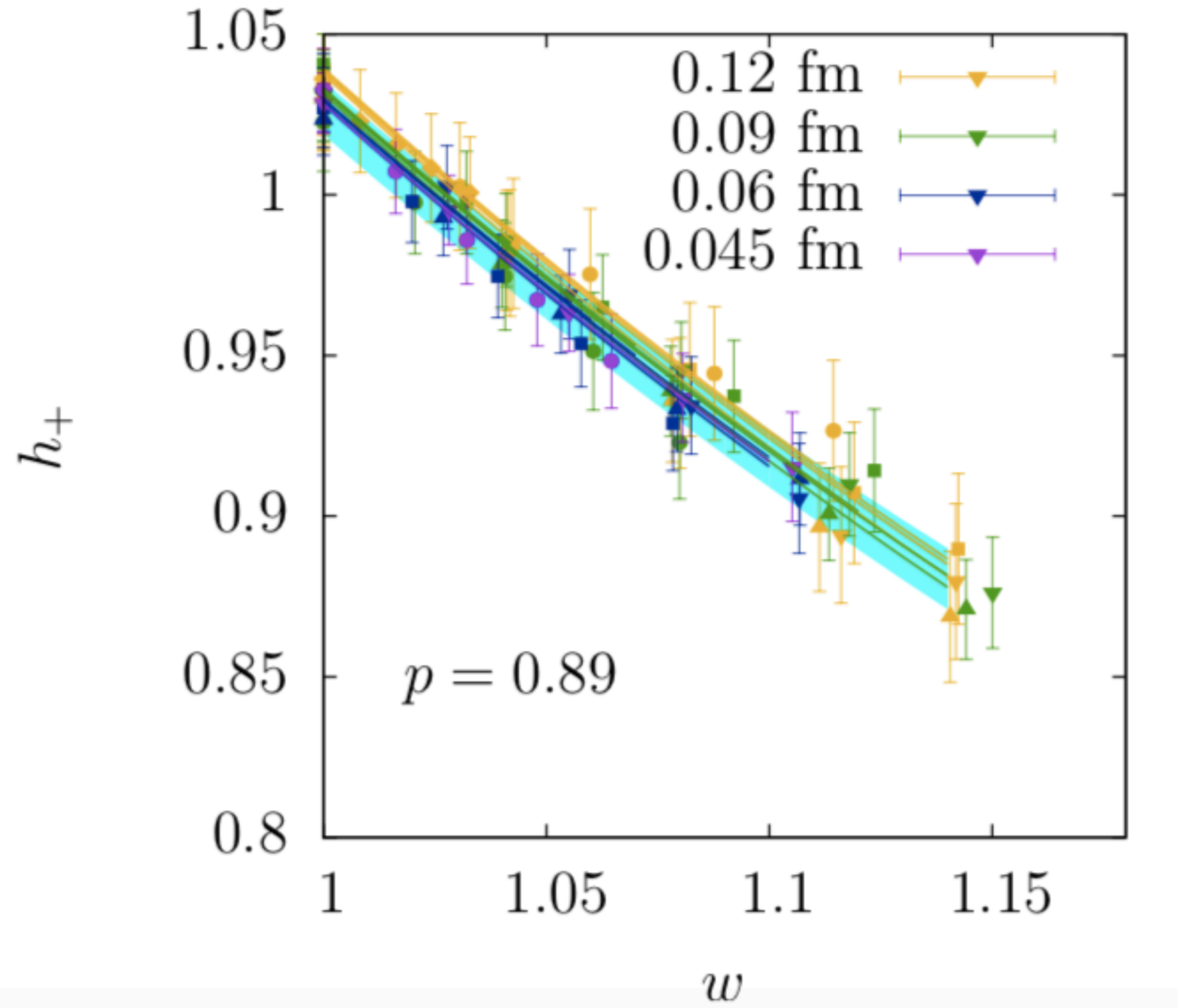


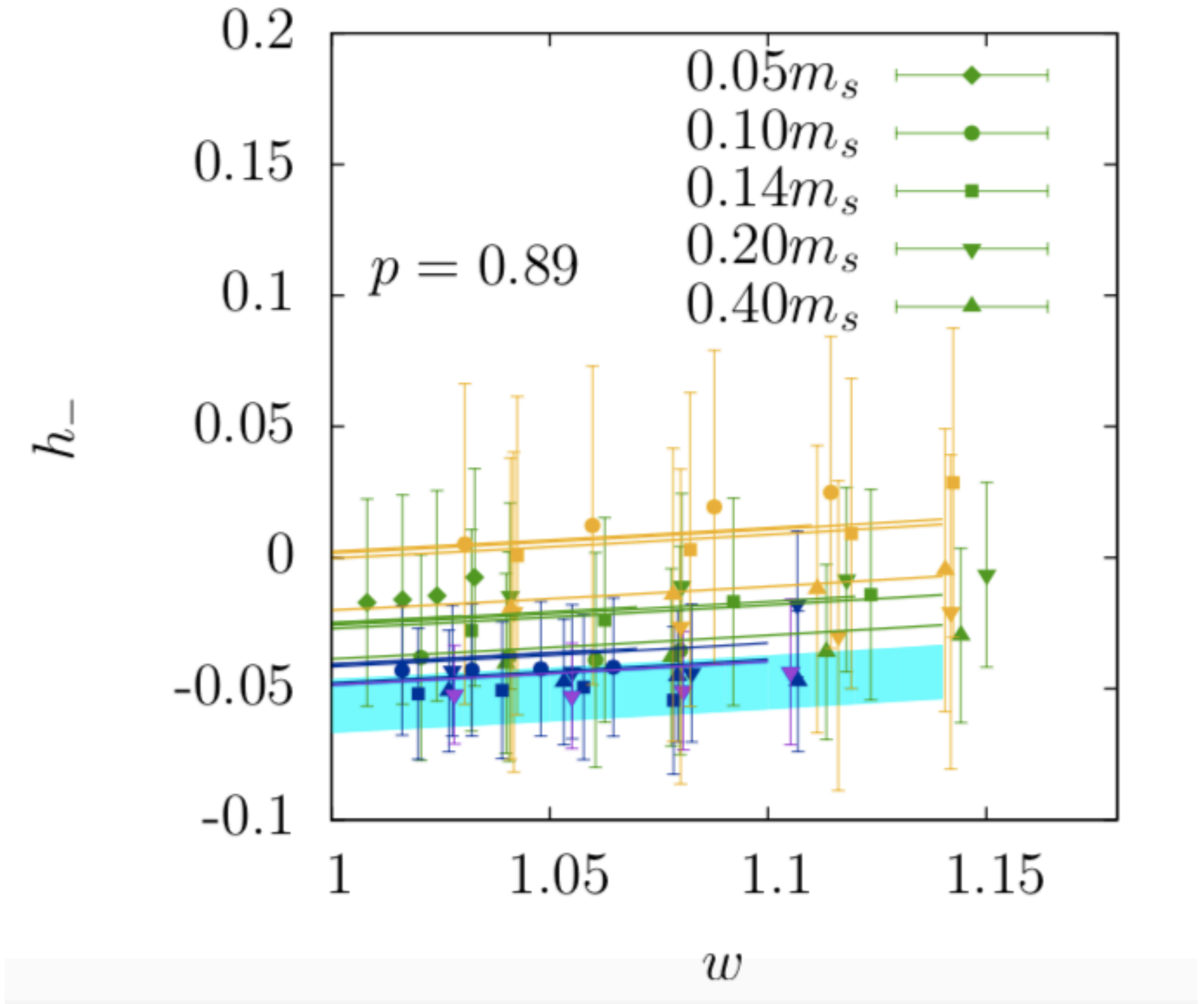
$B_{(s)} \to D_{(s)} \ell \nu$ eg. ETMC



$$\mathcal{G}(1) \equiv \mathcal{G}(1, m_b, m_c)$$
$$= \sigma_n \sigma_{n-1} \ldots \sigma_1 \sigma_0 \underbrace{\mathcal{G}(1, m_c, m_c)}_{=1}$$

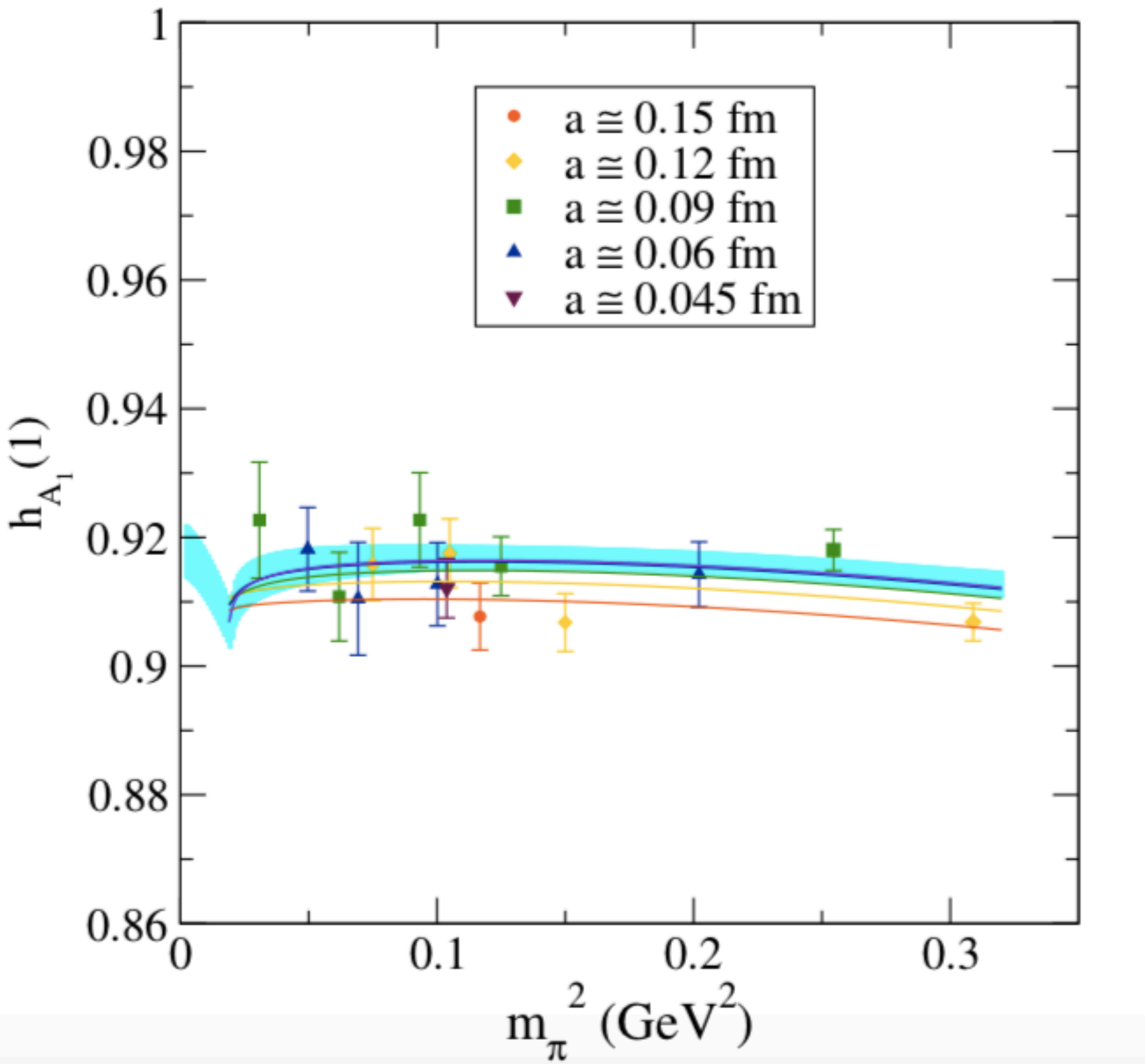
$B_{(s)} \to D_{(s)} \ell \nu$ eg. MILC/FNAL





HPQCD (NRQCD heavy) confirmed the MILC results

$B \to D^* \ell \nu$ FF (MILC/FNAL)



Intermezzo (little B-anomaly)

Results of new Belle angular analysis of $\bar{B} \to D^* \ell \nu$ [1702.01521] allow to show that $|V_{cb}|^{\rm excl}$ depends on parametrization of form factors.

$$\frac{d\Gamma(\bar{B} \to D^*(D\pi)\ell\nu)}{dw\, d\cos\theta_D\, d\cos\theta_\ell\, d\chi} \propto |V_{cb}|^2 \times f\Big(A_1(q^2), V(q^2), A_2(q^2), m_\ell A_0(q^2)\Big)$$
$$= |V_{cb}|^2 \widetilde{f}\Big(A_1(w), R_1(w), R_2(w), m_\ell R_0(w)\Big)_{w=\frac{m_B^2+m_{D*}^2-q^2}{2m_B m_{D*}}}$$

CLN [Caprini et al 1997]:

$$h_{A_1}(w) = h_{A_1}(1)\left[1 + 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$
$$R_2(w) = R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

$h_{A_1}(1)$ LQCD; Red numbers fixed by HQET and pheno.

BGL [Boyd et al 1997] do not do red step, otherwise parameterization is 'the same' expansion in $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$.

Intermezzo (little B-anomaly)

Results of new Belle angular analysis of $\bar{B} \to D^* \ell \nu$ [1702.01521] allow to show that $|V_{cb}|^{\rm excl}$ depends on parametrization of form factors. CLN [Caprini et al 1997]:

$$h_{A_1}(w) = h_{A_1}(1)\left[1 + 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$
$$R_2(w) = R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

$R_2(1)$: fit $>$ HQET by more than 2σ Refit [D.Bigi et al 1703.06124, Grinstein, Kobach 1703.08170]

$$|V_{cb}|^{\rm excl}_{\rm CLN} = (38.2 \pm 1.5) \times 10^{-3} \qquad |V_{cb}|^{\rm excl}_{\rm BGL} = (41.7^{+2.0}_{-2.1}) \times 10^{-3}$$
$$|V_{cb}|^{\rm incl}_{1S} = (42.0 \pm 0.5) \times 10^{-3} \qquad |V_{cb}|^{\rm incl}_{\rm kin} = (42.2 \pm 0.8) \times 10^{-3}$$

Both fits (using CLN or BGL) are good $\Rightarrow$ Inconclusive!
Way out: $|V_{cb}|$ from LQCD & Belle II data at small recoil.
See also uncertainties about $m_\ell R_0(w)$

In searching for NP...

$$\mathcal{H}_{\rm eff} = \sqrt{2} G_F V_{cb} \big[(1 + g_V)(\bar{c}\gamma_\mu b)(\bar{\ell}_L \gamma^\mu \nu_L) + (-1 + g_A)(\bar{c}\gamma_\mu \gamma_5 b)(\bar{\ell}_L \gamma^\mu \nu_L) + g_S(\bar{c}b)(\bar{\ell}_R \nu_L) + g_P(\bar{c}\gamma_5 b)(\bar{\ell}_R \nu_L) + g_T(\bar{c}\sigma_{\mu\nu} b)(\bar{\ell}_R \sigma^{\mu\nu} \nu_L) + g_{T5}(\bar{c}\sigma_{\mu\nu}\gamma_5 b)(\bar{\ell}_R \sigma^{\mu\nu} \nu_L) \big] + \text{h.c.}$$

$$\mathcal{H}_{\rm eff} = \frac{4 G_F}{\sqrt{2}} V_{cb} \big[(1 + g_{V_L})(\bar{c}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \nu_L) + g_{V_R}(\bar{c}_R \gamma_\mu b_R)(\bar{\ell}_L \gamma^\mu \nu_L) + g_{S_L}(\bar{c}_R b_L)(\bar{\ell}_R \nu_L) + g_{S_R}(\bar{c}_L b_R)(\bar{\ell}_R \nu_L) + g_{T_L}(\bar{c}_R \sigma_{\mu\nu} b_L)(\bar{\ell}_R \sigma^{\mu\nu} \nu_L) \big] + \text{h.c.}\,,$$

Intermezzo (HQE)

$$\langle D^*|\,\bar{c}b\,|\overline{B}\rangle = 0\,,$$

$$\langle D^*|\,\bar{c}\gamma^5 b\,|\overline{B}\rangle = -\sqrt{m_B m_{D^*}}\,h_P\,(\epsilon^*\cdot v)\,,$$

$$\langle D^*|\,\bar{c}\gamma^\mu b\,|\overline{B}\rangle = i\sqrt{m_B m_{D^*}}\,h_V\,\varepsilon^{\mu\nu\alpha\beta}\,\epsilon^*_\nu v'_\alpha v_\beta\,,$$

$$\langle D^*|\,\bar{c}\gamma^\mu\gamma^5 b\,|\overline{B}\rangle = \sqrt{m_B m_{D^*}}\,\big[h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu\big],$$

$$\langle D^*|\,\bar{c}\sigma^{\mu\nu} b\,|\overline{B}\rangle = -\sqrt{m_B m_{D^*}}\,\varepsilon^{\mu\nu\alpha\beta}\big[h_{T_1}\epsilon^*_\alpha(v+v')_\beta + h_{T_2}\epsilon^*_\alpha(v-v')_\beta + h_{T_3}(\epsilon^*\cdot$$

HQS

$$h_- = h_{A_2} = h_{T_2} = h_{T_3} = 0\,,$$

$$h_+ = h_V = h_{A_1} = h_{A_3} = h_S = h_P = h_T = h_{T_1} = \xi\,.$$

Intermezzo (HQE + ‘model’)

Bernlochner et al. 2017

$$\mathcal{L}_{\rm HQET} = \bar{h}_Q \, iv\cdot D \, h_Q \,, \qquad \mathcal{L}_{\rm power} = \frac{1}{2m_Q}\,\mathcal{L}_1 + \frac{1}{4m_Q^2}\,\mathcal{L}_2 + \cdots \,.$$

$$\mathcal{L}_1 = \bar{h}_Q \,(iD)^2 h_Q + Z(m_Q/\mu)\,\bar{h}_Q\, s_{\alpha\beta} G^{\alpha\beta} h_Q \,,$$

Good for ratios of FFs. Needs checks from LQCD

Intermezzo (HQE)

$$\mathcal{L}_{\rm HQET} = \bar{h}_Q \, iv\cdot D \, h_Q \,, \qquad \mathcal{L}_{\rm power} = \frac{1}{2m_Q}\,\mathcal{L}_1 + \frac{1}{4m_Q^2}\,\mathcal{L}_2 + \cdots \,.$$

$$\begin{aligned}
\bar{c}\, b &\to \bar{c}_{v'} \big(1 + \hat{\alpha}_s \, C_S\big) b_v \,, \\
\bar{c}\gamma^5 b &\to \bar{c}_{v'} \big(1 + \hat{\alpha}_s \, C_P\big) \gamma^5 b_v \,, \\
\bar{c}\gamma^\mu b &\to \bar{c}_{v'} \Big[\big(1 + \hat{\alpha}_s \, C_{V_1}\big)\gamma^\mu + \hat{\alpha}_s \, C_{V_2}\, v^\mu + \hat{\alpha}_s \, C_{V_3}\, v'^\mu\Big] b_v \,, \\
\bar{c}\gamma^\mu\gamma^5 b &\to \bar{c}_{v'} \Big[\big(1 + \hat{\alpha}_s \, C_{A_1}\big)\gamma^\mu + \hat{\alpha}_s \, C_{A_2}\, v^\mu + \hat{\alpha}_s \, C_{A_3}\, v'^\mu\Big] \gamma^5 b_v \\
\bar{c}\sigma^{\mu\nu} b &\to \bar{c}_{v'} \Big[\big(1 + \hat{\alpha}_s \, C_{T_1}\big)\sigma^{\mu\nu} + \hat{\alpha}_s \, C_{T_2}\, i(v^\mu\gamma^\nu - v^\nu\gamma^\mu) + \hat{\alpha}_s \, C_{T_3}\, i(v'^\mu\gamma^\nu - v'^\nu\gamma^\mu) \\
&\qquad + C_{T_4}(v'^\mu v^\nu - v'^\nu v^\mu)\Big] b_v \,,
\end{aligned}$$

Good for ratios of FFs. Needs checks from LQCD

Intermezzo (HQE + 'model')

Bernlochner et al. 2017

$$\frac{\langle D^{(*)}|\,\bar{c}\,\Gamma\,b\,|\bar{B}\rangle}{\sqrt{m_{D^{(*)}}m_B}} = -\xi(w)\left\{\mathrm{Tr}\big[\bar{H}^{(c)}_{v'}\,\Gamma\,H^{(b)}_{v}\big] + \varepsilon_c\,\mathrm{Tr}\big[\bar{H}^{(c,1)}_{v',v}\,\Gamma\,H^{(b)}_{v}\big] + \varepsilon_b\,\mathrm{Tr}\big[\bar{H}^{(c)}_{v'}\,\Gamma\,H^{(b,1)}_{v,v'}\big]\right\}$$

$$\hat{h}_V = 1 + \hat{\alpha}_s\,C_{V_1} + \varepsilon_c\big(\hat{L}_2 - \hat{L}_5\big) + \varepsilon_b\big(\hat{L}_1 - \hat{L}_4\big)\,,$$

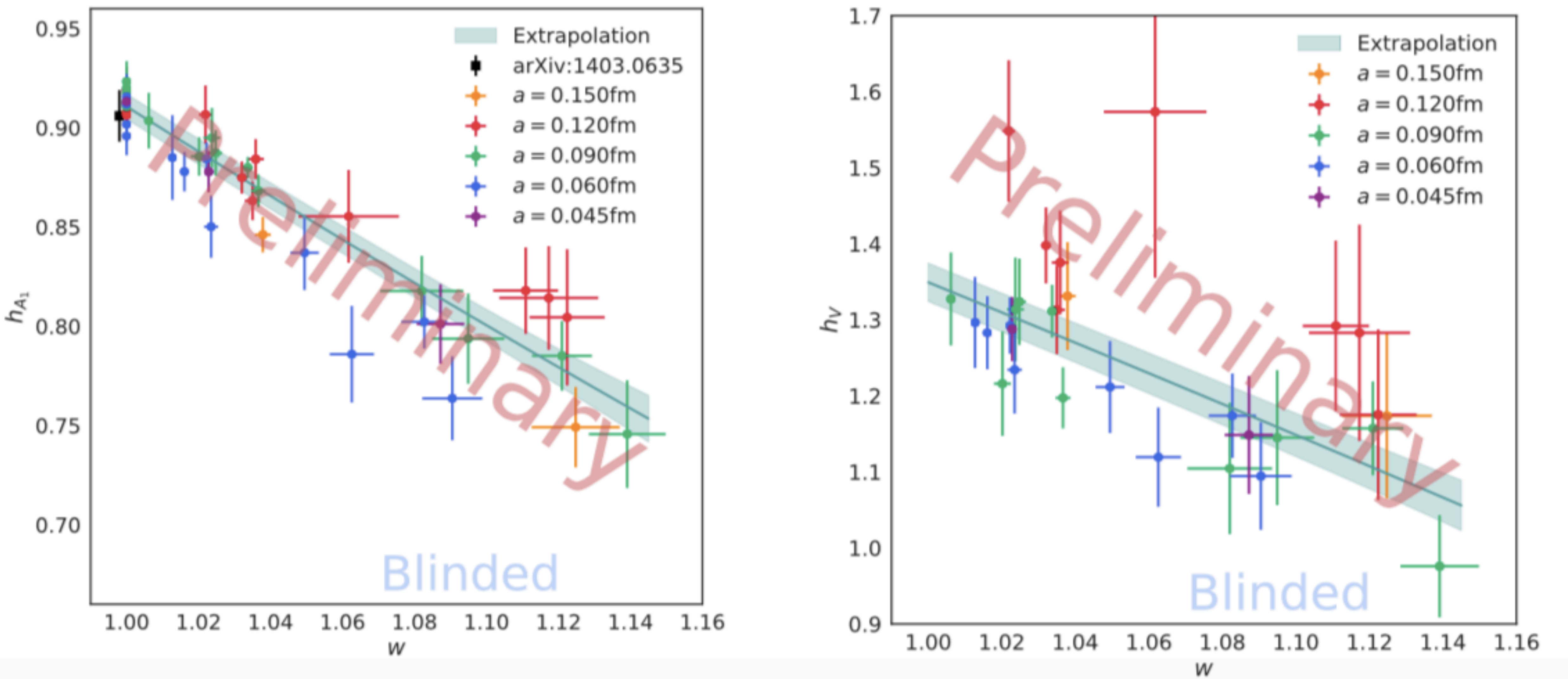
$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s\,C_{A_1} + \varepsilon_c\left(\hat{L}_2 - \hat{L}_5\,\frac{w-1}{w+1}\right) + \varepsilon_b\left(\hat{L}_1 - \hat{L}_4\,\frac{w-1}{w+1}\right),$$

$$\hat{h}_{A_2} = \hat{\alpha}_s\,C_{A_2} + \varepsilon_c\big(\hat{L}_3 + \hat{L}_6\big)\,,$$

$$\hat{h}_{A_3} = 1 + \hat{\alpha}_s\big(C_{A_1} + C_{A_3}\big) + \varepsilon_c\big(\hat{L}_2 - \hat{L}_3 + \hat{L}_6 - \hat{L}_5\big) + \varepsilon_b\big(\hat{L}_1 - \hat{L}_4\big)\,,$$

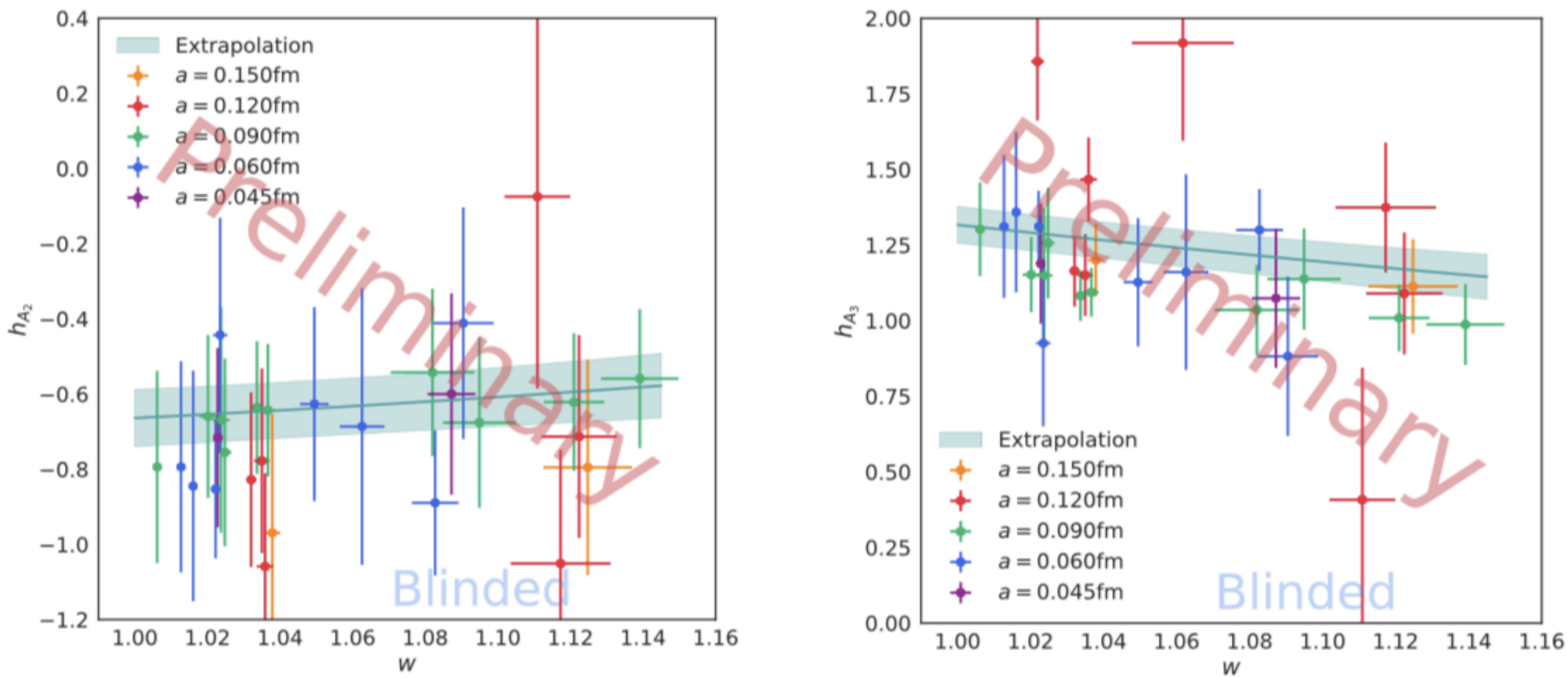
Good for ratios of FFs. Needs checks from LQCD

$B \to D^* \ell \nu$ FF (MILC/FNAL)



A. Vaquero @ Lattice 2018

$B \to D^* \ell \nu$ FF (MILC/FNAL)



A. Vaquero @ Lattice 2018

$R(J/\psi)$

$$R(J/\psi) = \frac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu})}{\mathcal{B}(B_c \to J/\psi\mu\bar{\nu})} = 0.71 \pm 0.25$$

High Energy Physics – Experiment

Measurement of the ratio of branching fractions $\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$

LHCb collaboration: R. Aaij, B. Adeva, M. Adinolfi, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, A. Alfonso Albero, S. Ali, G. Alkhazov, P. Alvarez Cartelle, A.A. Alves Jr, S. Amato, S. Amerio, Y. Amhis, L. An, L. Anderlini, G. Andreassi, M. Andreotti, J.E. Andrews, R.B. Appleby, F. Archilli, P. d'Argent, J. Arnau Romeu, A. Artamonov, M. Artuso, E. Aslanides, M. Atzeni, G. Auriemma, M. Baalouch, I. Babuschkin, S. Bachmann, J.J. Back, A. Badalov, C. Baesso, S. Baker, V. Balagura, W. Baldini, A. Baranov, R.J. Barlow, C. Barschel, S. Barsuk, W. Barter, F. Baryshnikov, V. Batozskaya, V. Battista, A. Bay, L. Beaucourt, J. Beddow, F. Bedeschi, I. Bediaga, A. Beiter, L.J. Bel, N. Beliy, V. Bellee, N. Belloli, K. Belous, I. Belyaev, E. Ben-Haim, G. Bencivenni, S. Benson, S. Beranek, et al. (738 additional authors not shown)

(Submitted on 15 Nov 2017 (v1), last revised 30 Mar 2018 (this version, v2))

A measurement is reported of the ratio of branching fractions $\mathcal{R}(J/\psi) = \mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$, where the τ^+ lepton is identified in the decay mode $\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau$. This analysis uses a sample of proton-proton collision data corresponding to $3.0\,\mathrm{fb}^{-1}$ of integrated luminosity recorded with the LHCb experiment at center-of-mass energies $7\,\mathrm{TeV}$ and $8\,\mathrm{TeV}$. A signal is found for the decay $B_c^+ \to J/\psi\tau^+\nu_\tau$ at a significance of 3 standard deviations, corrected for systematic uncertainty, and the ratio of the branching fractions is measured to be $\mathcal{R}(J/\psi) = 0.71 \pm 0.17\,(\mathrm{stat}) \pm 0.18\,(\mathrm{syst})$. This result lies within 2 standard deviations above the range of existing predictions in the Standard Model.

LHCb 1711.05623

$B_c \to J/\psi\ \ell\nu$ FF

$$
\begin{aligned}
-\mathrm{i}\,\langle J/\psi(p_2)|\,\gamma_\mu(1-\gamma_5)\,|B_c(p_1)\rangle =& \frac{2V(q^2)}{m_{B_c}+m_{J/\psi}}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_2^\alpha p_1^\beta \\
&+\mathrm{i}(m_{B_c}+m_{J/\psi})A_1(q^2)\epsilon^*_\mu \\
&-\mathrm{i}\frac{A_2(q^2)}{m_{B_c}+m_{J/\psi}}(\epsilon^*\cdot q)(p_1+p_2)_\mu \\
&-\mathrm{i}\frac{2m_{J/\psi}}{q^2}(A_3(q^2)-A_0(q^2))(\epsilon^*\cdot q)q_\mu
\end{aligned}
$$

$$
\begin{aligned}
-\mathrm{i}\,\langle J/\psi(p_2)|\,\sigma_{\mu\nu}\gamma_5\,|B_c(p_1)\rangle =& -\mathrm{i}A(q^2)\{\varepsilon^*_\mu(p_1+p_2)_\nu-(p_1+p_2)_\mu\varepsilon^*_\nu \\
&+\mathrm{i}B(q^2)\{\varepsilon^*_\mu q_\nu - q_\mu\varepsilon^*_\nu\} + 2\mathrm{i}C(q^2)\frac{\varepsilon^* q}{m_{B_c}^2-m_{J/\psi}^2}\{p_{2\mu}q_\nu - q_\mu p_{2\nu}
\end{aligned}
$$

$B_c \to J/\psi\, \ell\, \nu$ FF

$$
\begin{aligned}
-\mathrm{i}\, \langle J/\psi(p_2)|\, \gamma_\mu(1-\gamma_5)\, |B_c(p_1)\rangle = & \frac{2V(q^2)}{m_{B_c}+m_{J/\psi}} \varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} p_2^\alpha p_1^\beta \\
& + \mathrm{i}(m_{B_c}+m_{J/\psi}) A_1(q^2) \epsilon_\mu^* \\
& - \mathrm{i} \frac{A_2(q^2)}{m_{B_c}+m_{J/\psi}} (\epsilon^* \cdot q)(p_1+p_2)_\mu \\
& - \mathrm{i} \frac{2m_{J/\psi}}{q^2} (A_3(q^2) - A_0(q^2))(\epsilon^* \cdot q) q_\mu
\end{aligned}
$$

$$
T_1(q^2) = A(q^2) \qquad T_2(q^2) = A(q^2) - \frac{q^2}{m_{B_c}^2 - m_{J/\psi}^2} B(q^2)
$$

$$
T_3(q^2) = B(q^2) + C(q^2) \qquad \tilde{T}_3(q^2) = A(q^2) + \frac{q^2}{m_{B_c}^2 - m_{J/\psi}^2} C(q^2)
$$

$B_c \to J/\psi\, \ell\, \nu$ FF

Standard QCDSR *difficult*

- leading non-perturbative (power) correction ~ gluon condensate
- consistent with zero, ambiguous…

A way out

- fix QCDSR parameters in 2pt functions using the LQCD results
- plug them into 3pt functions and compute FFs
- check against lattice for $A_1(q^2)$ and $V(q^2)$

QHD

$$\Pi(q^2)_i \approx \frac{1}{\pi} \int_{(m_{q_1}+m_{q_2})^2}^{s_0^{\mathrm{eff.}}} \mathrm{d}s \frac{\mathrm{Im}[\Pi_i(s)]}{s-q^2}$$

1st hadron state contribution

QCD

with Leljak, Melic, Sumensari, *in preparation*

$B_c \to J/\psi\, \ell \nu$ FF

A way out

- fix QCDSR parameters in 2pt functions using the LQCD results

$$f_{J/\psi} = 418(8)(5)\ \mathrm{MeV}_{\mathrm{ETMC}}, 405(6)(2)\ \mathrm{MeV}_{\mathrm{HPQCD}} \qquad f_{B_c} = 427(6)(2)\ \mathrm{MeV}_{\mathrm{HPQCD}}$$

$$\langle 0 |\, \bar{c}\gamma_\nu c\, | J/\psi(p_2)\rangle = f_{J/\psi} m_{J/\psi} \epsilon_\nu$$

$$\langle B_c(p_1) |\, \bar{b} i\gamma_5 c\, | 0\rangle = -\frac{f_{B_c} m_{B_c}^2}{m_b + m_c}$$

Borelize DR and match to lattice values

$s_{J/\psi}$ [GeV^2]	$f_{J/\psi}$ [GeV]
15.5	0.385
16	0.402
16.5	0.407

s_{B_c} [GeV^2]	f_{B_c} [GeV]
52	0.406
53	0.424
54	0.439

For $m_b = 4.6\,\mathrm{GeV},\ m_c = z m_b\ [z \in (.28, 0.32)]$,

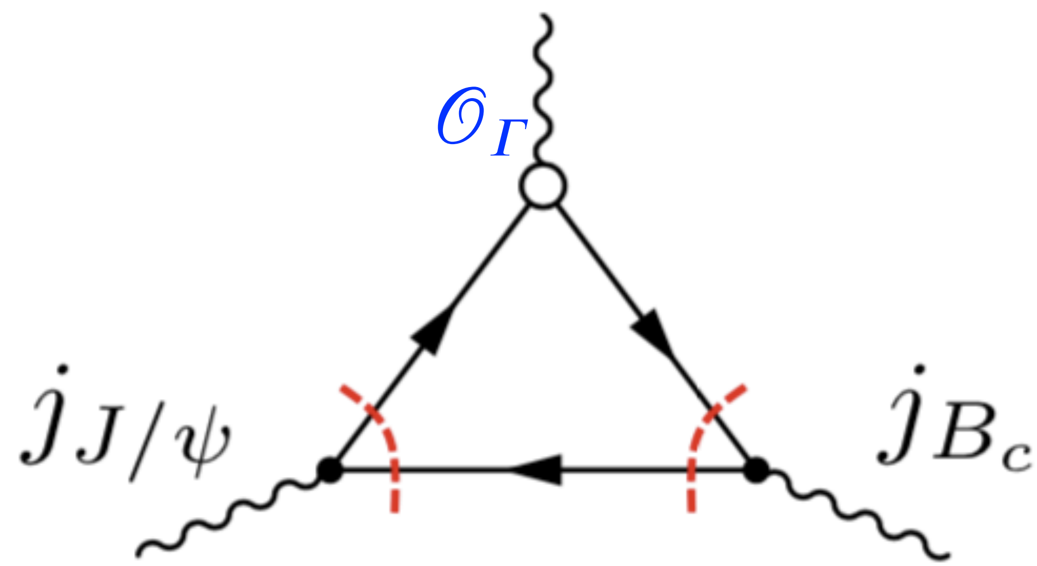
and $M^2_{J/\psi} \in (20, 25)\ \mathrm{GeV}^2,\ M^2_{B_c} \in (60, 80)\ \mathrm{GeV}^2$

with Leljak, Melic, Sumensari, *in preparation*

$B_c \to J/\psi\, \ell \nu$ FF

A way out

- plug parameters into 3pt functions and compute FFs



$$\Pi_{\mu\nu} = \sum_i \Pi^i(p_1^2, p_2^2, q^2)\Gamma^i_{\mu\nu}$$

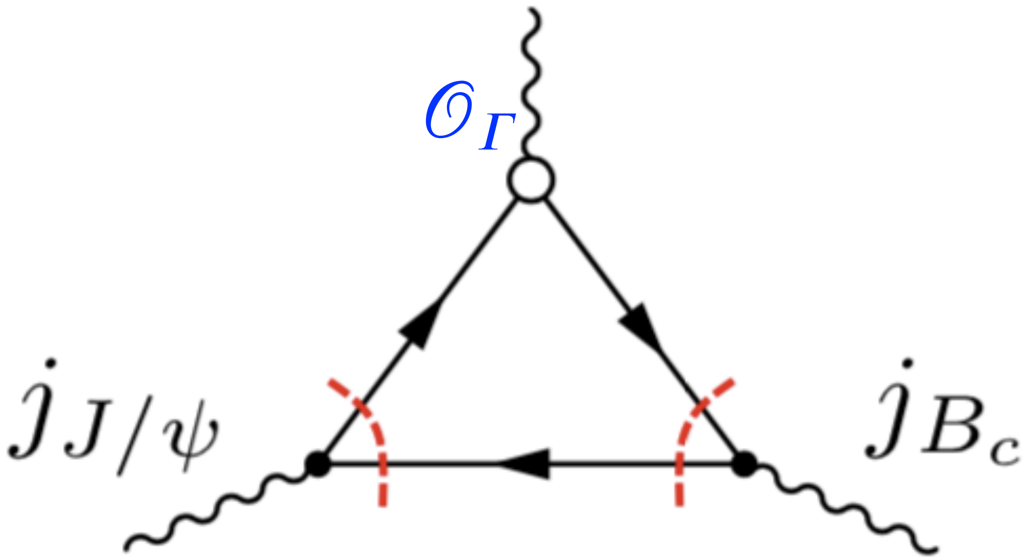
$$\Pi_i^{\rm ph}(p_1^2, p_2^2, q^2) = -\frac{1}{(2\pi)^2}\iint \frac{\rho_i^{\rm ph}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}{\rm d}s_1{\rm d}s_2$$

with Leljak, Melic, Sumensari, *in preparation*

$B_c \to J/\psi\, \ell \nu$ FF

A way out

- plug parameters into 3pt functions and compute FFs



$$\int_{s^0_{\mathrm{ph1}}} \int_{s^0_{\mathrm{ph2}}} \rho_i^{\mathrm{cont}}(s_1, s_2, q^2) \mathrm{e}^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \mathrm{d}s_1 \mathrm{d}s_2$$

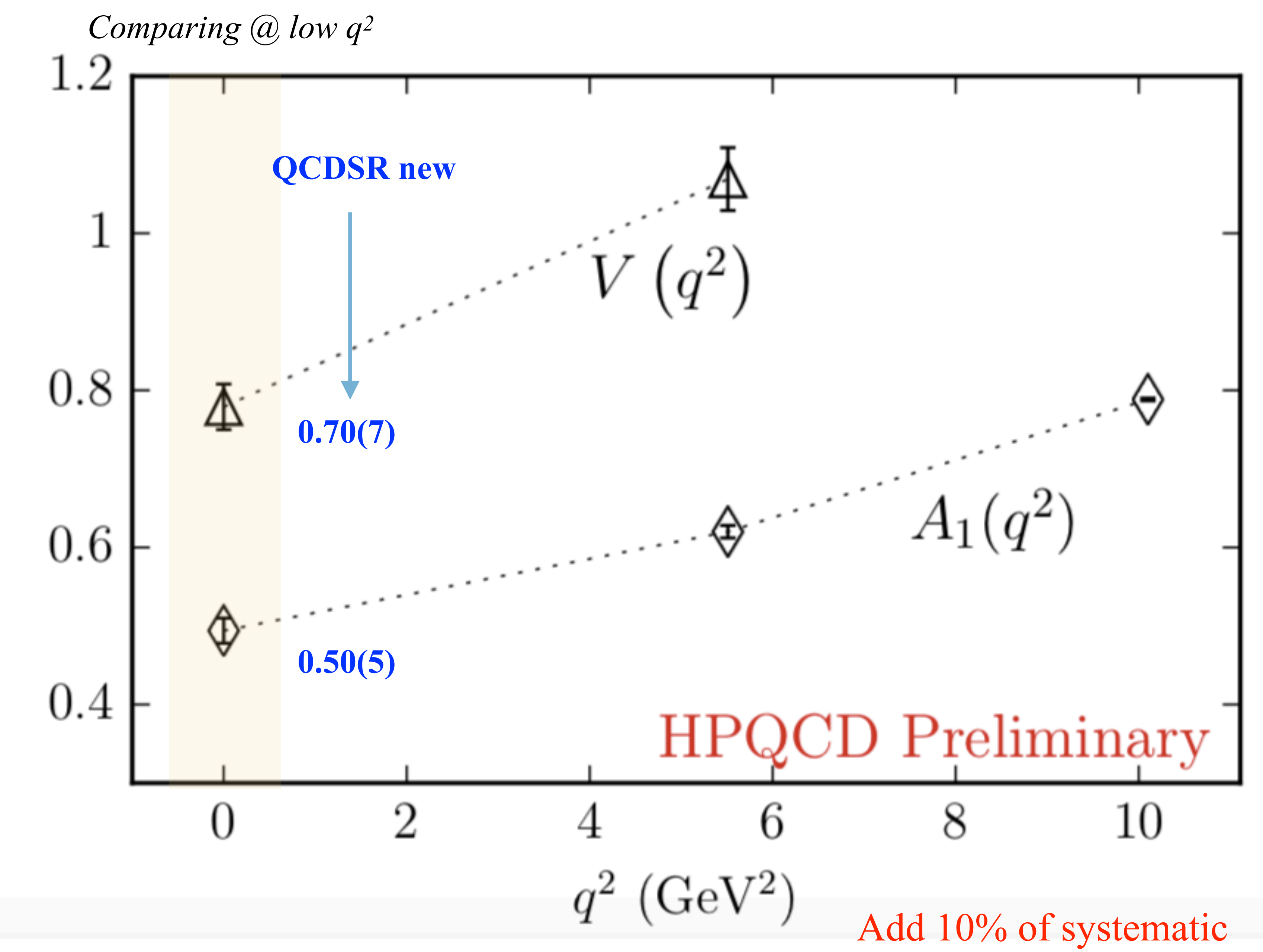
$$\approx \int_{s^0_{\mathrm{eff1}}} \int_{s^0_{\mathrm{eff2}}} \rho_i^{\mathrm{pert}}(s_1, s_2, q^2) \mathrm{e}^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \mathrm{d}s_1 \mathrm{d}s_2$$

with Leljak, Melic, Sumensari, *in preparation*

$B_c \to J/\psi\ \ell \nu$ FF

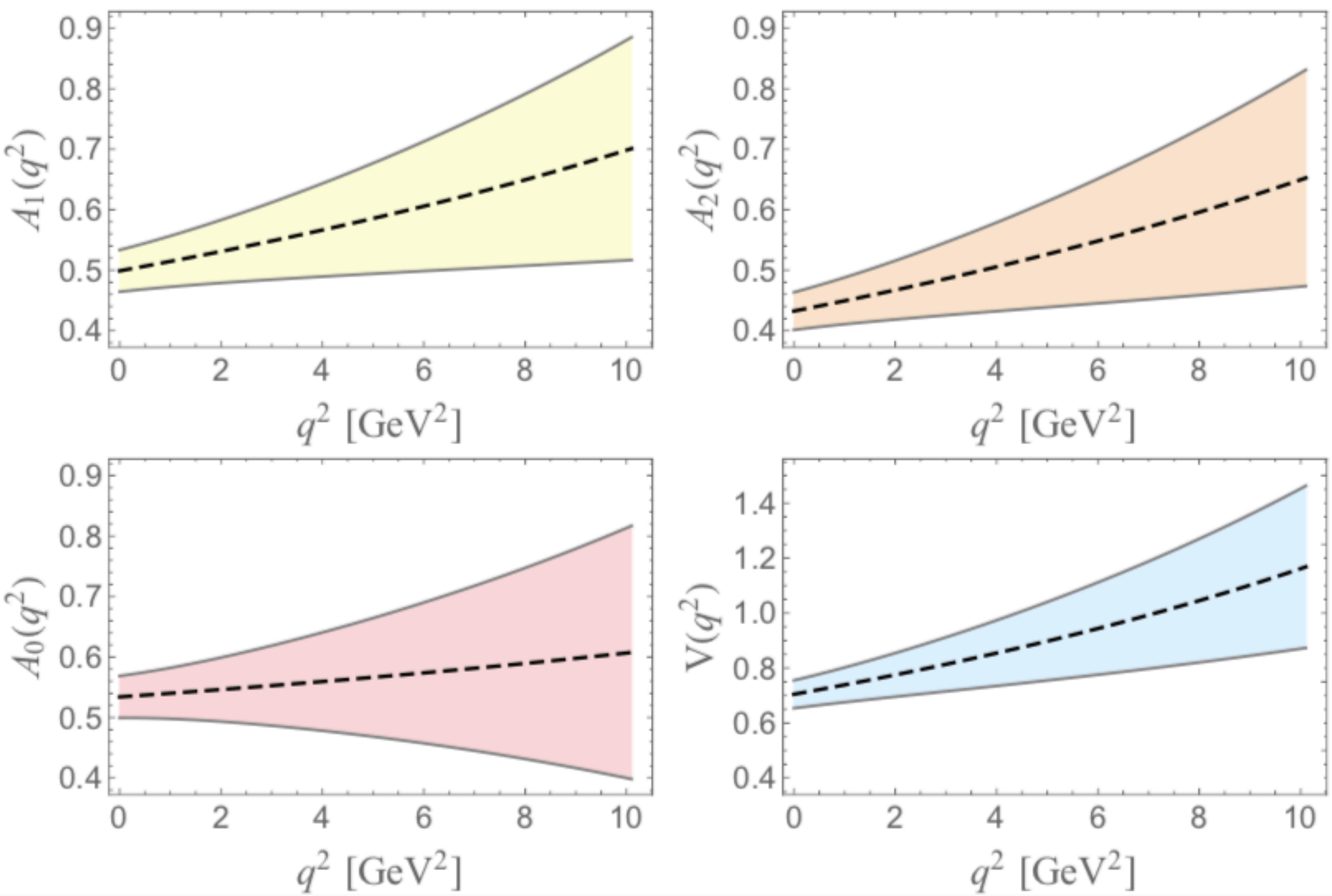
A way out

- check against lattice for $A_1(q^2)$ and $V(q^2)$



Add 10% of systematic error to LQCD values

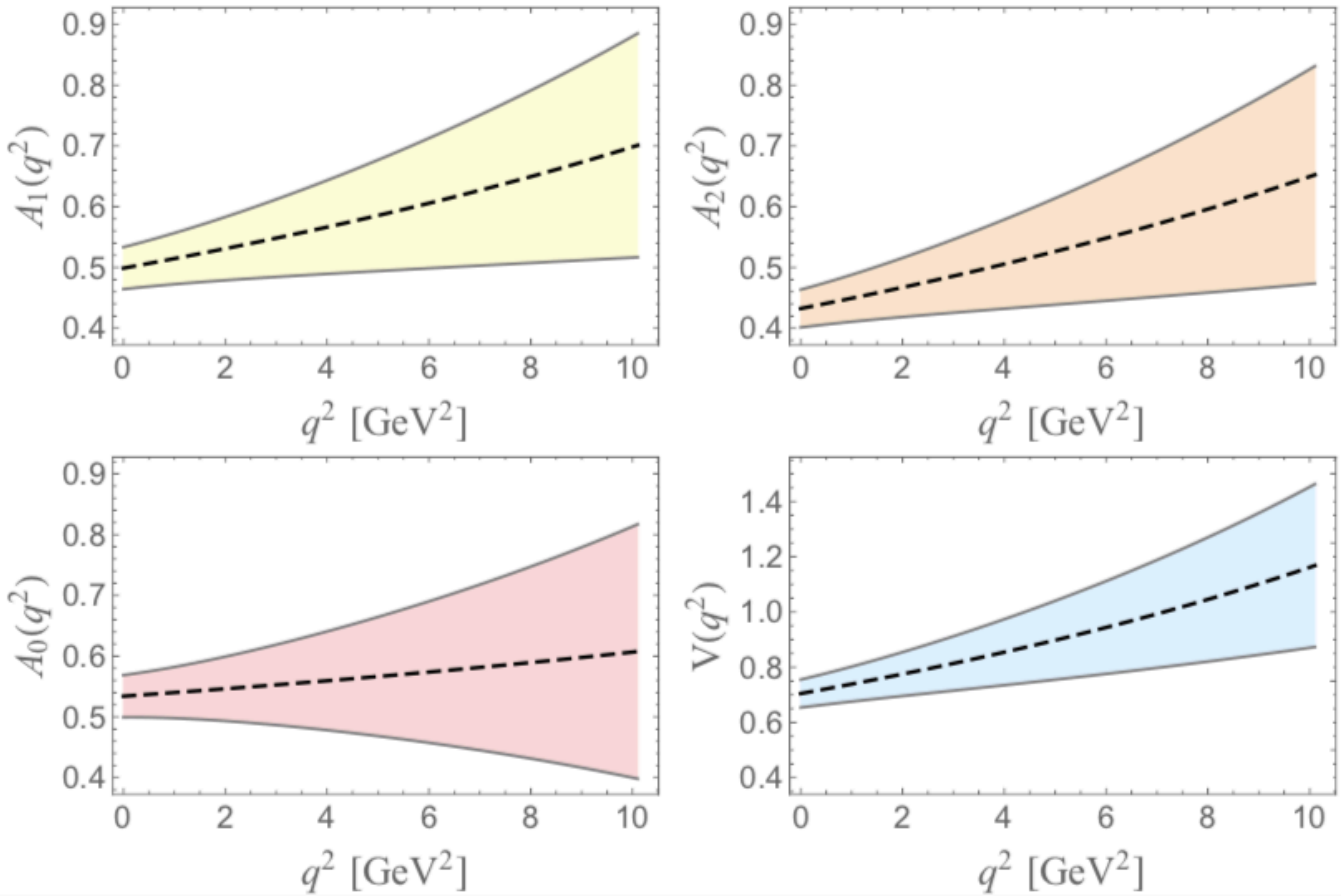
$B_c \to J/\psi\ \ell\nu$ FF



$$F_i(q^2) = \frac{1}{1 - q^2/m_{B_c}^2}\left(a_0 + a_1\, z \right)$$

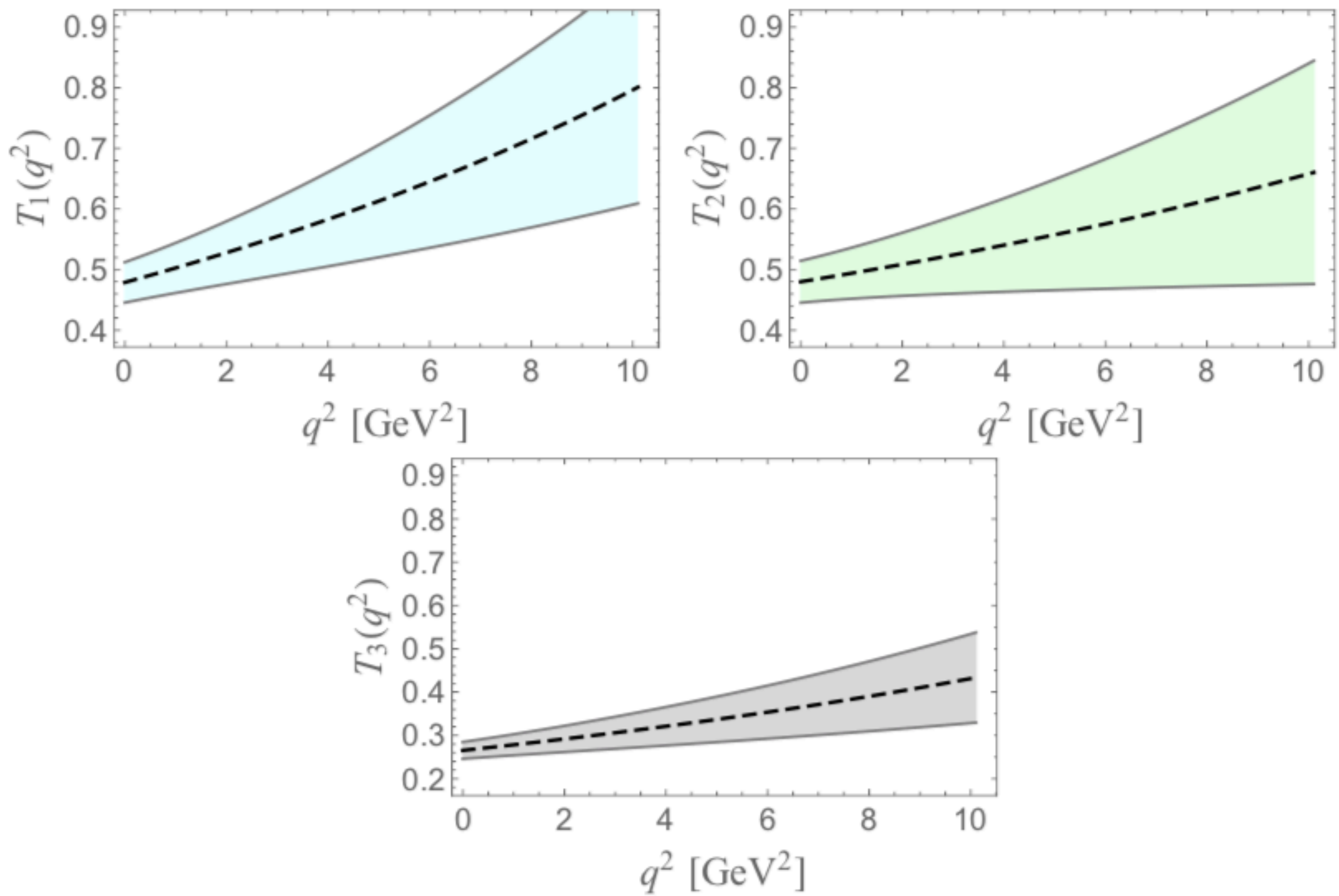
$$z(q^2) = \frac{\sqrt{(m_{B_c}+m_{J/\psi})^2 - q^2} - (m_{B_c}+m_{J/\psi})}{\sqrt{(m_{B_c}+m_{J/\psi})^2 - q^2} + (m_{B_c}+m_{J/\psi})}$$

$B_c \to J/\psi\ \ell\nu$ FF

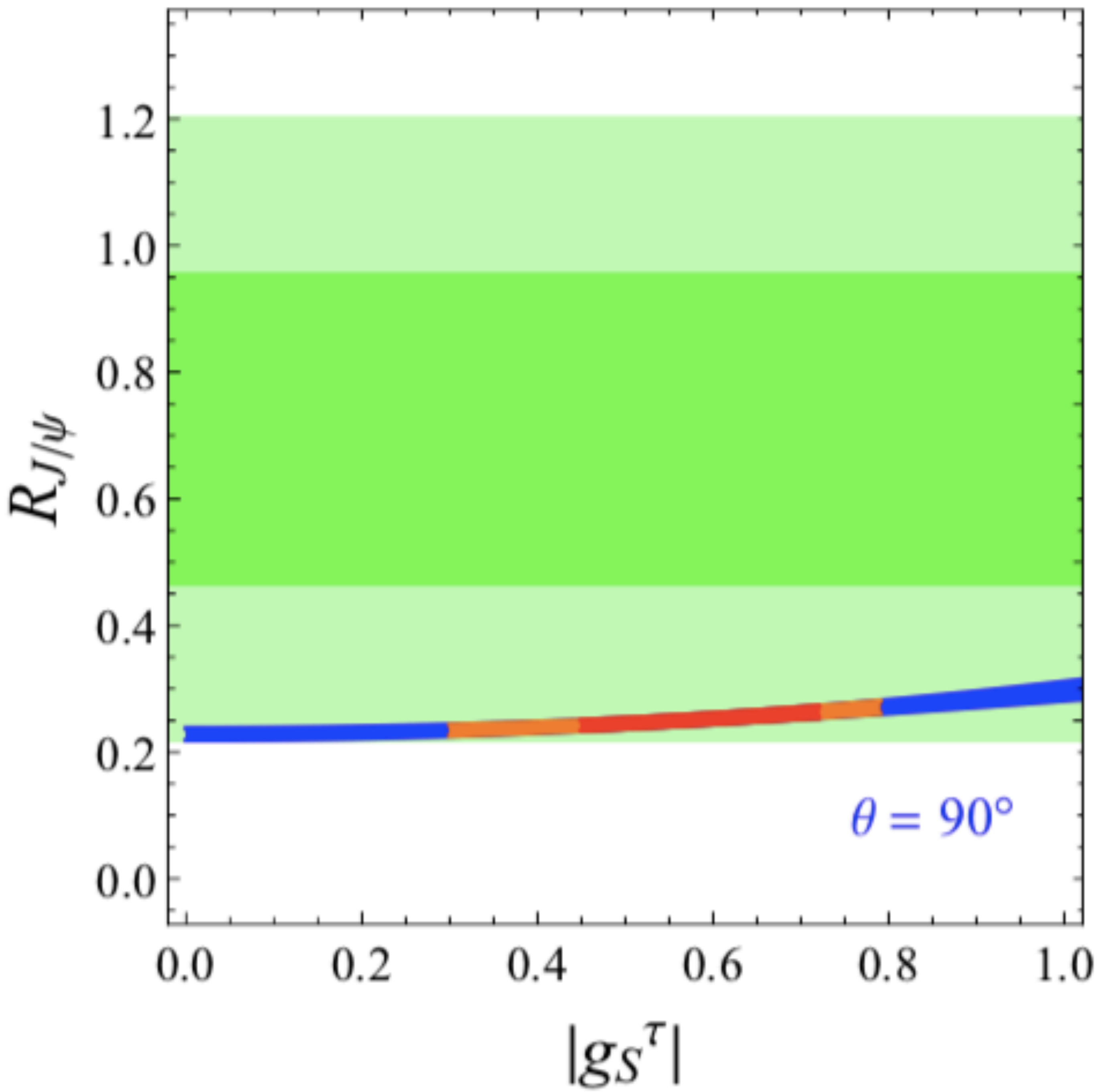


$$R(J/\psi)^{\rm SM} = 0.23 \pm 0.02 < 0.71 \pm 0.25 = R(J/\psi)^{\rm LHCb}$$

$B_c \to J/\psi\ \ell\nu$ FF



$B_c \to J/\psi\ \ell\nu$ in the BSM scenario discussed by Kosnik

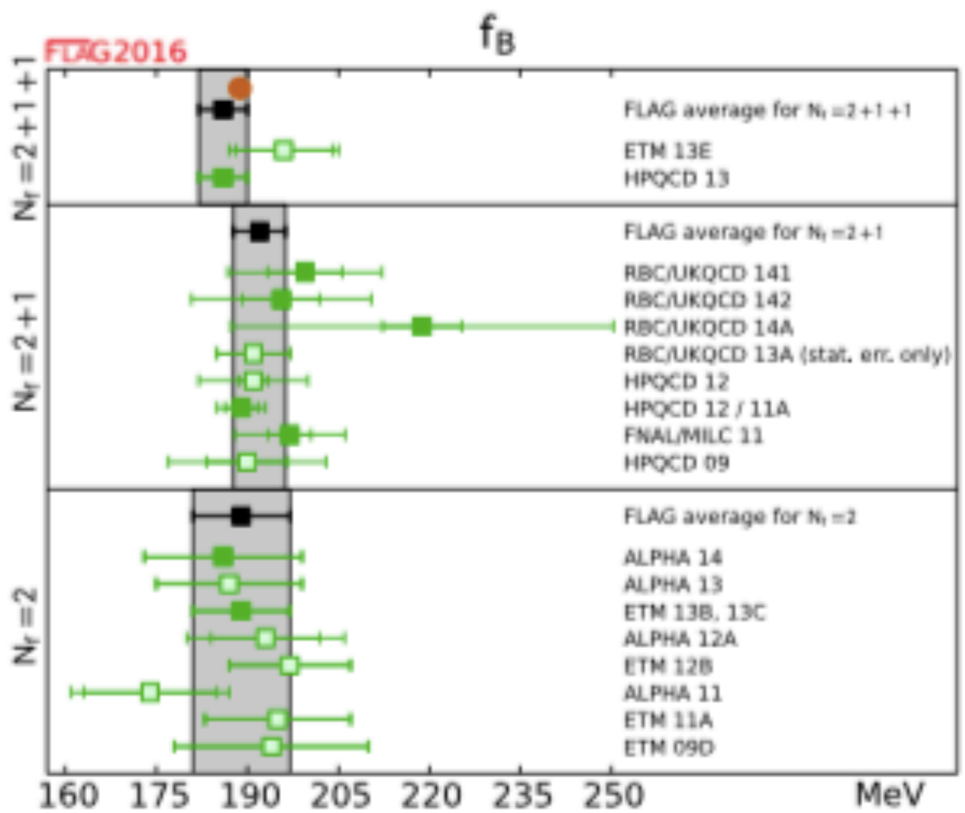


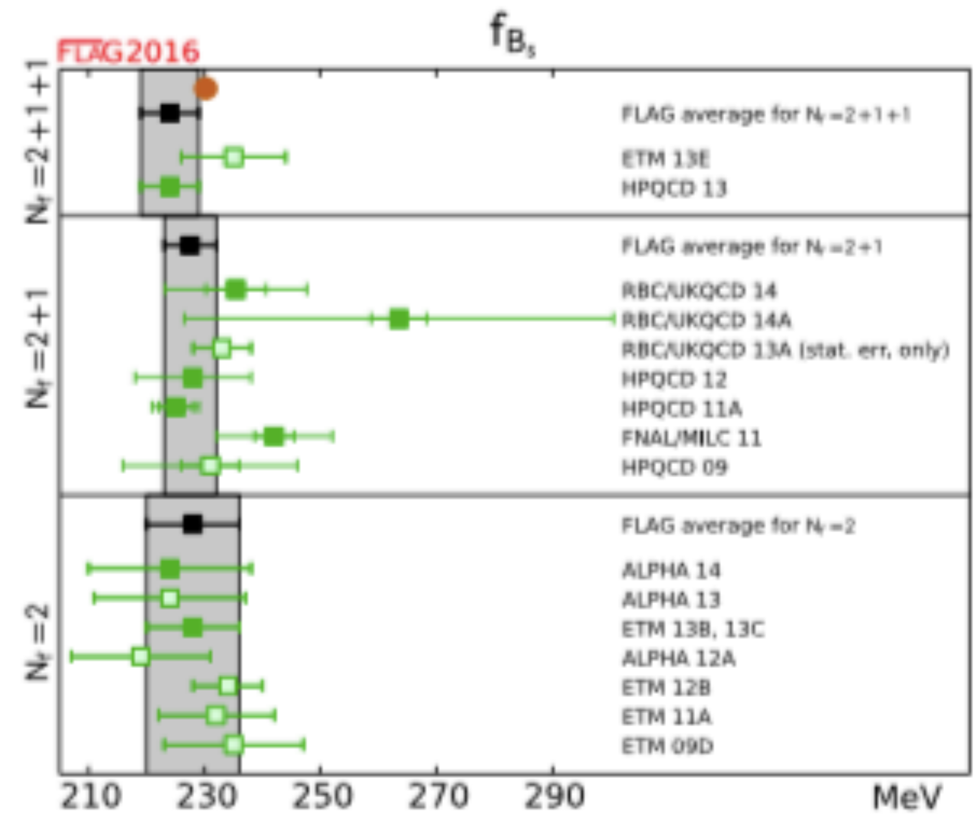
$$R(J/\psi)^{\mathrm{SM}} = 0.23 \pm 0.02 < 0.71 \pm 0.25 = R(J/\psi)^{\mathrm{LHCb}}$$

Summarizing…

- ✘ Decay constants computed on the lattices are accurate at the percent and even sub percent level

Summarizing…





N_f	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	188(7)	227(7)	1.206(23)
2+1	192.0(4.3)	228.4(3.7)	1.201(16)
2+1+1	186(4)	224(5)	1.205(7)
	196(6)	236(7)	1.207(7)
	189.4(1.4)	230.7(1.2)	1.2180(49)

[HPQCD arXiv:1711.09981; FNAL/MILC arXiv:1712.09262]

C. Pena, Moriond 2018

Summarizing…

- ✘ Decay constants computed on the lattices are accurate at the percent and even sub percent level
- ✘ Need to compute EM corrections [checks with other lattice regularizations]
- ✘ R_D in SM is under good theoretical control
- ✘ R_{D^*} in SM is not as good: missing better info on the shapes of FFs and $A_0(q^2)$
- ✘ For NP searches new FFs from HQE + model (but could be done on the lattice too)
- ✘ $R_{J/\psi}$ in SM is not reasonably controlled yet (attempt to constrain by QCDSR aided by LQCD results)
- ✘ Before declaring B-physics anomalies to be 5σ effects (and thus NP) all tiny hadronic errors should be tamed