Pionic couplings in lattice QCD

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Getting to grips with QCD
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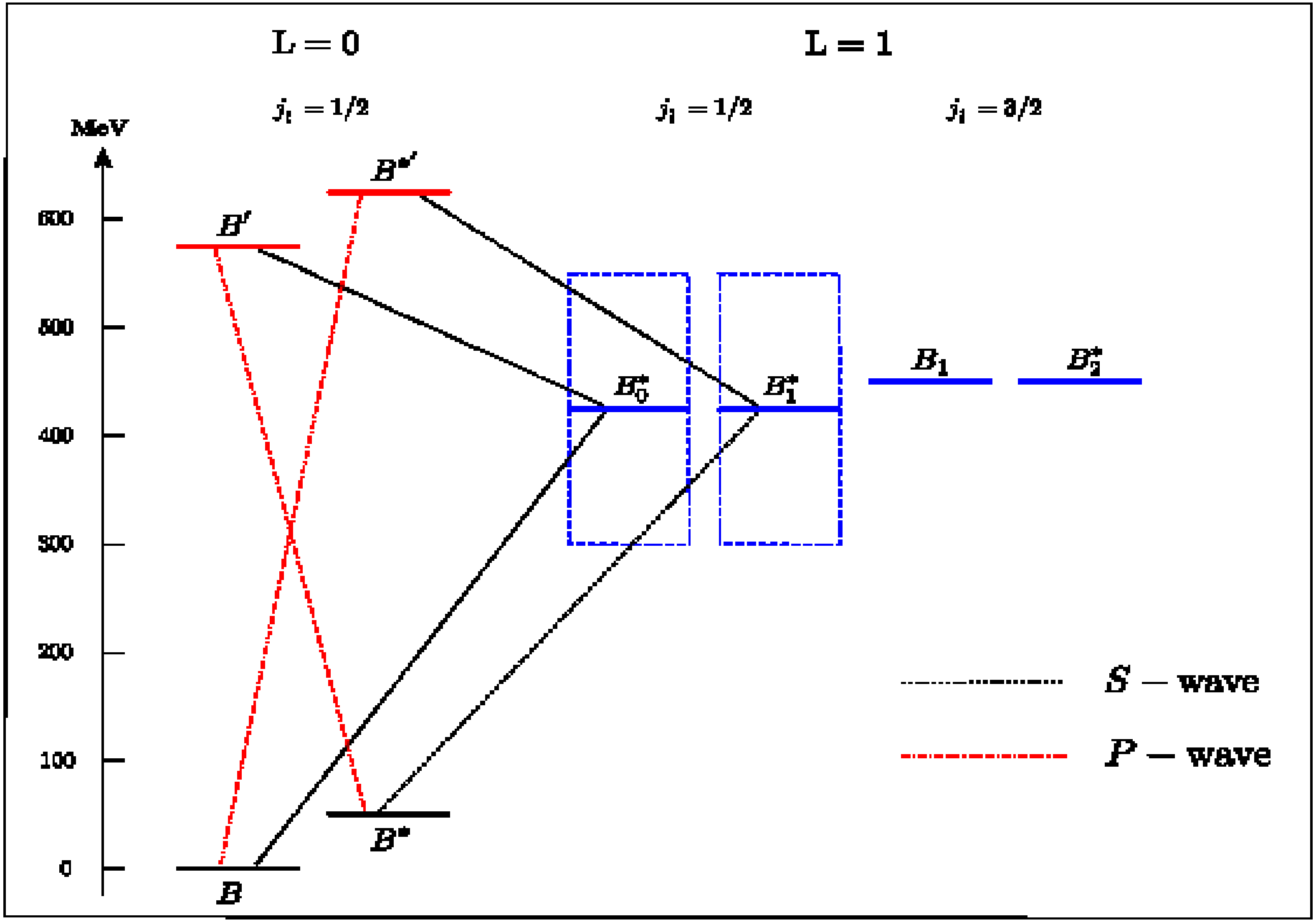
- $B^{*'} \to B\pi$ strong decay
- Pionic couplings to B_0^*
- Outlook

[B. B., J. Bulava, M. Donnellan and A. Gérardin, PRD**87**, 9, 094518 (2013)]
[B. Blossier, N. Garron and A. Gérardin, EPJC **75**, 103 (2015)]
[B. B. and A. Gérardin, PRD**94**, 7, 074504 (2016)]

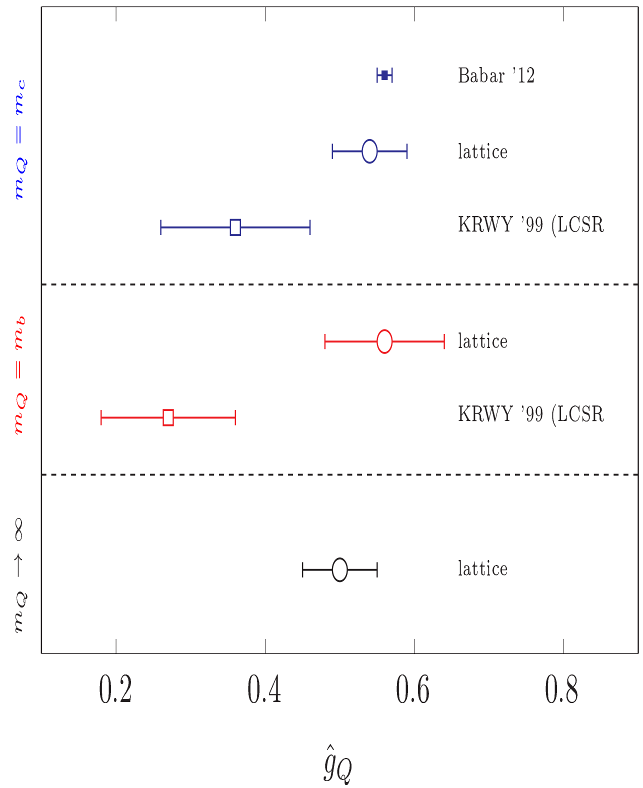
B meson spectroscopy



$B^{*'} \to B\pi$ strong decay

$D^* \to D\pi$: an ideal process to test analytical computations based on the soft pion theorem

$$\langle D(p')\pi(q)|D^*(p,\epsilon_\lambda)\rangle = g_{D^*D\pi}\, q\cdot\epsilon_\lambda, \quad g_{H^*H\pi} \equiv \frac{2\sqrt{m_H m_{H^*}}\,\hat{g}_Q}{f_\pi}$$



Claim: a negative radial excitation contribution to the hadronic side of LCSR might explain the discrepancy between $g^{\rm exp}_{D^*D\pi}$ and $g^{\rm LCSR}_{D^*D\pi}$ [D. Becirevic *et al*, '03].

Without any radial excitation:

$$g_{D^*D\pi} = \frac{f(M^2)}{f_D f_{D^*}}$$

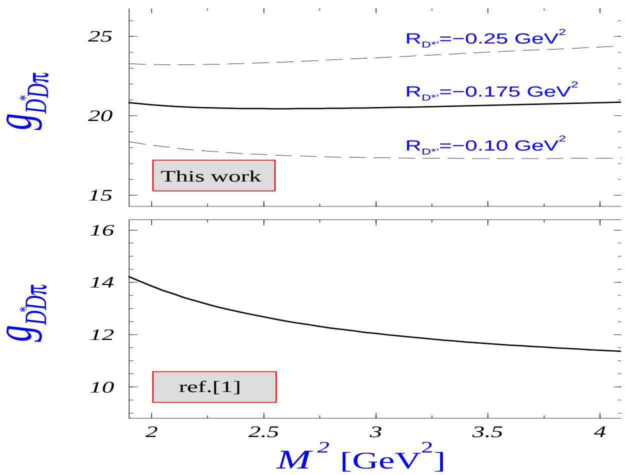
$$f(M^2) = \frac{m_c^2}{m_D^2 m_{D^*}} \; f_\pi \phi_\pi(1/2) \; M^2 \exp\left(\frac{m_D^2 + m_{D^*}^2}{2M^2}\right) \left[e^{-m_c^2/M^2} - e^{-s_0/M^2}\right] + \ldots$$

With radial excitation:

$$g_{D^*D\pi} = \frac{1}{f_D f_{D^*}} \left[f(M^2) - R_{D'} \exp\left(-\frac{m_{D'}^2 - m_D^2}{2M^2}\right) - R_{D^{*\prime}} \exp\left(-\frac{m_{D^{*\prime}}^2 - m_{D^*}^2}{2M^2}\right)\right]$$

$$R_{D'} = \left(\frac{m_{D'}}{m_D}\right)^2 f_{D'} f_{D^*} g_{D^*D'\pi} \qquad R_{D^{*\prime}} = \frac{m_{D^{*\prime}}}{m_{D^*}} f_D f_{D^{*\prime}} g_{D^{*\prime}D\pi}$$

Assuming $m_{D'} = m_{D^{*\prime}}$, $f_{D'} = f_{D^{*\prime}}$ and $g_{D^*D'\pi} = g_{D^{*\prime}D\pi} = g'$: $\frac{R_{D'}}{R_{D^{*\prime}}} = \frac{m_{D^{*\prime}} m_{D^*}}{m_D^2} \frac{f_{D^*}}{f_D}$



$$-0.25 \text{ GeV}^2 < R_{D^{*\prime}} < -0.1 \text{ GeV}^2$$
$$\Longrightarrow 17 \; < \; g_{D^*D\pi} \; < \; 25$$

Much better stability in Borel window

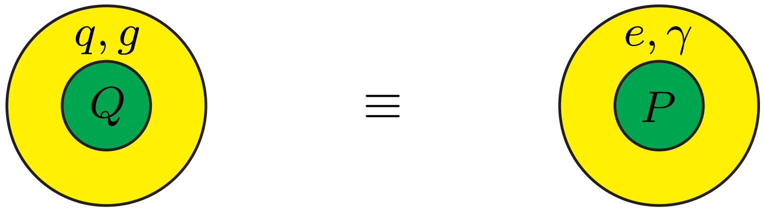
Our proposal: check on the lattice that statement in the heavy quark limit.

Heavy Quark Effective Theory

Effective theory "derived" by expanding in $\frac{\Lambda_{QCD}}{m_Q}$ the Lagrangian and currents of QCD.

$$\mathcal{L}_{\rm HQET} = \bar{h}_v (iv \cdot D) h_v + \mathcal{O}(1/m_Q) \equiv \mathcal{L}^{\rm stat}_{\rm HQET} + \mathcal{O}(\Lambda_{\rm QCD}/m_Q) \quad p_Q = m_Q\, v + k$$

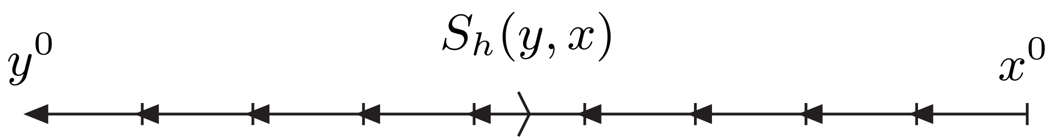
Symmetry $\mathrm{SU(2N_h)}$ for $\mathcal{L}^{\rm stat}_{\rm HQET}$: $\mathrm{flavor} \times \mathrm{spin}$



Heavy-light meson   Atom of hydrogen

$$L^{\rm stat,\, lat}_{\rm HQET}:\ S_h = a^3 \sum_x \bar{h}(x) \left[h(x) - \mathcal{U}_0^\dagger(x - \hat{0}) h(x - \hat{0}) \right]$$

Static propagator: $\mathcal{S}_h(y,x) = \delta_{\vec{x}\vec{y}} \frac{1+\gamma^0}{2} P_{\vec{x}}(y^0, x^0),\quad P_{\vec{x}}(y^0, x^0) = \prod_{x^0 - \hat{0}}^{y^0} \mathcal{U}_0^\dagger(\vec{x}, t)$



Static-light meson density distributions to extract pionic couplings

Transition amplitude under interest, with $q = p' - p$, $\mathcal{A}^\mu = \bar{d}\gamma^\mu\gamma_5 u$,
$T^{mn\,\mu} = \langle B_m(p)|\mathcal{A}^\mu|B_n^*(p',\lambda)\rangle$ and $\epsilon_\perp^\mu(p',\lambda) = \epsilon(p',\lambda)^\mu - \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu$:

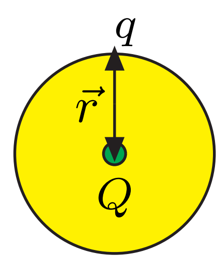
$$\begin{aligned} T^{mn\,\mu} &= 2m_{B_n^*} A_0^{mn}(q^2) \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu + (m_{B_m} + m_{B_n^*}) A_1^{mn}(q^2) \epsilon_\perp^\mu(p',\lambda) \\ &+ A_2^{mn}(q^2) \frac{\epsilon(p',\lambda)\cdot q}{m_{B_m} + m_{B_n^*}} \left[(p+p')^\mu + \frac{m_{B_m}^2 - m_{B_n^*}^2}{q^2} q^\mu \right] \end{aligned}$$

With $\langle B_m(p)|q_\mu \mathcal{A}^\mu|B_n^*(p',\lambda)\rangle = 2\,m_{B_n^*}\, A_0^{mn}(q^2)\; q\cdot\epsilon(p',\lambda)$, PCAC relation, LSZ reduction formula and $\sum_\lambda \epsilon_\mu(k,\lambda)\,\epsilon_\nu^*(k,\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$:

$$g_{H_n^* H_m \pi} = \frac{2\,m_{H_n^*} A_0^{mn}(0)}{f_\pi}\,,\; A_0^{mn}(q^2) = -\sum_\lambda \frac{\langle H_m(p)|q_\mu \mathcal{A}^\mu|H_n^*(p',\lambda)\rangle}{2m_{H_n^*}\, q_i}\, \epsilon_i^*(p',\lambda)$$

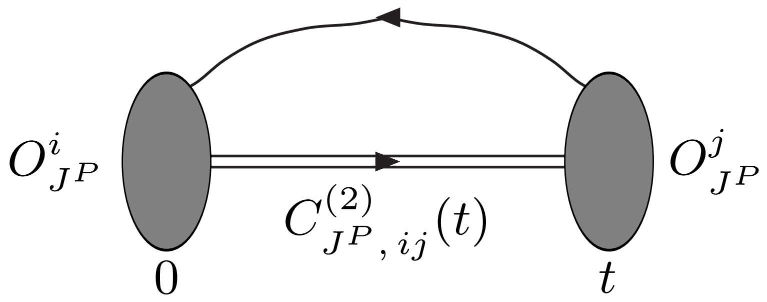
Back to the x space: $A_0^{mn}(q^2=0) = -\frac{q_0}{q_i}\int d^3r\, f_{\gamma_0\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}} + \int d^3r\, f_{\gamma_i\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}$

Axial density distributions $f_{\gamma_\mu\gamma_5}^{mn}(r)$ defined
in terms of 2-pt and 3-pt HQET correlation functions



Variational method: define an operator $O^n_{J^P}$ weakly coupled to other states than $|n\rangle$

[C. Michael, '85] [M. Lüscher and U. Wolff, '90] [B. B. *et al*, '09]



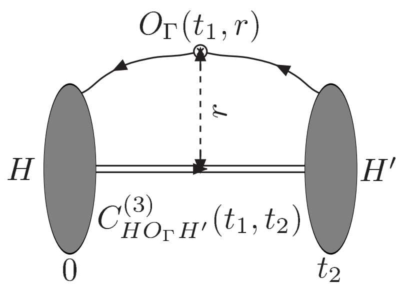
– Compute an $N \times N$ matrix of correlators

$$C^{(2)}_{P(V),\,ij}(t) = \sum_{\vec{x},\vec{y}} \langle \Omega | \mathcal{T}[O^i_{P(V)}(\vec{x},t) O^j_{P(V)}(\vec{y},0)] | \Omega \rangle$$

$$O^i_{P(V)}(\vec{x},t) = \sum_{\vec{z}} \bar{q}(\vec{x},t) [\Gamma \times \Phi(|\vec{x}-\vec{z}|)]^i_{P(V)} q(\vec{z},t)$$

– Solve the generalised eigenvalue problem

$$C^{(2)}_{P(V)}(t)\, v_{P(V),\,n}(t,t_0) = \lambda_{P(V),\,n}(t,t_0)\, C^{(2)}_{P(V)}(t_0)\, v_{P(V),\,n}(t,t_0), \;\; \lambda_n(t,t_0) \sim e^{-E_n\,(t-t_0)}$$



Two ratio methods, GEVP and sGEVP [J. Bulava *et al*, '11], built from 2-pt and 3-pt correlation functions to extract $f^{(mn)}_{\gamma_\mu\gamma_5}(r)$:

$$\mathcal{R}^{\rm GEVP}_{mn}(t,t_1;r) = f^{(mn)}_{\gamma_\mu\gamma_5}(r) + \mathcal{O}\left(e^{-\Delta_{N+1,m} t_2}, e^{-\Delta_{N+1,n} t_1}\right), \Delta_{nm} = E_n - E_m$$

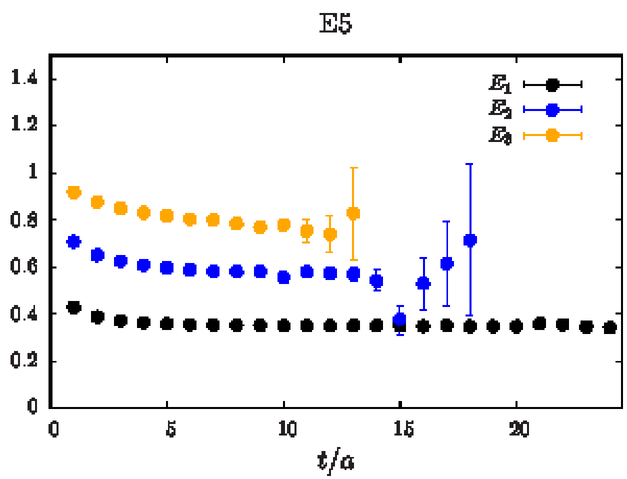
$$\mathcal{R}^{\rm sGEVP}_{mn}(t,t_0;r) = \underbrace{f^{(mn)}_{\gamma_\mu\gamma_5}(r) + \mathcal{O}\left(t\, e^{-\Delta_{N+1,n} t}\right)}_{n>m}, \;\; \underbrace{f^{(mn)}_{\gamma_\mu\gamma_5}(r) + \mathcal{O}\left(e^{-\Delta_{N+1,m} t}\right)}_{n<m}$$

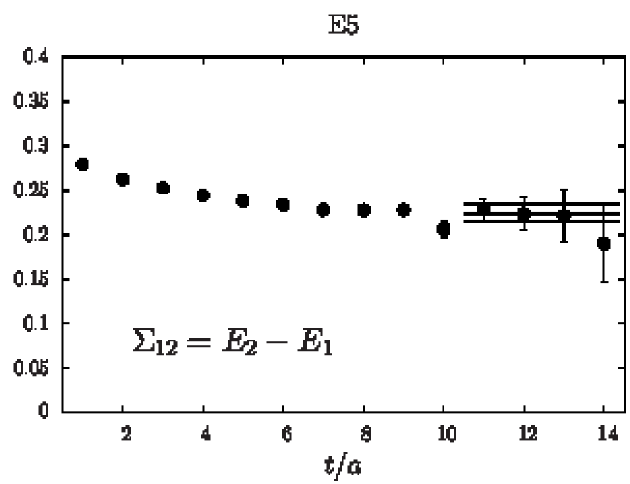
Results

Lattice set-up: $\mathcal{O}(a)$ improved Wilson-Clover (light quark), HYP2 (static quark)

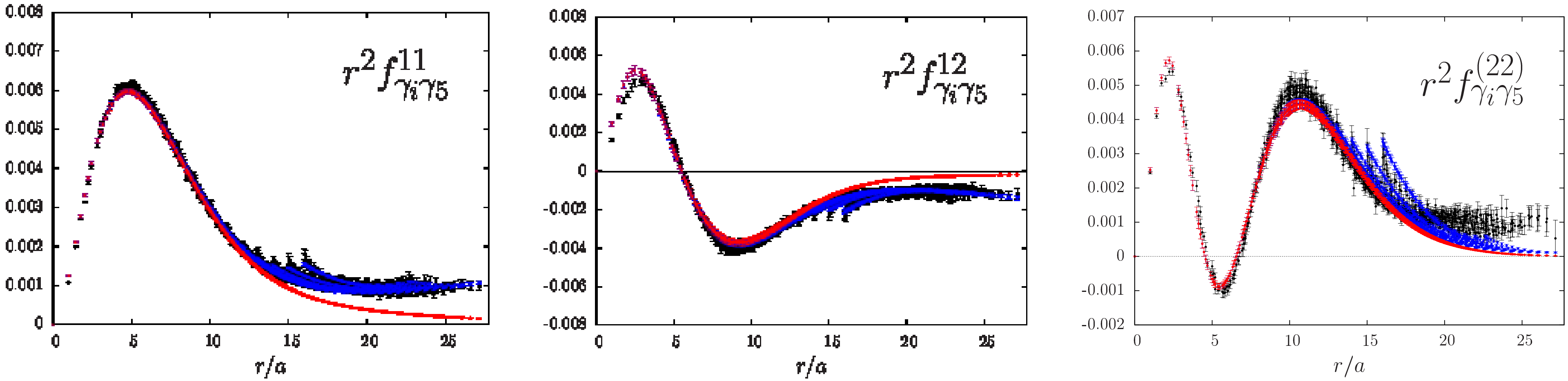
lattice	β	$L^3 \times T$	a[fm]	m_π[MeV]	Lm_π
A5	5.2	$32^3 \times 64$	0.075	330	4
B6		$48^3 \times 96$		280	5.2
D5	5.3	$24^3 \times 48$	0.065	450	3.6
E5		$32^3 \times 64$		440	4.7
F6		$48^3 \times 96$		310	5
N6	5.5	$48^3 \times 96$	0.048	340	4
Q1	6.2885	$24^3 \times 48$	0.06	-	-
Q2	6.2885	$32^3 \times 64$	0.06	-	-

Basis of interpolating fields (4×4 matrix of correlators) large enough to well isolate the ground state and the first excited state.



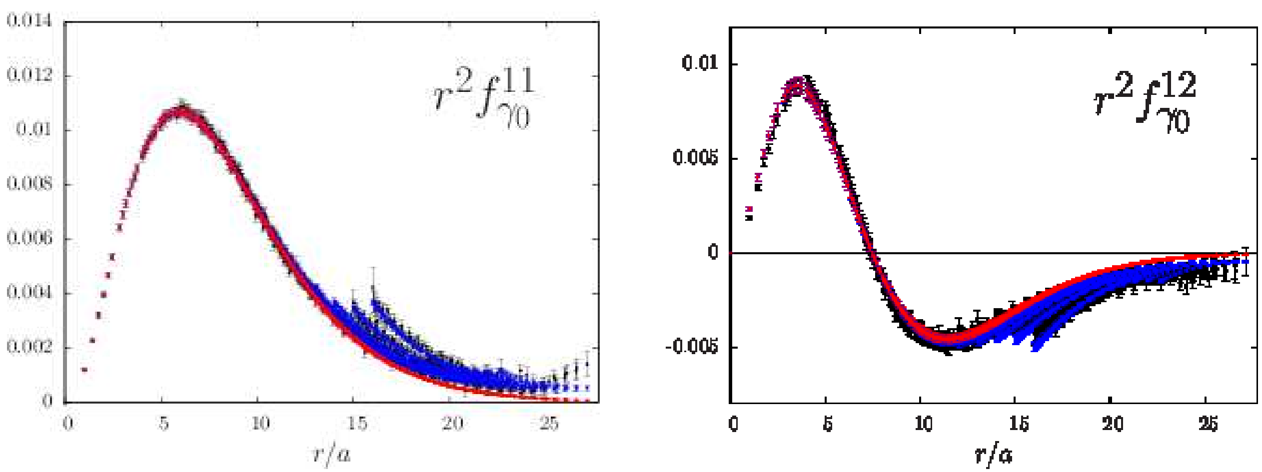


Spatial component of the axial density distributions: systematics from excited states, finite-volume effects and cut-off effects taken into account



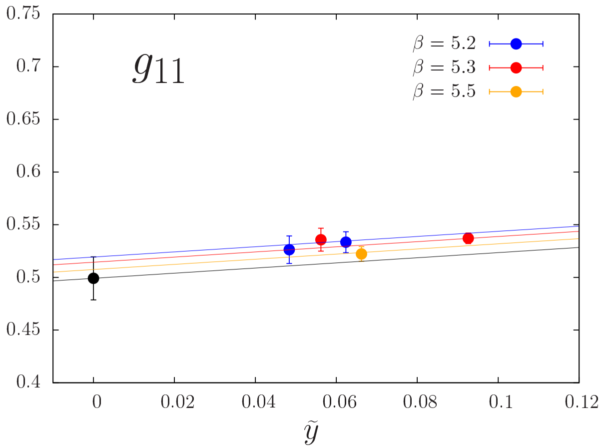
$f^{11}_{\gamma_i\gamma_5}(r)$: positive everywhere; $f^{12}_{\gamma_i\gamma_5}(r)$: there is a node; $f^{22}_{\gamma_i\gamma_5}(r)$: almost positive, negative part interpreted by relativistic effects

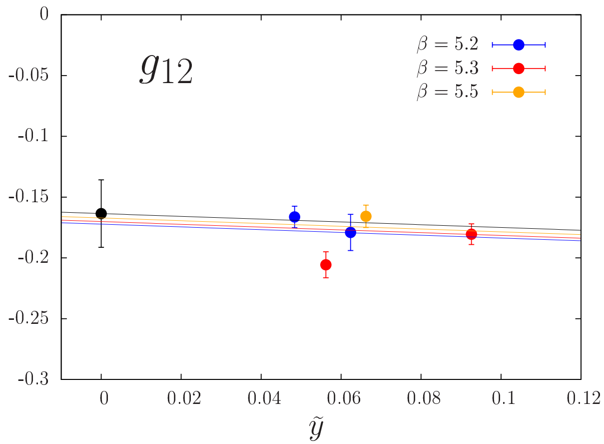
Techniques employed also for the charge density distribution $f^{mn}_{\gamma_0}(r)$

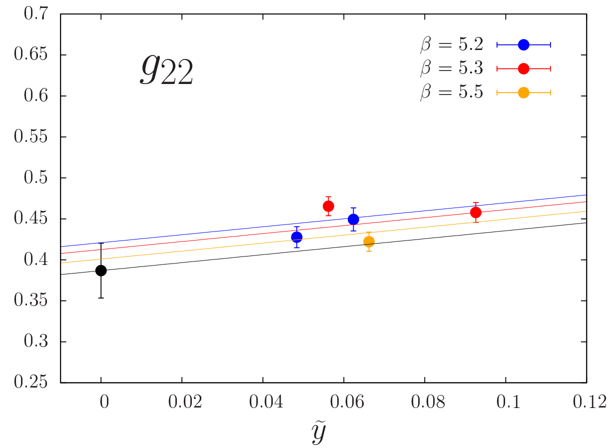


Including Z_V, $\int dr\, r^2\, f^{11}_{\gamma_0}(r)$ compatible with 1. $\int dr\, r^2 f^{12}_{\gamma_0}(r)$ compatible with 0.

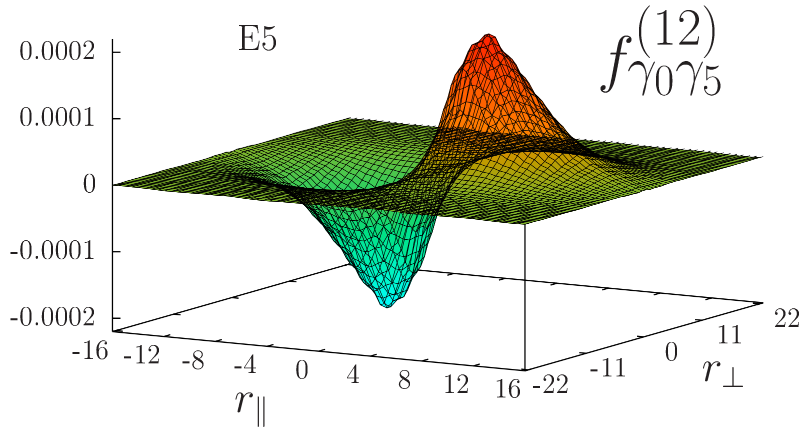
Summation over r of $f^{mn}_{\gamma_i\gamma_5}(r)$ to get g_{mn}. After renormalisation, a continuum and chiral extrapolation is possible: $\overline{g}_{nm}(a, m_\pi) = \overline{g}_{nm} + C_1\, a^2 + C_2\, m_\pi^2 / \left(8\pi f_\pi^2\right)$







Time component of the axial density distribution: systematics more tricky to estimate

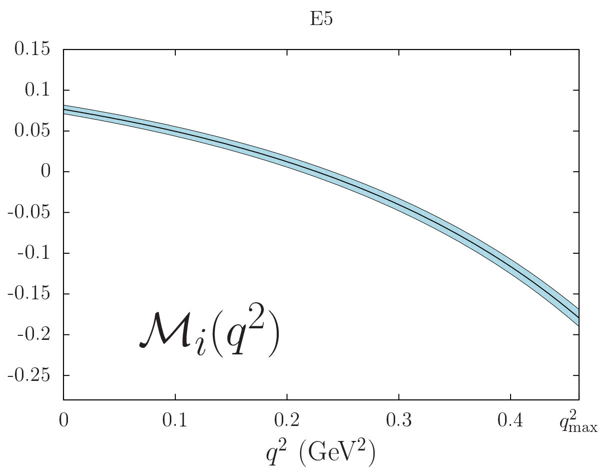


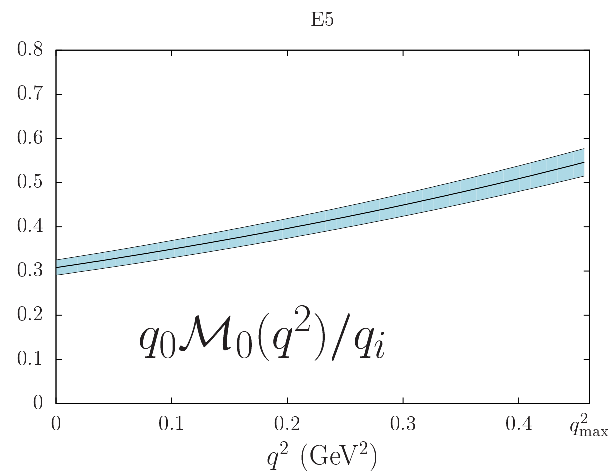
Distribution odd in $r_\parallel$, along the vector meson polarisation

Matrix elements obtained at q after a Fourier transform of the distributions to get $g_{B^{*\prime} B\pi}$

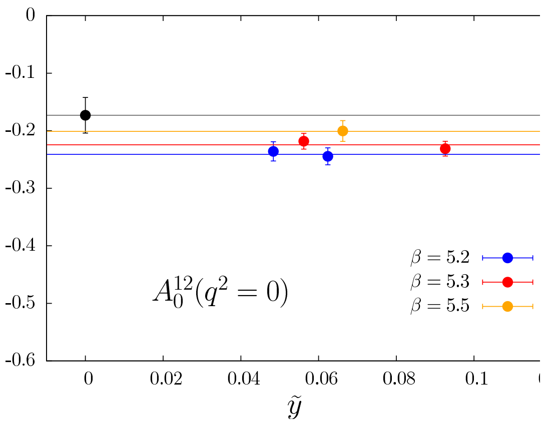
$$\mathcal{M}_i(q^2_{\rm max} - \vec{q}^{\,2}) = 4\pi \int_0^\infty \mathrm{d}r \; r^2 \, \frac{\sin(|\vec{q}|r)}{|\vec{q}|r} f^{(12)}_{\gamma_i\gamma_5}(\vec{r})$$

$$\frac{q_0}{q_i}\mathcal{M}_0(q^2_{\rm max} - \vec{q}^{\,2}) = -q_0 4i\pi \int_0^\infty \mathrm{d}r_\parallel \int_0^\infty \mathrm{d}r_\perp \; r_\perp \, f^{(12)}_{\gamma_0\gamma_5}(r_\parallel, r_\perp) \, \frac{\sin(|\vec{q}|\, r_\parallel)}{|\vec{q}|}$$





Lattice results and comparison with quark models (*à la* Bakamjian-Thomas/Godfrey-Isgur, Dirac) [A. Le Yaouanc, orivate communication]



$$A_0^{12}(q^2=0) = -\frac{q_0}{q_i}\mathcal{M}_0(q^2=0) + \mathcal{M}_i(q^2=0)$$

Extrapolation of $A_0^{12}(q^2=0)$ to the physical point:

$$A_0^{12}(0, m_\pi^2) = D_0 + D_1 a^2 + D_2 m_\pi^2/(8\pi f_\pi^2)$$

Normally, $\mathcal{O}(a)$ effects at $q^2 \neq q^2_{\rm max}$, not visible in our data.

$$A_0^{12}(0) = -0.173(31)_{\rm stat}(16)_{\rm syst}, \quad g_{B^{*\prime}B\pi} = -15.9(2.8)_{\rm stat}(1.4)_{\rm syst}$$

Quenched result ($m_q = m_s$): $A_0^{12}(0) = -0.143(14)$

	Latt		BT		D	
q^2	$q^2_{\rm max}$	0	$q^2_{\rm max}$	0	$q^2_{\rm max}$	0
$q_0\mathcal{M}_0(q^2)/q_i$	$0.402(54)(27)$	$0.237(27)(28)$	0.252	0.173	0.219	0.164
$\mathcal{M}_i(q^2)$	$-0.172(16)(6)$	$0.064(9)(13)$	-0.103	0.05	-0.223	-0.056

Lattice: $q_0 = 0.701(65)$ GeV
Bakamjian-Thomas with Godfrey-Isgur potential: $q_0 = 0.538$ GeV
Dirac: $q_0 = 0.576$ GeV
global sign of hadronic matrix elements fixed with conventions $f_B > 0$ and $f_{B^{*\prime}} > 0$

Qualitative agreement between lattice and quark models: $q_0\mathcal{M}_0/q_i$ dominates in $A_0^{12}(q^2)$ and explains why $A_0^{12}(q^2=0) < 0$.

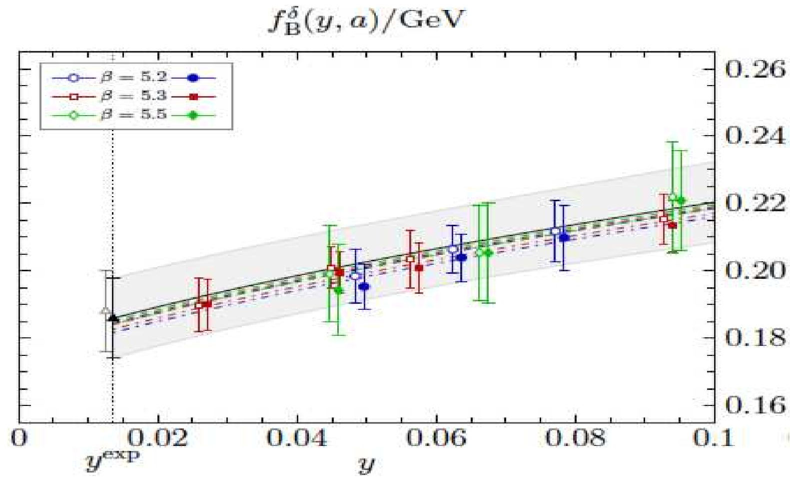
Pion couplings to B_0^*

Heavy Meson Chiral Perturbation Theory is often used to extrapolate lattice data in the heavy-light sector.

Example on f_B:

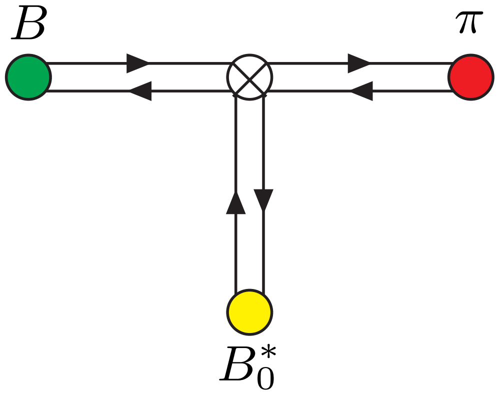
$$f_B\sqrt{\frac{m_B}{2}}(y,a,\delta) = A\left[1-\frac{3}{4}\frac{1+3\hat{g}^2}{2}(y\ln y - y^{\rm exp}\ln y^{\rm exp})\right] + C(y-y^{\rm exp}) + D^\delta a^2$$

[F. Bernardoni *et al*, ’14]



$$\mathcal{L}_{\mathrm{HM}\chi\mathrm{PT}} = \frac{f_\pi^2}{8}\mathrm{Tr}(\partial^\mu\Sigma\partial_\mu\Sigma^\dagger) + i\mathrm{Tr}(Hv\cdot\mathcal{D}\bar{H}) + i\mathrm{Tr}(Sv\cdot\mathcal{D}\bar{S})$$
$$+\ i\hat{g}\mathrm{Tr}(H\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{H}) + i\tilde{g}\mathrm{Tr}(S\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{S}) + ih\mathrm{Tr}(S\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{H})$$

$H : j^P = \frac{1}{2}^-$ heavy-light meson doublet $\quad S : j^P = \frac{1}{2}^+$ heavy-light meson doublet

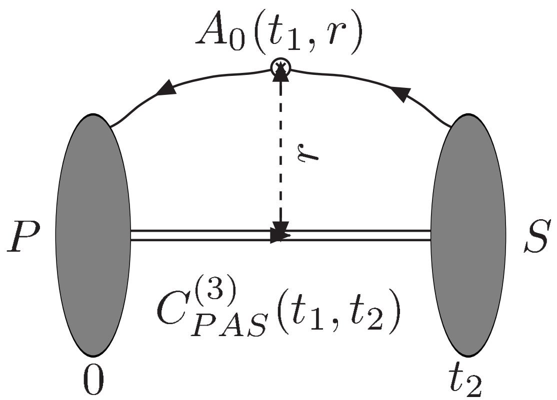


$$\Gamma(B_0^{*0} \to B^+\pi^-) = \frac{1}{8\pi} g^2_{B_0^* B\pi} \frac{|\vec{q}_\pi|}{m^2_{B_0^*}}$$

$$|\vec{q}_\pi| = \frac{\sqrt{[m^2_{B_0^*} - (m_B + m_\pi)^2][m^2_{B_0^*} - (m_B - m_\pi)^2]}}{2m_{B_0^*}}$$

HMχ PT: $\Gamma(B_0^* \to B^+\pi^-) = \frac{h^2}{8\pi f_\pi^2} \frac{m_B}{m^3_{B_0^*}} \left(m^2_{B_0^*} - m^2_B\right)^2 |\vec{q}_\pi|$

$$g_{B_0^* B\pi} = \sqrt{\frac{m_B}{m_{B_0^*}}} \left(m^2_{B_0^*} - m^2_B\right) \frac{h}{f_\pi}$$



Extract h from the density distribution [D. Becirevic *et al*, ’12]

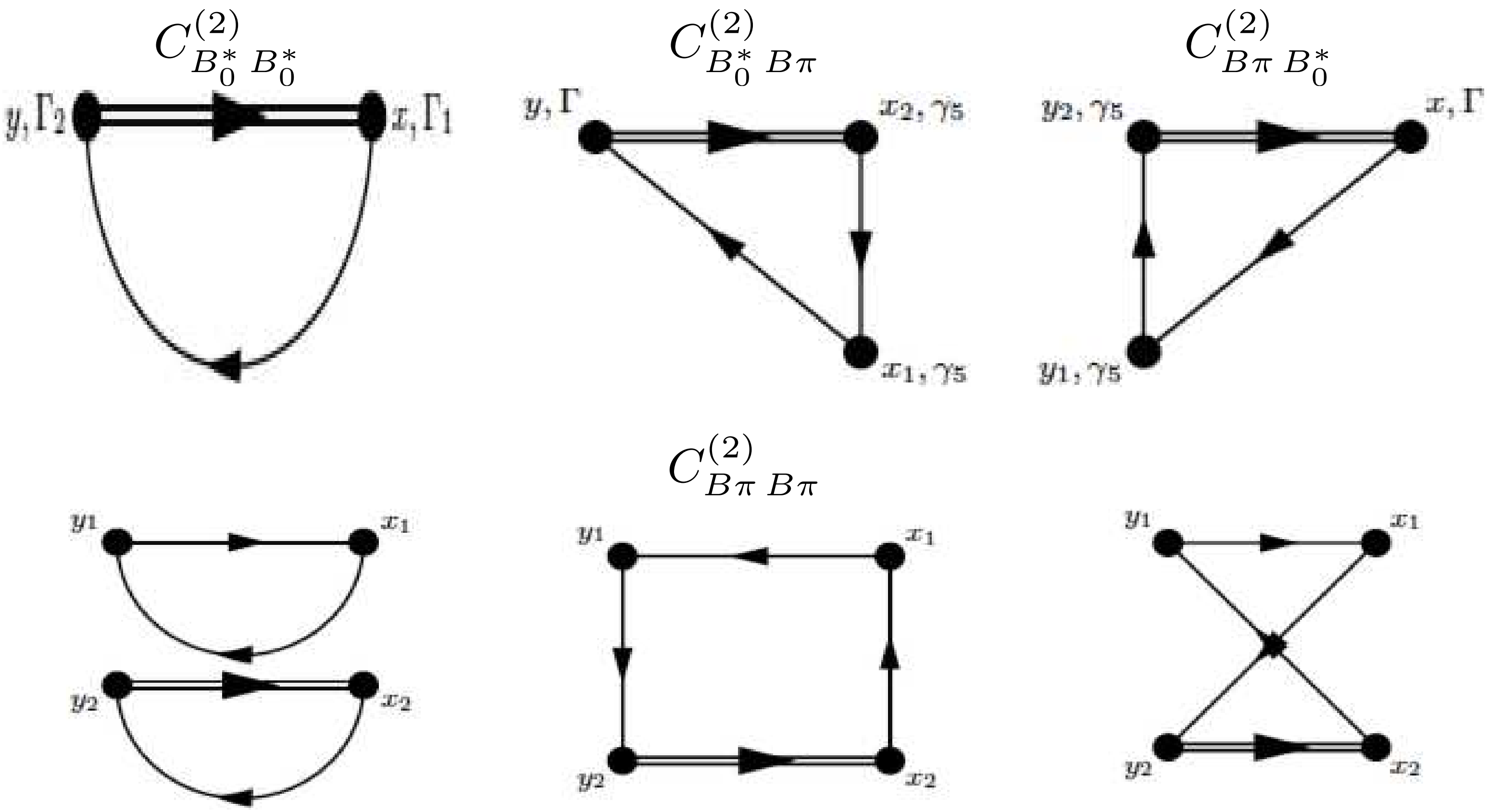
$$A_+(\delta^2 - q_\pi^2) = 4\pi \int_0^\infty r^2 dr \frac{\sin(q_\pi r)}{q_\pi r} f_{PAS}(r)$$

$$\delta = m_{B_0^*} - m_B \quad f_{PAS}(r) = \langle B|[\bar{q}\gamma_0\gamma_5 q](r)|B_0^*\rangle$$

$$\vec{q}_\pi = (0, 0, \delta)$$

We have followed another strategy, valid near thresholds [C. McNeile *et al*, ’01; ’03; ’04]

Correlation functions of B_0^* and $B\pi$



We consider the ratio $C^{(2)}_{B_0^*\,B\pi}(t)/\sqrt{C^{(2)}_{B_0^*\,B_0^*}(t)C^{(2)}_{B\pi\,B\pi}(t)}$

$$\langle \pi^+(q_\pi)B^-(p)|B_0^{*0}(p')\rangle = g_{B_0^*B\pi} = \sqrt{m_B m_{B_0^*}}\frac{m^2_{B_0^*} - m^2_B}{m^2_{B_0^*}}\frac{h}{f_\pi}$$

Fermi golden rule: $\Gamma(B_0^* \to B^-\pi^+) = 2\pi\,|\langle \pi^+(q_\pi)B^-(p)|B_0^{*0}(p')\rangle|^2\,\rho$

$\rho(E_\pi) = \frac{L^3}{(2\pi)^3}\,4\pi\vec{q}_\pi^{\,2}\,\frac{dq_\pi}{dE_\pi} = \frac{L^3}{2\pi^2}|\vec{q}_\pi|E_\pi$

$\frac{\Gamma(B_0^*\to B^-\pi^+)}{q_\pi} = \frac{1}{\pi}\left(\frac{L}{a}\right)^3(aE_\pi)\,|a\langle \pi^+(q_\pi)B^-(p)|B_0^{*0}(p')\rangle|^2$

$$C^{(2)}_{B_0^* \, B\pi}(t) = \sum_{t_1} \langle 0|\mathcal{O}^{B_0^*}|B_0^*\rangle x \langle B\pi|\mathcal{O}^{B\pi}|0\rangle e^{-m_{B_0^*} t_1} e^{-E_{B\pi}(t-t_1)} + \mathcal{O}(x^3) + \text{excited states}$$

Assumption: small overlaps $\langle 0|\mathcal{O}^{B_0^*}|B\pi\rangle$ and $\langle 0|\mathcal{O}^{B\pi}|B_0^*\rangle$

$x = |a\langle \pi^+(q_\pi)B^-(p)|B_0^{*0}(p')\rangle|$ $\quad \langle n|m\rangle = \delta_{mn}$

Close to the threshold $m_{B_0^*} \approx E_{B\pi}$:

$$C^{(2)}_{B_0^* \, B\pi}(t) = \langle 0|\mathcal{O}^{B_0^*}|B_0^*\rangle x \langle B\pi|\mathcal{O}^{B\pi}|0\rangle \times t e^{-m_{B_0^*} t} + \mathcal{O}(x^3) + \text{excited states}$$

$$R(t) = \frac{C^{(2)}_{B_0^* \, B\pi}(t)}{\left(C^{(2)}_{B_0^* \, B_0^*}(t) C^{(2)}_{B\pi \, B\pi}(t)\right)^{1/2}} \approx A + xt$$

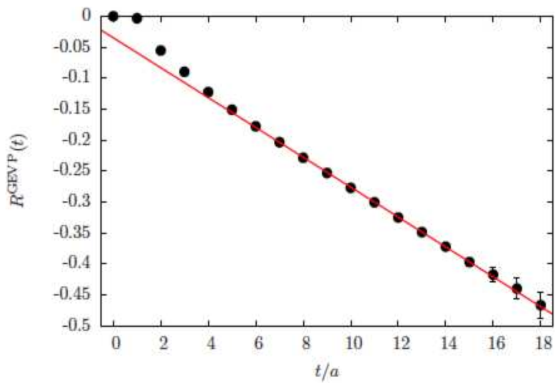
Further away from the threshold, $R(t)$ goes in $t \longrightarrow \frac{2}{\Delta}\sinh\left(\frac{\Delta}{2}t\right) = t + \frac{\Delta^2 t^3}{24} + \mathcal{O}(\Delta^4)$,
$\Delta = m_{B_0^*} - E_{B\pi}$

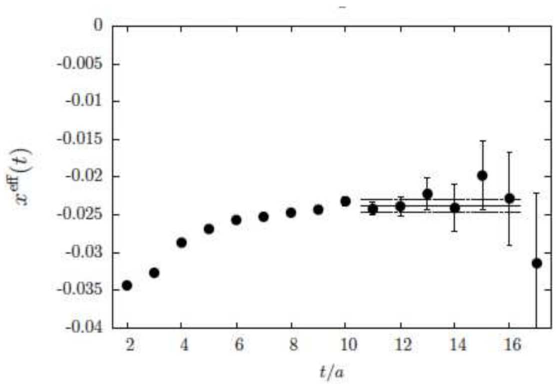
Excited states are suppressed by solving a GEVP.

CLS based

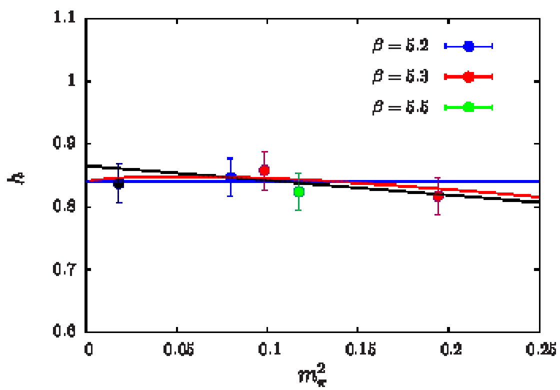
β	a[fm]	L/a	m_π[MeV]
5.2	0.075	48	280
5.3	0.065	32	440
		48	310
5.5	0.048	48	340

[$a = 0.065$ fm, $m_\pi = 440$ MeV]



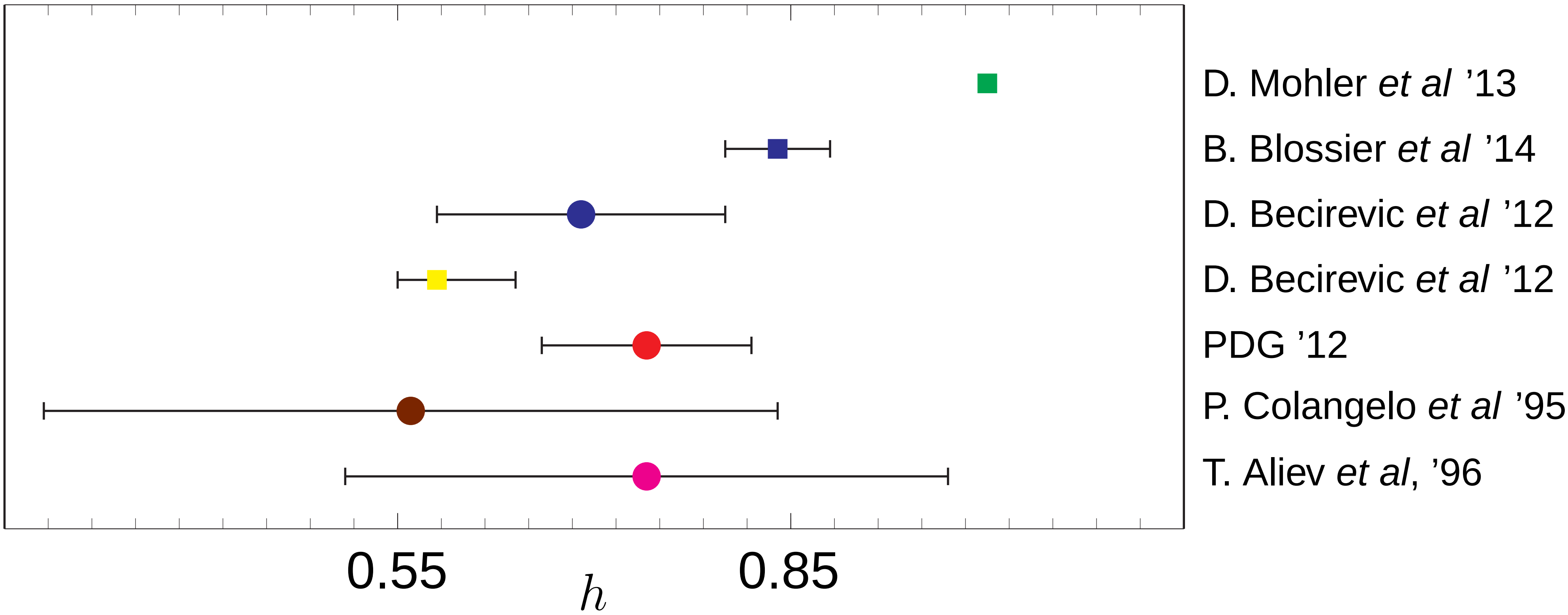


Several chiral extrapolations to get h (Cste, LO, NLO(H), NLO(H, S))



$$h = 0.84(3)(2)$$

Collection of results



Different ways to get h: $\Gamma(D_0^*)$, phase shift in $D\pi$ scattering state (small $1/m_c$ corrections), QCD sum rules, density distribution, transition at the threshold $m_{B_0^*} \approx E_{B\pi}$

Adler-Weisberger sum rule: $\sum_\delta |X_{\delta\to B}|^2 = 1 \quad \Gamma(\mathcal{I} \to \mathcal{F}\pi) = \frac{1}{2\pi f_\pi^2} \frac{|\vec{q}|^3}{2j_{\mathcal{I}}+1} |X_{\mathcal{I}\to\mathcal{F}}|^2$

With $\hat{g} \sim 0.5$, it is almost saturated by B^* and B_0^*.

h is pretty large, some care is required in the application of HMχPT for pion masses close to $m_{B_0^*} - m_B \sim 400$ MeV: B meson orbital excitations degrees of freedom can not be neglected in chiral loops.

Outlook

- Extract density distributions of the B meson is beneficial to get the form factors at $q^2 = 0$ associated to pionic couplings.

- The pionic coupling $g_{B_0^* B\pi}$ can be extracted from 2-pt correlation functions in a region of the near $B\pi$ threshold. Magnitude of h makes tricky chiral extrapolations in the HMχPT framework at $m_\pi \gtrsim 400$ MeV.

- The “near threshold" techniques might be beneficial to explore the decay $\psi(3770) \to D\bar{D}$