Light-cone distribution amplitude(s) of the B-meson

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“Light-Cone Distribution Amplitudes for Heavy-Quark Hadrons”

G. Bell, TF, Y.-M. Wang, M. Yip, JHEP 11 (2013) 191.

“B-Meson Light-Cone Distribution Amplitude:

Perturbative Constraints and Asymptotic Behaviour in Dual Space”

TF, B. O. Lange, Y.-M. Wang, Phys. Rev. D89 (2014) 114001.

“Systematic Parameterisation of the B-meson Light-Cone Distribution Amplitudes”’

TF, D. van Dyk et al., work in progress.

“Getting to Grips with QCD” (Summer Edition) – Primošten, 21.9.2018

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quark flavour physics and effective field theories

Motivation

B-Meson Light-Cone Distribution Amplitudes:

- Hadronic Input Functions for exclusive (energetic) B decays:
  - QCD Factorization Theorems [Beneke/Buchalla/Neubert/Sachrajda,...]
  - QCD/SCET Sum Rules [Khodjamirian/Mannel/Offen, DeFazio/TF/Hurth, ...]

- Needs: (logarithmic) Moments of B-Meson LCDAs (see below)

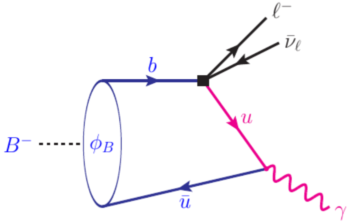
  $\leftrightarrow$ essential to control theoretical uncertainties

- Short-distance QCD Corrections / Resummation of Large Logs:
  - RG-Evolution from light-cone operators in HQET

    [Lange/Neubert, Descotes-Genon/Knodlseder/Offen, Kawamura et al., ...]
  - Non-trivial relations to HQET parameters

    [Lee/Neubert, Braun/Ivanov/Korchemsky]

(analogue of π-γ form factor)



$p^\mu = E_\gamma n^\mu \qquad (n^2 = 0)$

For large photon energy, $E_\gamma \sim m_b/2$:

- Described by a single $B \to \gamma$ transition form factor.
- Sensitive to light-cone projection $\omega = n \cdot k$ of spectator momentum in B-meson.

$$F^{B\to\gamma}(E_\gamma) \simeq [\text{kinematic factor}] \times \int \frac{d\omega}{\omega}\, \phi_B(\omega)$$

QCD Corrections:

- $+$ Radiative corrections [perturbative, factorizable]
- $+$ Power corrections in $1/m_b$, $1/E_\gamma$ expansion
  [partly non-factorizable (soft overlap – "Feynman mechanism")]

Beware:

Non-factorizable long-distance contributions still represent the phenomenlogical bottle-neck for many decay modes.

(i.e. requires more or less sophisticated hadronic modelling)

- Charmless non-leptonic $B \to \pi\pi, \pi K, \ldots$
- Rare radiative $B \to K^*\gamma, K^*\ell^+\ell^-, \ldots$
- $\ldots$

Recent progress in $B \to \gamma\ell\nu$

[Beneke, Rohrwild], [Braun/Khodjamirian], [Wang et al.], [Beneke/Braun]

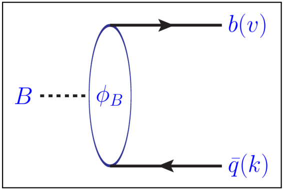
with new data (Belle-II) $\longrightarrow$ improved understanding of B-meson LCDAs

Outline

1. RGE for the LCDA $\phi_B^+(\omega, \mu)$ of the B-Meson
2. Factorization, Logarithmic Moments and $B \to \gamma\ell\nu$
3. OPE, Parton Model and HQET Parameters

Summary

1. RGE for the LCDA $\phi_B^+(\omega,\mu)$ of the B-Meson



Definition:

Fourier-Transform of Light-Cone Matrix Element in HQET $(n^2 = 0)$

$$m_B f_B^{(\mathrm{HQET})}\ \phi_B^+(\omega) = \int \frac{d\tau}{2\pi}\, e^{i\omega\tau}\, \langle 0|\, \bar{q}(\tau n)\ [\tau n, 0]\, \not{n}\gamma_5\ h_v^{(b)}(0)\, |\bar{B}(m_B v)\rangle$$

[there is another Dirac structure, leading to another LCDA, denoted as $\phi_B^-(\omega)$ (see backup)]

Comparison with Pion LCDA

Pion LCDA in full QCD:

$$(n \cdot p)\, f_\pi\, \phi_\pi(u) = \int \frac{d\tau}{2\pi}\, e^{i\bar{u}\,(n\cdot p)\,\tau}\, \langle 0|\, \bar{q}(\tau n)\, [\tau n, 0]\, \not{n}\gamma_5\, q(0)\, |\pi(p)\rangle$$

- Pion decay constant f_π (RG invariant)
- Momentum fractions $u, \bar{u} \in [0, 1]$ constrained to physical values.
- Pion LCDA normalized: $\int_0^1 du\, \phi_\pi(u) \equiv 1$

Comparison with Pion LCDA

B-Meson LCDA in HQET:

$$m_B \, f_B^{\rm (HQET)} \phi_B^+(\omega) = \int \frac{d\tau}{2\pi} \, e^{i\omega\tau} \, \langle 0 | \, \bar{q}(\tau n) \, [\tau n, 0] \, \not{n} \gamma_5 \, h_v^{(b)}(0) \, | \bar{B}(m_B v) \rangle$$

- Decay constant $f_B^{\rm HQET}(\mu) = C(\mu) \, f_B$ depends on RG scale
- Light-quark momentum $\omega \in [0, \infty)$ ($m_b \to \infty$ already performed!)
- B-meson LCDA *not* normalized: $\int_0^\infty d\omega \, \phi_B^+(\omega) \to \infty$ (see below)

Lange-Neubert Kernel

(analogue of Efremov-Radyushkin/Brodsky-Lepage)

- RGE for $\phi_B^+(\omega,\mu)$ as Integro-Differential Equation

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}\,\ln\frac{\mu}{\omega}+\gamma_+\right]\phi_B^+(\omega,\mu) - \omega\int_0^\infty d\eta\,\Gamma(\omega,\eta)\,\phi_B^+(\eta,\mu)$$

(Relatively) complicated solution in terms of convolution integrals with hypergeometric functions [Lange/Neubert]

"Can we find an integral transformation that diagonalizes the Lange-Neubert (LN) evolution kernel?"

Eigenfunction of LN Kernel

(analogue of Gegenbauer polynomials for pion LCDA)

. . . staring (sufficiently long) at the derivation from Lange/Neubert . . .

→ Identify Continuous Set of Eigenfunctions for (1-loop) LN-Kernel:

$$f_{\omega'}(\omega) \equiv \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \qquad \text{with Eigenvalues:} \quad \gamma_{\omega'} = -\left(\Gamma_{\rm cusp}\, \ln\frac{\mu}{\hat\omega'} + \gamma_+\right)$$

$J_1(z)$: Bessel function, and $\hat\omega' \equiv \omega'\, e^{-2\gamma_E}$

More formally:

Eigenfunctions can be understood as momentum-space representation of the eigenvectors of a generator of collinear conformal transformations:

$$S_+ = \tau^2 \partial_\tau + 2j\,\tau \qquad \textit{(conformal spin of light quark: } j = 1\textit{)}$$

[Braun/Manashov 1402.5822]

Dual representation of B-meson LCDA

(analogue of Gegenbauer expansion)

→ LCDA from convolution with Dual (spectral) Function

$$\phi_B^+(\omega,\mu) = \int_0^\infty \frac{d\omega'}{\omega'}\, f_{\omega'}(\omega)\, \rho_B^+(\omega',\mu) \quad \Leftrightarrow \quad \rho_B^+(\omega',\mu) = \int_0^\infty \frac{d\omega}{\omega}\, f_{\omega'}(\omega)\, \phi_B^+(\omega,\mu)$$

For comparison:

$$\phi_\pi(u,\mu) = 6u\bar{u} \sum_n C_n^{(3/2)}(2u-1)\, a_n(\mu) \quad \Leftrightarrow \quad a_n(\mu) \propto \int_0^1 du\, C_n^{(3/2)}(2u-1)\, \phi_\pi(u,\mu)$$

RGE in "Dual Space":

RGE <u>local</u> in ω': $$\frac{d\rho_B^+(\omega',\mu)}{d\ln\mu} = \gamma_{\omega'}\, \rho_B^+(\omega',\mu)$$

Solution of RGE in Dual Space

$$\rho_B^+(\omega', \mu) = U_{\omega'}(\mu, \mu_0)\, \rho_B^+(\omega', \mu_0) \qquad \surd$$

The RG factor is known from SCET [see e.g. Neubert et al. 2004]

$$U_{\omega'}(\mu, \mu_0) \quad = \quad e^{V(\mu,\mu_0)} \left(\frac{\mu_0}{\hat{\omega}'}\right)^{-g(\mu,\mu_0)}$$

with

$$g(\mu, \mu_0) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\, \Gamma_{\rm cusp}(\alpha)\,,$$

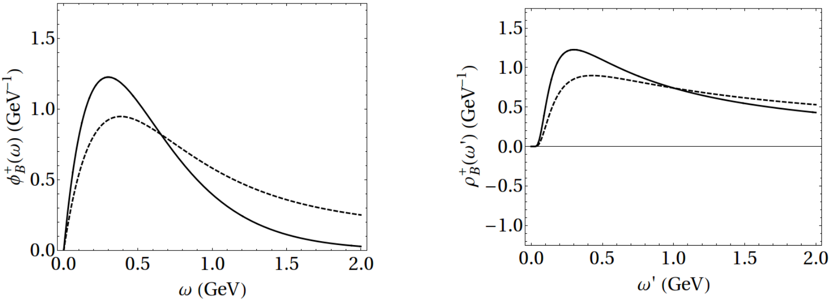
$$V(\mu, \mu_0) = -\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \left[\gamma_+(\alpha) + \Gamma_{\rm cusp}(\alpha) \int_{\alpha_s(\mu_0)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}\right]$$

Illustration of RGE behaviour

$g(\mu, \mu_0) = 0.3, \quad V(\mu, \mu_0) \to 1$

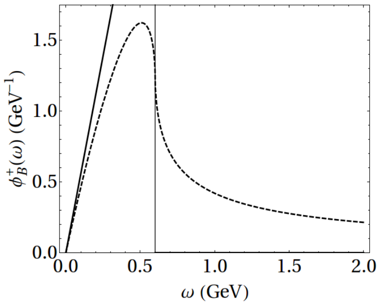
Example 1: Exponential Model $\quad (\omega_0 \sim \mathcal{O}(\Lambda_{\rm QCD}))$

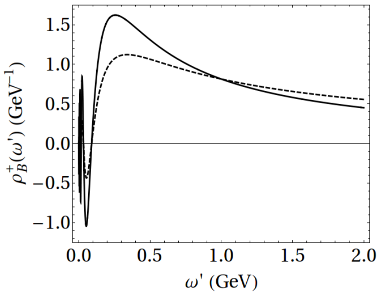
$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2} \exp\left(-\frac{\omega}{\omega_0}\right) \quad \leftrightarrow \quad \rho_B^+(\omega') = \frac{1}{\omega'} \exp\left(-\frac{\omega_0}{\omega'}\right)$$



Example 2: Free Parton Model $\quad (\bar{\Lambda} = M_B - m_b)$

$$\phi_B^+(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\,\theta(2\bar{\Lambda} - \omega) \quad \leftrightarrow \quad \rho_B^+(\omega') = \frac{1}{\bar{\Lambda}}\,J_2\left(2\sqrt{\frac{2\bar{\Lambda}}{\omega'}}\right)$$





2. Factorization, Logarithmic Moments and $B \to \gamma\ell\nu$

Leading $B \to \gamma$ Form Factor factorizes in **Hard**, **Jet** and **Soft** Dynamics:

(@large recoil)

$$F^{B\to\gamma}(E_\gamma) = \left[e^{V_h(\mu,\mu_h)} \left(\frac{\mu_h}{2E_\gamma} \right)^{-g(\mu,\mu_h)} H(E_\gamma, \mu_h) \right]$$
$$\times \int_0^\infty \frac{d\omega'}{\omega'} \left[e^{-2V_{\rm hc}(\mu,\mu_{\rm hc})} \left(\frac{\mu_{\rm hc}^2}{2E_\gamma \hat\omega'} \right)^{g(\mu,\mu_{\rm hc})} j(2E_\gamma\hat\omega', \mu_{\rm hc}) \right]$$
$$\times \left[e^{V(\mu,\mu_0)} \left(\frac{\mu_0}{\hat\omega'} \right)^{-g(\mu,\mu_0)} \rho_B^+(\omega', \mu_0) \right]$$

Notice:

- **hard scale:** set by twice the photon energy $2E_\gamma \sim m_b$
- **hard-collinear/jet scale=** geometric mean of hard and soft scale $\sqrt{2E_\gamma\hat\omega'} \sim \sqrt{m_b\mu_0}$
- In dual space, all RG factors are multiplicative!

  (i.e. jet function j in dual space obeys simple RGE, too)

Relations between Logarithmic Moments

- Jet Function has perturbative expansion in α_s and $\ln\omega'$
- Need Logarithmic Moments of $\rho_B^+(\omega', \mu_{\rm hc})$.

$$L_n(\mu) \equiv \int_0^\infty \frac{d\omega'}{\omega'} \ln^n\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega', \mu)$$

[for $n = 0, 1, 2$ identical to logarithmic moments of $\phi_B^+(\omega)$]

RGE: $$\frac{dL_n(\mu)}{d\ln\mu} = \Gamma_{\rm cusp}(\alpha_s)\, L_{n+1}(\mu) - \gamma_+(\alpha_s) L_n(\mu) - n\, L_{n-1}(\mu)$$

Formal solution: $$L_n(\mu) = e^V \sum_{m=0}^{\infty} \frac{g^m}{m!} \sum_{j=0}^{n} \frac{n!}{(n-j)!\, j!} \ln^{n-j}\left(\frac{\mu_0}{\mu}\right) L_{m+j}(\mu_0)$$

(requires truncation — or consider $\rho_B^+(\omega')$ as generating function)

Generic Parameterisations

Start with generalization of the exponential model:

$$\rho_B^+(\omega', \mu_0) = \frac{e^{-\omega_0/\omega'}}{\omega'} \sum_{k=0}^{\infty} c_k(\mu_0) \, \frac{(\omega_0/\omega')^k}{\Gamma(1+k)} \, .$$

Write this in RG-covariant form:

$$\rho_B^+(\omega', \mu) = \frac{e^{-\omega_0/\omega'}}{\omega'} \sum_{k=0}^{\infty} c_k(\mu) \, \frac{(\omega_0/\omega')^{k+g(\mu,\mu_0)}}{\Gamma(1+k+g(\mu,\mu_0))} \, .$$

with

$$c_k(\mu) = c_k(\mu_0) \, \frac{\Gamma(1+k+g(\mu,\mu_0))}{\Gamma(1+k)} \, e^{V(\mu,\mu_0)} \left(\frac{\mu_0 \, e^{2\gamma_E}}{\omega_0} \right)^{g(\mu,\mu_0)} \, .$$

This yields: $L_0(\mu) = \left\langle \frac{1}{\omega} \right\rangle_B = \frac{1}{\omega_0} \sum_{k=0}^{\infty} c_k(\mu)$

to be compared with $\langle \frac{1}{u} \rangle_\pi = 3 \sum_n a_n(\mu)$

Phenomenology of $B \to \gamma \ell \nu$

following [Beneke/Braun/Ji/Wei, arXiv:1804.04962]

Basic idea:

Model soft contributions by modifying the QCDF result for the spectral density of the $B \to \gamma^*$ form factor, which can be related to the light-cone sum rule prediction for the $B \to \rho$ form factor ...

$$F_{B\to\gamma}(E_\gamma) = F_{B\to\gamma}^{\mathrm{QCDF}}(E_\gamma) + \xi_{B\to\gamma}^{\mathrm{soft}}(E_\gamma)$$

with $(s = 2E_\gamma \omega)$

$$\xi_{B\to\gamma}^{\mathrm{soft}}(E_\gamma) = \frac{1}{\pi} \int_0^{s_0} \frac{ds}{s} \left[\frac{s}{m_\rho^2} \, e^{-(s-m_\rho^2)/M^2} - 1 \right] \mathrm{Im}\, F_{B\to\gamma^*}^{\mathrm{QCDF}}(E_\gamma, s)$$

- Prediction is strongly sensitive to first logarithmic moments L_0 and L_1.
- Dependence on remaining parameters is relatively mild.

$\Rightarrow$ Future data analysis should determine L_0 and L_1 in a correlated way !

3. OPE, Parton Model and HQET Parameters

Small and Large Values of ω' now **clearly separated** in Dual Function **!**

Small values of ω':

- Simple models / parametrizations
- Non-perturbative methods (QCD sum rules, [Lattice??])

  $\longleftrightarrow$ "quasi-LCDAs"
- Phenomenological constraints using Factorization (see above)

Large values of ω':

- Perturbative dynamics ($\rightarrow$ parton picture)
- Constraints from local OPE (HQET parameters)

OPE Constraints at Large Values of ω'

Consider Positive Moments of LCDA: [Lee/Neubert '05]

$$M_n(\Lambda_{\rm UV},\mu) := \int_0^{\Lambda_{\rm UV}} d\omega\, \omega^n\, \phi_B^+(\omega,\mu)$$

UV cut-off required $\leftrightarrow$ no simple relation to HQET parameters $(\bar\Lambda, \ldots)$ **!**

For $\boxed{\mu \sim \Lambda_{\rm UV} \gg \Lambda_{\rm hadr.}}$ perturbatively calculable (incl. power corr.):

$$M_0(\Lambda_{\rm UV},\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2\,\ln^2\frac{\Lambda_{\rm UV}}{\mu} + 2\,\ln\frac{\Lambda_{\rm UV}}{\mu} - \frac{\pi^2}{12}\right) + \frac{16\,\bar\Lambda}{3\,\Lambda_{\rm UV}}\,\frac{\alpha_s C_F}{4\pi}\left(\ln\frac{\Lambda_{\rm UV}}{\mu} - 1\right) + \ldots\,,$$

$$\frac{1}{\Lambda_{\rm UV}}\,M_1(\Lambda_{\rm UV},\mu) = 0 + \frac{\alpha_s C_F}{4\pi}\left(-4\,\ln\frac{\Lambda_{\rm UV}}{\mu} + 6\right) + \frac{4\bar\Lambda}{3\Lambda_{\rm UV}}\left[1 + \frac{\alpha_s C_F}{4\pi}\left(-2\,\ln^2\frac{\Lambda_{\rm UV}}{\mu} + 8\,\ln\frac{\Lambda_{\rm UV}}{\mu} - \frac{7}{4} - \frac{\pi^2}{12}\right)\right] + \ldots\,,$$

Positive Moments in Terms of Dual Function

$$M_n(\Lambda_{\rm UV},\mu) = \int\limits_0^\infty \frac{d\omega'}{\omega'} \int\limits_0^{\Lambda_{\rm UV}} d\omega\, \omega^n \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \rho_B^+(\omega',\mu)$$

- For the first two moments:

$$M_0(\Lambda_{\rm UV},\mu) = \Lambda_{\rm UV} \int_0^\infty \frac{d\omega'}{\omega'}\, J_2\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right) \rho_B^+(\omega')\,,$$

$$M_1(\Lambda_{\rm UV},\mu) = \frac{2\Lambda_{\rm UV}}{3}\, M_0(\Lambda_{\rm UV},\mu) - \frac{\Lambda_{\rm UV}^2}{3} \int\limits_0^\infty \frac{d\omega'}{\omega'}\, J_4\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right) \rho_B^+(\omega')\,.$$

Fixed-order Matching for $\rho_B^+(\omega')$:

→ Perturbative Expansion for $\rho_B^+(\omega',\mu)$ if $\boxed{\omega' \sim \mu \gg \bar\Lambda}$

$$\rho_B^+(\omega')_{\rm pert.} = C_0 \, \frac{1}{\bar\Lambda} \, J_2\left(2\sqrt{\frac{2\bar\Lambda}{\omega'}}\right) + (C_0 - C_1)\, \frac{4}{\bar\Lambda}\, J_4\left(2\sqrt{\frac{2\bar\Lambda}{\omega'}}\right) + \ldots$$

Model-independent Prediction: $(L = \ln \mu/\hat\omega')$

$$C_0 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2L^2 + 2L - 2 - \frac{\pi^2}{12}\right) + \mathcal{O}(\alpha_s^2)\,,$$

$$C_1 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2L^2 + 2L + \frac{5}{4} - \frac{\pi^2}{12}\right) + \mathcal{O}(\alpha_s^2)$$

→ reduces to the free parton result for $\alpha_s \to 0$ $\surd$

… further power corrections in $\bar\Lambda/\Lambda_{\rm UV}$, $\lambda_{1,2}/\Lambda_{\rm UV}^2$ etc. can be included

Interpolating between large and small values of ω'

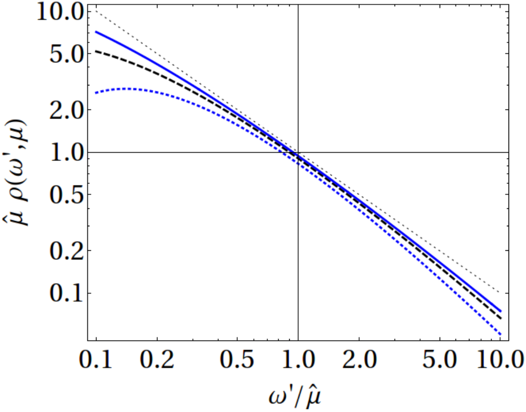
- Fixed-order calculation only works for $\hat{\omega}' \sim \mu$
- For $\hat{\omega}' \gg \mu \gg \bar{\Lambda}$ one has to *locally* resum large logarithms $L = \ln \mu/\hat{\omega}'$

Phenomenological procedure:

- Start from a given model for ϕ_B or ρ_B defined at reference scale μ_0
- Consider $1/\omega'$ expansion and adjust to perturbative result (requires to define an auxiliary parameter Ω where transition occurs)
- Implement local RG improvement

details for a concrete realization can be found in our paper [TF/Lange/Wang '13]

Interpolating between large and small values of ω'



log-log plot:

.... asymptotic $\mu/\hat{\omega}'$ behaviour

— $\mu = 10$ GeV

- - $\mu = 3$ GeV

- - $\mu = 1$ GeV

for exp. model at $\mu_0 = 1$ GeV

$\Omega := e^{\gamma_E}\ \mu_0$

⟵ non-perturbative ⟵ ⟶ perturbative ⟶

Summary

- B-meson LCDAs $\phi_B(\omega)$ as hadronic input for exclusive B-decays in QCD factorization or Light-cone sum rule approaches.
- Most direct phenomenological access from $B \to \gamma \ell \nu$
  $\longrightarrow$ constraints on logarithmic moments L_n

- QCD corrections generate "perturbative tail" at large momenta ω.
- Diagonalization of the RG equations in terms of Bessel functions.

- Model-independent relations to HQET parameters
  (parton model + radiative corrections)
- Complete picture requires additional non-perturbative input
  (lattice or sum rules)

Comparison $\phi_B^+(\omega)$ vs. $\phi_\pi(u)$

	$\phi_B^+(\omega)$	$\phi_\pi(u)$
definition	HQET on the LC	QCD on the LC
support	$\omega \in [0, \infty)$	$u \in [0, 1]$
1-loop evolution	Lange-Neubert	ERBL
eigenfunctions	$\sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)$	$C_n^{(3/2)}(2u-1)$
eigenvalues	$\gamma_{\omega'} = -\left(\Gamma_{\rm cusp} \ln \frac{\mu}{\hat{\omega}'} + \gamma_+\right)$	$\gamma_n = 1 - \frac{2}{(n+1)(n+2)} + 4 \sum_{m=2}^{n+1} \frac{1}{m}$
benchmark obs.	$B \to \gamma \ell \nu$	$\gamma\gamma^* \to \pi$
LO dependence	$\langle \omega^{-1} \rangle$	$\langle \bar{u}^{-1} \rangle$
positive moments	require cut-off	$\to$ Gegenbauer coeff.
special limits	partonic ($\omega', \mu \gg \Lambda_{\rm had}$) $\frac{1}{\bar\Lambda}\, J_2\left(2\sqrt{\frac{2\bar\Lambda}{\omega'}}\right)$	asymptotic ($\mu \gg \Lambda_{\rm had}$) $6u(1-u)$

Backup Slides

Definition of 2- and 3-particle LCDAs

[Kawamura et al. 2001; Braun/Ji/Manashov 1703.02446]

$$\langle 0|\bar{q}(nz)\,\Gamma\, h_v(0)|\bar{B}(v)\rangle = -\frac{iF_B(\mu)}{2}\,\mathrm{Tr}\left\{\gamma_5\,\Gamma\,\frac{1+\not{v}}{2}\left[\Phi_+ - \frac{\not{n}}{2}\,(\Phi_+ - \Phi_-)\right]\right\}$$

$$n^\mu\,\langle 0|\bar{q}(z_1 n)\,gG_{\mu\nu}(z_2 n)\,\Gamma\,h_v(0)|\bar{B}(v)\rangle =$$
$$\frac{F_B(\mu)}{2}\,\mathrm{Tr}\left\{\gamma_5\Gamma\frac{1+\not{v}}{2}\left[(\not{n}v_\mu - \gamma_\mu)\,(\Psi_A - \Psi_V) - i\sigma_{\mu\nu}n^\nu\,\Psi_V - n_\mu\,X_A + n_\mu\not{n}\,Y_A\right]\right\}$$

and

$$n^\mu\,\langle 0|\bar{q}(z_1 n)\,g\tilde{G}_{\mu\nu}(z_2 n)\,\Gamma\,h_v(0)|\bar{B}(v)\rangle =$$
$$\frac{F_B(\mu)}{2}\,\mathrm{Tr}\left\{\gamma_5\Gamma\frac{1+\not{v}}{2}\left[(\not{n}v_\mu - \gamma_\mu)\left(\tilde{\Psi}_A - \tilde{\Psi}_V\right) - i\sigma_{\mu\nu}n^\nu\tilde{\Psi}_V - n_\mu\tilde{X}_A + n_\mu\not{n}\tilde{Y}_A\right]\right\}$$

$f_B\sqrt{m_B} = F_B(\mu)\,(1+\mathcal{O}(\alpha_s))$ in HQET, $\tilde{\Psi}_A = -\Psi_V$, $\tilde{\Psi}_V = -\Psi_A$

(gauge links [Wilson lines] implicit)

Lange-Neubert Solution of RGE

Standard procedure: ("continous Mellin moments" / "logarithmic F.T.")

introduce: $$\varphi_B^+(\theta,\mu) = \int_0^\infty \frac{d\omega}{\omega} \left(\frac{\omega}{\mu}\right)^{-i\theta} \phi_B^+(\omega,\mu)$$

Explicit solution in θ-space: [Lange/Neubert]

$$\varphi_B^+(\theta,\mu) = e^{V-2\gamma_E g} \left(\frac{\mu}{\mu_0}\right)^{i\theta} \frac{\Gamma(1-i\theta)\Gamma(1+i\theta-g)}{\Gamma(1+i\theta)\Gamma(1-i\theta+g)} \varphi_B^+(\theta+ig,\mu_0)$$

with RG functions $V = V(\mu,\mu_0)$ and $g = g(\mu,\mu_0)$ given in pert. theory.

. . . staring at the solution for $\varphi_B^+(\theta,\mu)$ . . .

Def. $$\varphi_B^+(\theta,\mu) := \frac{\Gamma(1-i\theta)}{\Gamma(1+i\theta)} \int_0^\infty \frac{d\omega'}{\omega'} \rho_B^+(\omega',\mu) \left(\frac{\mu}{\omega'}\right)^{i\theta}$$