



Anomalies in B meson decays

Nejc Košnik



Getting to Grips with QCD, Primošten, Sep. 21 2018

Outline

- Standard model and lepton flavor universality
- “Ordinary” decay $B \rightarrow D^{(*)} \tau \nu$
- Rare decay $B \rightarrow K^{(*)} \mu^+ \mu^-$
- New Physics solution with two leptoquarks

Flavors in the Standard model

- Standard model contains 3 identical copies of fermionic matter

$$\mathcal{L} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

- Flavor symmetry of gauge interactions

$$U(3)^5 \sim U(3)_Q \otimes U(3)_L \otimes U(3)_u \otimes U(3)_d \otimes U(3)_e$$

$$\mathcal{L}_{\text{kin.}} = \sum_{i=1}^3 \sum_{f \in \{Q, u, d, L, e\}} \bar{f}_i i \not{D} f_i + \dots$$

an example of flavor transform: $Q_i \rightarrow e^{i\alpha_0} [\exp(i\lambda_a \alpha_a)]_{ij} Q_j$

- Yukawa couplings break the flavor symmetry. Only Higgs knows the leptons and quarks by flavor.

$$\mathcal{L}_{\text{Yukawa}} = -y_{ij}^{(u)} \bar{Q}_i \tilde{H} u_j - y_{ij}^{(d)} \bar{Q}_i H d_j - y_{ij}^{(e)} \bar{L}_i H e_j \quad U(1)_B \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

diagonalization of Yukawa matrices reveals physical masses

- The SM flavor puzzle: — huge hierarchies in the fermion masses
— structure of the CKM matrix
— neutrino sector?

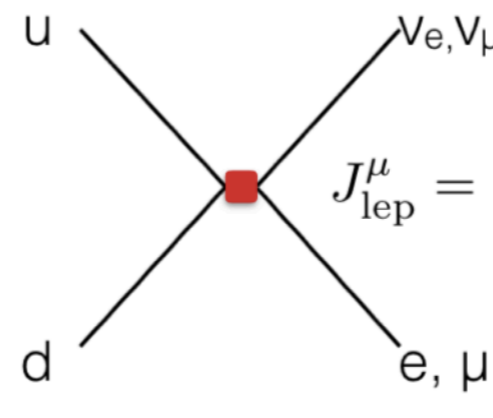
Lepton flavor universality

- Weak interactions are lepton flavor universal (contrary to the CKM mixing of quarks). The only distinguishing property between lepton generations are masses

$$\mathcal{L}_{\text{kin}} = -\frac{g}{\sqrt{2}} \left(\bar{\nu}_e \gamma^\mu \frac{1-\gamma^5}{2} e + \bar{\nu}_\mu \gamma^\mu \frac{1-\gamma^5}{2} \mu + \bar{\nu}_\tau \gamma^\mu \frac{1-\gamma^5}{2} \tau \right) W_\mu^+ + \dots$$

- Lepton flavor universality (LFU) first observed within the Fermi theory

$$G_F^e \approx G_F^\mu$$

$$J_{\text{quark}}^\mu = \bar{n} \gamma^\mu (1 - a \gamma_5) p$$


$$J_{\text{lep}}^\mu = \bar{\nu}_e \gamma^\mu (1 - \gamma_5) e + \bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu$$

$$\frac{G_F}{\sqrt{2}} J_{\text{lep}, \mu} J_{\text{had}}^\mu$$

- LFU tested in many observables

High energy probe at LEP: $Z \rightarrow ee, \mu\mu, \tau\tau$

$$\Gamma_{ll}^{SM} = \frac{GM_Z^3}{6\sqrt{2}\pi} \left((C_V^f)^2 + (C_A^f)^2 \right) = 83.42 \text{MeV}$$

$$C_V^\ell = -1$$

$$C_A^\ell = -1 + 4 \sin^2 \theta_W$$

$$\Gamma_{ee} = (83.94 \pm 0.14) \text{MeV}$$

$$\Gamma_{\mu\mu} = (83.84 \pm 0.20) \text{MeV}$$

$$\Gamma_{\tau\tau} = (83.68 \pm 0.24) \text{MeV}$$

Tests of LFU

- LFU ratios largely cancel out non-perturbative effects of QCD

$$\Gamma_{P \rightarrow \ell \nu} \sim G_F^2 |V_{ij}|^2 f_P^2 m_P m_\ell^2 \underbrace{\left(1 - \frac{m_\ell^2}{m_P^2}\right)}_{\text{chiral SM interaction}} \underbrace{\left(1 - \frac{m_\ell^2}{m_P^2}\right)}_{\text{phase space}}$$

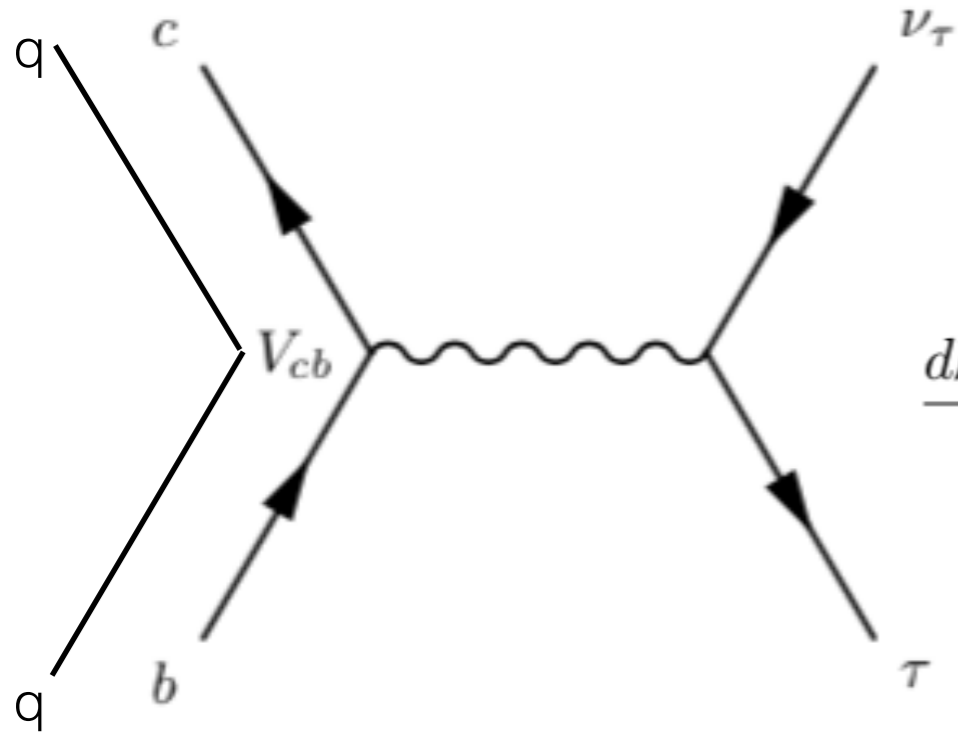
f_P = meson decay constant

- Tested to per-mille level in the first two generations

| | SM | exp. value |
|---|---------------------------------------|--------------------------------------|
| $R_{e/\mu}^\pi = \frac{\Gamma(\pi \rightarrow e\bar{\nu})}{\Gamma(\pi \rightarrow \mu\bar{\nu})}$ | $(1.2352 \pm 0.0001) \times 10^{-4}$ | $(1.2327 \pm 0.0023) \times 10^{-4}$ |
| $R_{e/\mu}^K = \frac{\Gamma(K \rightarrow e\bar{\nu})}{\Gamma(K \rightarrow \mu\bar{\nu})}$ | $(2.477 \pm 0.001) \times 10^{-5}$ | $(2.488 \pm 0.010) \times 10^{-5}$ |
| $R_{\tau/\mu}^K = \frac{\Gamma(K \rightarrow \tau\bar{\nu})}{\Gamma(K \rightarrow \mu\bar{\nu})}$ | $(1.1162 \pm 0.00026) \times 10^{-2}$ | $(1.101 \pm 0.016) \times 10^{-2}$ |
| $R_{\tau/\mu}^B = \frac{\Gamma(B \rightarrow \tau\nu)}{\Gamma(B \rightarrow \mu\bar{\nu})}$ | 223 | $\gtrsim 100$ |

The LFU ratio $R_{D^{(*)}}$

LFU in $b \rightarrow c \ell \nu$

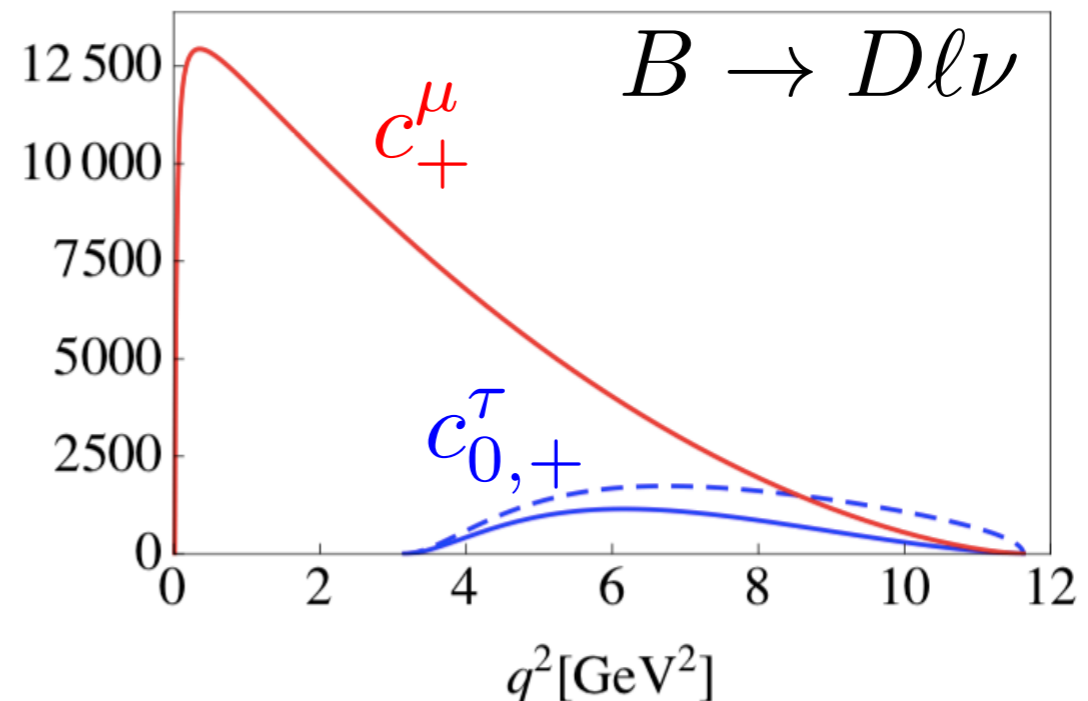


$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cb} (\bar{c} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu P_L \nu_\ell)$$

$$\frac{d\mathcal{B}(\bar{B} \rightarrow D \ell \bar{\nu}_\ell)}{dq^2} = \tau_{B^0} \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left[c_+^\ell(q^2) |F_+(q^2)|^2 + c_0^\ell(q^2) |F_0(q^2)|^2 \right]$$

| B⁺ DECAY MODES | Fraction (Γ_i/Γ) | Scal Confide |
|--|----------------------------------|-----------------|
| Semileptonic and leptonic modes | | |
| $\ell^+ \nu_\ell$ anything | [a] (10.99 ± 0.28) % | |
| $e^+ \nu_e X_c$ | (10.8 ± 0.4) % | |
| $D \ell^+ \nu_\ell$ anything | (8.4 ± 0.5) % | |
| $\bar{D}^0 \ell^+ \nu_\ell$ | [a] (2.20 ± 0.10) % | |
| $\bar{D}^0 \tau^+ \nu_\tau$ | (7.7 ± 2.5) × 10 ⁻³ | |

LFU is badly broken as $m_\mu < m_\tau \sim m_B$



$$R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu)}$$

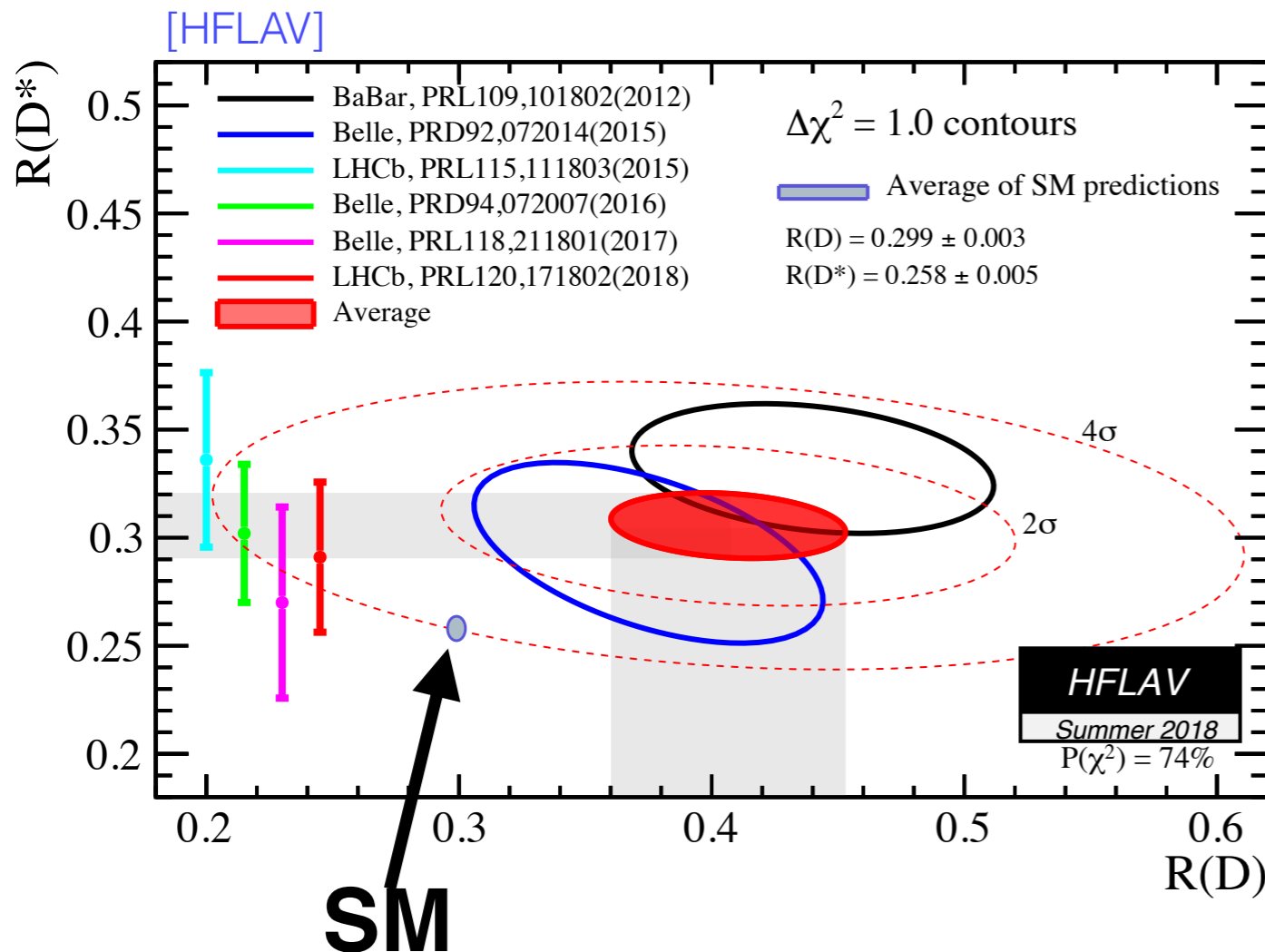
$b \rightarrow c \ell \nu$ universality, exp. Vs. SM

QCD nonperturbative effects cancel partially

$$R_D^{\text{SM}} = 0.29(1) \leftarrow \text{lattice form factors of MILC and HPQCD}$$

$$R_{D^*}^{\text{SM}} = 0.257(3) \leftarrow \text{heavy quark symmetry + power + } \alpha_s \text{ corrections}$$

fitted to measured Belle spectra of $\ell=e, \mu$
[\[Bernlochner et al, 1703.05330\]](#)



+ angular observables in $B \rightarrow D^* \tau \nu$

+ Similar trend in
 $R_{J/\psi}^{\text{SM}} < 0.71(17)(18)$

[\[Cohen et al, '18, LHCb\]](#)

See the afternoon talks
 by Andrey, Damir, Elena

World average excludes the SM point at 4σ

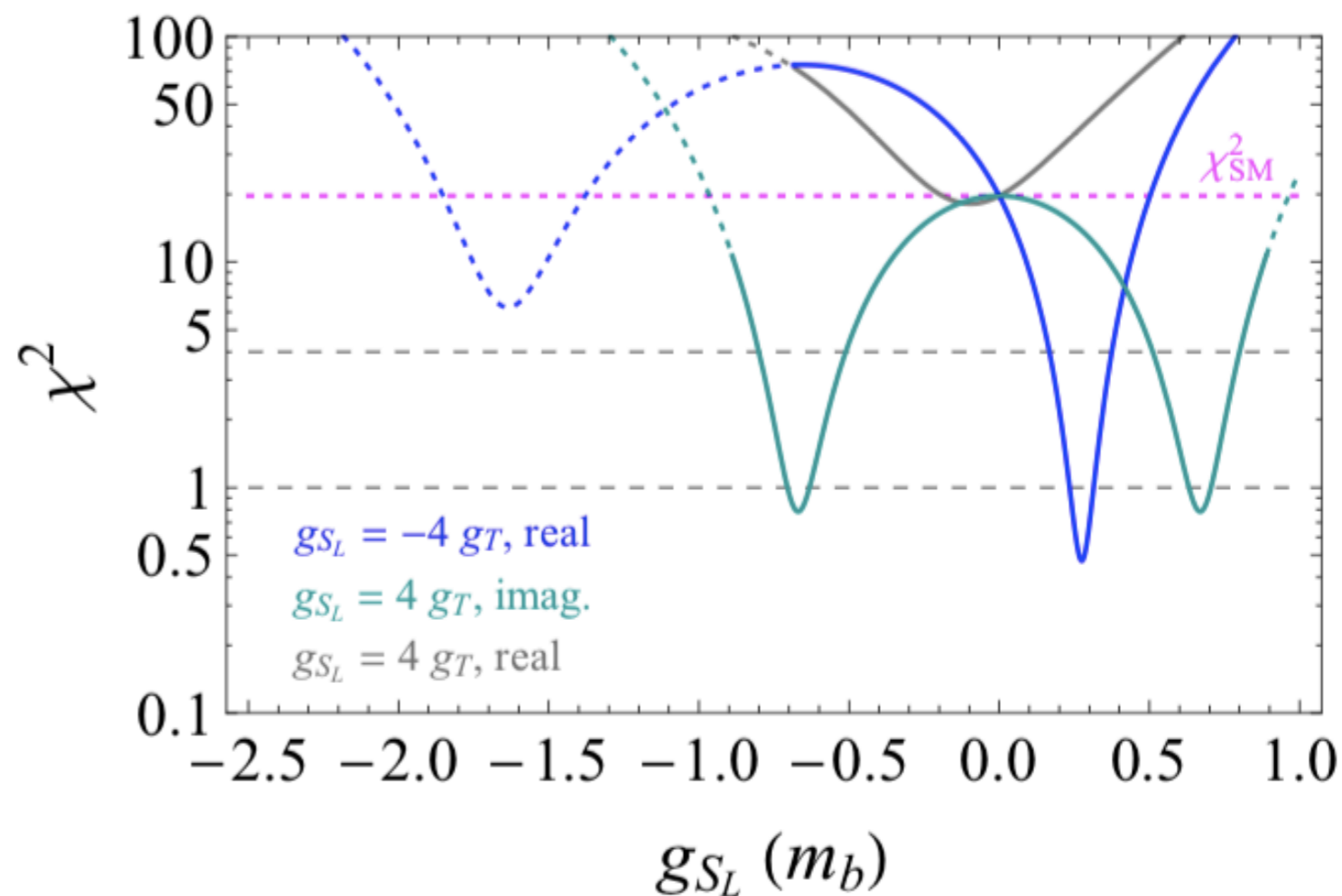
$b \rightarrow c \ell \nu$ universality, heavy NP

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cb} \left[(1 + g_{V_L}) (\bar{c} \gamma^\mu P_L b) (\bar{\tau} \gamma_\mu P_L \nu_\tau) + g_{S_L} (\bar{c} P_L b) (\bar{\tau} P_L \nu_\tau) + g_{S_R} (\bar{c} P_R b) (\bar{\tau} P_L \nu_\tau) \right. \\ \left. + g_T (\bar{c} \sigma_{\mu\nu} P_L b) (\bar{\tau} \sigma^{\mu\nu} P_L \nu_\tau) \right]$$

Assuming negligible contribution of NP to $b \rightarrow c e \nu$, $c \mu \nu$, motivated by $R_{D^{(*)}}^{\mu/e} \approx 1$

Particular choice of NP direction, implemented in the scalar LQ scenarios.

Indication of complex effective couplings in $g_S = 4 g_T$

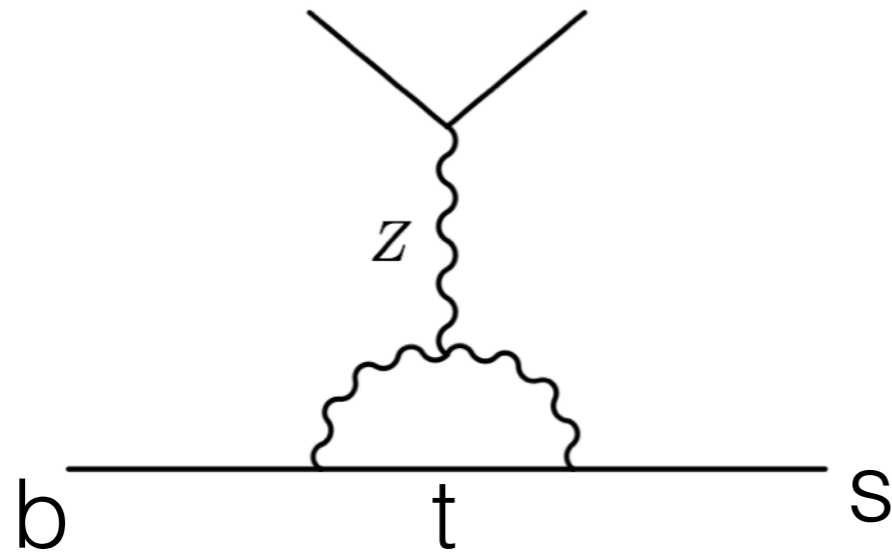


[Angelescu, Becirevic, Faroughy, Sumensari, 1808.08179]

The LFU ratio $R_{K^{(*)}}$ and related anomalies

LFU in $B \rightarrow K^{(*)} \ell^+ \ell^-$

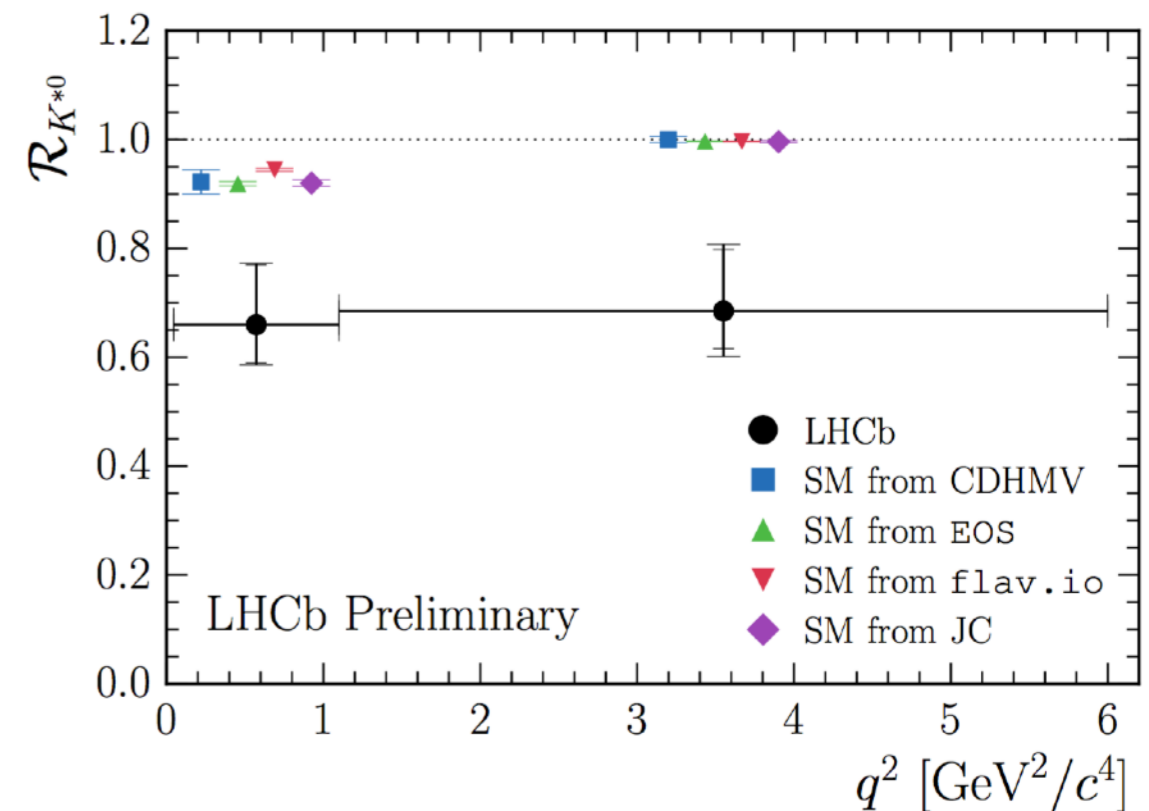
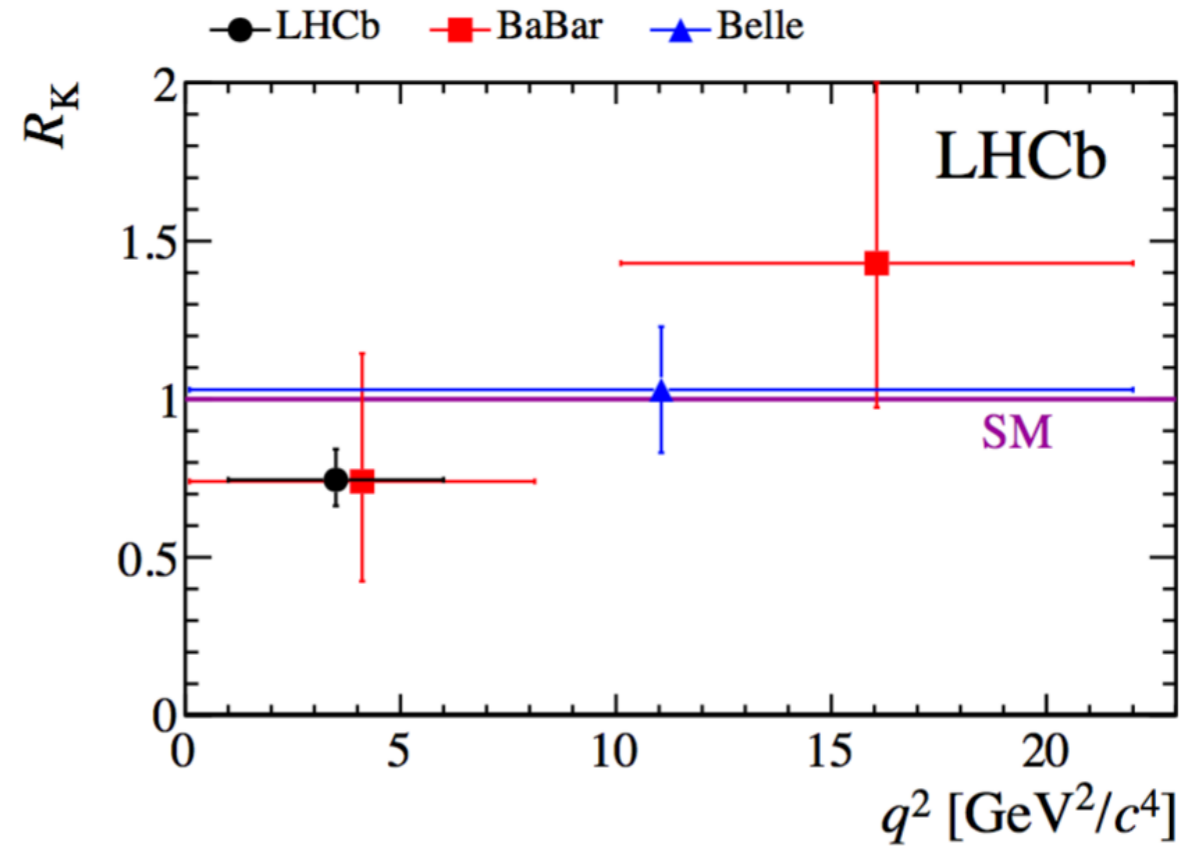
GIM suppressed, $\text{Br}(B \rightarrow K \ell \ell) \sim 4 \times 10^{-7}$
 Sensitive to New Physics



$$R_{K^{(*)}} = \frac{\mathcal{B}(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K^{(*)} e^+ e^-)}$$

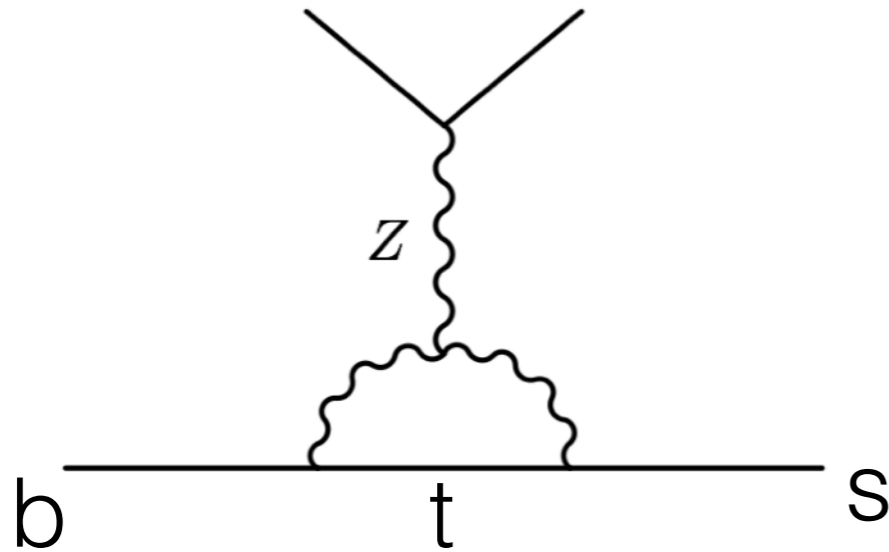
Hadronic uncertainties cancel out effectively:

| | LHCb | SM | dev. |
|-----------------------|------------|-----------|----------------|
| R_K | 0.745(97) | 1.00(1) | 2.6σ |
| $R_{K^* \text{ low}}$ | 0.660(113) | 0.906(28) | 2.1σ |
| R_{K^*} | 0.685(122) | 1.00(1) | 2.6σ |
| | | | $\sim 4\sigma$ |



[Kruger, Hiller '03]
 [LHCb '14, '16]
 [Bordone, Isidori, Pattori '16]

$B \rightarrow K^{(*)} \ell^+ \ell^-$



$$\mathcal{H}_{\text{eff}}^{b \rightarrow s \ell \ell} = -\frac{4G_F \lambda_t}{\sqrt{2}} \sum_{i=7,9,10} C_i(\mu) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu (\gamma^5) \ell) \quad \text{vector(axial)}$$

$$\mathcal{O}_7 = \frac{em_b}{(4\pi)^2} (\bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R) \quad \text{dipole}$$

Assume that NP modifies $C_{9,10}$ of muons

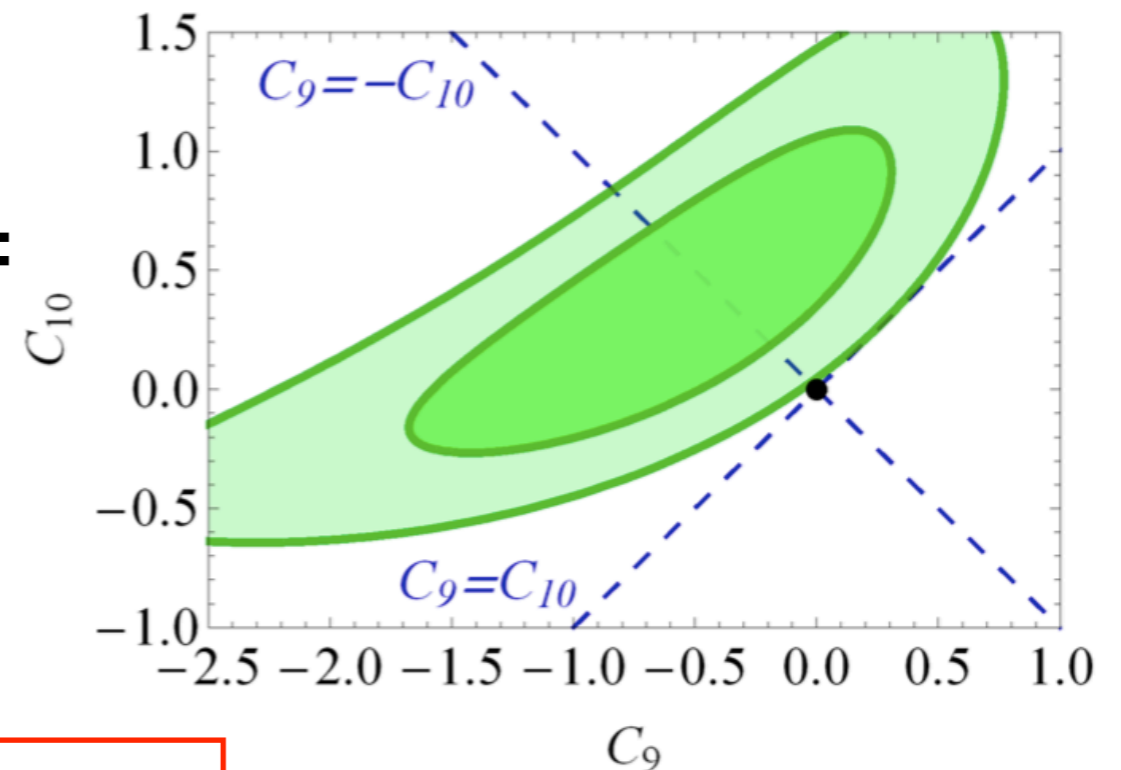
Consider theoretically reliable observables:

$R_K, R_{K^*},$

$B_s \rightarrow \mu\mu,$

$B \rightarrow K\mu\mu$

partial width at $q^2 > 15 \text{ GeV}^2$

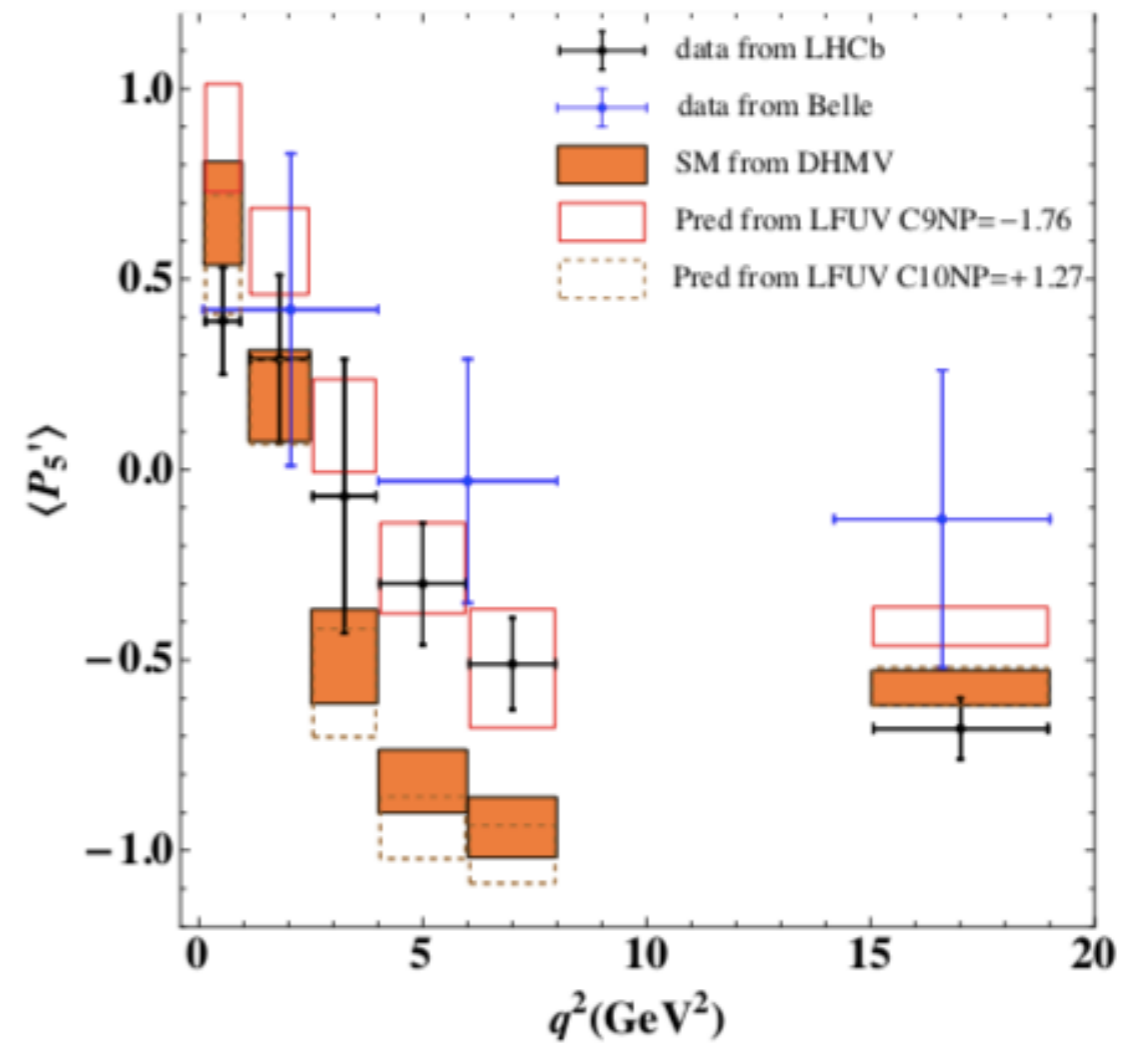
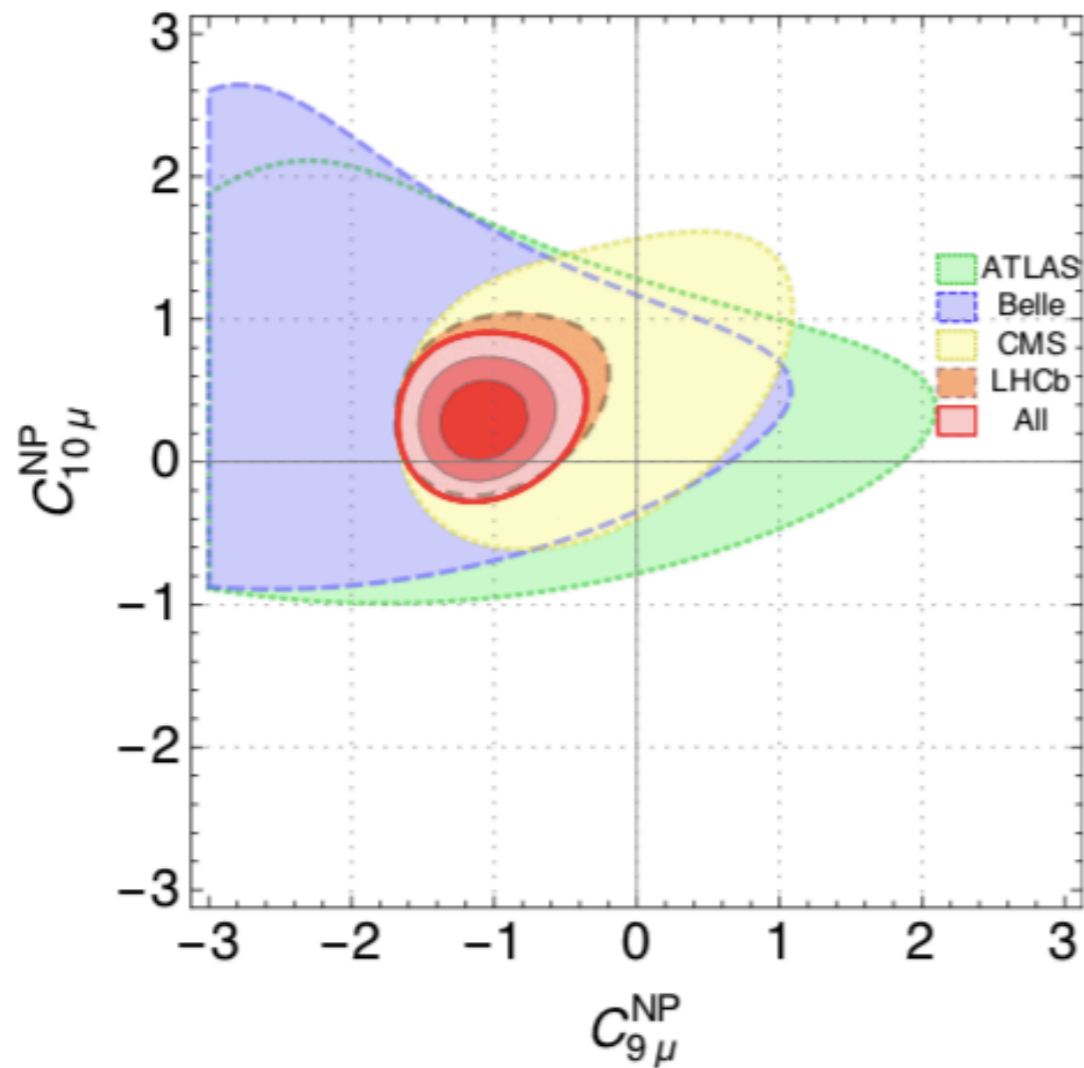


Anomaly in R_K, R_{K^*} is consistent with measurements of branching fractions in the $\mu\mu$ channel

$b \rightarrow s \ell^+ \ell^-$ global fits

Theoretically less reliable observables also show deviations which are consistent with the expectation from LFU anomaly

- spectra
- angular distributions
- e+e- data are included



[Capdevilla et al '17]

Model with two scalar leptoquarks from GUT

Implications for New Physics

- $R_{K^{(*)}} \sim 10\%$ modification of the 1-loop GIM-suppressed amplitude
- $R_{D^{(*)}} \sim 10\%$ effect on the tree-level W exchange V_{cb} .
- *Can the two effects have common NP origin?*

→ What are typical mass scales of NP addressing the two LFUV phenomena? (Assume NP contributes to the SM operator with coupling 1)

$R_{K^{(*)}} (b \rightarrow s\mu\mu)$

- SM: $\approx \frac{G_F V_{ts} V_{tb} \alpha}{(4\pi)} (\bar{s} \gamma^\mu P_L b) (\bar{\mu} \gamma_\mu \mu)$
- ⇒ 1-loop NP: $\frac{1}{(4\pi)^2 \Lambda^2} \Rightarrow \Lambda \approx 3 \text{ TeV}$
- ⇒ Tree-level NP: $C_9 \sim \frac{1}{\Lambda^2} \Rightarrow \Lambda \approx 30 \text{ TeV}$

$R_{D^{(*)}} (b \rightarrow c\tau\nu)$

- SM: $\frac{G_F V_{cb}}{\sqrt{2}} (\bar{c} b)_{V-A} (\bar{\ell} \nu)_{V-A}$
- ⇒ Tree-level NP: $\frac{1}{\Lambda^2} \Rightarrow \Lambda \approx 3 \text{ TeV}$

How to explain different sizes of the effects?

- 1 Different scales of NP for $R_{K^{(*)}}$ and $R_{D^{(*)}}$
- 2 Loop ($R_{K^{(*)}}$) Vs. tree ($R_{D^{(*)}}$)
- 3 Suppression of NP couplings in $R_{K^{(*)}}$ compared to $R_{D^{(*)}}$

Identifying a viable model

- Effective theory analysis focused on left-handed current operators

$$\frac{C_1}{\Lambda^2} (\bar{Q}_3 \gamma^\mu Q_3) (\bar{L}_3 \gamma_\mu L_3) + \frac{C_3}{\Lambda^2} (\bar{Q}_3 \sigma \gamma^\mu Q_3) \cdot (\bar{L}_3 \sigma \gamma_\mu L_3)$$

[Buttazzo et al '17, Bhattacharya et al '14, Feruglio et al '16]

- As a single mediator particle, vector leptoquark $U_1(3, 1, 2/3)$ is singled out [Buttazzo et al '17]
- UV complete setting needed (Pati-Salam, 4321 model) [Assad et al '17, Bordone et al '17,18, Greljo et al '18, Di Luzio et al '17, Blanke, Crivellin et al '18, Calibbi et al '17]
- Scalar LQs are not UV sensitive, however single LQ does not work
- Two scalar LQs can generate left-handed operators and solve the B -anomalies [Crivellin '17]
- Two scalar LQs can also contribute to alternative Lorentz structures, no UV complete model available

Two light scalar LQ model

- SM + 2 leptoquarks at 1 TeV, guided by $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$R_2(3, 2, 7/6) : \quad Y_R^{ij} \bar{Q}'_i \ell'_{Rj} R_2 + Y_L^{ij} \bar{u}'_{Ri} \tilde{R}'_2{}^\dagger L'_j$$

$$\text{mass basis} \rightarrow (V Y_R E_R^\dagger)^{ij} \bar{u}_{Li} \ell_{Rj} R_2^{\frac{5}{3}} + (Y_R E_R^\dagger)^{ij} \bar{d}_{Li} \ell_{Rj} R_2^{\frac{2}{3}} \\ + (U_R Y_L)^{ij} \bar{u}_{Ri} \nu_{Lj} R_2^{\frac{2}{3}} - (U_R Y_L)^{ij} \bar{u}_{Ri} \ell_{Lj} R_2^{\frac{5}{3}}$$

$$\text{Assumption:} \quad Y_R E_R^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_R^{b\tau} \end{pmatrix}, \quad U_R Y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_L^{c\mu} & y_L^{c\tau} \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_3(\bar{3}, 3, 1/3) : \quad Y^{ij} \bar{Q}'_i{}^C i_{\tau_2} (\tau_k S_3^k) L'_j$$

$$\text{mass basis} \rightarrow -(Y)^{ij} \bar{d}_{Li}^C \nu_{Lj} S_3^{\frac{1}{3}} + \sqrt{2} (V^* Y)^{ij} \bar{u}_{Li}^C \nu_{Lj} S_3^{-\frac{2}{3}} \\ \sqrt{2} Y^{ij} \bar{d}_{Li}^C \ell_{Lj} S_3^{\frac{4}{3}} - (V^* Y)^{ij} \bar{u}_{Li}^C \ell_{Lj} S_3^{\frac{1}{3}}$$

$$\text{GUT relation:} \quad Y = -Y_L = - \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_L^{c\mu} & y_L^{c\tau} \\ 0 & 0 & 0 \end{pmatrix}$$

⁰Free parameters: m_{R_2} , m_{S_3} , complex $y_R^{b\tau}$, $y_L^{c\mu}$, $y_L^{c\tau}$, θ

- $SU(5)$ unified gauge group
- fermions

$$\bar{\mathbf{5}}_i = (L, \bar{d}_R)_i, \quad \mathbf{10}_i = (\bar{e}_R, \bar{u}_R, Q)_i$$

- At unification scale M_{GUT} gauge bosons of $SU(5)$ (vector leptoquarks) couple leptons and quarks
- Scalar representations contain scalar leptoquarks:
 - ▶ **24** breaks $SU(5)$, $M_{X,Y} \sim g_{\text{GUT}} M_{\text{GUT}}$
 - ▶ **45** contains R_2 and S_3
 - ▶ **50** contains R'_2
- Yukawa couplings

$$a^{ij} \mathbf{10}_i \bar{\mathbf{5}}_j \mathbf{45} \quad b^{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{50} \quad (b = b^T, \text{ symmetric in flavor space})$$

- R_2 and R'_2 mix via $\mu \mathbf{45} \overline{\mathbf{50}} \mathbf{24}$ into light R_2 and heavy R_{2H} .

- Matching onto effective theory results in the couplings considered in flavor and LHC

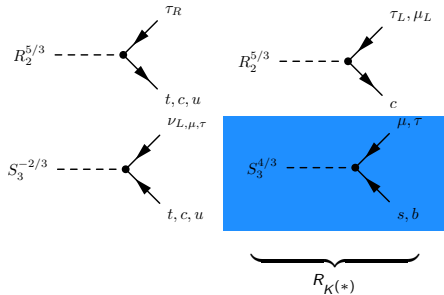
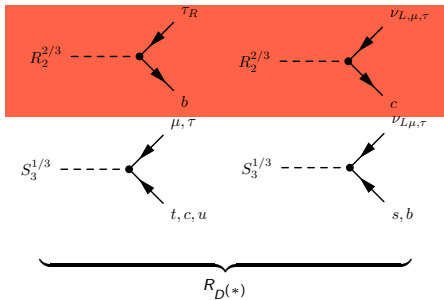
$$Y_R^{ij} \bar{Q}'_i \ell'_{Rj} R_2 + Y_L^{ij} \bar{u}'_{Ri} \tilde{R}_2^\dagger L'_j + Y^{ij} \bar{Q}'_i{}^C i\tau_2 (\tau_k S_3^k) L'_j$$

where

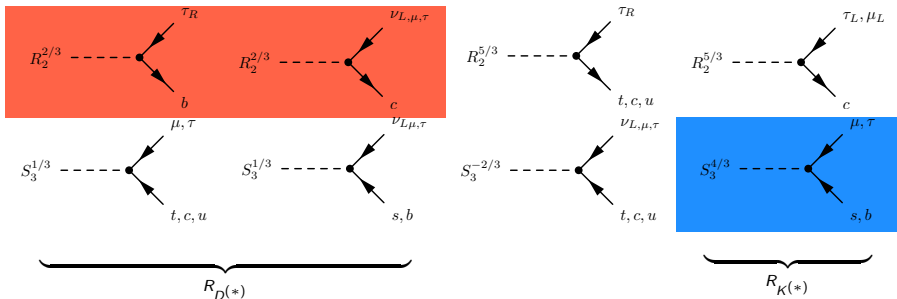
$$Y_L = a \cos \phi \quad Y_R = b \sin \phi, \quad Y = -\frac{a}{\sqrt{2}}$$

- Assume $\phi = \pi/2$ (assumption is not crucial to further analysis)
- Proton destabilizing diquark interactions $\bar{Q}\bar{Q}S_3$ ($\mathbf{10}_i, \mathbf{10}_j, \mathbf{45}$) can be forbidden. [Doršner et al, 2017]

Charged currents



Charged currents



$R_{D^{(*)}}$

Dominant contribution by R_2 via scalar and tensor interactions

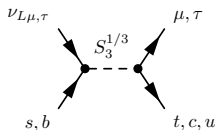
$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} V_{cb} [g_{S_L}(\bar{c}_R b_L)(\bar{\tau}_R \nu_L) + g_T(\bar{c}_R \sigma_{\mu\nu} b_L)(\bar{\tau}_R \sigma^{\mu\nu} \nu_L)]$$

$$g_{S_L} = 4g_T = \frac{y_L^{c\nu} y_R^{b\tau^*}}{4\sqrt{2} m_{R_2}^2 G_F V_{cb}} \quad (g_{S_L}(m_b) \approx 7g_T(m_b))$$

- Scalar and tensor $B \rightarrow D$ form factors from lattice QCD [HPQCD '15, MILC '15]
- $B \rightarrow D^*$ FFs extracted from exp. spectra using the heavy quark symm. [Bernlochner et al'18, HFLAV]

Charged currents constraints

- $b \rightarrow c\tau\nu$ is the *only* charged-current affected by R_2
 - ▶ $B_c \rightarrow \tau\nu$ taken into account
- Leptoquark state S_3 affects many charged currents via charge 1/3 component:

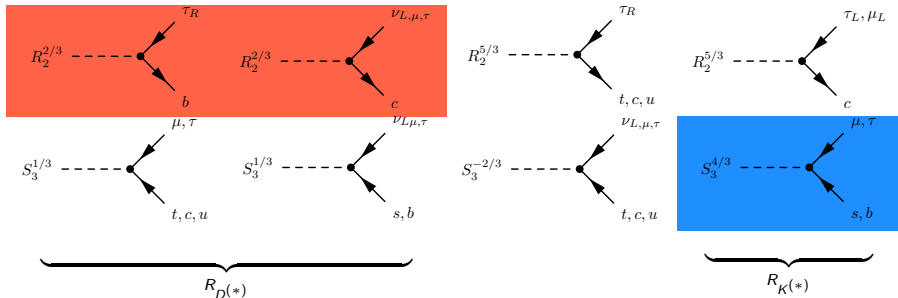


We consider the most relevant constraints:

- ▶ $R_{D^{(*)}}^{\mu/e} = \mathcal{B}(B \rightarrow D^{(*)} \mu \bar{\nu}) / \mathcal{B}(B \rightarrow D^{(*)} e \bar{\nu})$
- ▶ $B \rightarrow \tau\nu$
- ▶ $\mathcal{B}(K \rightarrow e\nu) / \mathcal{B}(K \rightarrow \mu\nu)$
- ▶ $\mathcal{B}(\tau \rightarrow K\nu) / \mathcal{B}(K \rightarrow e\nu)$

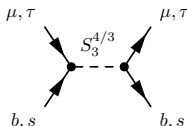
$R_{K(*)}$ and neutral currents

- Explained by the $S_3^{4/3}$ charge eigenstate



$R_{K^{(*)}}$ and neutral currents

- $S_3^{4/3}$ couples to left-handed fermions



- Left-handed current operators with μ and τ

$$\mathcal{H}_{\text{eff}}^{b \rightarrow sll} = -\frac{4G_F \lambda_t}{\sqrt{2}} \sum_{i=7,9,10} C_i(\mu) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\mu b_L) (\bar{l} \gamma^\mu (\gamma^5) l)$$

$$\delta C_9^{\mu\mu} = -\delta C_{10}^{\mu\mu} = \frac{\pi v^2}{\lambda_t \alpha_{\text{em}}} \frac{y^{b\mu} (y^{s\mu})^*}{m_{S_3}^2} = \frac{\pi v^2}{\lambda_t \alpha_{\text{em}}} \frac{\sin 2\theta (y_L^{c\mu})^2}{2m_{S_3}^2}$$

- $\sin 2\theta$ and large mass suppresses the S_3 effect in $R_{K^{(*)}}$
- Use constraint on $\delta C_9 = -\delta C_{10}$ determined from clean observables: $R_{K^{(*)}}$, $B_s \rightarrow \mu^+ \mu^-$

$$\delta C_9^{\mu\mu} \in (-0.85, -0.50)^1$$

¹In agreement with global analyses of $b \rightarrow s\mu\mu$ observables. [Capdevilla et al 17, ...]

- $B_s - \bar{B}_s$, Δm_s constraint, suppressed by θ [S_3]

$$\Delta m_s^{S_3} \sim \sin^2 2\theta \left[(y_L^{c\mu})^2 + (y_L^{c\tau})^2 \right]^2 / m_{S_3}^2$$

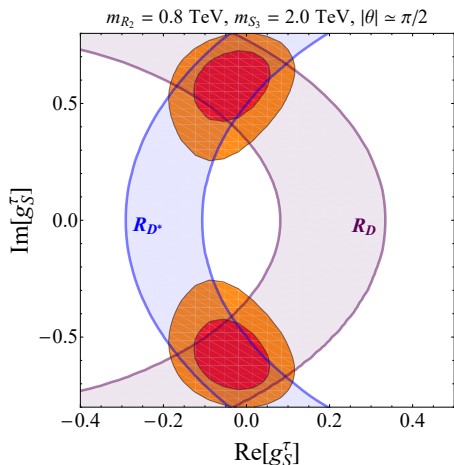
- $\mathcal{B}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}$ [S_3 and R_2]
- $Z \rightarrow \ell\ell$ constraints at LEP [S_3 and R_2]

$$\frac{g_V^\tau}{g_V^e} = 0.959(29), \quad \frac{g_A^\tau}{g_A^e} = 1.0019(15), \quad \frac{g_V^\mu}{g_V^e} = 0.961(61), \quad \frac{g_A^\mu}{g_A^e} = 1.0001(13)$$

- $\mathcal{B}(\tau \rightarrow \mu\phi) \sim \cos^4 \theta (y_L^{c\mu} y_L^{c\tau})^2 / m_{S_3}^4 < 8.4 \times 10^{-8}$, resolves $\theta = 0, \pi$ degeneracy [S_3]
- Small effect in $(g - 2)_\mu$
- We predict the S_3 contribution to $R_{\nu\nu}^{(*)} = \mathcal{B}(B \rightarrow K^{(*)}\nu\nu) / \mathcal{B}(B \rightarrow K^{(*)}\nu\nu)_{\text{SM}}$ and compare it with $R_{\nu\nu}^{(*)} < 2.7$ [Belle '17]

Flavor coupling analysis

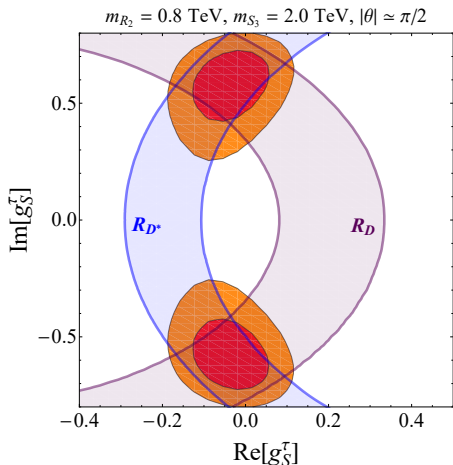
- Perform a fit with fixed $m_{R_2} = 0.8$ TeV, $m_{S_3} = 2$ TeV, variables $y_L^{c\mu}, y_L^{c\tau}$, $\text{Re } y_R^{b\tau}$, $\text{Im } y_R^{b\tau}$, mixing angle θ
- Larger mass m_{S_3} and small $\sin 2\theta$ suppress the effects in neutral currents relative to $R_{D^{(*)}}$
- Degeneracy of minima with $\theta \approx 0, \pi/2$ is broken by $\tau \rightarrow \mu\phi$ which selects $\theta \approx \pi/2$. $\Rightarrow S_3$ couplings to s-quark are suppressed



$$Y_{de} \approx - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_L^{c\mu} & y_L^{c\tau} \\ 0 & 0 & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & y_L^{c\mu} & y_L^{c\tau} \end{pmatrix}$$

Flavor coupling analysis

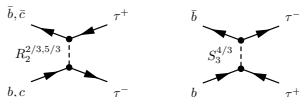
- Complex g_S^T needed in order to reconcile R_D and R_{D^*} . Imaginary g_S^T has strictly positive effect on $R_{D^{(*)}}$.
- SM point is excluded at 3.8σ (5 degrees of freedom)
- Both $R_{D^{(*)}}$ and $R_{K^{(*)}}$ anomalies are reduced to $< 1\sigma$ level



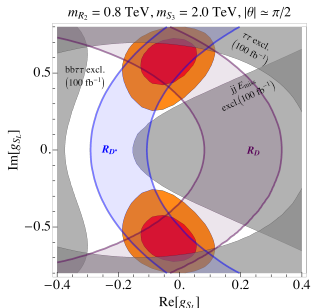
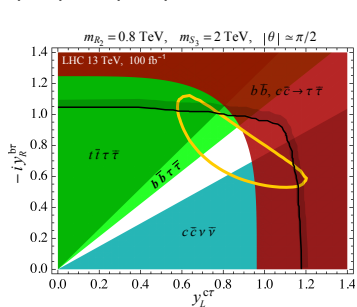
[Becirevic, Dorsner, Fajfer, Farougy, NK, Sumensari, 1806.05689]

LHC constraints

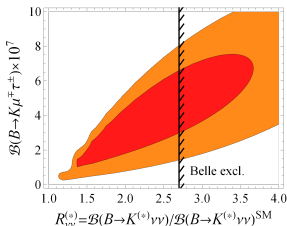
- t -channel $pp \rightarrow \tau^+ \tau^-$. Dominated by R_2 , S_3 only from b -quarks



- Require Yukawa couplings of the two LQs to remain perturbative to the unification scale 5×10^{15} GeV
- Constrained by searches for heavy $\tau\tau$ resonance [ATLAS, JHEP 1801, 055]
- Leptoquark pair production bounds the R_2 mass



- LFV and 2ν modes in reach of future high-intensity experiments
- Correlation between LFV observables



- $\mathcal{B}(B \rightarrow K \nu \bar{\nu}) \gtrsim 1.5 \times \mathcal{B}(B \rightarrow K \nu \bar{\nu})_{SM}$, genuine prediction.
- LFV decays
 - ▶ $\mathcal{B}(\tau \rightarrow \mu \gamma) > 1.5 \times 10^{-8}$
 - ▶ $\mathcal{B}(B \rightarrow K^* \tau \mu) \approx 1.9 \times \mathcal{B}(B \rightarrow K \mu \tau)$
 - ▶ $\mathcal{B}(B_s \rightarrow \tau \mu) \approx 0.9 \times \mathcal{B}(B \rightarrow K \mu \tau)$
 - ▶ All LFV limits will be improved by LHCb and Belle 2

Conclusion

- Presented model with two light leptoquarks, $R_2(3, 2, 7/6)$ and $S_3(\bar{3}, 3, 1/3)$
 - ▶ R_2 accommodates $R_{D^{(*)}}$ via complex Scalar and Tensor couplings
 - ▶ S_3 accommodates $R_{K^{(*)}}$ via real $C_9 = -C_{10}$
- $SU(5)$ GUT framework connects the couplings of R_2 and S_3

- Flavor fit
 - ▶ Light $m_{R_2} = 0.8$ TeV, requires complex coupling y_R^{bT}
 - ▶ Heavier $S_3 = 2.0$ TeV
- Complete resolution of $R_{K^{(*)}}$ and $R_{D^{(*)}}$

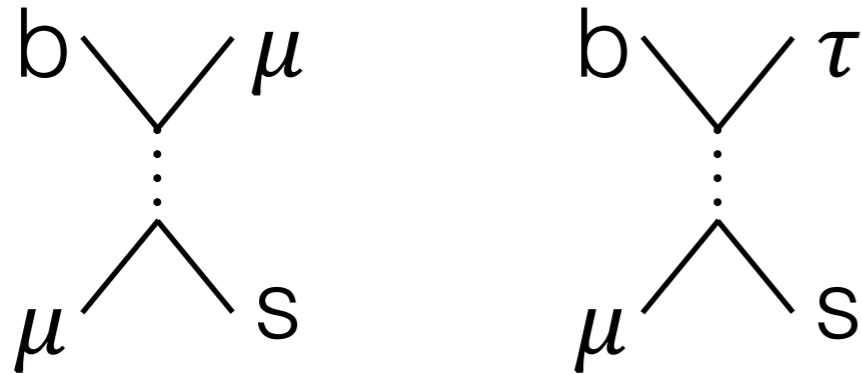
- Well-defined GUT completion, perturbative couplings to high scales
- Model consistent with LHC constraints, preferred region can be probed at HL-LHC

- Predicted enhanced LFV signals in $\tau\mu$ sector as well as enhanced $b \rightarrow s\nu\bar{\nu}$ modes
- Interesting target for future Belle 2 and LHCb searches

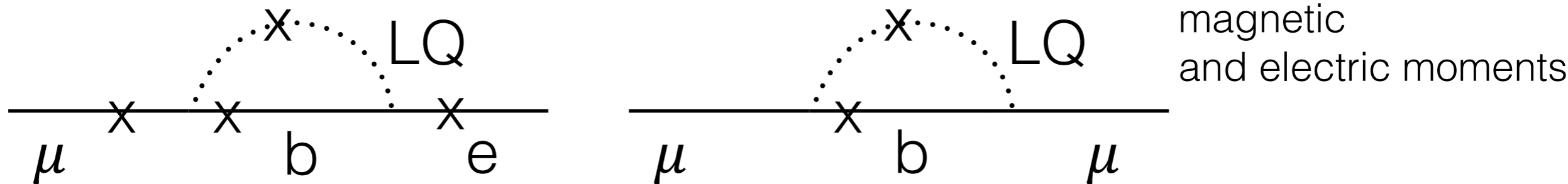
Backup

LQs effects at low energies - overview

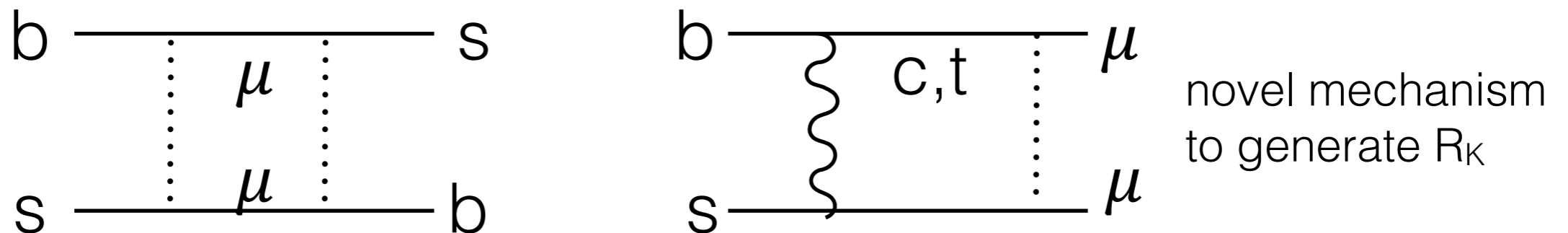
- Tree-level contributions: LL QQ operators



- Penguin-type: radiative processes, electro-magnetic moments



- Box-type: neutral meson mixing, $\tau \rightarrow 3\mu$, LL QQ operators



- For generic flavour pattern LFU and LFV is expected.

Theoretical motivation: Unification of quarks and leptons

□ At unification scale M_{GUT} gauge bosons of SU(5) X and Y couple to quarks and leptons - they are called leptoquark gauge bosons - **vector leptoquarks**

□ Scalar representations contain **scalar leptoquarks**. In order to break SU(5) to SU(3)xSU(2)xU(1) the Higgs mechanism is involved: **24**-dim scalar develops a VEV

$$M_{X,Y} \sim gM_{\text{GUT}}$$

□ EW breaking is achieved with additional **5**-dim scalar

doublet-triplet splitting

$$\mathbf{5} = \underbrace{(1, 2, 1/2)}_H \oplus \underbrace{(3, 1, -1/3)}_{S_1^*}$$

Higgs is a proof of existence of light scalar particles

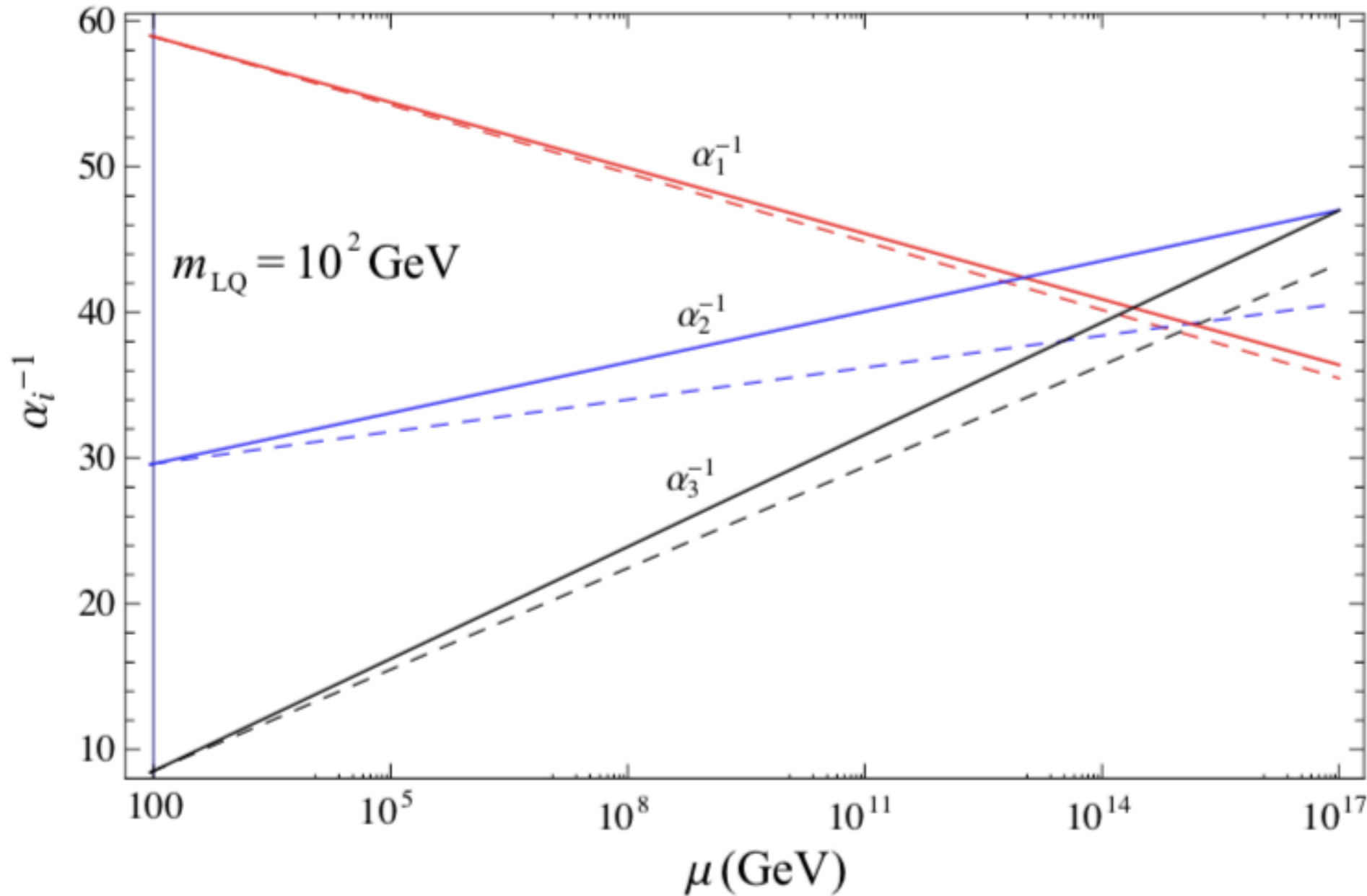
□ To reproduce realistic Yukawa couplings require additional scalar, e.g.,

$$\mathbf{45} = (8, 2, 1/2) \oplus (\bar{\mathbf{6}}, 1, -1/3) \oplus \underbrace{(3, 3, -1/3)}_{S_3} \oplus \underbrace{(\bar{\mathbf{3}}, 2, -7/6)}_{R_2} \oplus (3, 1, -1/3) \oplus (\bar{\mathbf{3}}, 1, 4/3) \oplus \underbrace{(1, 2, 1/2)}_H$$

$$\mathbf{50} = \underbrace{(3, 2, 7/6)}_{R_2} \oplus \dots$$

□ We know the SM Higgs is light, also other scalar LQs can be light. (Beware of proton decay!)

Theoretical motivation: Unification of quarks and leptons



[Dorsner et al, 1603.04993]

Full: SM

Dashed: SM + 3 scalars at 100 GeV:

(1,2,1/2) + (3,2,1/6) + (3,2,1/6)

Higgs-like

leptoquark

leptoquark

$$\alpha_3(m_Z) = 0.1176$$

$$\alpha_{\text{em}}^{-1}(m_Z) = 127.906$$

$$\sin^2 \theta_W(m_Z) = 0.23122$$

[Murayama, Yanagida '92]

Low energy perspective (EW scale)

- Above the weak scale, extend the SM spectrum with LQ fields. Respect $SU(3) \times SU(2) \times U(1)$
- Leptoquark (LQ) is a scalar or vector particle Φ with quantum numbers $(3, T, Y)$, with T and Y such that dim-4 couplings of $\Phi L Q$ are allowed
- “Discover” leptoquarks by writing all possible LQ bilinears of SM fields (easy exercise). Gauge + Lorentz symmetry

$$\bar{L} \gamma^\mu Q \sim (3, 1, -1/3) \rightarrow U_1^\mu$$

$$Q = Y + T_3$$

$$\bar{e}_R \gamma^\mu u_R \sim (3, 1, -1/3) \rightarrow U_1^\mu$$

$$\bar{L} \boldsymbol{\sigma} \gamma^\mu Q \sim (3, 3, -1/3) \rightarrow U_3^\mu$$

$$\bar{L}^c (i\sigma^2) \boldsymbol{\sigma} Q \sim (3, 3, 2/3) \rightarrow S_3$$

$$\bar{L} u_R \sim (3, 2, 7/6) \rightarrow R_2$$

$$\bar{e}_R Q \sim (3, 2, 7/6) \rightarrow R_2$$

⋮

- Vector LQs in general need a UV completion (Higgs mechanism). Some quantities may still be computed without the full theory. [[Crivellin et al.1807.02068](#)]

Baryon and lepton number

- In general, LQs exchange quarks for leptons and violate both B and L. Use freedom to consistently choose B and L numbers of a LQ field.

$$\bar{L}\gamma^\mu Q \sim (3, 1, -1/3) \rightarrow U_1^\mu$$

$$\bar{e}_R\gamma^\mu u_R \sim (3, 1, -1/3) \rightarrow U_1^\mu$$

$$\bar{L}\sigma\gamma^\mu Q \sim (3, 3, -1/3) \rightarrow U_3^\mu$$

$$\bar{L}^c(i\sigma^2)\sigma\gamma^\mu Q \sim (3, 3, 2/3) \rightarrow S_3$$

$$\bar{L}u_R \sim (3, 2, 7/6) \rightarrow R_2$$

$$\bar{e}_R Q \sim (3, 2, 7/6) \rightarrow R_2$$

⋮

← but also $\bar{Q}\sigma i\sigma^2 Q^c$

Double nature of S_3 : leptoquark and diquark!

- $F=3B+L$ fermion number

- $F=0$ LQs have well defined **$B = 1/3, L = -1$** (*genuine leptoquarks*)
- LQs which have diquark nature have $|F|=2$, not possible to assign B and L consistently: $B_{lq} = 1/3, L_{lq} = 1$
 $B_{qq} = -2/3, L_{qq} = 0$
- B-L is still conserved

Baryon and lepton number

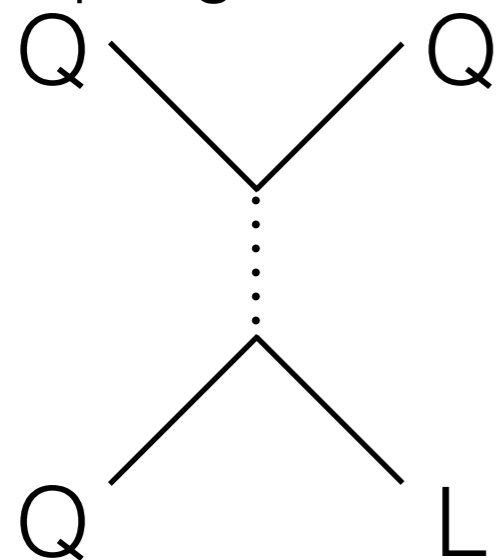
- Proton lifetime from Super Kamiokande

$$\tau(p \rightarrow e^+ \pi^0) > 1.6 \times 10^{34} \text{ years}$$

$$\tau(p \rightarrow \mu^+ \pi^0) > 7.7 \times 10^{33} \text{ years}$$

$$\Delta B = \Delta L = -1$$

- Light LQs cannot have large **lq** and **qq** couplings



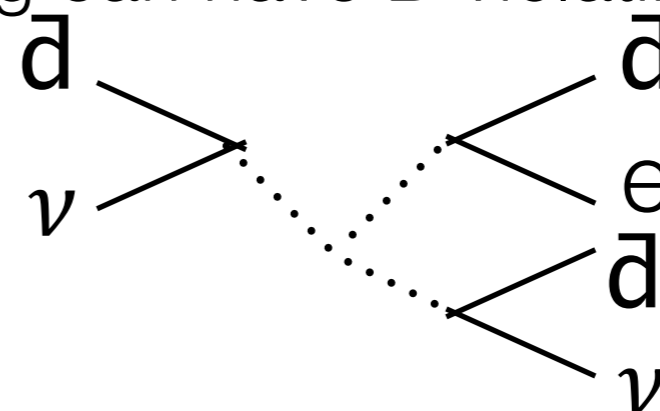
$$\sim \frac{y_{LQ} z_{QQ}}{M_{S_3}^2} (\bar{Q}^c i \sigma^2 \sigma Q) (\bar{L} \sigma i \tau^2 Q)$$

- Even LQs that appear B-conserving can have B-violating terms in the potential. e.g., $\sim R_2 (3, 2, 1/6)$

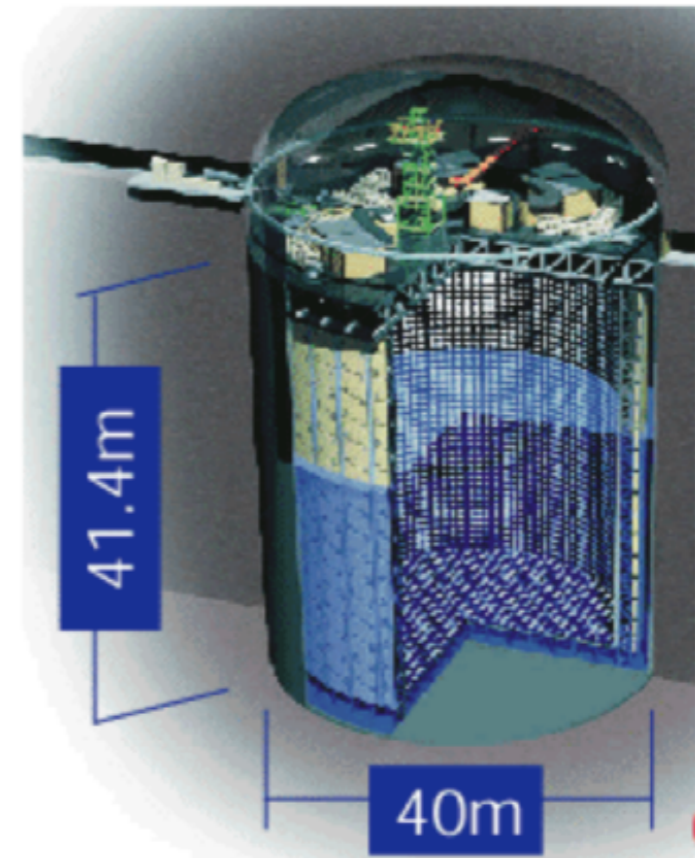
$$\lambda H^* \tilde{R}_2 \tilde{R}_2 \tilde{R}_2$$

$$B = 1, L = -3$$

(ex.: work out in isospin and color space)



$$p \rightarrow \pi^+ \pi^+ e^- \nu \nu$$



Catalogue

| $(SU(3), SU(2), U(1))$ | Spin | Symbol | Type | F |
|--|------|---------------|---|-----|
| $(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$ | 0 | S_3 | $LL(S_1^L)$ | -2 |
| $(\mathbf{3}, \mathbf{2}, 7/6)$ | 0 | R_2 | $RL(S_{1/2}^L), LR(S_{1/2}^R)$ | 0 |
| $(\mathbf{3}, \mathbf{2}, 1/6)$ | 0 | \tilde{R}_2 | $RL(\tilde{S}_{1/2}^L), \overline{LR}(\tilde{S}_{1/2}^L)$ | 0 |
| $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ | 0 | \tilde{S}_1 | $RR(\tilde{S}_0^R)$ | -2 |
| $(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$ | 0 | S_1 | $LL(S_0^L), RR(S_0^R), \overline{RR}(S_0^{\overline{R}})$ | -2 |
| ν_R $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | 0 | \bar{S}_1 | $\overline{RR}(\bar{S}_0^{\overline{R}})$ | -2 |
| $(\mathbf{3}, \mathbf{3}, 2/3)$ | 1 | U_3 | $LL(V_1^L)$ | 0 |
| $(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$ | 1 | V_2 | $RL(V_{1/2}^L), LR(V_{1/2}^R)$ | -2 |
| $(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | 1 | \tilde{V}_2 | $RL(\tilde{V}_{1/2}^L), \overline{LR}(\tilde{V}_{1/2}^R)$ | -2 |
| $(\mathbf{3}, \mathbf{1}, 5/3)$ | 1 | \tilde{U}_1 | $RR(\tilde{V}_0^R)$ | 0 |
| $(\mathbf{3}, \mathbf{1}, 2/3)$ | 1 | U_1 | $LL(V_0^L), RR(V_0^R), \overline{RR}(V_0^{\overline{R}})$ | 0 |
| ν_R $(\mathbf{3}, \mathbf{1}, -1/3)$ | 1 | \bar{U}_1 | $\overline{RR}(\bar{V}_0^{\overline{R}})$ | 0 |

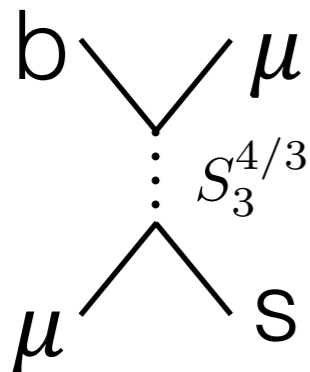
Singlet fermions ν_R included, indicated by $\bar{}$

[Dorsner et al, 1603.04993]

[Buchmueller et al, PLB 177; PLB 191]

Transition to mass basis

□ Consider $b \rightarrow s\mu\mu$ process in the presence of S_3 ($3^*, 3, 1/3$)



Lagrangian in the interaction basis

$$Y^{ij} \bar{Q}'_i{}^C i\tau_2 (\tau_k S_3^k) L'_j$$

$$\rightarrow - (Y)^{ij} \bar{d}_{Li}^C \nu_{Lj} S_3^{\frac{1}{3}} + \sqrt{2} (V^* Y)^{ij} \bar{u}_{Li}^C \nu_{Lj} S_3^{-\frac{2}{3}}$$

$$+ \sqrt{2} Y^{ij} \bar{d}_{Li}^C \ell_{Lj} S_3^{\frac{4}{3}} - (V^* Y)^{ij} \bar{u}_{Li}^C \ell_{Lj} S_3^{\frac{1}{3}} \quad (\text{neutrinos in the flavour basis})$$

Transition to the mass eigenstate of fermions is achieved by unitary rotations

$$\mathcal{L}_{\text{upYukawa}} = -y_{(u)}^{ij} \bar{Q}'^i \tilde{H} u_R'^j$$

$$\rightarrow -\bar{u}'_L \left(\frac{y_{(u)} v}{\sqrt{2}} \right) u'_R$$

$$\rightarrow -\bar{u}_L \underbrace{\left(U_L y_{(u)} U_R^\dagger \right)}_{\text{diag}(y_u, y_c, y_t)} (v/\sqrt{2}) u_R$$

$$u_{L,R} = U_{L,R} u'_{L,R}$$

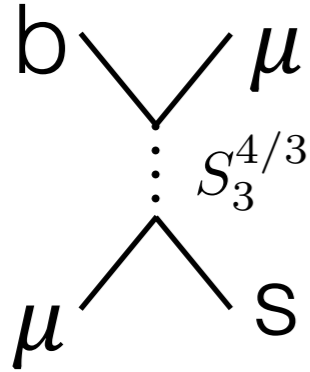
$$V = U_L D_L^\dagger$$

□ LQs interacting with Q doublets have relative CKM rotation between u_L and d_L couplings, e.g.

$$Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Y_{s\mu} & 0 \\ 0 & Y_{b\mu} & 0 \end{pmatrix} \Rightarrow V^* Y = \begin{pmatrix} 0 & V_{us}^* Y_{s\mu} + V_{ub}^* Y_{s\mu} & 0 \\ 0 & V_{cs}^* Y_{s\mu} + V_{cb}^* Y_{s\mu} & 0 \\ 0 & V_{ts}^* Y_{s\mu} + V_{tb}^* Y_{s\mu} & 0 \end{pmatrix}$$

Operator basis

□ Consider $b \rightarrow s\mu\mu$ process in the presence of S_3 ($3^*, 3, 1/3$)



Tree-level matching results in the Fierzed basis

$$\mathcal{A} = \frac{2iY^{b\mu}Y^{s\mu*}}{m_{S_3}^2} (\bar{b}_L^C \mu_L) (\bar{\mu}_L s_L^C) \Rightarrow \mathcal{L}_{\text{eff}} = \frac{2Y^{b\mu}Y^{s\mu*}}{m_{S_3}^2} (\bar{b}_L^C \mu_L) (\bar{\mu}_L s_L^C)$$

n.b.

1. Fierz identities for fields
2. Bilinear transpose to get rid of C operators

$$\psi^c \equiv C\bar{\psi}^T, \quad \bar{\psi}^c = -\psi^T C^\dagger$$

$$C^\dagger = C^{-1}, \quad C^T = -C$$

$$\gamma^{\mu T} = -C^\dagger \gamma^\mu C$$

$$= -\frac{Y^{b\mu}Y^{s\mu*}}{m_{S_3}^2} (\bar{b}_L^C \gamma^\mu s_L^C) (\bar{\mu}_L \gamma_\mu \mu_L)$$

$$= \frac{Y^{b\mu}Y^{s\mu*}}{m_{S_3}^2} (\bar{s}_L \gamma^\mu b_L) (\bar{\mu}_L \gamma_\mu \mu_L)$$

□ Effective Hamiltonian for $b \rightarrow s\mu\mu$

$$\mathcal{H}_{\text{eff}}^{b \rightarrow sll} = -\frac{4G_F \lambda_t}{\sqrt{2}} \sum_{i=7,9,10} C_i(\mu) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\mu b_L) (\bar{l} \gamma^\mu (\gamma^5) l)$$

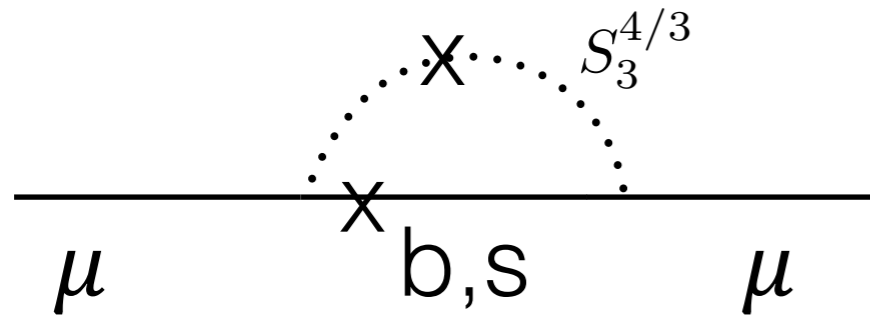
$$\delta C_9^{\mu\mu} = -\delta C_{10}^{\mu\mu} = \frac{\pi v^2}{\lambda_t \alpha_{\text{em}}} \frac{Y^{b\mu}Y^{s\mu*}}{m_{S_3}^2}$$

Simplest V-A tree-level attempt to solve R_K

Many related flavor violating processes depend on the combination of Y's

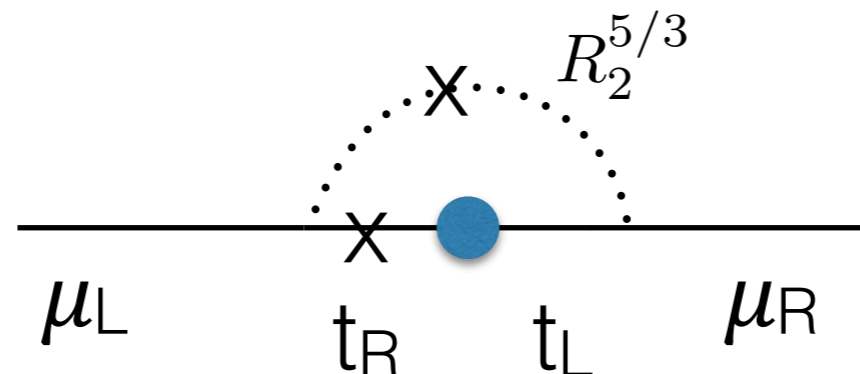
g-2 in simple S_3 model

□ Penguin diagram with same couplings



$$\delta a_\mu = -\frac{3m_\mu^2}{32\pi^2 m_{S_3}^2} (|Y^{s\mu}|^2 + |Y^{b\mu}|^2)$$

- Wrong sign contribution! Constrains individual Yukawas
- Chiral LQs contributions are mass suppressed ($\bar{\mu}_R \sigma^{\mu\nu} q_\nu \mu_L$) and with fixed sign
- Non-chiral LQs allow for freedom in sign and enhancement with quark mass! E.g. R_2



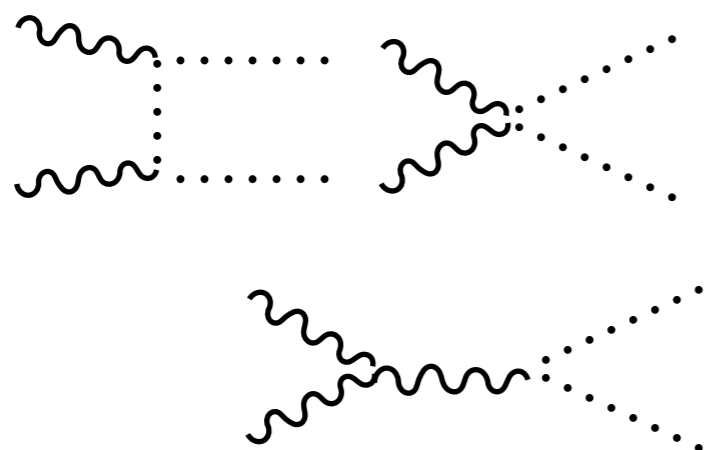
recall!

$$\bar{L}u_R \sim (3, 2, 7/6) \rightarrow R_2$$

$$\bar{e}_R Q \sim (3, 2, 7/6) \rightarrow R_2$$

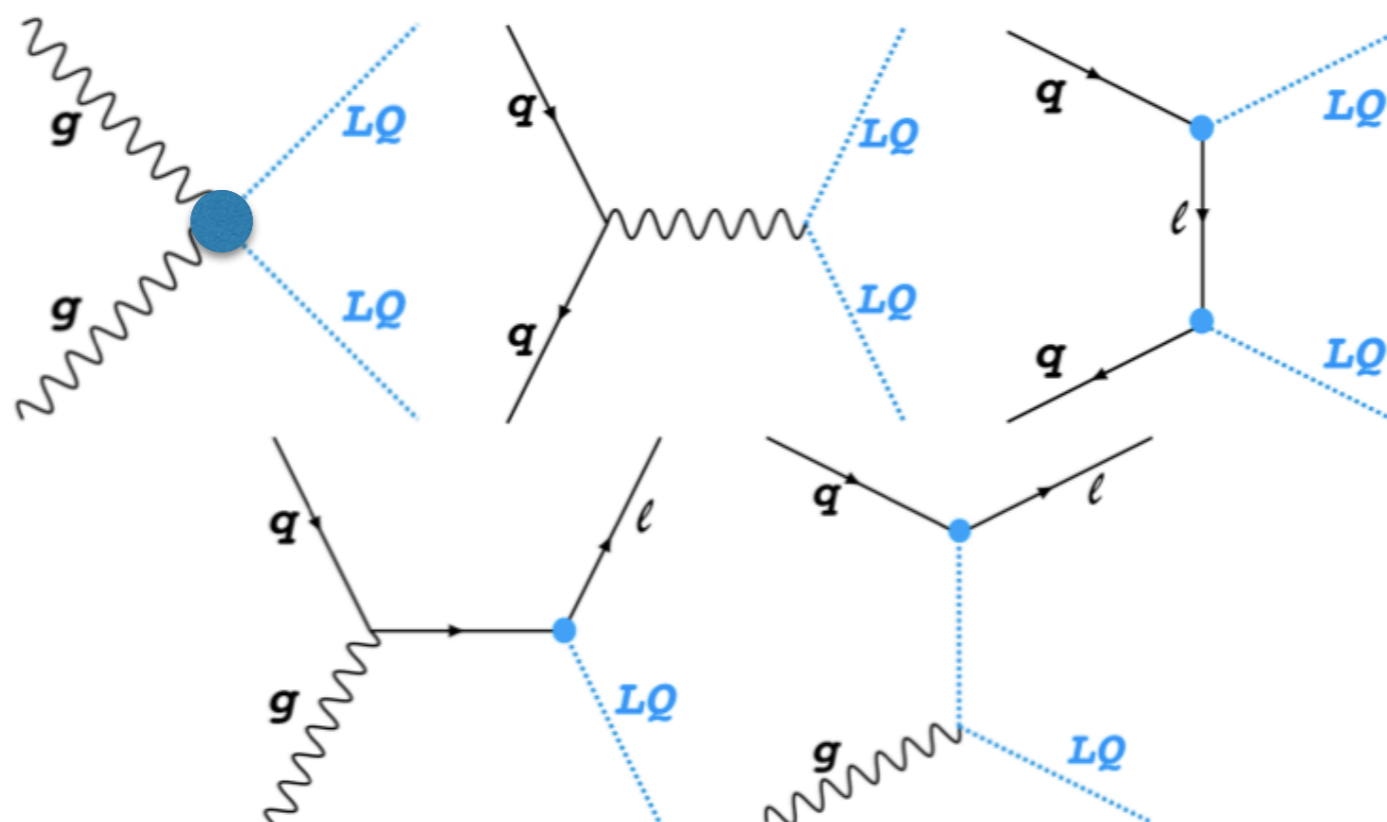
Direct searches at the LHC

- LQs are color triplet states - model independent production cross section



$$\hat{\sigma}[g g \rightarrow LQ \bar{LQ}] = \frac{\alpha_3^2 \pi}{96 \hat{s}} \left[\beta(41 - 31\beta^2) + (18\beta^2 - \beta^4 - 17) \log \frac{1 + \beta}{1 - \beta} \right]$$

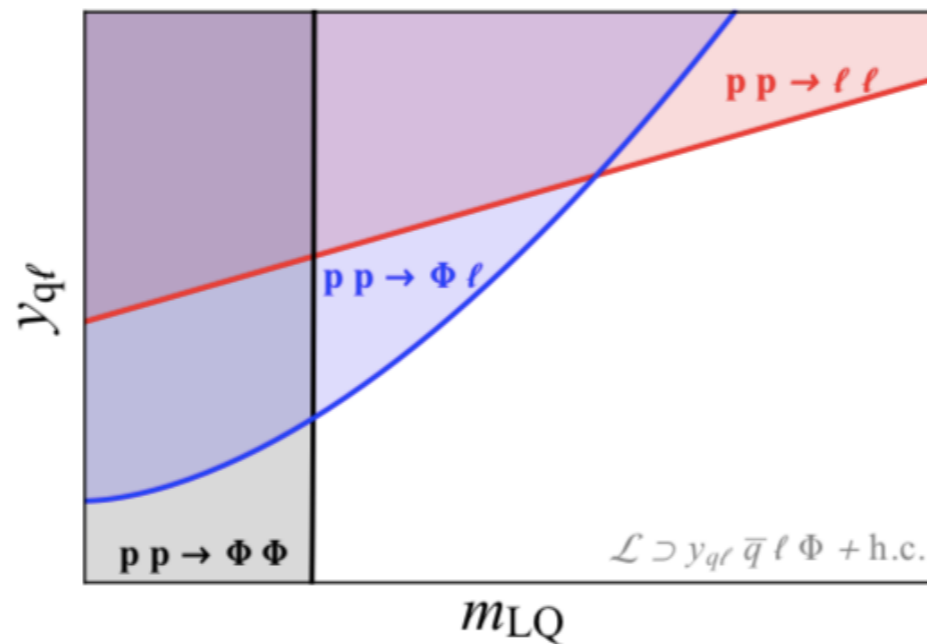
- Need large energies to produce a heavy pair of LQs
- With large Yukawa couplings also single LQ can be produced



[Dorsner, Greljo, 1801.07641]

Direct searches at the LHC

- LQs produced decay to jets+leptons or jets+neutrinos

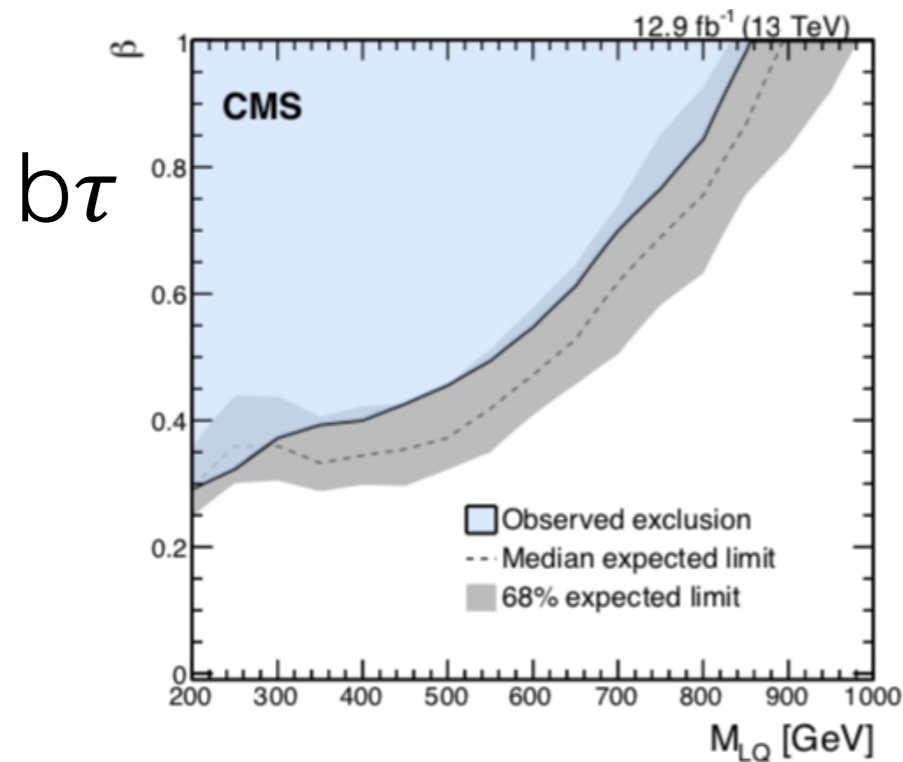


[Dorsner, Greljo, 1801.07641]

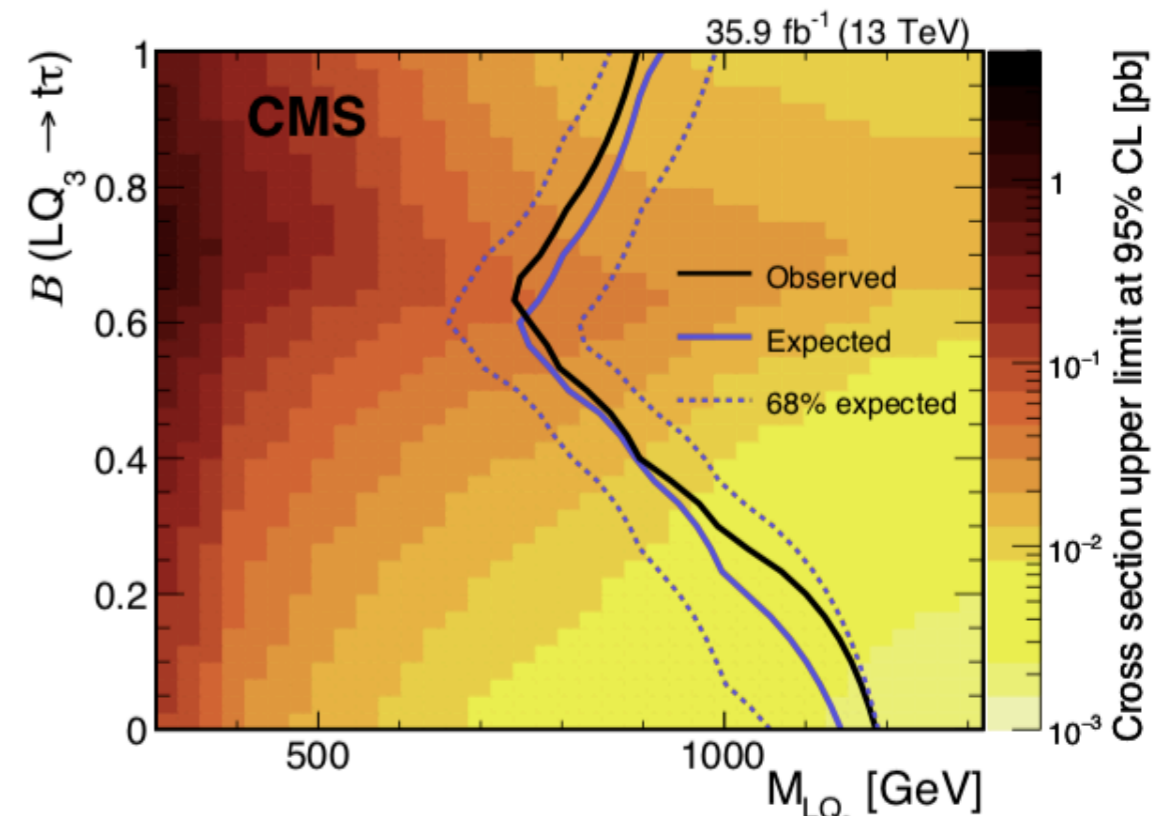
$$\Gamma_S = \frac{|y|^2}{16\pi} m_{LQ}$$

(assume prompt decay)

- Current bounds from pair-production (3rd gen. LQ), $M_{LQ} > 1$ TeV



[CMS 1703.03995]

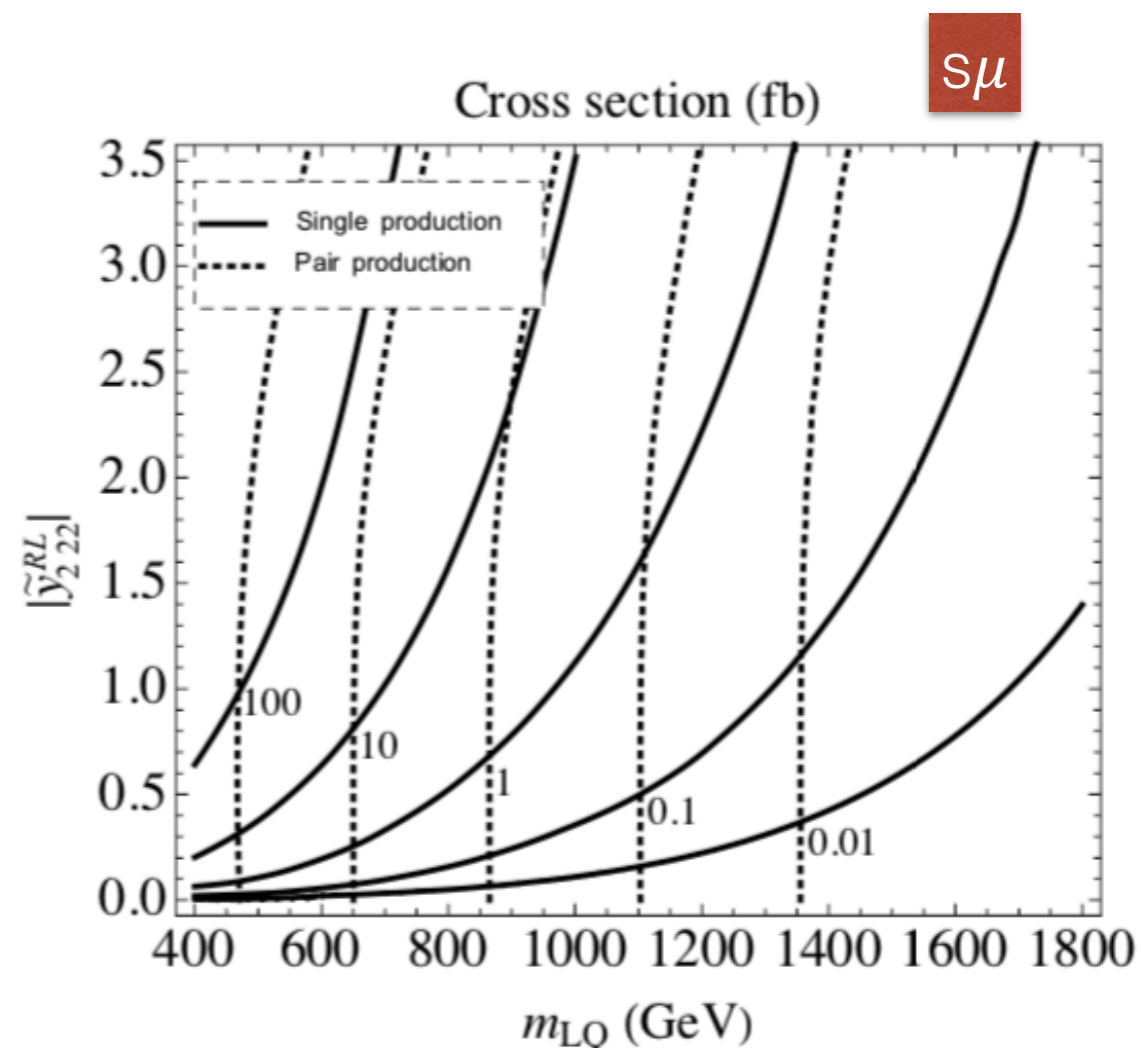
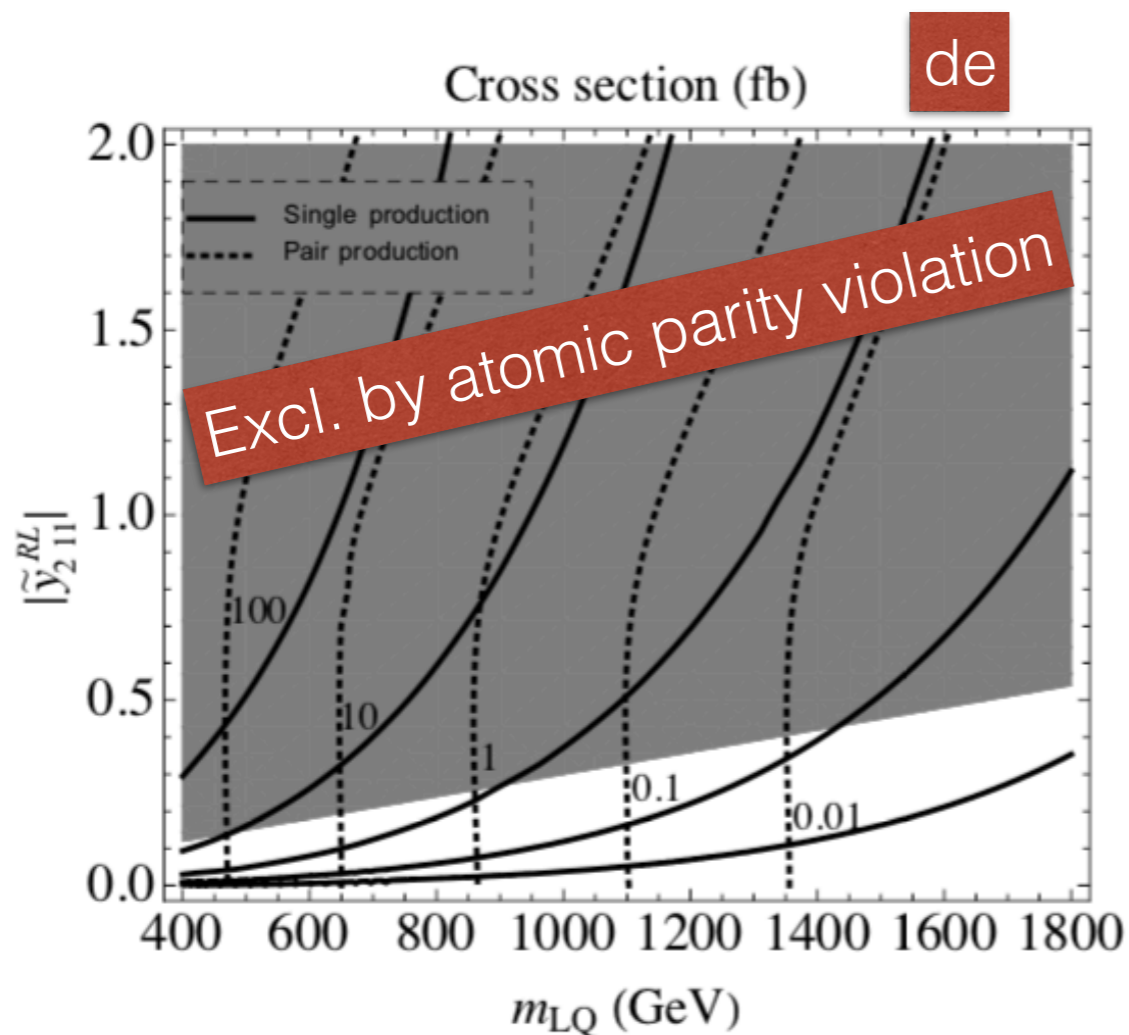


[CMS 1803.02864]

Interplay of low and high energy processes

$$\sigma_{\text{pair}}(y_i, m_{\text{LQ}}) = a_0(m_{\text{LQ}}) + a_2(m_{\text{LQ}})|y_i|^2 + a_4(m_{\text{LQ}})|y_i|^4,$$

$$\sigma_{\text{single}}(y_i, m_{\text{LQ}}) = a(m_{\text{LQ}})|y_i|^2,$$



$$\sqrt{s} = 8 \text{ TeV}$$

leading order σ

$$-\bar{d}_R \tilde{y}_2^{RL} \tilde{R}_2^T i\sigma^2 L$$

..... pair production
—— single LQ production

APV and LFV in light generations

Atomic parity violation (in ^{133}Cs)

$$\mathcal{L}_{PV} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d} \bar{e} \gamma^\mu \gamma^5 e (\delta C_{1u} \bar{u} \gamma_\mu u + \delta C_{1d} \bar{d} \gamma_\mu d) \quad \delta C_{1q} = c_{qq;ee}^{LL} - c_{qq;ee}^{LR} + c_{qq;ee}^{RL} - c_{qq;ee}^{RR}$$

Experimentally: $\delta Q_W(Z, N) = -2(2Z + N)\delta C_{1u} - 2(Z + 2N)\delta C_{1d} \Rightarrow |\delta C_{1u(d)}| \lesssim 10^{-3}$
 $Q_W - Q_W^{SM} \equiv \delta Q_W = 0.65(43)$

| LQ | $d_j \rightarrow d_i \ell^- \ell'^+$ decays | $u_j \rightarrow u_i \ell^- \ell'^+$ decays |
|---------------|---|--|
| S_3 | $c^{LL} = -\frac{v^2}{2m_{LQ}^2} y_{3j\ell'}^{LL} y_{3i\ell}^{LL*}$ | $c^{LL} = -\frac{v^2}{2m_{LQ}^2} (V^T y_3^{LL})_{j\ell'} (V^T y_3^{LL})_{i\ell}^*$ |
| R_2 | $c^{LR} = \frac{v^2}{4m_{LQ}^2} y_{2\ell_j}^{LR} y_{2\ell'i}^{LR*}$ | $c^{LR} = \frac{v^2}{4m_{LQ}^2} (y_2^{LR} V^\dagger)_{\ell_j} (y_2^{LR} V^\dagger)_{\ell'i}^*$ $c^{RL} = \frac{v^2}{4m_{LQ}^2} y_{2i\ell'}^{RL} y_{2j\ell}^{RL*}$ $g^{LL} = 4h^{LL} = -\frac{v^2}{4m_{LQ}^2} y_{2j\ell}^{RL*} (y_2^{LR} V^\dagger)_{\ell'i}^*$ $g^{RR} = 4h^{RR} = -\frac{v^2}{4m_{LQ}^2} y_{2i\ell'}^{RL} (y_2^{LR} V^\dagger)_{\ell_j}$ |
| \tilde{R}_2 | $c^{RL} = \frac{v^2}{4m_{LQ}^2} \tilde{y}_{2i\ell'}^{RL} \tilde{y}_{2j\ell}^{RL*}$ | |
| \tilde{S}_1 | $c^{RR} = -\frac{v^2}{4m_{LQ}^2} \tilde{y}_{1j\ell'}^{RR} \tilde{y}_{1i\ell}^{RR*}$ | |
| S_1 | | $c^{LL} = -\frac{v^2}{4m_{LQ}^2} (V^T y_1^{LL})_{j\ell'} (V^T y_1^{LL})_{i\ell}^*$ $c^{RR} = -\frac{v^2}{4m_{LQ}^2} y_{1j\ell'}^{RR} y_{1i\ell}^{RR*}$ $g^{LL} = -4h^{LL} = \frac{v^2}{4m_{LQ}^2} y_{1j\ell'}^{RR} (V^T y_1^{LL})_{i\ell}^*$ $g^{RR} = -4h^{RR} = \frac{v^2}{4m_{LQ}^2} (V^T y_1^{LL})_{j\ell'} y_{1i\ell}^{RR*}$ |

APV and LFV in light generations

μ -e conversion in nuclei (Au, Ti)

$$\Gamma_{\text{conversion}} = 2G_F^2 \left| A_R^* D + (2g_{LV}^{(u)} + g_{LV}^{(d)})V^{(p)} + (g_{LV}^{(u)} + 2g_{LV}^{(d)})V^{(n)} \right. \\ \left. + (G_S^{(u,p)} g_{LS}^{(u)} + G_S^{(d,p)} g_{LS}^{(d)} + G_S^{(s,p)} g_{LS}^{(s)})S^{(p)} \right. \\ \left. + (G_S^{(u,n)} g_{LS}^{(u)} + G_S^{(d,n)} g_{LS}^{(d)} + G_S^{(s,n)} g_{LS}^{(s)})S^{(n)} \right|^2 \\ + (L \leftrightarrow R),$$

nuclear matrix elements

[Kitano et al, hep-ph/0203110]

$$\text{BR}_{\mu e}^{(\text{Ti})} < 4.3 \times 10^{-12}$$

[SINDRUM]

$$\text{BR}_{\mu e}^{(\text{Au})} < 7 \times 10^{-13}$$



$$c_{qq,e\mu}^{XY} \lesssim 10^{-8}$$

$$+ \text{Br}(\mu \rightarrow e\gamma) < 4.2 \cdot 10^{-13}$$

[MEG]

$$+ Z \rightarrow ee$$

[LEP]

$$+ \text{EWPO}$$



LQs in e, d, u are extremely well constrained

| LQ | $d_j \rightarrow d_i \ell^- \ell'^+$ decays | $u_j \rightarrow u_i \ell^- \ell'^+$ decays |
|---------------|---|---|
| S_3 | $c^{LL} = -\frac{v^2}{2m_{LQ}^2} y_{3j\ell'}^{LL} y_{3i\ell}^{LL*}$ | $c^{LL} = -\frac{v^2}{2m_{LQ}^2} (V^T y_3^{LL})_{j\ell'} (V^T y_3^{LL})_{i\ell}^*$ |
| R_2 | $c^{LR} = \frac{v^2}{4m_{LQ}^2} y_{2\ell_j}^{LR} y_{2\ell'_i}^{LR*}$ | $c^{LR} = \frac{v^2}{4m_{LQ}^2} (y_2^{LR} V^\dagger)_{\ell_j} (y_2^{LR} V^\dagger)_{\ell'_i}^*$ |
| | | $c^{RL} = \frac{v^2}{4m_{LQ}^2} y_{2i\ell'}^{RL} y_{2j\ell}^{RL*}$ |
| | | $g^{LL} = 4h^{LL} = -\frac{v^2}{4m_{LQ}^2} y_{2j\ell}^{RL*} (y_2^{LR} V^\dagger)_{\ell'_i}^*$ |
| | | $g^{RR} = 4h^{RR} = -\frac{v^2}{4m_{LQ}^2} y_{2i\ell'}^{RL} (y_2^{LR} V^\dagger)_{\ell_j}$ |
| \tilde{R}_2 | $c^{RL} = \frac{v^2}{4m_{LQ}^2} \tilde{y}_{2i\ell'}^{RL} \tilde{y}_{2j\ell}^{RL*}$ | |
| \tilde{S}_1 | $c^{RR} = -\frac{v^2}{4m_{LQ}^2} \tilde{y}_{1j\ell'}^{RR} \tilde{y}_{1i\ell}^{RR*}$ | |
| S_1 | | $c^{LL} = -\frac{v^2}{4m_{LQ}^2} (V^T y_1^{LL})_{j\ell'} (V^T y_1^{LL})_{i\ell}^*$ |
| | | $c^{RR} = -\frac{v^2}{4m_{LQ}^2} y_{1j\ell'}^{RR} y_{1i\ell}^{RR*}$ |
| | | $g^{LL} = -4h^{LL} = \frac{v^2}{4m_{LQ}^2} y_{1j\ell'}^{RR} (V^T y_1^{LL})_{i\ell}^*$ |
| | | $g^{RR} = -4h^{RR} = \frac{v^2}{4m_{LQ}^2} (V^T y_1^{LL})_{j\ell'} y_{1i\ell}^{RR*}$ |

[Dorsner et al, 1603.04993]