# Uncovering structure of a nucleon in hard electro- and neutrino-production processes

Lech Szymanowski

National Centre for Nuclear Research (NCBJ) Warsaw

Getting to Grips with QCD - Summer Edition workshop
18-22 September 2018, Primosten, Croatia

in collaboration with:

B. Pire (CPhT, Ecole Polytechnique, Palaiseau), J. Wagner (NCNR, Warsaw)

Content:

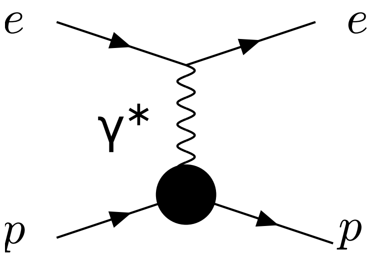
Introduction & DVCS & TCS & DDVCS & UPC:

M. Diehl - Phys.Rept. 388 (2003),
M. Guidal, H. Moutarde, M. Vanderhaeghen - Rept.Prog.Phys. 76 (2013),
P. Kroll, H. Moutarde, F. Sabatie - Phys. Rev. D87 (2013),
E.-C. Aschenauer, S. Fazio, K. Kumericki, D. Mueller-JHEP 1309 (2013) 093

Neutrino-production of a charmed ($D$ or $D^*$) and light ($\pi$) meson:

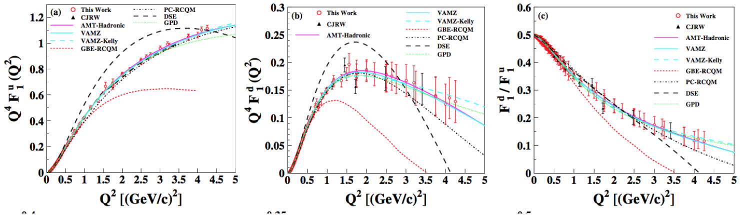
B. Pire, L.Sz. PRL 115 (2015)
B. Pire, L. Sz, J. Wagner Phys. Rev. D 95 (2017) 094001 & 114029
B. Pire, L. Sz Phys. Rev. D 96 (2017) 114008

# Elastic Scattering $e\,p \to e\,p$



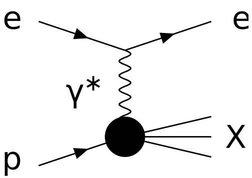

$$\langle p'|\, J^\mu(0)\, |p\rangle = \bar{u}(p')\left[ F_1(t)\,\gamma^\mu + F_2(t)\,\frac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m}\right] u(p),$$

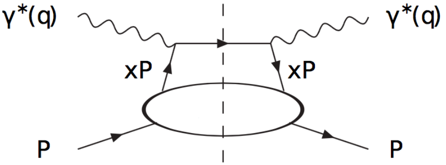
# Elastic Scattering $e\,p \to e\,p$




Up-and down-quark contributions to the nucleon electromagnetic form ...
arxiv.org/pdf/1701.01662 Quattan et al

# Deep Inelastic Scattering $e\,p \to e\,X$

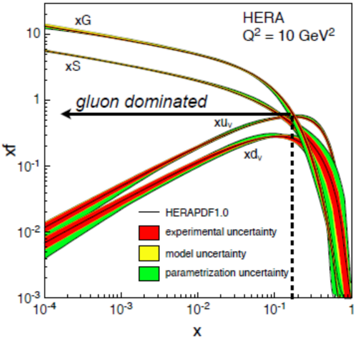


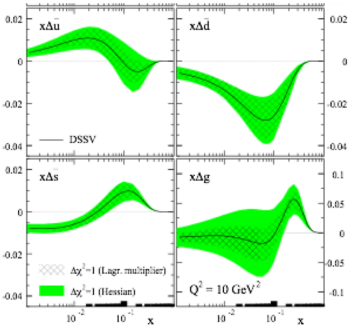




In the Björken limit i.e. when the photon virtality $Q^2 = -q^2$ and the squared hadronic c.m. energy $(p+q)^2$ become large, with the ratio $x_B = \frac{Q^2}{2p\cdot q}$ fixed, the cross section factorizes into a hard partonic subprocess calculable in the perturbation theory, and a parton distributions.

Collinear factorization in QCD

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\,\langle p|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+ q(\frac{1}{2}z)\,|p\rangle\Big|_{z^+=0,\,\mathbf{z}=0} = q(x)\,\bar{u}(p')\gamma^+ u(p)$$

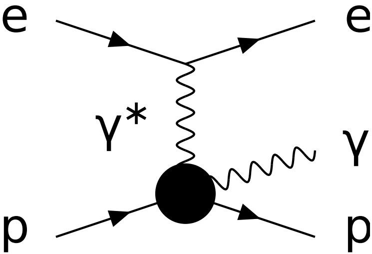



from EIC at Quarkonium 2016
https://indico.hep.pnnl.gov/event/0/session/24/contribution/87/material/slides/0.pdf

# DVCS

The simplest and best known process is Deeply Virtual Compton Scattering: $e\,p \to e\,p\,\gamma$




$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\frac{1}{2}z)\, \gamma^+ q(\frac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, \mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[ H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right],$$

# DVCS

Factorization into GPDs and perturbative coefficient function - on the level of amplitude.

$$\begin{aligned} \text{DIS}: \quad & \sigma = \text{PDF} \otimes \text{partonic cross section} \\ \text{DVCS}: \quad & \mathcal{M} = \text{GPD} \otimes \text{partonic amplitude} \end{aligned}$$

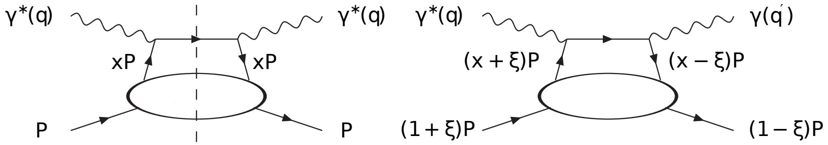



Figure: Deep Inelastic Scattering cross section is given by the imaginary part of diagram (a). Amplitude of Deeply Virtual Compton Scattering is given by diagram (b).

$$W^{\mu\nu} \sim \Im T^{\mu\nu}$$

# Symmetric variables

$$P = \frac{p + p'}{2} \quad , \quad \bar{q} = \frac{q + q'}{2}$$

Generalized Bjorken variable:

$$\xi = \frac{-\bar{q}^2}{2\bar{q} \cdot P} \approx \frac{x_B}{2 - x_B} \quad , \quad x_B = \frac{Q^2}{2q \cdot p}$$

momentum transfer between proton initial and final state:

$$t = (p' - p)^2$$

In the convenient reference frame, where P has only positive time- and z-components, and light vector are defined as:

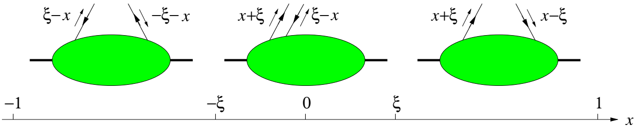
$$v_+ = (1, 0, 0, 1)\frac{1}{\sqrt{2}} \quad , \quad v_- = (1, 0, 0, -1)\frac{1}{\sqrt{2}}$$

$(-2\xi)$ has an interpretation of the fraction of momentum transport in "+" direction.

# GPD definition.

$$\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+ q(\frac{1}{2}z)\,|p\rangle\Big|_{z^+=0,\,\mathbf{z}=0} \\
&= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right], \\
F^g &= \frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\,G^{+\mu}(-\frac{1}{2}z)\,G_\mu{}^+(\frac{1}{2}z)\;|p\rangle\Big|_{z^+=0,\,\mathbf{z}=0} \\
&= \frac{1}{2P^+}\left[H^g(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) + E^g(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],
\end{aligned}$$

- interpretation, ERBL, DGLAP




- Factorization scale dependance,
- Three variables $x, \xi, t$ .

# GPD - properties,

- Forward limit:

$$H^q(x,0,0) = q(x)\,, \quad \text{for} \quad x>0\,,$$
$$H^q(x,0,0) = -\bar{q}(x)\,, \quad \text{for} \quad x<0\,,$$
$$H^g(x,0,0) = xg(x)\,,$$

similarly for polarized disributions and PDFs.

- Reduction to form factors:

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t),$$

where the Dirac and Pauli form factors

$$\langle p'|\,\bar{q}(0)\gamma^\mu q(0)\,|p\rangle = \bar{u}(p')\left[F_1^q(t)\,\gamma^\mu + F_2^q(t)\,\frac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m}\right]u(p),$$

- Ji sum rule:

$$\lim_{t\to 0}\int_{-1}^{1} dx\; x\,[H_f(x,\xi,t)+E_f(x,\xi,t)] = 2J_f$$

where $J_f$ is fraction of the proton spin carried by quark $f$ (including spin and orbital angular momentum).

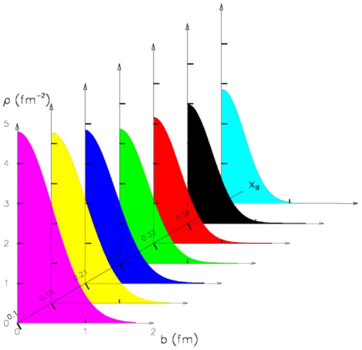
# Impact parameter representation

M. Burkardt PRD 62 (2000)

At $\xi = 0 \quad \Rightarrow \quad -t = \Delta_\perp^2$ :

$$H(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} H(x, 0, -\mathbf{\Delta}_\perp)$$

can be interpreted as probability of finding a parton with longitudinal momentum fraction $x$ at a given $\mathbf{b}_\perp$.

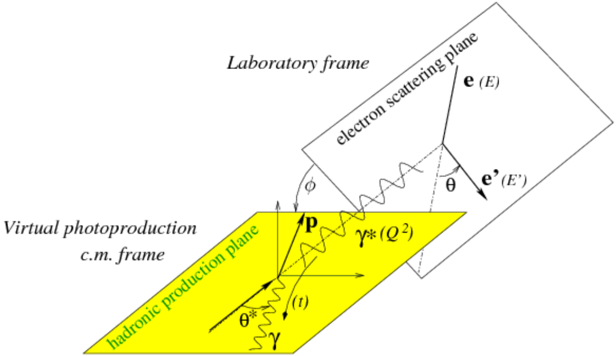


# GPDs

- GPDs enter factorization theorems for hard exlusive reactions (DVCS, deeply virtual meson production, TCS etc.), in a similar manner as PDFs enter factorization theorems for inclusive (DIS, etc.)
- GPDs are functions of $x, t, \xi, \mu_F^2$
- First moment of GPDs enters the Ji's sum rule for the angular momentum carried by partons in the nucleon,
- 2+1 imaging of nucleon,
- Deeply Virtual Compton Scattering (DVCS) is a golden channel for GPDs extraction,

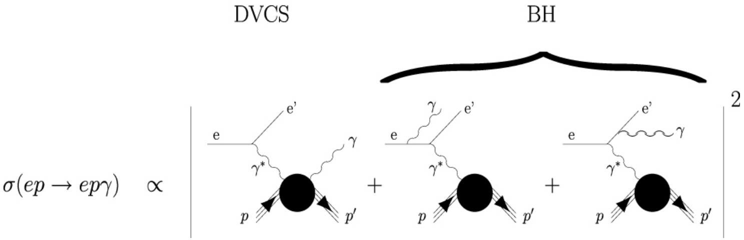
# DVCS - variables

Four variables needed to describe $ep \longrightarrow ep\gamma$ at fixed beam energy. Usually : $Q^2, x_B, t$ and $\phi$:



# DVCS and BH

Figure 12 from Michel Guidal et al 2013 Rep. Prog. Phys. 76 066202



# Observables

The $lp \to lp\gamma$ cross section on an unpolarized target for a given beam charge, $e_l$ in units of the positron charge and beam helicity $h_l/2$ can be written as :

$$d\sigma^{h_l,e_l}(\phi) = d\sigma_{\mathrm{UU}}(\phi)\left[1 + h_l A_{\mathrm{LU,DVCS}}(\phi) + e_l h_l A_{\mathrm{LU,I}}(\phi) + e_l A_{\mathrm{C}}(\phi)\right] ,$$

If both longitudinally polarized positively and negatively charged beams are available (HERMES):

$$A_{\mathrm{C}}(\phi) = \frac{1}{4 d\sigma_{\mathrm{UU}}(\phi)}\left[(d\sigma^{\overset{+}{\rightarrow}} + d\sigma^{\overset{+}{\leftarrow}}) - (d\sigma^{\overset{-}{\rightarrow}} + d\sigma^{\overset{-}{\leftarrow}})\right] . \tag{1}$$

$$A_{\mathrm{LU,I}}(\phi) = \frac{1}{4 d\sigma_{\mathrm{UU}}(\phi)}\left[(d\sigma^{\overset{+}{\rightarrow}} - d\sigma^{\overset{+}{\leftarrow}}) - (d\sigma^{\overset{-}{\rightarrow}} - d\sigma^{\overset{-}{\leftarrow}})\right] , \tag{2}$$

$$A_{\mathrm{LU,DVCS}}(\phi) = \frac{1}{4 d\sigma_{\mathrm{UU}}(\phi)}\left[(d\sigma^{\overset{+}{\rightarrow}} - d\sigma^{\overset{+}{\leftarrow}}) + (d\sigma^{\overset{-}{\rightarrow}} - d\sigma^{\overset{-}{\leftarrow}})\right] . \tag{3}$$

If an experiment only has access to one value of $e_l$ such as in Jefferson Lab, one can only measure the beam spin asymmetry $A_{\mathrm{LU}}^{e_l}$

$$A_{\mathrm{LU}}^{e_l}(\phi) = \frac{d\sigma^{\overset{e_l}{\rightarrow}} - d\sigma^{\overset{e_l}{\leftarrow}}}{d\sigma^{\overset{e_l}{\rightarrow}} + d\sigma^{\overset{e_l}{\leftarrow}}} , \tag{4}$$

# Models & Fits

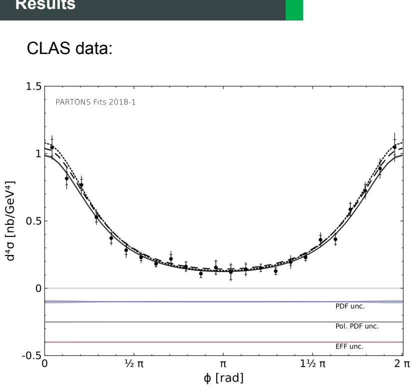
Topic for another seminar...

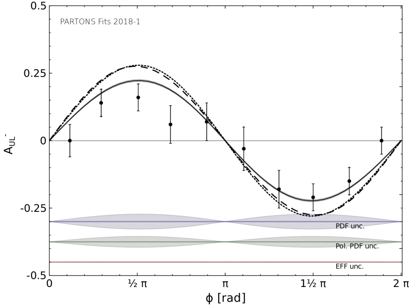
- A lot of data, but not enough to fit 4 GPDs (function of 3 variables) for every quark flavour ... and gluons
- GPDs must satisfy certain principles
- Few models on the market (Goloskokov-Kroll, VGG, Kumericki-Mueller ...), most of them describe data well (small problems with Hall A), only one describes all data - including small $x$.
- still much more data needed to determine GPDs (mostly the imaginary part of CFF $H$ determined)
- PARTONS - modern platform devoted to study GPDs (Herve Moutarde, Saclay)

# DATA vs models

## Results

CLAS data:



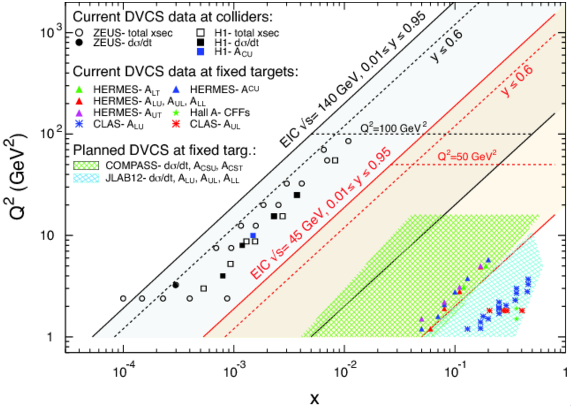




Phys. Rev. Lett. 115(21), 212003 (2015)
$x_{Bj} = 0.244,\ t = -0.15\ \mathrm{GeV}^2,\ Q^2 = 1.79\ \mathrm{GeV}^2$

Phys. Rev. D91(5), 052014 (2015)
$x_{Bj} = 0.257,\ t = -0.23\ \mathrm{GeV}^2,\ Q^2 = 2.02\ \mathrm{GeV}^2$

# FUTURE

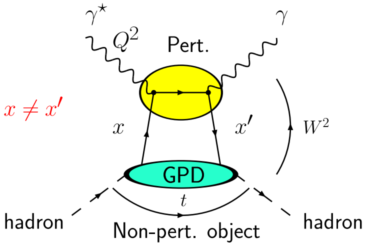
- JLAB - 12 GeV . Plans for Hall A and CLAS to measure beam spin and target spin asymmetries with much higher luminosity, smaller $x_B$ and higher $Q^2$. Also CLAS plan to measure DVCS on neutron - necessary to make GPD flavour separation.
- COMPASS - recoil detector to ensure exclusivity - plans to measure mixed charge-spin asymmetries with $160$GeV muon beam.
- EIC (!)

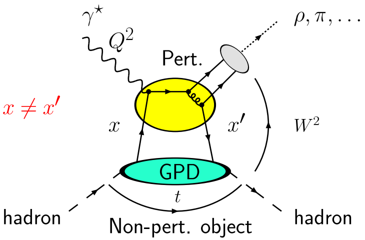


# DVCS - what else, and why

- Difficult: exlusivity, 3 variables, GPD enter through convolutions, only GPD$(\xi, \xi, t)$ accesible through DVCS at LO!
- universality,
- flavour separation,

# Processes




- Meson production - additional information (and difficulties),

# So, in addition to spacelike DVCS ...

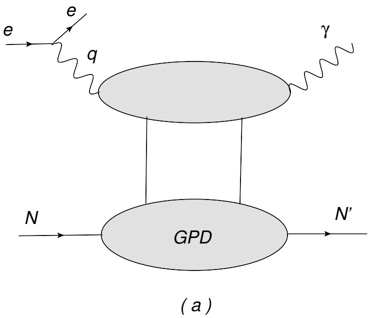



Figure: Deeply Virtual Compton Scattering (DVCS) : $lN \to l'N'\gamma$

# we can also study timelike DVCS

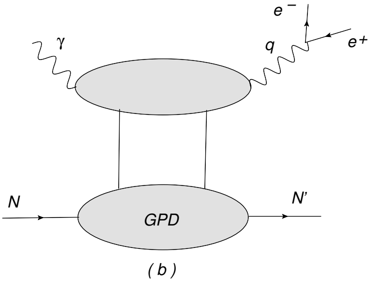



Figure: Timelike Compton Scattering (TCS): $l^- N \to l^- N' l'^+ l'^-$

Why TCS:

- universality of the GPDs
- another source for GPDs (special sensitivity on real part of GPD $H$),
- spacelike-timelike crossing,
- first step towards DDCVS,

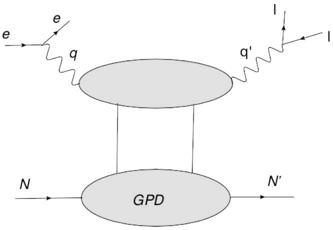



Figure: Double Deeply Virtual Compton Scattering (DDVCS): $\gamma N \to l^+ l^- N'$

$$\gamma^*(q_{in}) N(p) \to \gamma^*(q_{out}) N'(p')$$

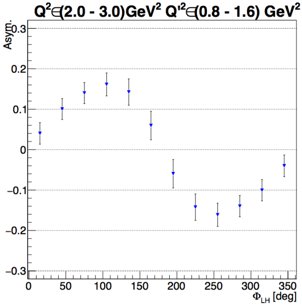
variables, describing the processes of interest in this generalized Bjorken limit, are the scaling variable $\xi$ and skewness $\eta > 0$:

$$\xi = -\frac{q_{out}^2 + q_{in}^2}{q_{out}^2 - q_{in}^2}\eta\,, \quad \eta = \frac{q_{out}^2 - q_{in}^2}{(p+p')\cdot(q_{in}+q_{out})}\,.$$

- DDVCS: $q_{in}^2 < 0\,, \quad q_{out}^2 > 0\,, \quad \eta \neq \xi$
- DVCS: $q_{in}^2 < 0\,, \quad q_{out}^2 = 0\,, \quad \eta = \xi > 0$
- TCS: $q_{in}^2 = 0\,, \quad q_{out}^2 > 0\,, \quad \eta = -\xi > 0$

# Remark on DDVCS

A DDVCS experiment using a modified CLAS12 detector and 11 GeV electron beam in Hall-B was introduced as a letter of intent to Jefferson Lab PAC44




Projected statistical uncertainties, based on Bethe- Heitler cross section, on the beam spin asymetry calculated from the VGG model, for one example of $Q'^2$ bin in the proposed CLAS12 DDVCS experiment

# Coefficient functions and Compton Form Factors for DDVCS

CFFs are the GPD dependent quantities which enter the amplitudes and now depend on two variables: skewness $\xi$ and and skewness $\eta$.

$$\mathcal{A}^{\mu\nu}(\xi,\eta,t) = -e^2 \frac{1}{(P+P')^+}\,\bar{u}(P')\left[ g_T^{\mu\nu}\left(\mathcal{H}(\xi,\eta,t)\,\gamma^+ + \mathcal{E}(\xi,\eta,t)\,\frac{i\sigma^{+\rho}\Delta_\rho}{2M}\right) + i\epsilon_T^{\mu\nu}\left(\widetilde{\mathcal{H}}(\xi,\eta,t)\,\gamma^+\gamma_5 + \widetilde{\mathcal{E}}(\xi,\eta,t)\,\frac{\Delta^+\gamma_5}{2M}\right)\right]u(P)\,,$$

,where:

$$\begin{aligned}
\mathcal{H}(\xi,\eta,t) &= +\int_{-1}^{1} dx\left(\sum_q T^q(x,\xi,\eta)H^q(x,\eta,t) + T^g(x,\xi,\eta)H^g(x,\eta,t)\right) \\
\widetilde{\mathcal{H}}(\xi,\eta,t) &= -\int_{-1}^{1} dx\left(\sum_q \widetilde{T}^q(x,\xi,\eta)\widetilde{H}^q(x,\eta,t) + \widetilde{T}^g(x,\xi,\eta)\widetilde{H}^g(x,\eta,t)\right).
\end{aligned}$$

# LO

- DVCS vs TCS

$$
\begin{aligned}
{}^{DVCS}T^q &= -e_q^2 \tfrac{1}{x+\eta-i\varepsilon} - (x \to -x) = &({}^{TCS}T^q)^* \\
{}^{DVCS}\tilde{T}^q &= -e_q^2 \tfrac{1}{x+\eta-i\varepsilon} + (x \to -x) = &-({}^{TCS}\tilde{T}^q)^*
\end{aligned}
$$

$$
{}^{DVCS}Re(\mathcal{H}) \sim P \int \frac{1}{x \pm \eta} H^q(x,\eta,t)\,, \qquad {}^{DVCS}Im(\mathcal{H}) \sim i\pi H^q(\pm\eta,\eta,t)
$$

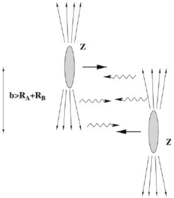
- DDVCS

$$
{}^{DDVCS}T^q = -e_q^2 \frac{1}{x+\xi-i\varepsilon} - (x \to -x)
$$

$$
{}^{DDVCS}Re(\mathcal{H}) \sim P \int \frac{1}{x \pm \xi} H^q(x,\eta,t)\,, \qquad {}^{DDVCS}Im(\mathcal{H}) \sim i\pi H^q(\pm\xi,\eta,t)
$$

But this is only true at LO. At NLO all GPDs hidden in the convolutions.

# Ultraperipheral collisions




$$\sigma = \int \frac{dn(k)}{dk} \sigma_{\gamma p}(k) dk$$

$\sigma_{\gamma p}(k)$ is the cross section for the $\gamma p \to p l^+ l^-$ process and $k$ is the $\gamma$'s energy, and $\frac{dn(k)}{dk}$ is an equivalent photon flux.

$$\frac{dn}{dk} = \frac{2Z^2 \alpha_{EM}}{\pi k} \left[ \omega^{pA} K_0(\omega^{pA}) K_1(\omega^{pA}) - \frac{{\omega^{pA}}^2}{2} \left( K_1^2(\omega^{pA}) - K_0^2(\omega^{pA}) \right) \right] \quad (5)$$

# The TCS differential cross section at UPC

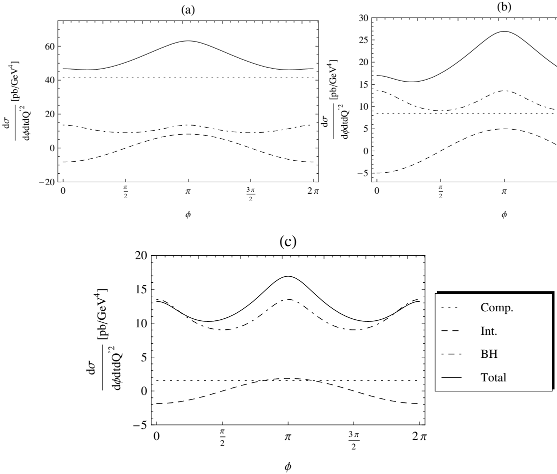
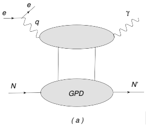



Figure: The differential cross sections (solid lines) for $t = -0.2\text{GeV}^2$, $Q'^2 = 5\text{GeV}^2$ and integrated over $\theta = [\pi/4, 3\pi/4]$, as a function of $\varphi$, for $s = 10^7\text{GeV}^2$ (a), $s = 10^5\text{GeV}^2$(b), $s = 10^3\text{GeV}^2$ (c) with $\mu_F^2 = 5\text{GeV}^2$. We also display the Compton (dotted), Bethe-Heitler (dash-dotted) and Interference (dashed) contributions.




DVCS

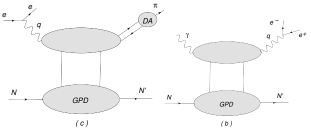
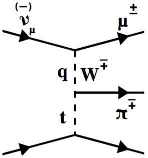


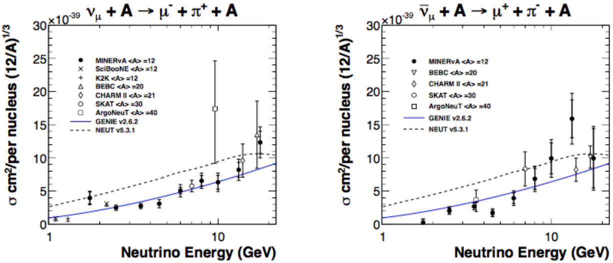

Figure: DVMP and TCS

- Various exclusive processes give information about GPDs: DVCS, TCS, DDVCS, DVMP, HVMP
- Also neutrino production of light mesons considered: allows for flavour separation, different combination of GPDs due to the charged current coupling structure. Smaller cross sections, less intense beams but process in the reach of the i.e. MINERVA experiments
  $\rightarrow$ [Kopeliovich,Schmidt, Siddikov] PRD 86

# Neutrino production of charmed meson

MINERvA (Fermilab) $\nu$ on nuclei, $E_\nu = 1 - 10$ GeV




from PRL 113, 261802 (2014) Measurement of Coherent Production of $\pi^\pm$ in Neutrino and Anti-Neutrino Beams on Carbon from $E_\nu$ of 1.5 to 20 GeV

# Neutrino production of charmed meson

- Here we consider $D$ pseudo scalar charmed meson production - heavy quark production allows to extend the range of validity of collinear factorization, the heavy quark mass playing the role of the hard scale.

- Factorization theorem with HEAVY quark: $\rightarrow$ [J. C. Collins, PRD58]
  - Independently of the relative sizes of the heavy quark masses and $Q$
  - Size of the errors is a power of $\Lambda/\sqrt{Q^2+M_D^2}$ when $\sqrt{Q^2+M_D^2}$ is the large scale.

- Sensitivity to transversity GPDs. $\rightarrow$ [Pire,Sz.] PRL 115

# Transversity

- The transverse spin structure of the nucleon - that is the way quarks and antiquarks spins share the polarization of a nucleon, when it is polarized transversely to its direction of motion - is almost completely unknown. Poorly known PDF, TMDs, GPDs.
- Lattice result and SIDIS analysis suggest that transversity distributions are not small
- Transversity GPDs coupled to chiral-odd twist 3 pi-meson DA may explain $\pi$ electroproduction data at JLab [Goloskokov, Kroll], [Ahmad, Goldstein, Liuti]
- One can consider a 3-body final state process [Ivanov, Pire, Sz., Teryaev], [Enberg, Pire, Sz.], [El Beiyad et al.], [Boussarie, Pire, Sz., Wallon]

  $\rightarrow$ Leading twist process

$$\gamma N \to \rho \rho N' \qquad \gamma N \to \pi \rho N' \qquad \gamma N \to \gamma \rho N'$$

# Neutrino-production of charmed meson

We consider the exclusive production of a pseudoscalar $D-$meson through the reactions on a proton (p) or a neutron (n) target:

$$\begin{aligned}
\nu_l(k)p(p_1) &\to l^-(k')D^+(p_D)p'(p_2)\,,\\
\nu_l(k)n(p_1) &\to l^-(k')D^+(p_D)n'(p_2)\,,\\
\nu_l(k)n(p_1) &\to l^-(k')D^0(p_D)p'(p_2)\,,\\
\bar{\nu}_l(k)p(p_1) &\to l^+(k')D^-(p_D)p'(p_2)\,,\\
\bar{\nu}_l(k)p(p_1) &\to l^+(k')\bar{D}^0(p_D)n'(p_2)\,,\\
\bar{\nu}_l(k)n(p_1) &\to l^+(k')D^-(p_D)n'(p_2)\,,
\end{aligned}$$

in the kinematical domain where collinear factorization leads to a description of the scattering amplitude in terms of nucleon GPDs and the $D-$meson distribution amplitude, with the hard subprocesses:

$$W^+d \to D^+d \quad , \quad W^+d \to D^0u \quad , \quad W^-\bar{d} \to D^-\bar{d} \quad , \quad W^-\bar{d} \to \bar{D}^0\bar{u}\,,$$

convoluted with chiral-even or chiral-odd quark GPDs, and the hard subprocesses:

$$W^+g \to D^+g \quad , \quad W^-g \to D^-g\,,$$

convoluted with gluon GPDs.

## Feynman diagrams

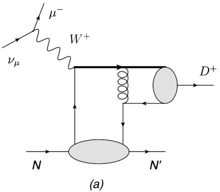



Figure: Feynman diagrams for the factorized amplitude for the $\nu_\mu N \to \mu^- D^+ N'$ process; the thick line represents the heavy quark.

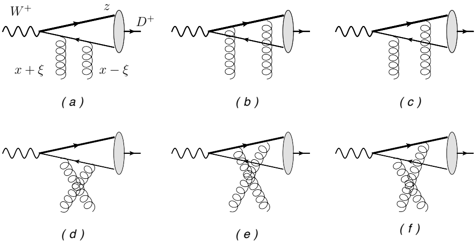


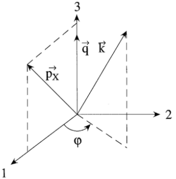

Figure: Feynman diagrams for the factorized amplitude for the $W^+ N \to D^+ N'$ process involving the gluon GPDs; the thick line represents the heavy quark.

# Neutrino-production of charmed meson

Standard notations of deep exclusive leptoproduction:

- $P = \frac{(p_1+p_2)}{2}$, $\quad \Delta = p_2 - p_1$, $\quad t = \Delta^2$, $\quad x_B = \frac{Q^2}{2p_1 \cdot q}$,
- $y = \frac{p_1 \cdot q}{p_1 \cdot k}$ and $\epsilon \simeq 2(1-y)/[1+(1-y)^2]$.
- $n$ are light-cone vectors and $\xi = -\frac{\Delta \cdot n}{2P \cdot n}$ is the skewness variable.
- The azimuthal angle $\varphi$ is defined in the initial nucleon rest frame as:

$$sin\ \varphi = \frac{\vec{q} \cdot [(\vec{q} \times \vec{p}_D) \times (\vec{q} \times \vec{k}_\nu)]}{|\vec{q}||\vec{q} \times \vec{p}_D||\vec{q} \times \vec{k}_\nu|},$$



# Neutrino-production of charmed meson

- $\nu N \to \mu^- D^+ N$ differential cross section:

$$\frac{d^4\sigma(\nu N \to l^- N' D)}{dy\, dQ^2\, dt\, d\varphi} = \tilde{\Gamma}\Big\{ \frac{1+\sqrt{1-\varepsilon^2}}{2}\sigma_{--} + \varepsilon\sigma_{00}$$
$$+\sqrt{\varepsilon}(\sqrt{1+\varepsilon}+\sqrt{1-\varepsilon})(\cos\varphi\ \mathrm{Re}\sigma_{-0} + \sin\varphi\ \mathrm{Im}\sigma_{-0})\ \Big\},$$

with

$$\tilde{\Gamma} = \frac{G_F^2}{(2\pi)^4}\frac{1}{32y}\frac{1}{\sqrt{1+4x_B^2 m_N^2/Q^2}}\frac{1}{(s-m_N^2)^2}\frac{Q^2}{1-\epsilon},$$

and the "cross-sections" $\sigma_{lm} = \epsilon_l^{*\mu} W_{\mu\nu}\epsilon_m^\nu$ are product of amplitudes for the process $W(\epsilon_l)N \to DN'$, averaged (summed) over the initial (final) hadron polarizations.

- transverse amplitude $W_T q \to Dq'$ gets its leading term in the collinear QCD framework as a convolution of chiral odd leading twist GPDs with a coefficient function of order $\frac{m_c}{Q^2}$ or $\frac{M_D}{Q^2}$ (to be compared to the $O(\frac{1}{Q})$ longitudinal amplitude)

# GPD Models

- Chiral even GPDs: Goloskokov-Kroll model
- Transversity GPDs

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\,\langle p_2,\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\, i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\,|p_1,\lambda\rangle\Big|_{z^+=\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p_2,\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde H_T^q\,\frac{P^+\Delta^i - \Delta^+P^i}{m_N^2} + E_T^q\,\frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m_N} + \tilde E_T^q\,\frac{\gamma^+P^i - P^+\gamma^i}{m_N}\right]u(p_1,\lambda).$$

The GPD $H_T(x,\xi,t)$ is equal to the transversity PDF in the $\xi = t = 0$ limit. G-K provide parametrization (with some lattice input) for $H_T(x,\xi,t)$ and for the combination $\bar E_T(x,\xi,t) = 2\tilde H_T(x,\xi,t) + E_T(x,\xi,t)$. Since $\tilde E_T(x,\xi,t)$ is odd under $\xi \to -\xi$, most models find it vanishingly small. We will put it to zero. We consider 3 models:

- model 1 : $\tilde H_T(x,\xi,t) = 0; E_T(x,\xi,t) = \bar E_T(x,\xi,t)$.
- model 2 : $\tilde H_T(x,\xi,t) = H_T(x,\xi,t); E_T(x,\xi,t) = \bar E_T(x,\xi,t) - 2H_T(x,\xi,t)$.
- model 3 : $\tilde H_T(x,\xi,t) = -H_T(x,\xi,t); E_T(x,\xi,t) = \bar E_T(x,\xi,t) + 2H_T(x,\xi,t)$.

# Distribution amplitudes

- Usual heavy-light meson DA reads :

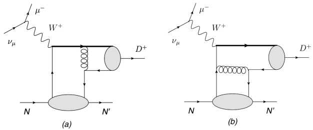
$$\langle D^+(P_D)|\bar{c}_\beta(y)d_\gamma(-y)|0\rangle = i\frac{f_D}{4}\int_0^1 dz e^{i(z-\bar{z})P_D\cdot y}[(\hat{P}_D - M_D)\gamma^5]_{\gamma\beta}\phi_D(z),$$

with $z = \frac{k^+}{P_D^+}$, $\int_0^1 dz\ \phi_D(z) = 1$, $f_D = 0.223$ GeV, $\bar{z} = 1 - z$ and $\hat{p} = p_\mu\gamma^\mu$.

- We will parametrize $\phi_D(z)$:

$$\phi_D(z) = 6z(1-z)(1 + C_D(2z-1))$$

with $C_D \approx 1.5$, which has a maximum around $z = 0.7$.
$\rightarrow$ [T. Kurimoto, H. n. Li and A. I. Sanda, Phys. Rev. D 65]




The transverse amplitude is then written as ($\tau = 1 - i2$):

$$T_T = \frac{-i2C_q\xi(2M_D - m_c)}{\sqrt{2}(Q^2 + M_D^2)} \bar{N}(p_2)\left[\mathcal{H}_T i\sigma^{n\tau} + \tilde{\mathcal{H}}_T \frac{\Delta^\tau}{m_N^2} + \mathcal{E}_T \frac{\hat{n}\Delta^\tau + 2\xi\gamma^\tau}{2m_N} - \tilde{\mathcal{E}}_T \frac{\gamma^\tau}{m_N}\right] N(p_1),$$

with $C_q = \frac{2\pi}{3} C_F \alpha_s V_{dc}$, in terms of transverse form factors that we define as :

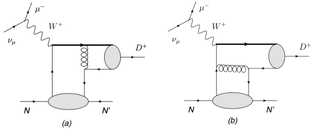
$$\mathcal{F}_T = f_D \int \frac{\phi_D(z)dz}{\bar{z}} \int \frac{F_T^d(x,\xi,t)dx}{(x - \xi + \beta\xi + i\epsilon)(x - \xi + \alpha\bar{z} + i\epsilon)},$$

where $F_T^d$ is any d-quark transversity GPD, $\alpha = \frac{2\xi M_D^2}{Q^2 + M_D^2}$, $\beta = \frac{2(M_D^2 - m_c^2)}{Q^2 + M_D^2}$.

- $T_T$ vanishes when $m_c = 0 = M_D$.

For chiral-even GPDs due to the collinear kinematics and the leading twist CF

For chiral-odd GPDs due to the odd number of $\gamma$ matrices in the Dirac trace.




The quark contribution to longitudinal amplitude of leading twist is a slight modification of the calculation in:

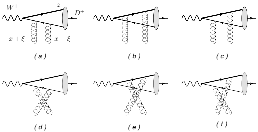
B. Z. Kopeliovich, I. Schmidt and M. Siddikov, Phys. Rev. D **86** and D **89**
G. R. Goldstein, O. G. Hernandez, S. Liuti and T. McAskill, AIP Conf. Proc. **1222**

$$T_L^q = \frac{-iC_q}{2Q}\bar{N}(p_2)\left[\mathcal{H}_L\hat{n} - \tilde{\mathcal{H}}_L\hat{n}\gamma^5 + \mathcal{E}_L\frac{i\sigma^{n\Delta}}{2m_N} - \tilde{\mathcal{E}}_L\frac{\gamma^5\Delta.n}{2m_N}\right]N(p_1),$$

with the chiral-even form factors defined by

$$\mathcal{F}_L = f_D\int\frac{\phi_D(z)dz}{\bar{z}}\int dx\frac{F^d(x,\xi,t)}{x-\xi+\alpha\bar{z}+i\epsilon}\left[\frac{x-\xi+\gamma\xi}{x-\xi+\beta\xi+i\epsilon}+\frac{Q^2}{Q^2+zM_D^2}\right],$$

with $\gamma = \frac{2M_D(M_D-2m_c)}{Q^2+M_D^2}$, $\beta = \frac{2(M_D^2-m_c^2)}{Q^2+M_D^2}$




The gluonic contribution to the amplitude reads:

$$
\begin{aligned}
T_L^g \quad &= \quad \frac{iC_g}{2}\int_{-1}^{1} dx \frac{-1}{(x+\xi-i\epsilon)(x-\xi+i\epsilon)}\int_0^1 dz f_D\phi_D(z)\cdot \\
&\Big[\bar{N}(p_2)[H^g\hat{n}+E^g\frac{i\sigma^{n\Delta}}{2m}]N(p_1)\mathcal{M}_H^S \\
&+\bar{N}(p_2)[\tilde{H}^g\hat{n}\gamma^5+\tilde{E}^g\frac{\gamma^5 n.\Delta}{2m}]N(p_1)\mathcal{M}_H^A\Big] \\
\equiv \quad &\frac{-iC_g}{2Q}\bar{N}(p_2)\left[\mathcal{H}^g\hat{n}+\mathcal{E}^g\frac{i\sigma^{n\Delta}}{2m}+\tilde{\mathcal{H}}^g\hat{n}\gamma^5+\tilde{\mathcal{E}}^g\frac{\gamma^5 n.\Delta}{2m}\right]N(p_1)\,,
\end{aligned}
$$

where the last line defines the gluonic form factors $\mathcal{H}^g$, $\tilde{\mathcal{H}}^g$, $\mathcal{E}^g$, $\tilde{\mathcal{E}}^g$ and $C_g = T_f\frac{\pi}{3}\alpha_s V_{dc}$ with $T_f=\frac{1}{2}$ and the factor $\frac{-1}{(x+\xi-i\epsilon)(x-\xi+i\epsilon)}$ comes from the conversion of the strength tensor to the gluon field.

# The longitudinal cross section $\sigma_{00}$

$$\sigma_{00} = \frac{1}{Q^2}\Big\{ \quad [\,|C_q\mathcal{H}_L + C_g\mathcal{H}_g|^2 + |C_q\tilde{\mathcal{H}}_L - C_g\tilde{\mathcal{H}}_g|^2](1-\xi^2)$$
$$+\frac{\xi^4}{1-\xi^2}[\,|C_q\tilde{\mathcal{E}}_L - C_g\tilde{\mathcal{E}}_g|^2 + |C_q\mathcal{E}_L + C_g\mathcal{E}_g|^2]$$
$$-2\xi^2\mathcal{R}e[C_q\mathcal{H}_L + C_g\mathcal{H}_g][C_q\mathcal{E}_L^* + C_g\mathcal{E}_g^*]$$
$$-2\xi^2\mathcal{R}e[C_q\tilde{\mathcal{H}}_L - C_g\tilde{\mathcal{H}}_g][C_q\tilde{\mathcal{E}}_L^* - C_g\tilde{\mathcal{E}}_g^*]\Big\}.$$

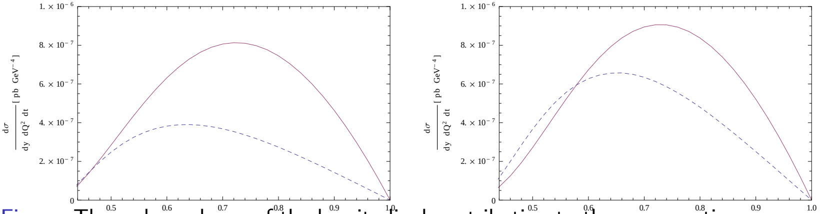



Figure: The $y$ dependence of the longitudinal contribution to the cross section $\frac{d\sigma(\nu N \to l^- N D^+)}{dy\, dQ^2\, dt}$ (in pb GeV$^{-4}$) for $Q^2 = 1$ GeV$^2$, $\Delta_T = 0$ and $s = 20$ GeV$^2$ for a proton (left panel) and neutron (right panel) target : total (quark and gluon, solid curve) and quark only (dashed curve) contributions. GLUONS IMPORTANT!!!

$$\sigma_{--} = \frac{16\xi^2 C_q^2 (m_c - 2M_D)^2}{(Q^2+M_D^2)^2}\left\{(1-\xi^2)|\mathcal{H}_T|^2 + \frac{\xi^2}{1-\xi^2}|\mathcal{E}'_T|^2 - 2\xi \mathcal{R}e[\mathcal{H}_T \mathcal{E}'^*_T]\right\}$$

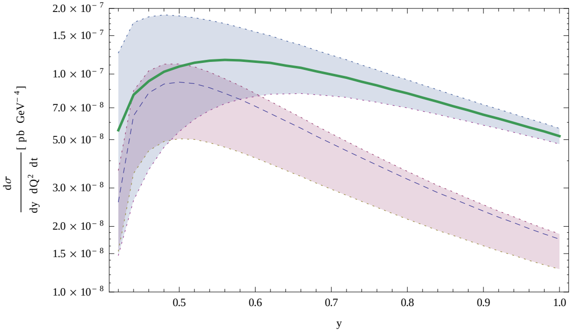



Figure: The $y$ dependence of the transverse contribution to the cross section $\frac{d\sigma(\nu N \to l^- N D^+)}{dy\, dQ^2\, dt}$ (in pb GeV$^{-4}$) for $Q^2 = 1$ GeV$^2$, $\Delta_T = 0$ and $s = 20$ GeV$^2$ for a proton (dashed curve) and neutron (solid curve) target.

# The interference cross section $\sigma_{-0}$

Vanishes at zeroth order in $\Delta_T$, the term linear in $\Delta_T/m_N$ reads
$\lambda = \tau^* = 1 + i2$

$$
\begin{aligned}
\sigma_{-0} &= \frac{\xi\sqrt{2}C_q}{m}\frac{2M_D - m_c}{Q(Q^2+M_D^2)}\Bigg\{ \\
&- i\mathcal{H}_T^*[C_q\tilde{\mathcal{E}}_L - C_g\tilde{\mathcal{E}}_g]\xi\epsilon^{pn\Delta\lambda} + i\mathcal{E}'^*_T\epsilon^{pn\Delta\lambda}[C_q\tilde{\mathcal{H}}_L - C_g\tilde{\mathcal{H}}_g] \\
&+ 2\tilde{\mathcal{H}}_T^*\Delta^\lambda\{C_q\mathcal{H}_L + C_g\mathcal{H}_g - \frac{\xi^2}{1-\xi^2}[C_q\mathcal{E}_L + C_g\mathcal{E}_g]\} \\
&+ \mathcal{E}_T^*\Delta^\lambda\{(1-\xi^2)[C_q\mathcal{H}_L + C_g\mathcal{H}_g] - \xi^2[C_q\mathcal{E}_L + C_g\mathcal{E}_g]\} \\
&- \mathcal{H}_T^*\Delta^\lambda[C_q\mathcal{E}_L + C_g\mathcal{E}_g] + \mathcal{E}'^*_T\Delta^\lambda\xi[C_q\mathcal{H}_L + C_g\mathcal{H}_g + C_q\mathcal{E}_L + C_g\mathcal{E}_g]\Bigg\}
\end{aligned}
$$

In our kinematics, $\Delta^1 = \Delta^x = \Delta_T$, $\Delta^y = 0$, $\epsilon^{pn\Delta\lambda} = -i\Delta_T$.

$$
\begin{aligned}
< cos\ \varphi > &= \frac{\int cos\ \varphi\ d\varphi\ d^4\sigma}{\int d\varphi\ d^4\sigma} = K_\epsilon \frac{\mathcal{R}e\sigma_{-0}}{\sigma_{00}}\ , \\
< sin\ \varphi > &= K_\epsilon \frac{\mathcal{I}m\sigma_{-0}}{\sigma_{00}}
\end{aligned}
$$

- with $K_\epsilon = \frac{\sqrt{1+\varepsilon}+\sqrt{1-\varepsilon}}{2\sqrt{\epsilon}}$
- Simple approximate results:

$$
\begin{aligned}
< cos\varphi > &\approx \frac{K\mathcal{R}e[\mathcal{H}_D(2\tilde{\mathcal{H}}_T^\phi + \mathcal{E}_T^\phi + \bar{\mathcal{E}}_T^\phi)^* - \mathcal{E}_D\mathcal{H}_T^{\phi *}]}{8|\mathcal{H}_D^2| + |\tilde{\mathcal{E}}_D^2|}\ , \\
< sin\varphi > &\approx \frac{K\mathcal{I}m[\mathcal{H}_D(2\tilde{\mathcal{H}}_T^\phi + \mathcal{E}_T^\phi + \bar{\mathcal{E}}_T^\phi)^* - \mathcal{E}_D\mathcal{H}_T^{\phi *}]}{8|\mathcal{H}_D^2| + |\tilde{\mathcal{E}}_D^2|}\ , \\
K &= -\frac{\sqrt{1+\varepsilon}+\sqrt{1-\varepsilon}}{2\sqrt{\epsilon}}\ \frac{2\sqrt{2}\xi m_c}{Q}\ \frac{\Delta_T}{m_N}
\end{aligned}
$$

# Azimuthal dependence

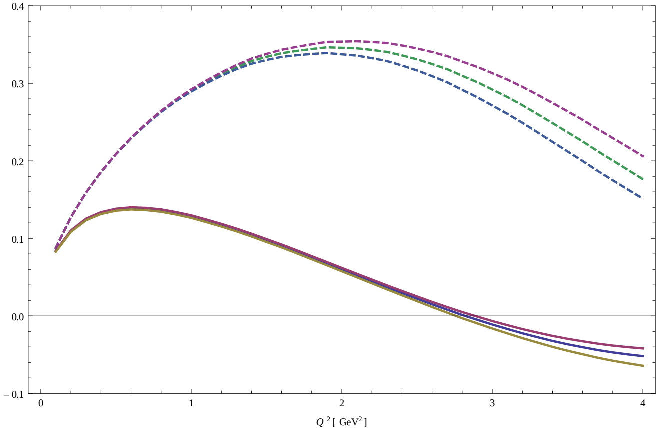



Figure: The $Q^2$ dependence of the $< cos\ \varphi >$ (solid curves) and $< sin\ \varphi >$ (dashed curves) moments normalized by the total cross section, for $\Delta_T = 0.5$ GeV, $y = 0.7$ and $s = 20$ GeV$^2$. The three curves correspond to the three models explained in the text, and quantify the theoretical uncertainty of our estimates.

# Light meson production - importance of gluon contribution.

J. Wagner, B. Pire, LSz Phys. Rev. D **95** (2017)

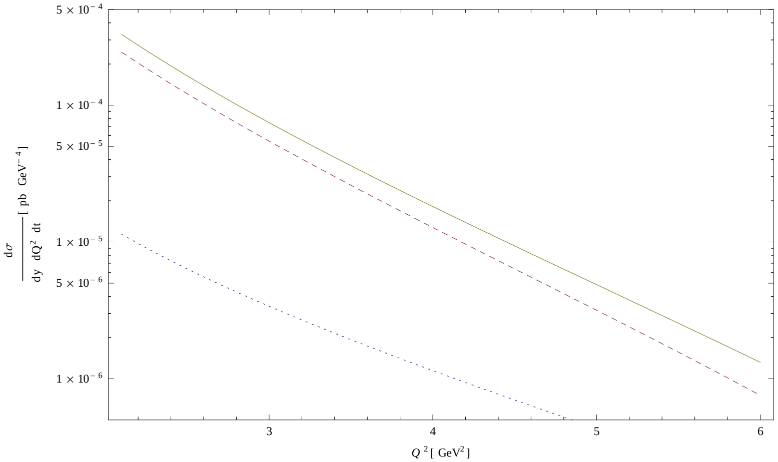



Figure: The $Q^2$ dependence of the quark contribution (dotted curve) compared to the gluon contribution (dashed line) and to the total (quark and gluon, solid curve) longitudinal cross section $\frac{d\sigma(\nu N \to l^- N \pi^+)}{dy\, dQ^2\, dt}$ (in pb GeV$^{-4}$) for $\pi^+$ production on a proton target for $y = 0.7, \Delta_T = 0$ and $s = 20$ GeV$^2$.

# Summary

- Collinear QCD factorization allows to calculate neutrino production of $D-$mesons in terms of GPDs down to $Q^2 = 0$.

- Chiral-odd and chiral-even GPDs contribute to the amplitude for different polarization states of the W

- The azimuthal dependence of the cross section allows to get access to chiral-odd GPDs

  most sensitive for transversity GPD is $\bar{\nu}\, p \to l^+ \bar{D}^0 n$

- The behaviour of the proton and neutron target cross sections for $D^+$, $D^-$ and $D^0$ production with $\nu$ and $\bar{\nu}$ enables to separate the u and d quark contributions.

- Within the reach of planned medium and high energy neutrino facilities and experiments such as MINER$\nu$A and MINOS+.

- Gluon contribution very important -> consequences for light mesons!