# Are we entering a paradigm shift in turbulence?

A 19th century problem with a 21st century solution


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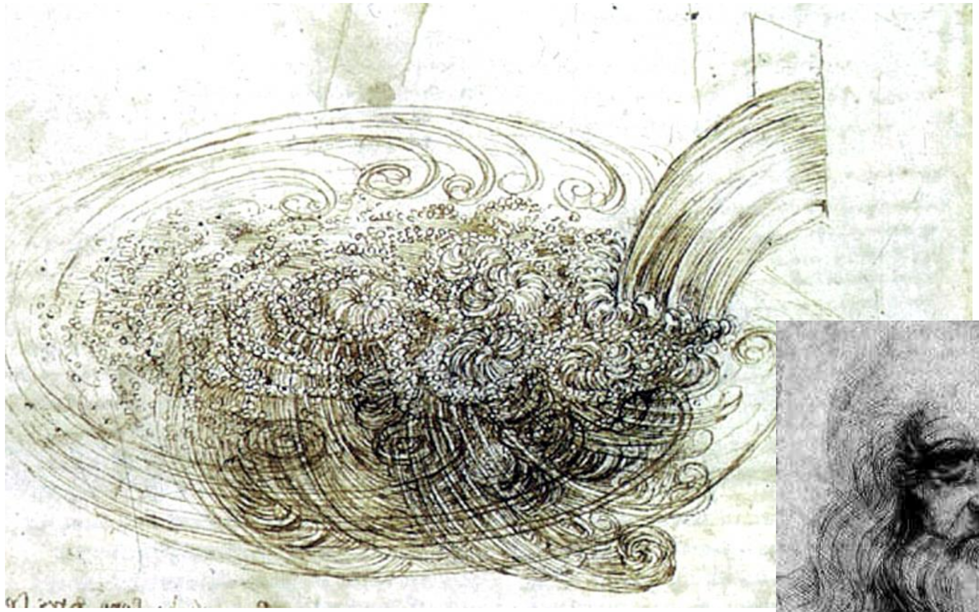
# Motivation



Flow visualization ~500 years ago

**Leonardo da Vinci**

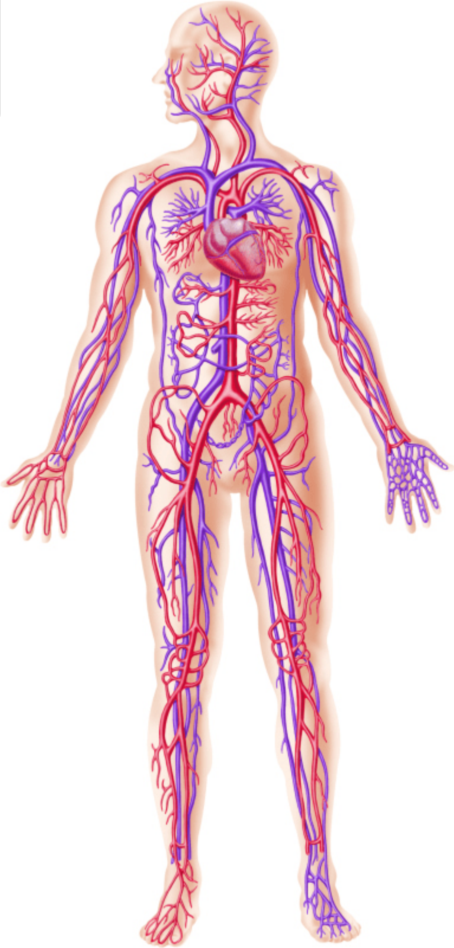
Flow visualization ~130 years ago

**Vincent van Gogh's 'Starry night'**

# Motivation

Turbulence is the most important unsolved problem of classical physics.

**Richard Feynman**



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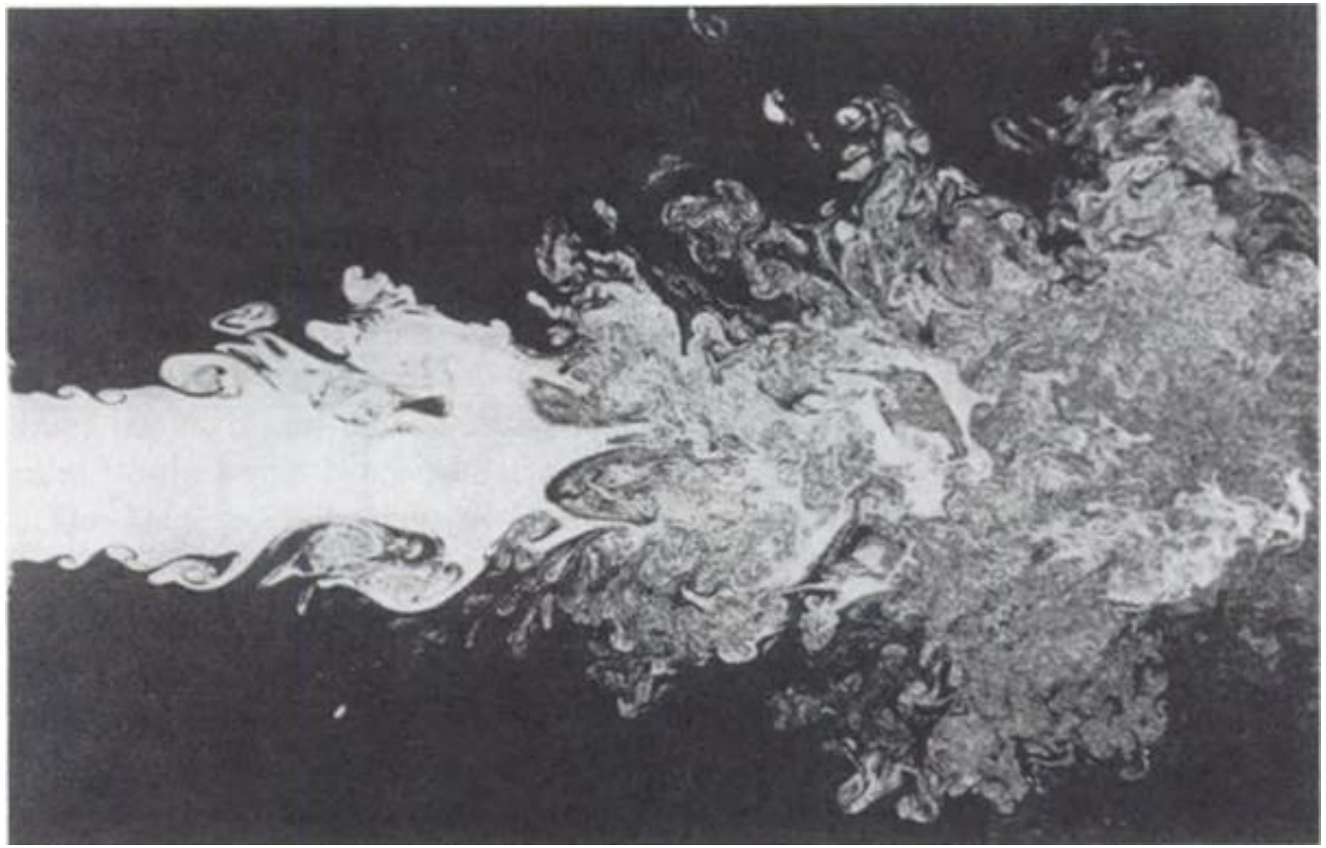
# What is the challenge?

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t} \qquad \Rightarrow \qquad \underbrace{\rho\,(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u})}_{\text{Acceleration}} = \underbrace{-\nabla p}_{\text{Pressure}} + \underbrace{\nu \triangle \vec{u}}_{\text{Viscosity}}$$



**Why is turbulence so difficult?**

Highly non-linear

4-dimensional

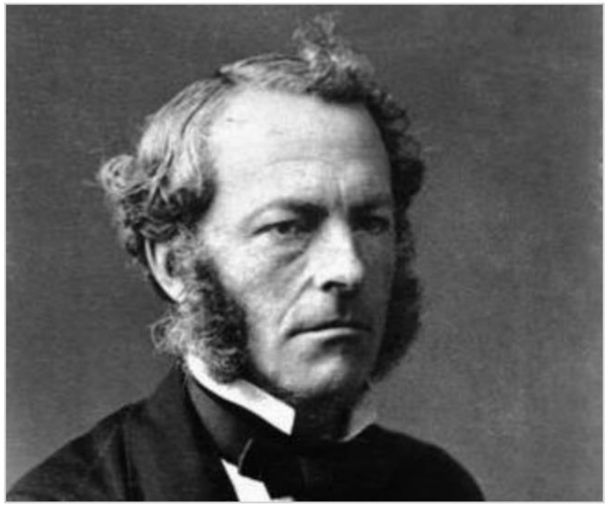
Wide range of scales

# Clay Mathematics Institute Millenium Problems

CMI

ABOUT PROGRAMS MILLENNIUM PROBLEMS PEOPLE PUBLICATIONS EVENTS EUCLID

## Navier–Stokes Equation



Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

Image: Sir George Gabriel Stokes (13 August 1819–1 February 1903). Public Domain

**This problem is:** Unsolved

**Rules:**

Rules for the Millennium Prizes

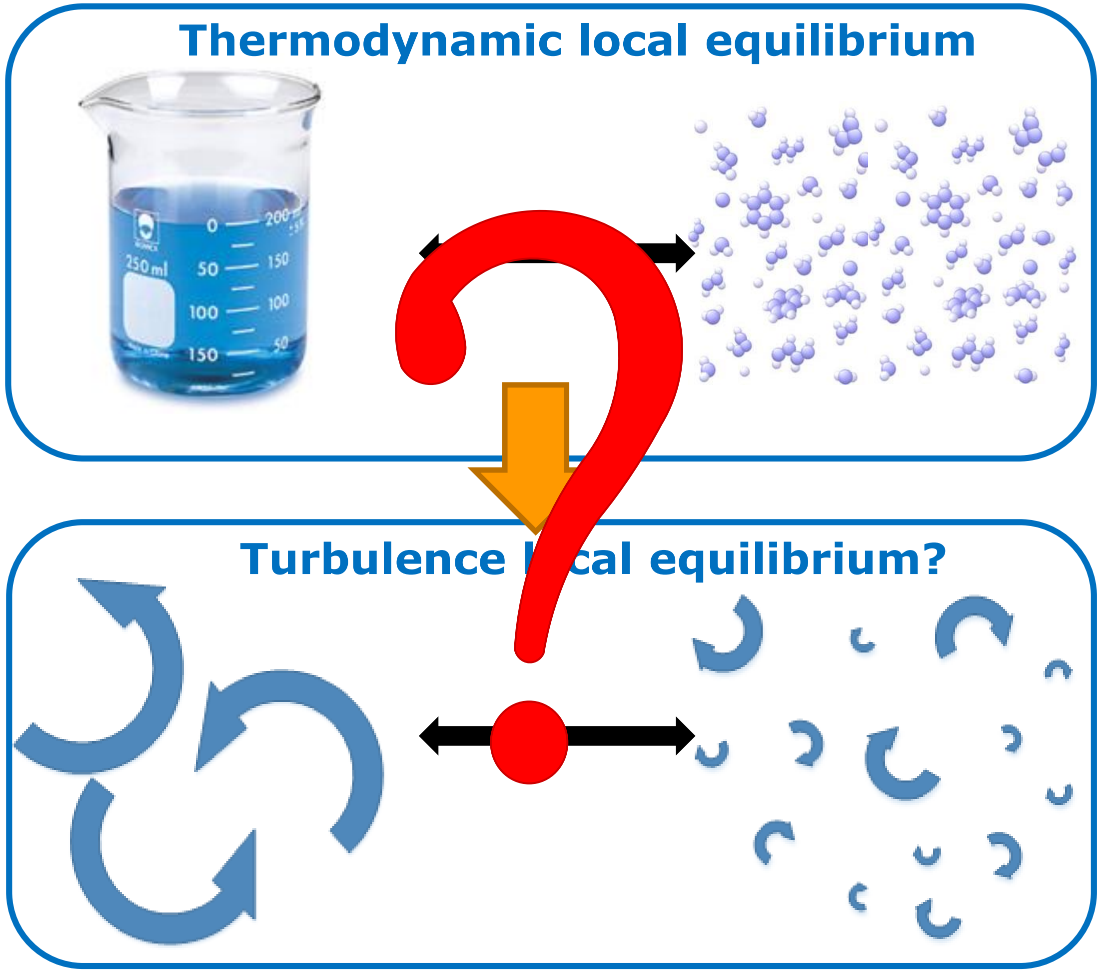
**Related Documents:**

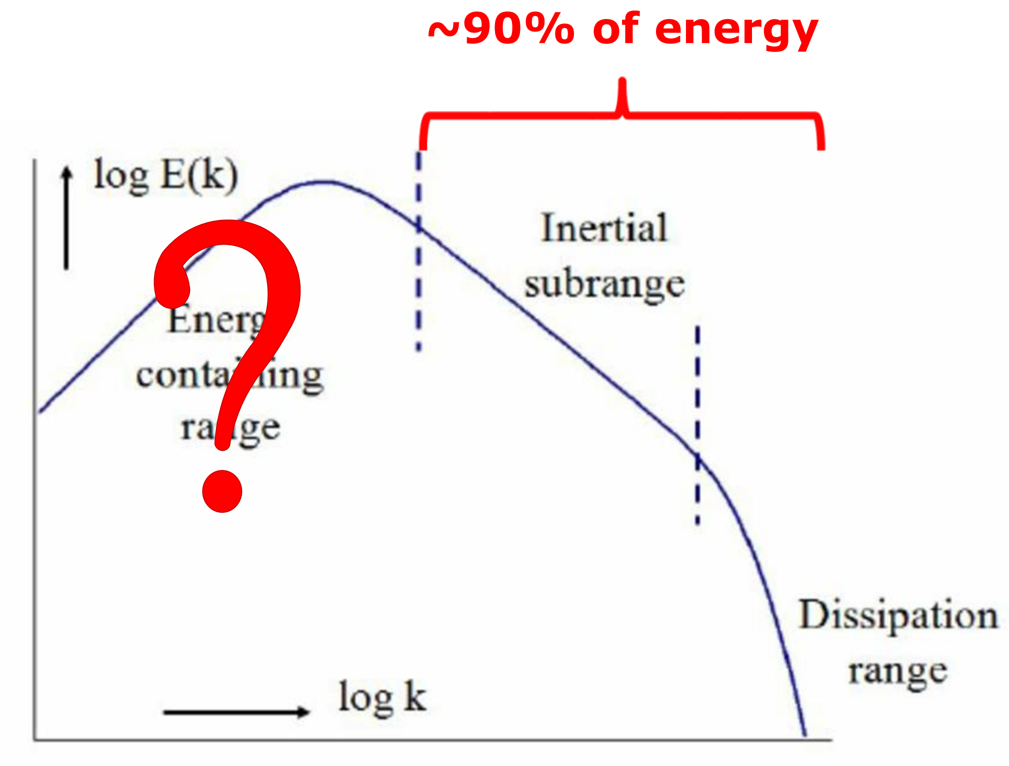
Official Problem Description

**Related Links:**

Lecture by Luis Cafarelli

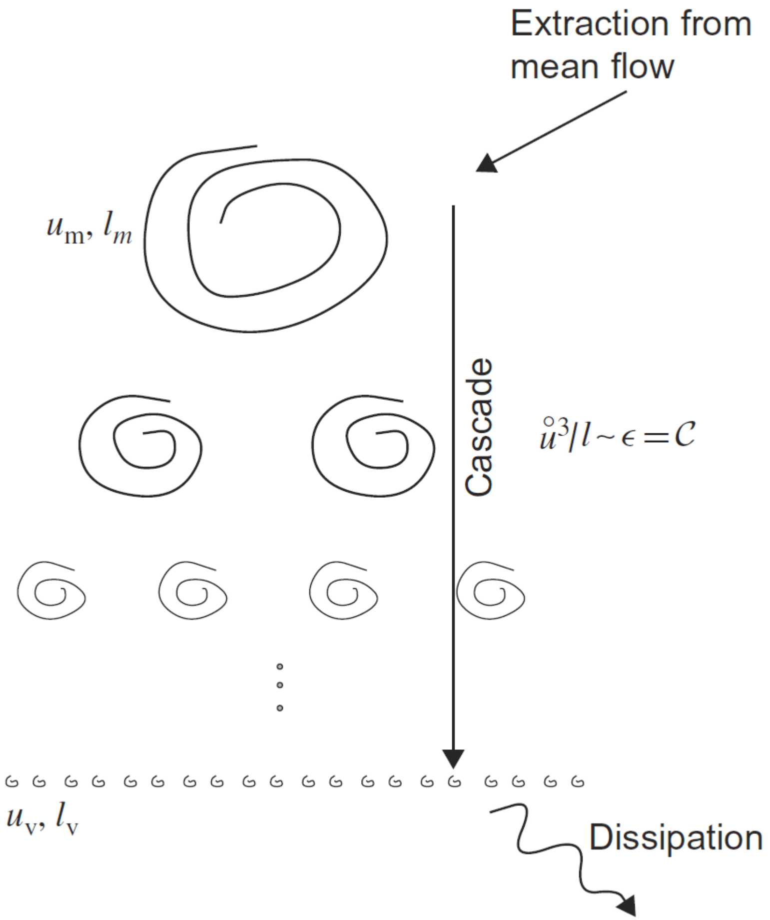
# Current ‘consensus’ – Kolmogorov turbulence




Collier *et al.* (2014)

# Local interactions

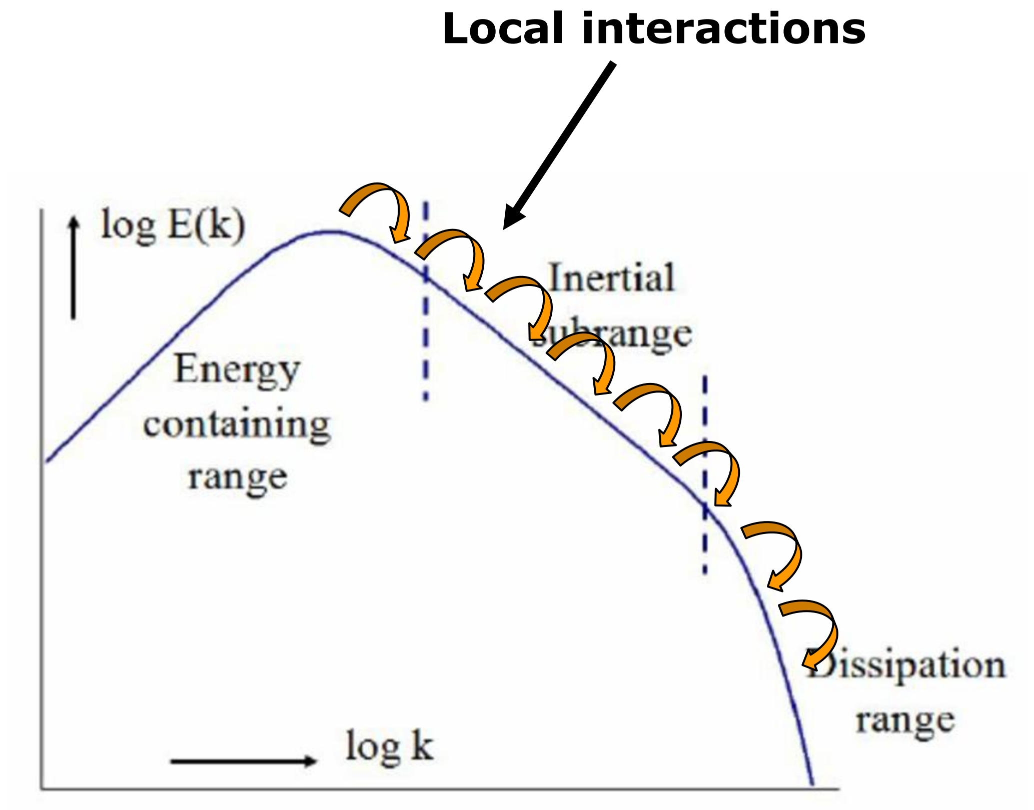



Cushman-Roisin & Beckers (2011)

*Big whorls have little whorls,*
*Which feed on their velocity;*
*Little whorls have smaller whorls,*
*And so on unto viscosity.*

**L. F. Richardson (1881-1953)**

# Kolmogorov turbulence

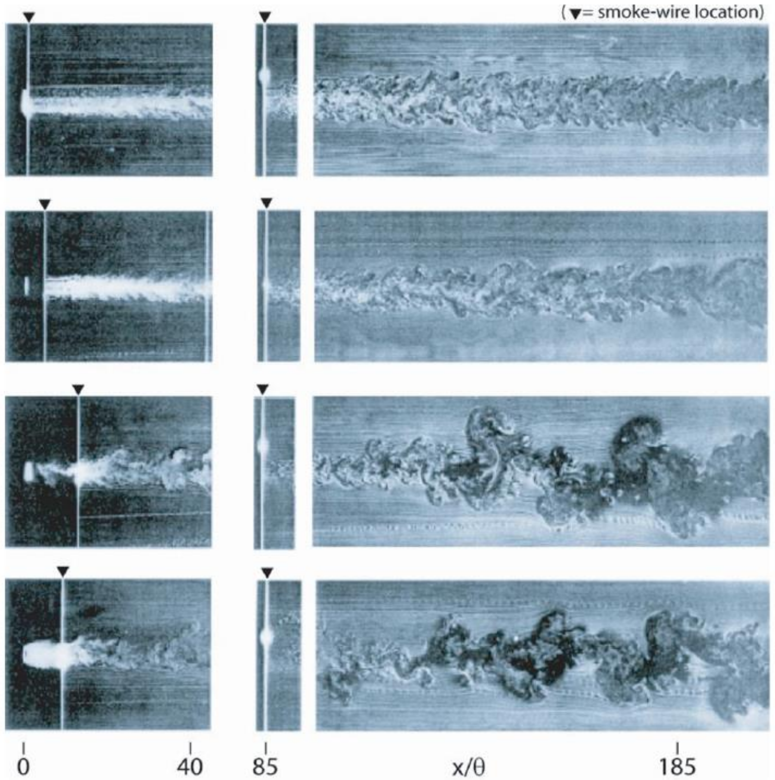



**Local interactions**

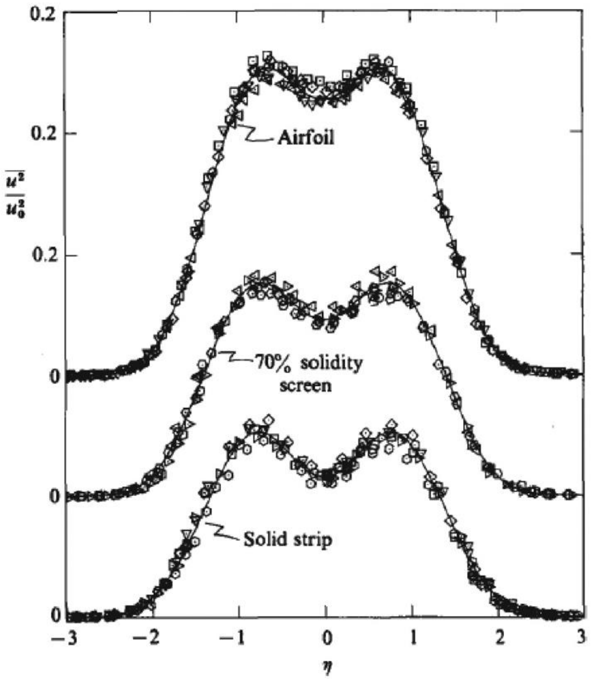
$\Downarrow$

**Equilibrium small/interm. scales**

# Kolmogorov turbulence




Cannon *et al.* 1993




Wygnanski *et al.* 1986

**Local eractions**

⇓

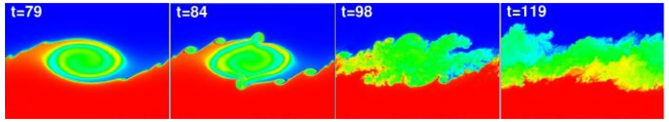
**Equilibrium s ll/interm. scales**

⇓

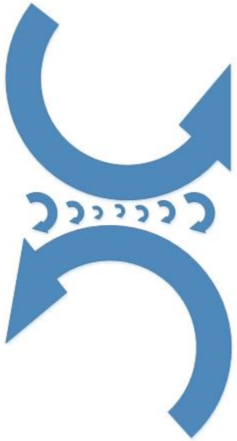
**Universal s all/interm. scales**

# Kolmogorov turbulence

Implicit to all of these was the 'local equilibirum' hypothesis put forth by Batchelor (1953)




~~Local interactions~~



Local (small scale) equilibrium ?

Empirical foundation

Robust understanding

# Why experiments cannot 'prove' a theory

Ingeniøren, 23. Sept. 2017

# Et af turbulensens mysterier er opklaret

**Ifølge en anekdote mente Werner Heisenberg, at selv ikke gud kunne forklare turbulens – men nu er det præcist vist, hvordan energien spredes fra store strømhvirvler til mindre og mindre strømhvirvler.**

Cardesa et al.

(2017), Science



FLUID MECHANICS

## The turbulent cascade in five dimensions

José I. Cardesa,* Alberto Vela-Martín, Javier Jiménez

**To the naked eye, turbulent flows exhibit whirls of many different sizes. To each size, or scale, corresponds a fraction of the total energy resulting from a cascade in five dimensions: scale, time, and three-dimensional space. Understanding this process is critical to strategies for modeling geophysical and industrial flows. By tracking the flow regions containing energy in different scales, we have detected the statistical predominance of a cross-scale link whereby fluid lumps of energy at scale Δ appear within lumps of scale 2Δ and die within those of scale Δ/2. Our approach uncovers the energy cascade in a simple water-like fluid, offering insights for turbulence models while paving the way for similar analyses in conducting fluids, quantum fluids, and plasmas.**

Perhaps no other area of physics research has borne the influence of a rhyming verse more than turbulence, where Richardson's "Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity" (*1*) is embedded in the seminal theory of Kolmogorov, Onsager, von Weizsäcker, and Heisenberg (*2*–*5*). The last three physicists transcribed the phenomenology in terms of wave numbers, which were to become the predominant tool in theoretical studies of the energy cascade (*6*–*10*). Consequently, scale and wave number became almost interchangeable concepts. A crucial point in the development of theories was the scale locality of the cascade, understood in terms of how close wave numbers are when energy is exchanged between them (*11*). Since the advent of computer simulations, the locality of these wave number interactions has been controversial, with studies claiming evidence in favor of (*12*) or against (*13*) it. Rigorous explanations proposed for these discrepancies (*14*, *15*) advocate for the classic scale-local view of the cascade. The debate, however, has turned predominantly around the equivalence between wave number and scale, ruling out any possibility of attributing the ongoing cascade to specific whirls visible where the flow actually evolves: the real space. Furthermore, computer simulations of industrial and atmospheric flows are carried out on numerical grids representing physical space and rely heavily on the modeling of the interaction between the resolved (large) and subgrid (small) scales (*16*).

Studies of the interscale energy transfer based on real-space quantities share one of two limitations. They either focus on a subset of the source or sink terms responsible for the changes in energy at a point (*17*, *18*), or they make no use of time, thus precluding any dynamical information or knowledge of causality (*19*–*23*). Often both limitations are combined. A noteworthy exception found a delay in the peak of the correlation between energy at two different scales when following the larger-scale flow (*24*), suggesting that eddy structures transfer their energy to smaller scales. In the wake of that study, we aimed to follow individual eddy structures. This has become possible with modern data-storage facilities where flow simulations are preserved in a movie-like manner. Such data sets have enabled the verification of phenomenological descriptions that eventually feed into dynamical models.

We analyzed data from a direct numerical simulation of turbulence in a triply periodic cube, obtained by solving the Navier-Stokes equations for an incompressible fluid by means of a deterministically forced and statistically steady pseudo-spectral code (*25*). An important length scale in turbulent flows, η, is given by $\eta = \nu^{3/4}/\varepsilon^{1/4}$, where ν is the kinematic viscosity and ε is the mean rate of kinetic energy dissipation. This small-scale length is associated with the tiniest whirls of turbulence. Our $(2\pi)^3$ computational domain spanned $(1516\eta)^3$ in space and lasted 2090 small-scale time units $\tau = \sqrt{\upsilon/\varepsilon}$. Expressed in terms of large-scale length and time units $L_{int}$ and $T_{int}$ (*25*), respectively, the simulation spanned $(5.3L_{int})^3$ and $66T_{int}$, with snapshots stored every 0.078τ. Although previous simulations have surpassed our Reynolds number $L_{int}/\eta = 284$, our long yet temporally resolved data set with a sizeable scale separation allowed us to statistically characterize a phenomenon by tracking many flow regions throughout their life.

The tracked flow regions that we now introduce in detail underpin our definition of whirls or eddies. We isolated a range of scales by filtering the velocity fields with a spatial band-pass filter. Owing to the homogeneity of the flow, we used an isotropic filter to simplify the concept of scale to a single scalar Δ. We set the center of the filter band at the chosen scale Δ and used bands of constant width on a logarithmic scale (*15*). The upper and lower edges of the band resulted from subtracting two low-pass Gaussian filters (*25*). We focused on four scales from the geometric sequence Δ/η = (30, 60, 120, 240). This led to four time series of the dynamics of the flow, one for each scale. The object of our study was a scalar quantity, the kinetic energy, which evolved in time *t*, scale Δ, and three-dimensional space (*x*, *y*, *z*). The kinetic energy at a scale is half the sum of the squared filtered velocity components. The flow structures in Fig. 1 are geometrically connected regions of space where the energy is above a given threshold [movie S1 (*25*)]. We chose the threshold systematically in the same way for all scales on the basis of the percolation properties of the energy at that scale (*25*, *26*). We further time-tracked these flow objects by using a technique developed for the tracking of coherent structures in turbulent channel flows (*27*). Whereas generally an object was born small as the underlying energy exceeded the threshold and died small as its intensity decreased, an object often merged

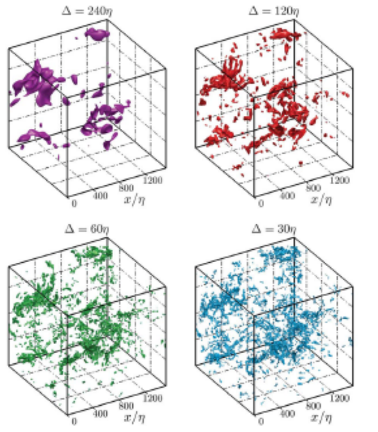



**Fig. 1. Energy-eddies at four different scales Δ for the same instant in a numerical simulation of turbulence in a periodic cube.** A time sequence is shown in movie S1 (*25*). The flow structures observed are the spatially connected regions of the flow where the energy at scale Δ is above a certain threshold (*25*).

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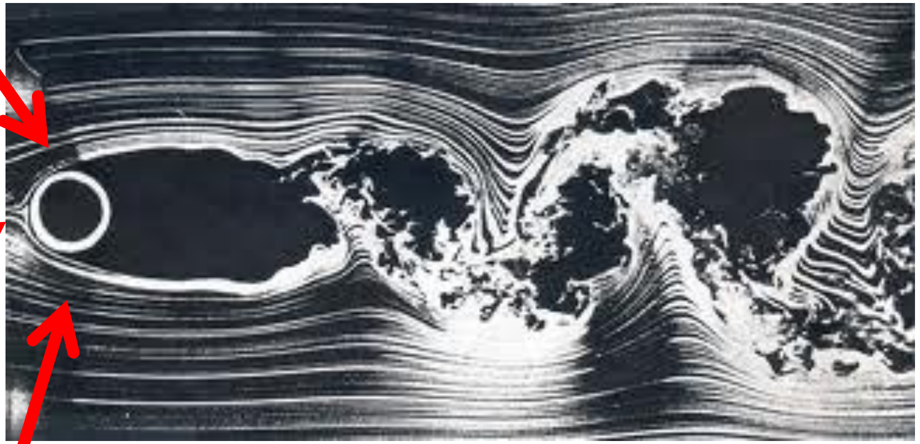
# Which flows do not ‘work’ well?

Separation

Stagnation

Temporal/spatial accelerations



$$\underbrace{\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right)}_{\text{Acceleration}} = -\nabla p + \nu \Delta \vec{u}$$

Friday, October 11th
3:30 PM Bowen Hall Rm 222

**Toward Predictive Simulations of Turbulent Multiphase Combustion Processes in Advanced Combustion Systems**

Dr. Joseph C. Oefelein
Sandia National Laboratories, Combustion Research Facility
Livermore, CA

Progress in the application of the Large Eddy Simulation (LES) technique for prediction of high-pressure high-Reynolds-number turbulent multiphase combustion processes in transportation, propulsion, and power devices will be presented. The primary objectives are to enable predictive simulations over a wide range of operating conditions, to establish a hierarchy of validated benchmark cases relevant to device-scale systems, and to establish technical performance metrics that define model implementation requirements and accuracy in the context of LES. Our approach involves four basic steps. 1) Establish complementary links between basic and applied research programs. 2) Establish close coordination with related target experiments designed for model validation. 3) Build our theoretical-numerical capabilities through development of advanced sub-models for LES. 4) Maximize the benefits of high-performance massively-parallel computing through close collaborations with key DOE computational facilities. Model development aimed at the treatment of high-pressure fuel injection processes and turbulent mixed-mode combustion will be described with emphasis on systematic validation and future needs.

Joe Oefelein received a Doctorate in Mechanical Engineering from Penn State University in May 1997. He worked as a Research Associate in the Department of Mechanical Engineering at Stanford University with Professor W. C. Reynolds from 1997 to December 2000. After completing his postdoctoral studies, he accepted a permanent position at the Sandia National Laboratories, Combustion Research Facility (CRF), where he is now employed as a Distinguished Member of Technical Staff. Joe has extensive experience in the development and application of the large-eddy-simulation (LES) technique and related sub-models, with emphasis on treatment of turbulent combustion, high-pressure supercritical phenomena, and multiphase flows. He also has significant experience in advanced CFD methods and massively-parallel high-performance computing.

Social Period outside of Bowen Hall Rm 222 following the seminar.

*For inquiries, please contact the Dept. of Mechanical & Aerospace Engineering at 609-258-0315*

***ALL VISITORS ARE WELCOME!***

# Let's dig into some equations…

$$\underbrace{\frac{\partial E(k,t)}{\partial t}}_{\substack{\text{time rate of} \\ \text{change of energy}}} = \underbrace{-\frac{\partial \varepsilon_k(k,t)}{\partial k}}_{\substack{\text{non-linear} \\ \text{transport} \\ \text{of energy}}} \underbrace{-2\nu k^2 E(k,t)}_{\substack{\text{dissipation} \\ \text{of energy}}}$$

$$\frac{\partial E}{\partial t} \approx 0$$

***Equilibrium assumed***

***for small/intermediate scales***

***Cornerstone*** *K41 result* $E = C_k \varepsilon^{2/3} k^{-5/3}$ + *Central assumption* $\partial E/\partial t \approx 0$

$$\frac{1}{E}\frac{\partial E}{\partial t} \propto \frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial t}$$

# How to advance our understanding?

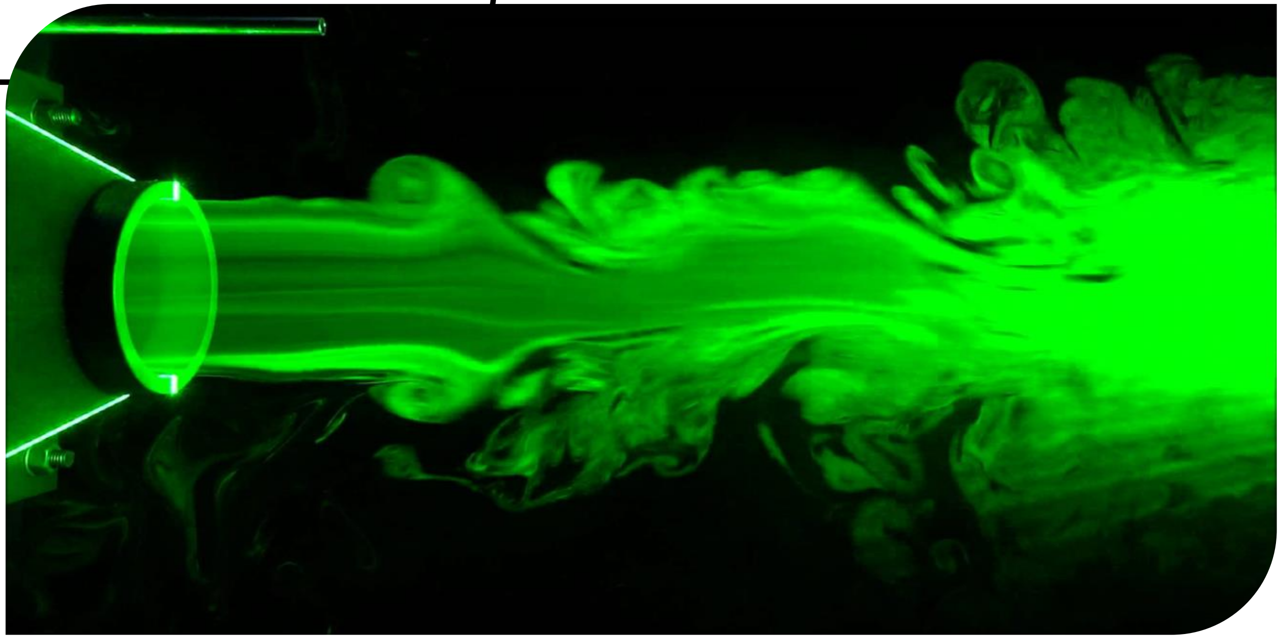
- Relax the assumptions as much as possible
- Dedicated measurements with the governing equations as a starting point
- Test deviations from equilibrium and locality of interactions

# The round jet – the ultimate testbed

**Stationary jet in local equilibrium**

**‘Large and slow’ scales – resolveable!**

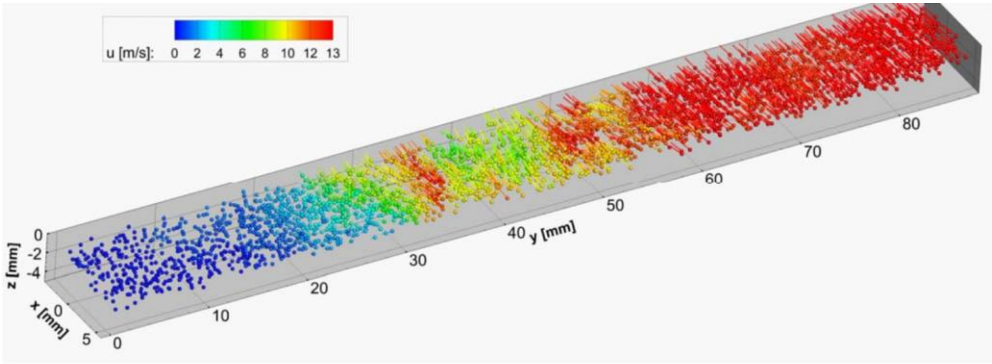
**Can easily be pushed out of equilibrium, arbitrarily and quantifiably**



# Two-fold strategy

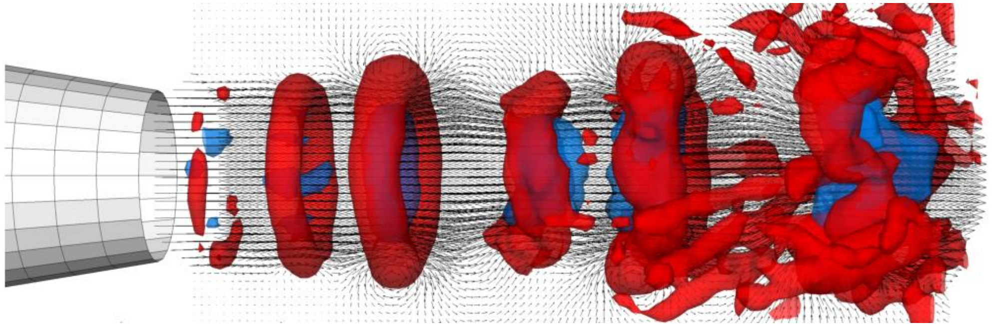
**HIGH RESOLUTION (HR) MEASUREMENTS**

**Central objective:** Quantify degree of non-equilibrium




**FULL-FIELD (FF) MEASUREMENTS**

**Central objective:** Local/non-local interactions?



# Two-fold strategy

**HIGH RESOLUTION (HR) MEASUREMENTS**

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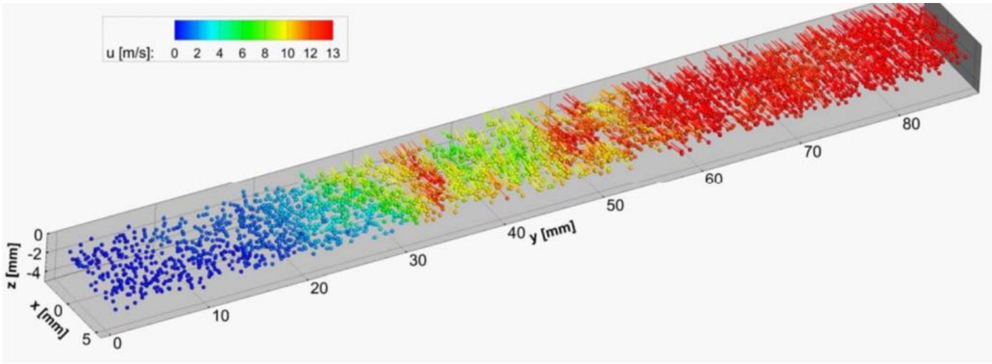



$$\frac{1}{E}\frac{\partial E}{\partial t} \propto \frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial t} \Rightarrow \frac{u'^2}{\varepsilon}\frac{1}{E}\frac{\partial E}{\partial t} \propto \frac{u'^2}{\varepsilon^2}\frac{\partial \varepsilon}{\partial t}$$

$$\Downarrow$$

$$\frac{D\overline{\varepsilon}}{Dt} = \frac{\partial \overline{\varepsilon}}{\partial t} + U_j\frac{\partial \overline{\varepsilon}}{\partial x_j} = D_{\overline{\varepsilon}} + P_{\overline{\varepsilon}} + T_{\overline{\varepsilon}}$$

# Dissipation transport

$$\frac{D\varepsilon}{Dt} = D_\varepsilon + P_\varepsilon + T_\varepsilon$$

**Common solution for turbulence models:**

$$D_\varepsilon \propto \frac{\varepsilon}{u'^2/\varepsilon} \qquad P_\varepsilon \propto \frac{P_{TKE}}{u'^2/\varepsilon}$$

$$A = \frac{\partial \varepsilon}{\partial t} = \nu \frac{\partial}{\partial t} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$B = \nu u_1 \frac{\partial}{\partial x_1} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$+\nu u_2 \frac{\partial}{\partial x_2} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$C = \nu \langle u_j s_{ij} \partial_j r_{ij} \rangle - \nu \langle u_i s_{ij} \partial_i r_{ij} \rangle$$

$$\Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$+\nu u_3 \frac{\partial}{\partial x_3} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$D = -2\nu \Big\{ \Big[ \Big\langle 2\frac{\partial u_1}{\partial x_1} \frac{\partial^2 \langle u_1 u_1 \rangle}{\partial x_1^2} \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\Big) \frac{\partial^2 \langle u_1 u_2 \rangle}{\partial x_2^2} \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\Big) \frac{\partial^2 \langle u_1 u_3 \rangle}{\partial x_3^2} \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\Big) \frac{\partial^2 \langle u_1 u_2 \rangle}{\partial x_1^2} \Big\rangle + \Big\langle 2\frac{\partial u_2}{\partial x_2} \frac{\partial^2 \langle u_2 u_2 \rangle}{\partial x_2^2} \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\Big) \frac{\partial^2 \langle u_2 u_3 \rangle}{\partial x_3^2} \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\Big) \frac{\partial^2 \langle u_1 u_3 \rangle}{\partial x_1^2} \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\Big) \frac{\partial^2 \langle u_2 u_3 \rangle}{\partial x_2^2} \Big\rangle + \Big\langle 2\frac{\partial u_3}{\partial x_3} \frac{\partial^2 \langle u_3 u_3 \rangle}{\partial x_3^2} \Big\rangle \Big] \Big\}$$

$$-\frac{4\nu}{\rho} \Big[ \Big\langle 2\frac{\partial u_1}{\partial x_1}\frac{\partial^2 p}{\partial x_1^2} \Big\rangle + \Big\langle 2\frac{\partial u_2}{\partial x_2}\frac{\partial^2 p}{\partial x_2^2} \Big\rangle + \Big\langle 2\frac{\partial u_3}{\partial x_3}\frac{\partial^2 p}{\partial x_3^2} \Big\rangle +$$

$$\Big\langle \Big(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\Big) \frac{\partial^2 p}{\partial x_1 \partial x_2} \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\Big) \frac{\partial^2 p}{\partial x_1 \partial x_3} \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\Big) \frac{\partial^2 p}{\partial x_2 \partial x_3} \Big\rangle \Big]$$

$$F = \nu^2 \frac{\partial^2}{\partial x_1^2} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$+\nu^2 \frac{\partial^2}{\partial x_2^2} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$\Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$+\nu^2 \frac{\partial^2}{\partial x_3^2} \Big\{ 2 \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_3}\Big)^2 \Big\rangle \Big] +$$

$$+ \Big[ \Big\langle \Big(\frac{\partial u_1}{\partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_1}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_2}{\partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial u_3}{\partial x_2}\Big)^2 \Big\rangle \Big] +$$

$$2 \Big[ \Big\langle \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_1}{\partial x_3}\frac{\partial u_3}{\partial x_1} \Big\rangle + \Big\langle \frac{\partial u_2}{\partial x_3}\frac{\partial u_3}{\partial x_2} \Big\rangle \Big] \Big\}$$

$$G = -\nu^2 \Big\{ 4 \Big[ \Big\langle \Big(\frac{\partial^2 u_1}{\partial x_1^2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_2}{\partial x_2^2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_3}{\partial x_3^2}\Big)^2 \Big\rangle \Big]$$

$$+ \Big[ \Big\langle \Big(\frac{\partial^2 u_1}{\partial x_2^2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_1}{\partial x_3^2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_2}{\partial x_1^2}\Big)^2 \Big\rangle +$$

$$\Big\langle \Big(\frac{\partial^2 u_2}{\partial x_3^2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_3}{\partial x_1^2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_3}{\partial x_2^2}\Big)^2 \Big\rangle \Big]$$

$$+ \Big[ \Big\langle \Big(\frac{\partial^2 u_2}{\partial x_1 \partial x_2}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_3}{\partial x_1 \partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_1}{\partial x_2 \partial x_1}\Big)^2 \Big\rangle +$$

$$\Big\langle \Big(\frac{\partial^2 u_3}{\partial x_2 \partial x_3}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_1}{\partial x_3 \partial x_1}\Big)^2 \Big\rangle + \Big\langle \Big(\frac{\partial^2 u_2}{\partial x_3 \partial x_2}\Big)^2 \Big\rangle \Big]$$

$$\Big[ 2 \Big\langle \frac{\partial^2 u_1}{\partial x_2^2}\frac{\partial^2 u_2}{\partial x_1 \partial x_2} \Big\rangle + 2 \Big\langle \frac{\partial^2 u_1}{\partial x_3^2}\frac{\partial^2 u_3}{\partial x_1 \partial x_3} \Big\rangle + 2 \Big\langle \frac{\partial^2 u_2}{\partial x_1^2}\frac{\partial^2 u_1}{\partial x_2 \partial x_1} \Big\rangle +$$

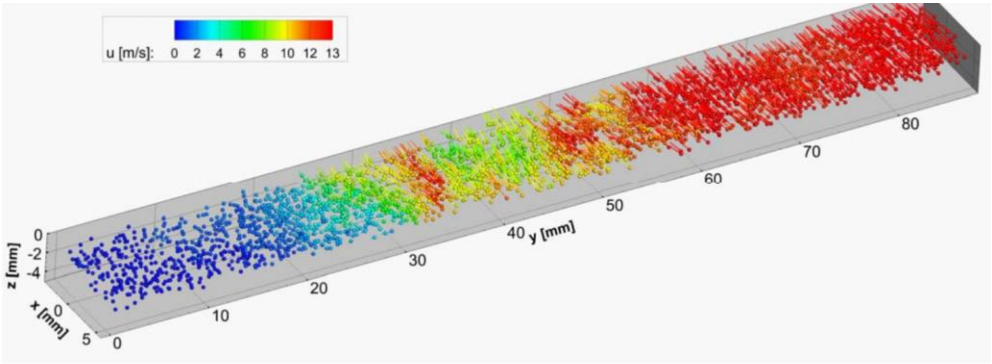
$$\Big[ 2 \Big\langle \frac{\partial^2 u_2}{\partial x_3^2}\frac{\partial^2 u_3}{\partial x_2 \partial x_3} \Big\rangle + 2 \Big\langle \frac{\partial^2 u_3}{\partial x_1^2}\frac{\partial^2 u_1}{\partial x_3 \partial x_1} \Big\rangle + 2 \Big\langle \frac{\partial^2 u_3}{\partial x_2^2}\frac{\partial^2 u_2}{\partial x_3 \partial x_2} \Big\rangle \Big] \Big] \Big\}$$

$$H = \nu^2 \Big[ \Big\langle s_{ij} \frac{\partial^2 r_{ij}}{\partial x_j^2} \Big\rangle - \Big\langle s_{ij} \frac{\partial^2 r_{ij}}{\partial x_i^2} \Big\rangle \Big]$$

# Two-fold strategy

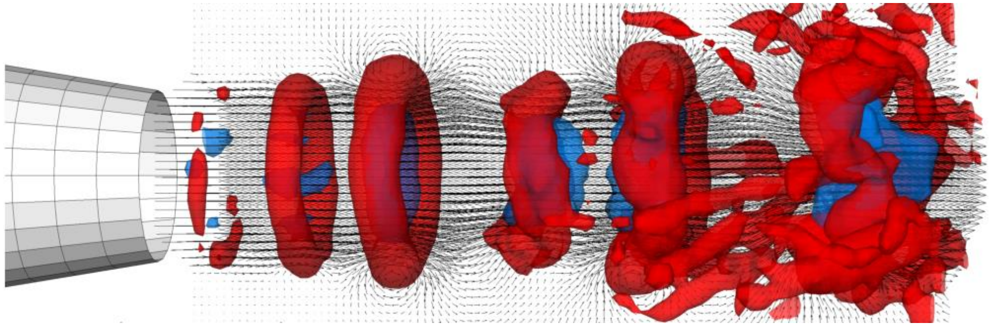
**HIGH RESOLUTION (HR) MEASUREMENTS**

**Central objective:** Quantify degree of non-equilibrium




**FULL-FIELD (FF) MEASUREMENTS**

**Central objective:** Local/non-local interactions?



# Two-fold strategy

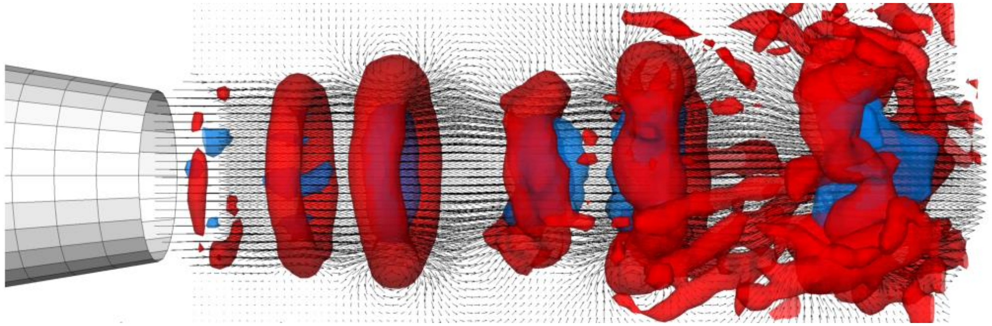
***"What part of modeling is in serious need of work? Foremost, I would say, is the mechanism that sets the level of dissipation in a turbulent flow, particularly in changing circumstances."***

**- John L. Lumley, 1992**

$$\int \ldots \int_{\text{all space, time}} R_{i,j}(\mathbf{x}, \mathbf{x}', t, t')\ \phi_j(\mathbf{x}', t')d\mathbf{x}'dt' \ = \ \lambda\ \phi_i(\mathbf{x}, t)$$

**FULL-FIELD (FF) MEASUREMENTS**

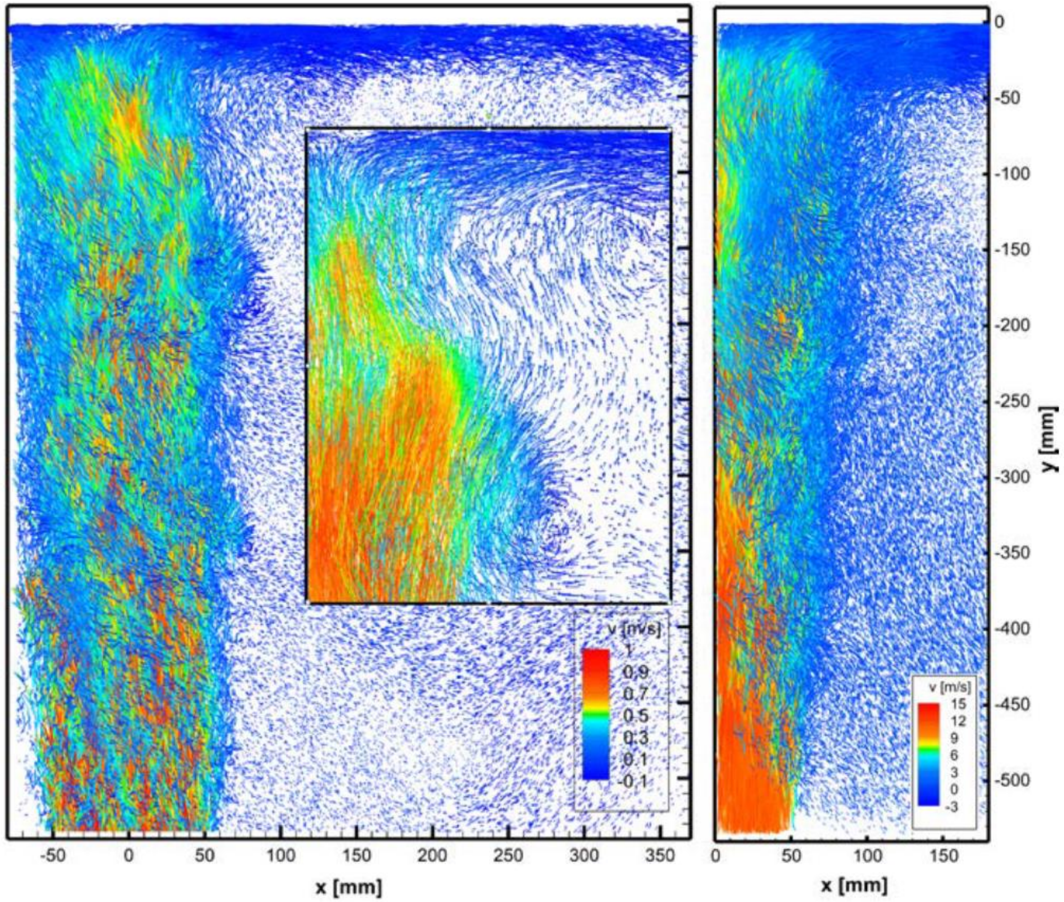
**Central objective:** Local/non-local interactions?



# Full-field experiment

Example from Schanz, Gesemann & Schröder (2016)

$$\int \ldots \int_{\text{all space, time}} R_{i,j}(\mathbf{x}, \mathbf{x}', t, t')\ \phi_j(\mathbf{x}', t')d\mathbf{x}'dt' \ = \ \lambda\ \phi_i(\mathbf{x}, t)$$



# Full-field experiment

**The Lumley decomposition has the potential to provide 4D eigenmodes which are solutions to the Navier-Stokes equations.**

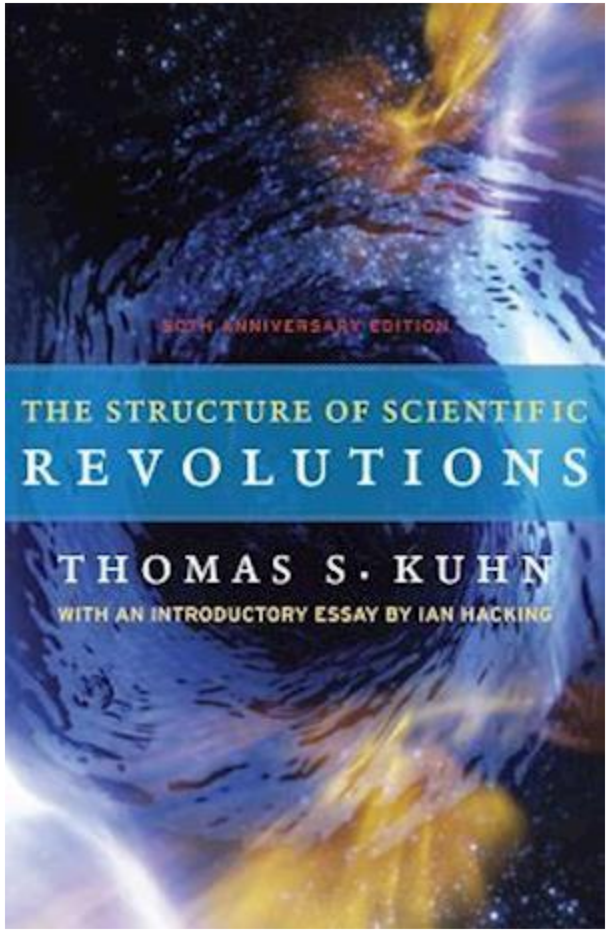
$$\int \dots \int_{\text{all space, time}} R_{i,j}(\mathbf{x}, \mathbf{x}', t, t')\ \phi_j(\mathbf{x}', t') d\mathbf{x}' dt' \ = \ \lambda\ \phi_i(\mathbf{x}, t)$$

⇓

# Governing equations

# Results

# The structure of scientific revolution




**The paradigm**, in Kuhn's view, is **not simply the current theory, but the entire worldview in which it exists**, and all of the implications which come with it.

**A scientific revolution occurs**, according to Kuhn, **when scientists encounter anomalies that cannot be explained by the universally accepted paradigm** within which scientific progress has thereto been made.

> "What man sees depends both upon what he looks at and also upon what his previous visual-conception experience has taught him to see."
> ― Thomas S. Kuhn, The Structure of Scientific Revolutions

> "Max Planck, surveying his own career in his Scientific Autobiography, sadly remarked that "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it."
> ― Thomas S. Kuhn, The Structure of Scientific Revolutions

European Research Council

Established by the European Commission

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 803419).

Jupiter photographed by NASA's Juno spacecraft

**Thank you for your attention! ☺**