Implications of Symmetries in the Scalar Sector (CP Properties and Mass Degeneracies)

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Two Higgs Doublet Models

Several motivations

- New sources of CP violation

SM cannot account for BAU

- Possibility of having spontaneous CP violation

EW symmetry breaking and CP violation same footing

T. D. Lee 1973, Kobayashi and Maskawa 1973

- Strong CP Problem, Peccei-Quinn

- Supersymmetry?

LHC important role

Motivation for three Higgs doublets

Three fermion generations may suggest three doublets

Interesting scenario for dark matter

Possibility of having a discrete symmetry and still having spontaneous CP violation

Rich phenomenology

Motivation for imposing discrete symmetries

Symmetries reduce the number of free parameters leading to (testable) predictions

Symmetries help to control HFCNC

Example: NFC, no HFCNC due to Z_2 symmetry(ies)

Example: MFV suppression of HFCNC, BGL models

Symmetries are needed to stabilise dark matter

Symmetries of the 2 Higgs Doublet Model

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)$$
$$+\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} . \qquad (2.1)$$

11 independent parameters

If all the parameters are real CP is explicitly conserved:

most general CP transformation $\quad \Phi_i \xrightarrow{\text{CP}} U_{ij} \Phi_j^*$

with U a unitary matrix which we can choose as the identity matrix when all parameters are real

However, there is still the possibility of Spontaneous Symmetry Breaking

T. D. Lee 1973

The above equation together with the assumption that the vacuum is CP invariant leads to

$U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \Phi_i | 0 \rangle \qquad \mathcal{L}(U\phi) = \mathcal{L}(\phi) \qquad \text{CP}|0\rangle = |0\rangle$

G. C. Branco, J. M. Gerard and W. Grimus 1984

CP is violated spontaneously by vevs of the form $(\rho_1 e^{i\theta}, \rho_2)$, in the region of parameters of the potential where ρ_1 and ρ_2 are different from zero and $\quad e^{i\theta} \neq 1$

List of all possible Symmetries of the 2HDM

The complete list of such symmetries is known:

symmetry	transformation law		
$\mathbb{Z}_2$	$\Phi_1 \to \Phi_1$	$\Phi_2 \to -\Phi_2$	
U(1)	$\Phi_1 \to \Phi_1$	$\Phi_2 \to e^{2i\theta}\Phi_2$	
SO(3)	$\Phi_a \to U_{ab}\Phi_b$	$U \in \mathrm{U}(2)/\mathrm{U}(1)_\mathrm{Y}$	(for $a,\, b = 1, 2$)
GCP1	$\Phi_1 \to \Phi_1^*$	$\Phi_2 \to \Phi_2^*$	
GCP2	$\Phi_1 \to \Phi_2^*$	$\Phi_2 \to -\Phi_1^*$	
GCP3	$\Phi_1 \to \Phi_1^* \cos\theta + \Phi_2^* \sin\theta$	$\Phi_2 \to -\Phi_1^* \sin\theta + \Phi_2^* \cos\theta$	(for $0 < \theta < \frac{1}{2}\pi$)
Π_2	$\Phi_1 \to \Phi_2$	$\Phi_2 \to \Phi_1$	

Deshpande and Ma 1978, Ivanov 2007, Ferreira, Haber and Silva 2009, Ferreira, Haber, Maniatis, Nachtmann and Silva 2011, Battye, Brawn, Pilaftsis 2011, Pilaftsis 2011

There are three possible Higgs family symmetries (first three rows) and three classes of CP symmetries with different U matrices (next three rows)

There are seven additional accidental symmetries of the 2HDM scalar potential

Battye, Brawn, Pilaftsis 2011, Pilaftsis 2012

which are not exact symmetries since they are violated by the U(1) gauge kinetic term of the scalar potential, as well as by the Yukawa couplings, therefore, not considered here.

List of all possible Symmetries of the 2HDM (cont.)

Starting from a generic scalar potential given by Eq. (2.1) if the scalar potential respects one of the symmetries listed in Table 1, the coefficients of the scalar potential are constrained according to Table 2, in the basis where the symmetry is manifest

symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	Re λ_5	Im λ_5	λ_6	λ_7
$\mathbb{Z}_2$	-	-	0	-	-	-	-	-	-	0	0
U(1)	-	-	0	-	-	-	-	0	0	0	0
SO(3)	-	m_{11}^2	0	-	λ_1	-	$\lambda_1 - \lambda_3$	0	0	0	0
GCP1	-	-	real	-	-	-	-	-	0	real	real
GCP2	-	m_{11}^2	0	-	λ_1	-	-	-	-	-	$-\lambda_6$
GCP3	-	m_{11}^2	0	-	λ_1	-	-	$\lambda_1 - \lambda_3 - \lambda_4$	0	0	0
Π_2	-	m_{11}^2	real	-	λ_1	-	-	-	0	-	λ_6^*
$\mathbb{Z}_2 \oplus \Pi_2$	-	m_{11}^2	0	-	λ_1	-	-	-	0	0	0
U(1)$\oplus\Pi_2$	-	m_{11}^2	0	-	λ_1	-	-	0	0	0	0

In all these cases the imposed symmetry leads to explicit CP is conservation

In all cases GCP1, and also 2 and 3 there is invariance under hermitian conjugation

Ferreira, Haber and Silva 2009

Possibility of spontaneous CP violation with Z_2 softly broken

Branco and Rebelo 1985

Natural 2HDM mass degeneracies

Analysis of explicit expressions of the neutral scalar masses

or

Consider all possible symmetries of the 2HDM

Mass degenerate neutral scalars can only arise naturally in the 2HDM in the case of the IDM with Z_5 = 0

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\},$$

exact $\mathbb{Z}_2$ symmetry $H_1 \to +H_1$ and $H_2 \to -H_2$

$Y_3 = Z_6 = Z_7 = 0$ preserved by the vacuum

Physical scalar mass spectrum

$$m_h^2 = Z_1 v^2 , \qquad m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2 ,$$

$$m_A^2 = m_{H^\pm}^2 + \tfrac{1}{2}(Z_4 - Z_5) v^2 , \qquad m_H^2 = m_A^2 + Z_5 v^2 .$$

$m_H = m_A$, due to $Z_5 = 0$.

Natural 2HDM mass degeneracies (cont.)

$Y_3 = Z_6 = Z_7 = 0$. **together with** $Z_5 = 0$

exact continuous unbroken U(1) symmetry $H_1 \to H_1 \qquad H_2 \to e^{i\theta} H_2$

It is this symmetry that is responsible for the mass degenerate states H and A

One can now define eigenstates of U(1) charge:

$$\phi^{\pm} = \frac{1}{\sqrt{2}}\left[H \pm iA\right]$$

Physical mass spectrum of the mass degenerate IDM:

$$m_h^2 = Z_1 v^2 \,,$$

$$m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2 \,,$$

$$m_{\phi^\pm}^2 = Y_2 + \tfrac{1}{2}(Z_3 + Z_4) v^2$$

Natural 2HDM mass degeneracies (cont.)

Although $\phi^\pm$ are mass degenerate states, they can be physically distinguished on an event by event basis.

The relevant interaction terms of $\phi^\pm$ are

$$\mathscr{L}_{\rm int} = \left[\tfrac{1}{2}g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{4c_W^2} Z_\mu Z^\mu\right]\phi^+\phi^- + \frac{ig}{2c_W} Z^\mu \phi^- \overleftrightarrow{\partial}_\mu\, \phi^+ - \frac{g}{\sqrt{2}}\left[iW_\mu^+ H^- \overleftrightarrow{\partial}^\mu\, \phi^+ + {\rm h.c.}\right]$$
$$+\frac{eg}{\sqrt{2}}\left(A^\mu W_\mu^+ H^-\phi^+ + A^\mu W_\mu^- H^+\phi^-\right) - \frac{g^2 s_W^2}{\sqrt{2}c_W}\left(Z^\mu W_\mu^+ H^-\phi^+ + Z^\mu W_\mu^- H^+\phi^-\right)$$
$$-v(Z_3+Z_4)h\phi^+\phi^- - \tfrac{1}{2}\left[Z_2(\phi^+\phi^-)^2 + (Z_3+Z_4)h^2\phi^+\phi^-\right] - Z_2 H^+H^-\phi^+\phi^-\,.$$

For example, Drell-Yan production via a virtual s-channel W^+ exchange can produce H^+ in association with ϕ^-, whereas virtual s-channel W^- exchange can produce H^- in association with ϕ^+. Thus, the sign of the charged Higgs boson reveals the U(1)-charge of the produced neutral scalar. The origin of this correlation lies in the fact that, by construction, H^+ and ϕ^+ both reside in H_2, whereas H^- and ϕ^- both reside in $H_2^\dagger$.

Models with three Higgs doublets

There is not yet a full study of all possible symmetries

e.g. Ivanov et al

Two Particular Scenarios will be briefly discussed in what follows

A CP-conserving multi-Higgs Model with irremovable complex coefficients

Ivanov and Silva 2015

Three Higgs doublet models with S_3 Symmetry

A CP-conserving multi-Higgs Model with irremovable complex coefficients

I.P. Ivanov and J.P. Silva, Phys. Rev. D **93**, 095014 (2016) [arXiv:1512.09276],

The IS potential can be written in the following way

$$V = V_0 + V_1$$

where

$$\begin{aligned} V_0 &= -m_{11}^2(\phi_1^\dagger\phi_1) - m_{22}^2(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2[(\phi_2^\dagger\phi_2)^2 + (\phi_3^\dagger\phi_3)^2] + \lambda_3'(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\ &\quad + \lambda_3(\phi_1^\dagger\phi_1)[(\phi_2^\dagger\phi_2) + (\phi_3^\dagger\phi_3)] + \lambda_4'(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) + \lambda_4[(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + (\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1)], \quad (7) \end{aligned}$$

$$\begin{aligned} V_1 &= \lambda_5(\phi_3^\dagger\phi_1)(\phi_2^\dagger\phi_1) + \tfrac{1}{2}\lambda_6[(\phi_2^\dagger\phi_1)^2 - (\phi_1^\dagger\phi_3)^2] + \lambda_8(\phi_2^\dagger\phi_3)^2 \\ &\quad + \lambda_9(\phi_2^\dagger\phi_3)[(\phi_2^\dagger\phi_2) - (\phi_3^\dagger\phi_3)] + \text{h.c.} \end{aligned}$$

with λ_5, λ_6 real and λ_8, λ_9 complex. This potential is fixed by the following CP symmetry:

$$\phi_i \to W_{ij}\phi_j^*, \quad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}$$

There is a region of parameter space where only the first doublet acquires a vev different from zero and the above symmetry is not broken

After a change of basis the IS potential can be written in the following way

$$\mathcal{V}_{\rm IS} = \mathcal{V}_{\rm RIDM} + Z_3'(H_2^\dagger H_2)(H_3^\dagger H_3) + Z_4'(H_2^\dagger H_3)(H_3^\dagger H_2) + \left[Z_8(H_2^\dagger H_3)^2 + Z_9(H_2^\dagger H_3)(H_2^\dagger H_2 - H_3^\dagger H_3) + \text{h.c.}\right]$$

where

$$\begin{aligned}\mathcal{V}_{\rm RIDM} = {} & Y_1 H_1^\dagger H_1 + Y_2\left(H_2^\dagger H_2 + H_3^\dagger H_3\right) + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \tfrac{1}{2}Z_2(H_2^\dagger H_2 + H_3^\dagger H_3)^2 \\ & + Z_3(H_1^\dagger H_1)\left(H_2^\dagger H_2 + H_3^\dagger H_3\right) + Z_4\left[(H_1^\dagger H_2)(H_2^\dagger H_1) + (H_1^\dagger H_3)(H_3^\dagger H_1)\right] \\ & + \tfrac{1}{2}Z_5\left\{(H_1^\dagger H_2)^2 + (H_2^\dagger H_1)^2 + (H_1^\dagger H_3)^2 + (H_3^\dagger H_1)^2\right\}\,.\end{aligned}$$

Z_8 and Z_9 are potentially complex

Whenever all the coefficients of the potential are real CP is conserved with a simple CP transformation where U the unitary matrix can be chosen as the identity

If Z_8 and/or Z_9 are complex CP is conserved:

$$H_i \to X_{ij} H_j^\star \,, \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

This was called a CP4 symmetry by Ivanov and Silva since it must be applied four times in order to yield the identity

There is no CP2 symmetry in this case since there is no possible change of basis in which all scalar potential parameters are real

This symmetry is responsible for the degeneracies in the IS model

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left[v + h_{\rm SM} + iG^0\right] \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left[H + iA\right] \end{pmatrix}, \quad H_3 = \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}}\left[h + ia\right] \end{pmatrix}$$

These are already the physical fields and there is pairwise degeneracy among the fields of the second and third doublets

It is convenient to re-express the neutral scalar fields in terms of complex fields P and Q and their conjugates:

$$P \equiv \frac{H + ih}{\sqrt{2}}, \qquad Q \equiv \frac{A - ia}{\sqrt{2}}, \qquad P^\dagger \equiv \frac{H - ih}{\sqrt{2}}, \qquad Q^\dagger \equiv \frac{A + ia}{\sqrt{2}},$$

the fields P, Q and the corresponding conjugate fields $P^\dagger$ and $Q^\dagger$ are each eigenstates of CP4.

In particular, under a CP4 transformation, $P \to iP$, $Q \to iQ$, $P^\dagger \to -iP^\dagger$, and $Q^\dagger \to -iQ^\dagger$

P and the corresponding conjugate state $P^\dagger$ are mass-degenerate, but are otherwise unrelated fields (and similarly for Q and $Q^\dagger$)

The distinction between the IS scalar potential with Z_8 and Z_9 real or complex is physical $Z_5 \neq 0$,

The presence of the terms:

$$\delta\mathscr{L}_{4h} \ni \tfrac{1}{2}\,\mathrm{Im}\,Z_8\big[(PQ - P^\dagger Q^\dagger)(P^2 - Q^2 - P^{\dagger\,2} + Q^{\dagger\,2})\big]$$

$$+\tfrac{1}{2}i\,\mathrm{Im}\,Z_9\big[(PQ - P^\dagger Q^\dagger)(P^2 + Q^2 + P^{\dagger\,2} + Q^{\dagger\,2})\big]\,.$$

signals a CP4 symmetric IS scalar potential that does not respect a conventional CP symmetry $H_i \to H_i^\star$

The Z coupling to the P and Q fields is of the form:

$$\frac{g}{2c_W}\,Z^\mu(Q\overleftrightarrow{\partial}_\mu P + Q^\dagger\overleftrightarrow{\partial}_\mu P^\dagger)$$

this ZPQ interaction would permit the decay, if kinematically available:

$$Z \to PQ, P^*Q^*$$

suppose that $M_Q < \frac{1}{4}m_Z < M_P$ **In this example** P and P^* would be virtual

One possible decay of the virtual P or P^* **makes use of the existence of the four scalar interaction given above. If this interaction is present, the decay**

$$Z \to QQQQ^*,\ Q^*Q^*Q^*Q$$

is allowed and provides unambiguous evidence that either Z_8 and/or Z_9 **possess a nonzero imaginary part**

An Observable distinction between real and complex Z_8 and Z_9 (cont)

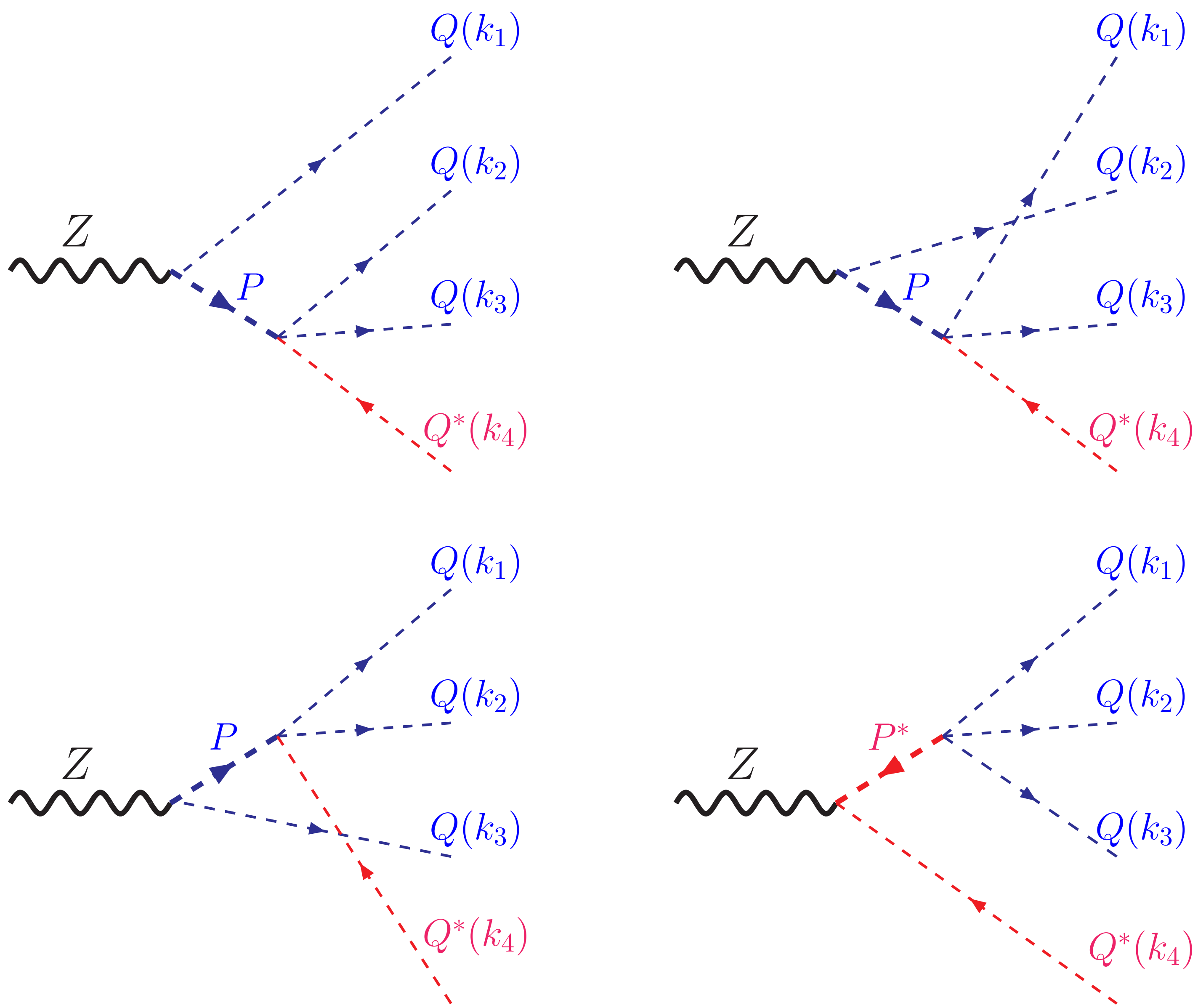

Figure 2: Feynman diagrams for $Z \to QQQQ^*$

Finally let us look at the special case $Z_5 = 0$

if $Z_5 = 0$, then it is possible to change the basis of scalar fields of the IS model,

in such a way that $$\operatorname{Im} Z_8 = \operatorname{Im} Z_9 = 0.$$

without changing the form of the potential

if $Z_5 = 0$, then $M_P = M_Q$. **and**

the decay $Z \to QQQQ^*, QQ^*Q^*Q^*$ is no longer an experimental observable.

However, there is a

four-scalar $|P|^2|Q|^2$ interaction that contributes to $Z \to QQPP^*$

The ZZZ and ZWW vertices

the CP violating effects cancel at loop level in the effective ZZZ and ZWW vertices

Nevertheless, any CP-violating observable of the IS model must vanish. For example, the contributions to the CP-violating form factors of the effective ZZZ and ZWW vertices generated in the IS model must exactly cancel. As a check of this statement, we confirmed this cancellation up to three-loop order in the IS model with no real Higgs basis.

Three Higgs doublet models with S_3 Symmetry

The Scalar potential

S_3 is the permutation group involving three objects, ϕ_1, ϕ_2, ϕ_3

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \mathrm{hc}]$$

$$V_4 = A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \mathrm{hc}]\}$$

$$+ \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \mathrm{hc}] + \sum_{i \neq j \neq k \neq i, j<k} \{\frac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \mathrm{hc}]$$

$$+ \frac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \mathrm{hc}] + \frac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \mathrm{hc}]\}$$

Derman, 1979

here all fields appear on equal footing

this representation is not irreducible, for instance, the combination

$$\phi_1 + \phi_2 + \phi_3$$

remains invariant, it splits into two irreducible representations,

doublet and singlet: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad h_S$

Decomposition into these two irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

This definition does not treat equally

$$\phi_1, \phi_2, \phi_3$$

they could be interchanged

Notice similarity with tribimaximal mixing:

Harrison, Perkins and Scott, 1999

$$(F =) \quad \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The scalar potential in terms of fields from irreducible representations

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2),$$

$$\begin{aligned} V_4 = {} & \lambda_8 (h_S^\dagger h_S)^2 + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) + \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 \\ & + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] \\ & + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] \end{aligned}$$

Das and Dey

no symmetry under the interchange of h_1 and h_2

however there is symmetry for $h_1 \to -h_1$

equivalent doublet representation $\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

now there is symmetry for $\chi_1 \leftrightarrow \chi_2$

In the special case $\lambda_4 = 0$ **the potential has SO(2) symmetry:**

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Danger: massless scalar!

Constraining the potential by the vevs

Possibility of SCPV - real parameters

Let us start with real vacua (no CP violation)

Three minimisation conditions:

can be solved to give μ_0^2 and μ_1^2 in terms of the quartic coefficients:

$$\mu_0^2 = \frac{1}{2w_S}\left[\lambda_4(w_2^2 - 3w_1^2)w_2 - (\lambda_5+\lambda_6+2\lambda_7)(w_1^2+w_2^2)w_S - 2\lambda_8 w_S^3\right], \qquad (4.2a)$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1+\lambda_3)(w_1^2+w_2^2) + 6\lambda_4 w_2 w_S + (\lambda_5+\lambda_6+2\lambda_7)w_S^2\right], \qquad (4.2b)$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1+\lambda_3)(w_1^2+w_2^2) - 3\lambda_4(w_2^2-w_1^2)\frac{w_S}{w_2} + (\lambda_5+\lambda_6+2\lambda_7)w_S^2\right]. \qquad (4.2c)$$

Eqs (4.2b) and (4.2c) obtained dividing by w_1 and w_2 respectively

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Consistency requires:

- for $w_1 = 0$ the corresponding derivative is zero - no clash
- or else $\lambda_4(3w_2^2 - w_1^2)w_S = 0$ i. e., $\lambda_4 = 0$ or $w_1 = \pm\sqrt{3}w_2$ or $w_S = 0.$
- for $w_S = 0.$ special condition: $\lambda_4 w_2(3w_1^2 - w_2^2) = 0,$ i. e., in addition: $\lambda_4 = 0$ or $w_2 = \pm\sqrt{3}w_1,$ or else $w_2 = 0.$

SSB, real vacua, residual symmetries

Derman, Tsao Phys. Rev. D20 (1979) 1207:

(x, x, x) S_3 ; (x, x, y) S_2 ; (x, y, z) = (x, -x, 0) S_2 $\lambda_4 \neq 0$

Translation into doublet singlet notation

(x, x, x) $\rightarrow$ (0,0, ω_S) $w_1 = 0$ (also verifies $w_1 = \pm\sqrt{3}w_2$)

(x, -x, 0) $\rightarrow$ $(\omega_1, 0, 0)$ $w_S = 0$ together with $w_2 = 0.$

(x, 0, -x) $\rightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together $w_2 = \sqrt{3}w_1$

(0,x, -x) $\rightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together with $w_2 = -\sqrt{3}w_1$

(x, x, y) translates into $(0, w_2, w_S)$; consistency condition: $w_1 = 0.$

(x, y, x) translates into $(w_1, -\frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = -\sqrt{3}w_2$

(y, x, x) translates into $(w_1, \frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = \sqrt{3}w_2$

For $\lambda_4 = 0$ SO(2) symmetry implies (x, y, z) possible solution

Vacuum	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	Comment
R-0	$0, 0, 0$	$0, 0, 0$	Not interesting
R-I-1	x, x, x	$0, 0, w_S$	$\mu_0^2 = -\lambda_8 w_S^2$
R-I-2a	$x, -x, 0$	$w, 0, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_1^2$
R-I-2b	$x, 0, -x$	$w, \sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-I-2c	$0, x, -x$	$w, -\sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-II-1a	x, x, y	$0, w, w_S$	$\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1b	x, y, x	$w, -w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1c	y, x, x	$w, w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-2	$x, x, -2x$	$0, w, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2,\ \lambda_4 = 0$
R-II-3	$x, y, -x-y$	$w_1, w_2, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right), \lambda_4 = 0$
R-III	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	$\mu_0^2 = -\frac{1}{2}\lambda_a (w_1^2 + w_2^2) - \lambda_8 w_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right) - \frac{1}{2}\lambda_a w_S^2,$ $\lambda_4 = 0$

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7,$$

$$\lambda_b = \lambda_5 + \lambda_6 - 2\lambda_7.$$

Complex vacua

Table 2: Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3\sin 2\rho_1/\sin 2\rho_2}$, $\psi = \sqrt{[3+3\cos(\rho_2-2\rho_1)]/(2\cos\rho_2)}$. With the constraints of Table 4 the vacua labelled with an asterisk (*) are in fact real.

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho}-\frac{y}{2}, -xe^{i\rho}-\frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho}\pm ix, re^{i\rho}\mp ix, \frac{3}{2}re^{-i\rho}-\frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho}\pm ix, re^{-i\rho}\mp ix, \frac{3}{2}re^{i\rho}-\frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho}+x, -re^{i\rho}+x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho}+x, -re^{-i\rho}+x, -re^{i\rho}+re^{-i\rho}+x$
C-IV-c	$\sqrt{1+2\cos^2\sigma_2}\hat{w}_2$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho}+r\sqrt{3(1+2\cos^2\rho)}+x$, $re^{i\rho}-r\sqrt{3(1+2\cos^2\rho)}+x, -2re^{i\rho}+x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho}+x, (r_2-r_1)e^{i\rho}+x, -r_2 e^{i\rho}+x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2}+re^{i\rho_1}\xi+x, re^{i\rho_2}-re^{i\rho_1}\xi+x$, $-2re^{i\rho_2}+x$
C-IV-f	$\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1}+re^{i\rho_2}\psi+x$, $re^{i\rho_1}-re^{i\rho_2}\psi+x, -2re^{i\rho_1}+x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

Constraints

Vacuum	Constraints
C-I-a	$\mu_1^2 = -2\left(\lambda_1 - \lambda_2\right)\hat{w}_1^2$
C-III-a	$\mu_0^2 = -\frac{1}{2}\lambda_b\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \frac{4\cos\sigma_2\hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-b	$\mu_0^2 = -\frac{1}{2}\lambda_b\hat{w}_1^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\lambda_b\hat{w}_S^2,$ $\lambda_4 = 0$
C-III-c	$\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$
C-III-d,e	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{(\hat{w}_1^2 - \hat{w}_2^2)^2}{\hat{w}_S^2} - \epsilon\lambda_4\frac{(\hat{w}_1^2 - \hat{w}_2^2)(\hat{w}_1^2 - 3\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \epsilon\lambda_4\frac{\hat{w}_S(\hat{w}_1^2 - \hat{w}_2^2)}{4\hat{w}_2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_7 = \frac{\hat{w}_1^2 - \hat{w}_2^2}{\hat{w}_S^2}(\lambda_2 + \lambda_3) - \epsilon\frac{(\hat{w}_1^2 - 5\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}\lambda_4$
C-III-f,g	$\mu_0^2 = -\frac{1}{2}\lambda_b\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\lambda_b\hat{w}_S^2, \lambda_4 = 0$
C-III-h	$\mu_0^2 = -2\lambda_b\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \mp\frac{2\cos\sigma_2\hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-i	$\mu_0^2 = \frac{16\left(1-3\tan^2\sigma_1\right)^2}{(1+9\tan^2\sigma_1)^2}(\lambda_2 + \lambda_3)\frac{\hat{w}_2^4}{\hat{w}_S^2} \pm \frac{6\left(1-\tan^2\sigma_1\right)(1-3\tan^2\sigma_1)}{(1+9\tan^2\sigma_1)^{\frac{3}{2}}}\lambda_4\frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{2(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_5 + \lambda_6)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\frac{4(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_1 - \lambda_2)\hat{w}_2^2 \mp \frac{\left(1-3\tan^2\sigma_1\right)}{2\sqrt{1+9\tan^2\sigma_1}}\lambda_4\hat{w}_2\hat{w}_S$ $-\frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_S^2,$ $\lambda_7 = -\frac{4\left(1-3\tan^2\sigma_1\right)\hat{w}_2^2}{(1+9\tan^2\sigma_1)\hat{w}_S^2}(\lambda_2 + \lambda_3) \mp \frac{\left(5-3\tan^2\sigma_1\right)\hat{w}_2}{2\sqrt{1+9\tan^2\sigma_1}\hat{w}_S}\lambda_4$

Vacuum	Constraints
C-IV-a*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_1^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-b	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)^2}{\hat{w}_S^2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-c	$\mu_0^2 = 2\cos^2\sigma_2\left(1 + \cos^2\sigma_2\right)\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\left(1 + \cos^2\sigma_2\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left[2\left(1 + \cos^2\sigma_2\right)\lambda_1 - \left(2 + 3\cos^2\sigma_2\right)\lambda_2 - \cos^2\sigma_2\lambda_3\right]\hat{w}_2^2$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = -\frac{2\cos\sigma_2\hat{w}_2}{\hat{w}_S}\left(\lambda_2 + \lambda_3\right), \lambda_7 = \frac{\cos^2\sigma_2\hat{w}_2^2}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-d*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-e	$\mu_0^2 = \frac{\sin^2(2(\sigma_1 - \sigma_2))}{\sin^2(2\sigma_1)}\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\frac{1}{2}\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_1 - \lambda_2\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\sin(2(\sigma_1 - \sigma_2))\hat{w}_2^2}{\sin 2\sigma_1\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-f	$\mu_0^2 = -\frac{(\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1)\cos(\sigma_2 - \sigma_1)}{2\cos^2\sigma_1}\lambda_4\frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{2\cos\sigma_1}\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{\cos\sigma_1}\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2$ $-\frac{3\cos 2\sigma_1 + 2\cos(2(\sigma_1 - \sigma_2)) + \cos 2\sigma_2 + 4}{4\cos(\sigma_1 - \sigma_2)\cos\sigma_1}\lambda_4\hat{w}_2\hat{w}_S - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = -\frac{\cos\sigma_1\hat{w}_S}{2\cos(\sigma_2 - \sigma_1)\hat{w}_2}\lambda_4, \lambda_7 = -\frac{\cos(\sigma_2 - \sigma_1)\hat{w}_2}{2\cos\sigma_1\hat{w}_S}\lambda_4$
C-V*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8\hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0, \lambda_7 = 0$

The case of $\lambda_4 = 0$

Potential has additional continuous SO(2) symmetry

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Derman (1979), "unnatural"

Spontaneous breaking of this SO(2) symmetry leads to massless particles

Possible solution: break the symmetry softly, the most general quadratic potential can be written:

$$V = \mu_0^2 h_S^\dagger h_S + \mu_1^2(h_1^\dagger h_1 + h_2^\dagger h_2) + \mu_2^2(h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2}\nu^2(h_2^\dagger h_1 + h_1^\dagger h_2)$$
$$+ \mu_3^2(h_S^\dagger h_1 + h_1^\dagger h_S) + \mu_4^2(h_S^\dagger h_2 + h_2^\dagger h_S)$$

Complex vacua, Spontaneous CP Violation

Table 1: Spontaneous CP violation

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

Next we present a few illustrative examples. Important tool:

most general CP transformation

$$\Phi_i \xrightarrow{\text{CP}} U_{ij}\Phi_j^*$$

together with assumption that vacuum is invariant

$$\text{CP}|0\rangle = |0\rangle$$

leads to the following condition

$\mathcal{L}(U\phi) = \mathcal{L}(\phi)$

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

G. C. Branco, J. M. Gerard and W. Grimus (1984)

Vacuum C-I-a

$$x, \;\; xe^{\frac{2\pi i}{3}}, \;\; xe^{-\frac{2\pi i}{3}}$$

geometrical phases

G. C. Branco, J. M. Gerard and W. Grimus (1984)

calculable non-trivial phases, fixed by symmetry of V, no explicit dependence on parameters of the potential

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

CP is conserved

For new models with geometrical phases and the possibility of having CP violation with geometrical phases see

Ivo de Medeiros Varzielas, JHEP 1208 (2012) 055

Vacuum C-III-c

$$\hat{w}_1 e^{\imath\sigma_1}, \hat{w}_2 e^{\imath\sigma_2}, 0 \qquad \lambda_4 = 0$$

SO(2) rotation

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \qquad \tan 2\theta = \frac{\hat{w}_1^2 - \hat{w}_2^2}{2\hat{w}_1\hat{w}_2\cos\sigma}.$$

$$(ae^{i\delta_1}, ae^{i\delta_2}, 0)$$

followed by overall phase rotation

$$(ae^{i\delta}, ae^{-i\delta}, 0) \qquad U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

symmetry for interchange: $h_1' \leftrightarrow h_2'$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ae^{i\delta} \\ ae^{-i\delta} \\ 0 \end{pmatrix}^* = \begin{pmatrix} ae^{i\delta} \\ ae^{-i\delta} \\ 0 \end{pmatrix}$$

CP is conserved

$$U = e^{i(\delta_1+\delta_2)} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

Very simple and powerful relation. However, in some cases construction of matrix U may not be obvious

Simple Alternative Test

- Go to a basis where only one Higgs field acquires a vev different from zero and real

- If the coefficients of scalar potential can be made real by rephasing the fields with zero vev, there is no CP violation

- Notice that there is still the freedom of using a U(n-1) transformation acting on the fields with zero vev

Inspect the potential

C-III-c vacuum

$$\begin{pmatrix} h_1' \\ h_2' \\ h_S' \end{pmatrix} = \begin{pmatrix} \frac{1}{N_1}(\hat{w}_1 & \hat{w}_2 & \hat{w}_S) \\ \frac{1}{N_2}(\hat{w}_2 & -\hat{w}_1 & 0) \\ \frac{1}{N_3}(\hat{w}_1 & \hat{w}_2 & X) \end{pmatrix} \begin{pmatrix} e^{-i\sigma_1} & 0 & 0 \\ 0 & e^{-i\sigma_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix}$$

CONCLUSIONS

Symmetries play an important rôle in multi-Higgs models

- reduction of the number of free parameters

- experimental predictions

Connections can be established between Symmetries and:

- mass degeneracies

- CP violation