Flavour symmetries in the symmetric limit (and the neutrino normal hierarchy)

Andrea Romanino, SISSA

Reyimuaji, R 1801.10530, JHEP
Reyimuaji, R to appear
The content of this talk
Q1: can lepton flavour be accounted for by a flavour
symmetry constraining light neutrino Majorana masses
close to its symmetric limit?
Q1: can lepton flavour be accounted for by a flavour symmetry *constraining* light neutrino Majorana masses close to its symmetric limit?

A1: yes, but only if neutrinos are *inverted hierarchical* or *unconstrained* (anarchical)

If NH is confirmed, symmetry breaking must play a leading role in the understanding of lepton flavour
Light neutrino majorana masses may originate from high-scale physics

**Q2:** is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?
Light neutrino Majorana masses may originate from high-scale physics

**Q2:** is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?

**A2:** not necessarily

Conditions for the equivalence in the case of see-saw I
**Q3**: can lepton flavour be accounted for by a flavour symmetry constraining a see-saw lagrangian close to its symmetric limit?
Q3: can lepton flavour be accounted for by a flavour symmetry constraining a see-saw lagrangian close to its symmetric limit?

A3: yes, and neutrinos can be normally hierarchical if the high-scale and low-scale actions of the flavour symmetry are not equivalent
Flavour symmetries
Flavour symmetries

- $G_f$ flavour group acting on “i”, $\mathcal{L}$ invariant

- $\mathcal{L}_f = \lambda_{ij} \psi_i \psi_j h$
Flavour symmetries

- $G_f$ flavour group acting on “i”, $\mathcal{L}$ invariant

\[
\mathcal{L}_f = \lambda_{ij} \psi_i \psi_j h \\
\downarrow \\
\begin{align*}
m_{ij}^0 &= \lambda_{ij} v
\end{align*}
\]
Flavour symmetries

- $G_f$ flavour group acting on “i”, $\mathcal{L}$ invariant

\[ \mathcal{L}_f = \lambda_{ij} \psi_i \psi_j h + \frac{\lambda_{ijk}}{\Lambda_f} \phi_k \psi_i \psi_j h + \ldots \]

$\downarrow$

$m_{ij}^0 = \lambda_{ij} \nu$

$\Phi_k$ scalar “flavon”
(SM invariant)
spontaneously breaks $G_f$
Flavour symmetries

- $G_f$ flavour group acting on “i”, $\mathcal{L}$ invariant

$$\mathcal{L}_f = \lambda_{ij} \psi_i \psi_j h + \frac{\lambda_{ijk}}{\Lambda_f} \phi_k \psi_i \psi_j h + \ldots$$

$$m_{ij}^0 = \lambda_{ij} v, \quad m_{ij}^1 = \lambda_{ijk} \frac{\langle \phi_k \rangle}{\Lambda} v + \ldots$$

$\phi_k$ scalar “flavon”

(SM invariant)

spontaneously breaks $G_f$
Flavour symmetries

- $G_f$ flavour group acting on “i”, $\mathcal{L}$ invariant

\[ \mathcal{L}_f = \lambda_{ij} \psi_i \psi_j h + \frac{\lambda_{ijk}}{\Lambda_f} \phi_k \psi_i \psi_j h + \ldots \]

$\phi_k$ scalar “flavon” (SM invariant) spontaneously breaks $G_f$

\[ m_{ij}^0 = \lambda_{ij} \nu \quad m_{ij}^1 = \lambda_{ijk} \frac{\langle \phi_k \rangle}{\Lambda} \nu + \ldots \]

\[ m_{ij} = m_{ij}^0 + m_{ij}^1 \]
Flavour symmetries

- $G_f$ flavour group acting on “i”, $L$ invariant

$$\mathcal{L}_f = \lambda_{ij} \psi_i \psi_j h + \frac{\lambda_{ijk}}{\Lambda_f} \phi_k \psi_i \psi_j h + \ldots$$

$$m^0_{ij} = \lambda_{ij} v \quad m^1_{ij} = \lambda_{ijk} \frac{\langle \phi_k \rangle}{\Lambda} v + \ldots$$

$\Phi_k$ scalar “flavon” (SM invariant) spontaneously breaks $G_f$

$$m_{ij} = m^0_{ij} + m^1_{ij}$$

vanishes for $\phi_k = 0$

“symmetric limit”
In our case
Flavour group

- $G_f$ flavour group
  - any: discrete/continuous, abelian/non-abelian, global/gauge, etc
  - includes all “hidden” factors
  - unitary representation, commuting with Poincaré and $G_{SM}$
Flavour group

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Flavour representation

$$g \in G_f : \begin{cases} l_i & \rightarrow U_l(g)_{ij} l_j \\ e^c_i & \rightarrow U_{ec}(g)_{ij} e^c_j \end{cases}$$
Flavour group

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Flavour representation

$$g \in G_f : \begin{cases} l_i & \rightarrow & U_l(g)_{ij} l_j \\ e^c_i & \rightarrow & U_{e^c}(g)_{ij} e^c_j \end{cases} \quad e^c \leftrightarrow \overline{e_R}$$
Symmetric limit (low-scale)

\[
\lambda^E_{ij} e^c_i l_j h \quad \rightarrow \quad m_{Eij}^0 e^c_i e_j
\]

\[
\frac{c_{ij}}{2\Lambda} l_i l_j h h \quad \rightarrow \quad m_{\nu ij}^0 \frac{\nu_i \nu_j}{2}
\]
Symmetric limit (low-scale)

\[ \lambda_{ij}^E e_i^c l_j h \rightarrow m_{Eij}^0 e_i^c e_j \]

\[ \frac{c_{ij}}{2\Lambda} l_i l_j h h \rightarrow \frac{m_{\nu ij}^0}{2} \nu_i \nu_j \]

\[ U_{e_\ell}^T m_{E}^0 U_\ell (g) = m_{E}^0 \]

\[ U_{\ell}^T m_{\nu}^0 U_\ell (g) = m_{\nu}^0 \]
Symmetric limit (low-scale)

$$\lambda_{ij}^E e_i^c l_j h \rightarrow m_0^{Eij} e_i^c e_j$$

$$c_{ij} \frac{l_i l_j h h}{2\Lambda} \rightarrow m_{\nu ij} \frac{\nu_i \nu_j}{2}$$

$$U_e^T(g) m_0^E U_l(g) = m_0^E$$

$$U_l^T(g) m_0^\nu U_l(g) = m_0^\nu$$
Symmetric limit (low-scale)

\[
\begin{align*}
\lambda_{ij}^E e_i^c l_j h & \to \begin{array}{c}
m_{Eij}^0 e_i^c e_j
\end{array} \\
\frac{c_{ij}}{2\Lambda} l_i l_j h h & \to \frac{m_{\nu ij}^0}{2} \nu_i \nu_j
\end{align*}
\]

\[
U_{\rho\rho}^T m_E^0 U_{\rho\rho}^l (g) = m_E^0
\]

\[
U_l^T m_\nu^0 U_l (g) = m_\nu^0
\]

Symmetry breaking

\[
m_E = m_{E}^0 + m_E^1
\]

\[
m_\nu = m_{\nu}^0 + m_\nu^1
\]

invariant under \( G_f \)
Symmetric limit (low-scale)

\[
\lambda^E_{ij} e^c_i l_j h \quad \rightarrow \quad m^0_{Eij} e^c_i e_j
\]

\[
\frac{c_{ij}}{2\Lambda} l_i l_j hh \quad \rightarrow \quad \frac{m^0_{\nu ij}}{2} \nu_i \nu_j
\]

\[
U_{\phi}(g)^T m^0_E U_l(g) = m^0_E
\]

\[
U_l(g)^T m^0_\nu U_l(g) = m^0_\nu
\]

Symmetry breaking

\[
m_E = m^0_E + m^1_E
\]

\[
m_\nu = m^0_\nu + m^1_\nu
\]

invariant under \( G_f \)

not invariant under \( G_f \) generated by \( \phi \)
The symmetric limit provides an approximate description of lepton flavour

- $m_E^0 \neq 0$ and $m_\nu^0 \neq 0$

$m_E = m_E^0 + m_E^1$

$m_\nu = m_\nu^0 + m_\nu^1$
The symmetric limit provides an approximate description of lepton flavour

- $m^0_E \neq 0$ and $m^0_\nu \neq 0$

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\begin{align*}
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approximate description of lepton observables
The symmetric limit provides an approximate description of lepton flavour

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approximate description of lepton observables

moderate correction necessary for an accurate description
The symmetric limit provides an approximate description of lepton flavour

- $m^0_E \neq 0$ and $m^0_{\nu} \neq 0$

\[ m^0_E = m^0_E + m^1_E \]
\[ m^0_\nu = m^0_\nu + m^1_\nu \]

approximate description of lepton observables

moderate correction necessary for an accurate description

e.g. \[ m^0_D = \begin{pmatrix} 0 & 0 \\ 0 & m_b \end{pmatrix} \quad m^0_U = \begin{pmatrix} 0 & 0 \\ 0 & m_t \end{pmatrix} \]
\[ m^0_E = \begin{pmatrix} 0 & 0 \\ 0 & m_\tau \end{pmatrix} \quad V^0_{\text{CKM}} = 1 \]
\[ (\theta_C \text{ undetermined}) \]
The LO pattern of lepton flavour is determined by symmetry breaking

\[ m_E = m_E^0 + m_E^1 \]

\[ m_\nu = m_\nu^0 + m_\nu^1 \]
The LO pattern of lepton flavour is determined by symmetry breaking

\[ m_E = m^0_E + m^1_E \]
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e.g. \( m_E = 0 \) or \( m_\nu = 0 \) in the symmetric limit
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\[ m_E = m_0^E + m_1^E \]

\[ m_\nu = m_0^\nu + m_1^\nu \]

e.g. \( m_E = 0 \) or \( m_\nu = 0 \) in the symmetric limit

fully determine the PMNS matrix
The LO pattern of lepton flavour is determined by symmetry breaking

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\[ m_\nu = m_\nu^0 + m_\nu^1 \]

e.g. \( m_E = 0 \) or \( m_\nu = 0 \) in the symmetric limit

fully determine the PMNS matrix

\[ m_\nu^0 = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix} \]
\[ m_\nu^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ m_E^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ m_E^1: H_1 \text{ invariant} \]

\[ m_\nu^1: H_2 \text{ invariant} \]

e.g. \( G = A_4 \)
The symmetric limit provides an approximate description of lepton flavour

- $m_E \neq 0$ and $m_\nu \neq 0$

\[
\begin{align*}
  m_E &= \left[ m_{E}^0 \right] + m_{E}^1 \\
  m_\nu &= \left[ m_{\nu}^0 \right] + m_{\nu}^1
\end{align*}
\]

approximate description of lepton observables
The symmetric limit provides an approximate description of lepton flavour

- \( m_E \neq 0 \) and \( m_\nu \neq 0 \)

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approximate description of lepton observables

Neutrino masses

<table>
<thead>
<tr>
<th>NH/IH</th>
<th>(a 0 0)</th>
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</tr>
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<tbody>
<tr>
<td>NH or IH</td>
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The symmetric limit provides an approximate description of lepton flavour

- $m_E \neq 0$ and $m_\nu \neq 0$

$$m_E = m_E^0 + m_E^1$$

$$m_\nu = m_\nu^0 + m_\nu^1$$

approximate description of lepton observables

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approximate description of lepton observables

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Charged lepton masses

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<th>Hierarchy needed</th>
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PMNS matrix

\[
\begin{pmatrix}
X & X & 0 \\
X & X & X \\
X & X & X \\
\end{pmatrix}
\]
or

\[
\begin{pmatrix}
X & X & X \\
X & X & X \\
X & X & X \\
\end{pmatrix}
\]

\( (X \neq 0 \text{ generic}) \)
Q1: can lepton flavour be accounted for by a flavour symmetry constraining light neutrino Majorana masses close to its symmetric limit?
Gf U1 Ue leading, in the symmetric limit, to lepton masses and mixings in the above form

- A complete and concise classification is possible, as the predictions in the symmetric limit only depend on the structure of the decomposition of the representations in irreducible components (irreps) and in particular on their
  - **Type** (real, pseudoreal, complex)
  - **Dimension**
  - **Equivalence**
  - **Notation**
    - “n”: dimension n **complex** or **pseudoreal** irrep
    - “n”: dimension n **real** irrep
    - “n, n’, n””: dimension n **inequivalent** irreps
$G_f U_l U_e$ leading, in the symmetric limit, to lepton masses and mixings in the above form

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- Only 6 cases
- Only $d = 1$ (abelian) irreps and no pseudoreal irreps on $l$ (on $e^c$ only if $m_e, \mu = 0$)
- Neutrinos are either inverted hierarchical or unconstrained (anarchical)
- If NH confirmed, lepton flavour at low-scale can only be accounted for by SB
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Leading, in the symmetric limit, to lepton masses and mixings in the above form

- Only 6 cases
- Only $d = 1$ (abelian) irreps and no pseudoreal irreps on $l$ (on $e^c$ only if $m_\mu = 0$)
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1+1+1

• “1” = real one-dimensional: \( f \rightarrow \pm f \)

• 1+1+1: \( U(g)_{ij} = \pm 1_{ij} \)

• any \( m_\nu \) is trivially invariant

• neutrino masses and mixing completely unconstrained

• (anarchy)
SU(5) and SO(10)

- **SU(5):** assume $U_5 = U_l$ and $U_{10} = U_{ec}$, require $(V_{CKM})_0 = 1$ or $V_{12}$
  - only unconstrained (anarchical) neutrinos are allowed

- **SO(10):** assume $U_l = U_{ec} = U_{16}$
  - no solutions
Features of the proof

• in 2 steps: masses first, then mixings

• no need to write down any mass matrix, texture: the flavour pattern is directly determined by the irrep decomposition

• in particular, the form of the PMNS matrix is given by

\[ U = H_E P_E V D^{-1} P_\nu^{-1} H_\nu^{-1} \]

• \( V \) commutes with \( U_1 \)

• \( D \) maximal rotation, if \( U_1 \) contains conjugated complex irreps (Dirac substructure)

• \( P \) permutations possibly needed to bring mass eigenvalues in standard ordering

• \( H \) ambiguity in case of degeneracies (fixed by symmetry breaking effects)
Q2: is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?
Equivalence in the symmetric limit

\[ \mathcal{L}_{\text{low-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j hh \]
Equivalence in the symmetric limit

\[
\mathcal{L}_{\text{low-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h \\
\mathcal{L}_{\text{high-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \lambda_{ij}^N \nu_i^c l_j h + \frac{M_{ij}}{2} \nu_i^c \nu_j^c
\]
Equivalence in the symmetric limit

\[ \mathcal{L}_{\text{low-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h \]

\[ \mathcal{L}_{\text{high-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \lambda_{ij}^N \nu_i^c l_j h + \frac{M_{ij}}{2} \nu_i^c \nu_j^c \]

\[ \begin{align*}
    l_i & \rightarrow U_l(g)_{ij} l_j \\
    e_i^c & \rightarrow U_{e^c}(g)_{ij} e_j^c \\
    \nu_i^c & \rightarrow U_{\nu^c}(g)_{ij} \nu_j^c
\end{align*} \]
Equivalence in the symmetric limit

\[ \mathcal{L}_{\text{low-scale}} = \lambda_{ij} e_i^c l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h \quad \text{from} \quad \begin{cases} l_i \to U_{l_i} g_{ij} l_j \\ e_i^c \to U_{e_i^c} (g)_{ij} e_j^c \\ \nu_i^c \to U_{\nu_i^c} (g)_{ij} \nu_j^c \end{cases} \]

\[ \mathcal{L}_{\text{high-scale}} = \lambda_{ij} e_i^c l_j h^* + \lambda_{ij} N_{ij} \nu_i^c l_j h + \frac{M_{ij}}{2} \nu_i^c \nu_j^c \]

Are the neutrino mass matrices obtained from \textit{invariant} \( \mathcal{L}_{\text{low-scale}} \)
the same as those obtained through see-saw from \( \mathcal{L}_{\text{high-scale}} \)?
Equivalence in the symmetric limit

\[ \mathcal{L}_{\text{low-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h \quad \text{from} \quad \left\{ \begin{array}{l} l_i \rightarrow U_l (g)_{ij} l_j \\ e_i^c \rightarrow U_{e^c} (g)_{ij} e_j^c \\ \nu_i^c \rightarrow U_{\nu^c} (g)_{ij} \nu_j^c \end{array} \right. \]

\[ \mathcal{L}_{\text{high-scale}} = \lambda_{ij}^E e_i^c l_j h^* + \lambda_{ij}^N \nu_i^c l_j h + \frac{M_{ij}}{2} \nu_i^c \nu_j^c \]

Are the neutrino mass matrices obtained from \textit{invariant} \( \mathcal{L}_{\text{low-scale}} \)
the same as those obtained through see-saw from \( \mathcal{L}_{\text{high-scale}} \)?

Given an invariant \( m_\nu \), is it always possible to find invariant \( m_N \)
and \( M \) such that \( m_\nu = - m_N^T M^{-1} m_N \)? (converse is always true)
Equivalence in the symmetric limit

• Given a low-scale representation does an equivalent high-scale version always exist?
Equivalence in the symmetric limit

- Given a low-scale representation does an equivalent high-scale version always exists? **YES**
Equivalence in the symmetric limit

• Given a low-scale representation does an equivalent high-scale version always exist?  **YES**

• Is the low-scale version of a representation always equivalent to the high-scale version?
Equivalence in the symmetric limit

• Given a low-scale representation does an equivalent high-scale version always exists? **YES**

• Is the low-scale version of a representation always equivalent to the high-scale version? **NO**
Equivalence in the symmetric limit

- Given a low-scale representation does an equivalent high-scale version always exists? **YES**

- Is the low-scale version of a representation always equivalent to the high-scale version? **NO**

- Necessary and sufficient conditions for LS to be equivalent to HS

  *Require $M \neq 0$ in the symmetric limit*

  1. $U_{vc}$ vectorlike
  2. The vectorlike part of $U_l$ is contained in $U_{vc}$
Equivalence in the symmetric limit

- Given a low-scale representation does an equivalent high-scale version always exists? **YES**
- Is the low-scale version of a representation always equivalent to the high-scale version? **NO**
- **Necessary and sufficient conditions** for LS to be equivalent to HS

Require $M \neq 0$ in the symmetric limit

1. $U_{vc}$ vectorlike
   - real, or pairs of complex conjugated, or pairs of equivalent pseudoreal
2. The vectorlike part of $U_I$ is contained in $U_{vc}$
Example: the vectorlike part of $U_I$ is not in $U_{\nu c}$

**low-scale**

$U_I = 1+1+1$

$m_{ei} = (A 0 0)$

$U_{\nu c} = 1+1+1$

$m_{vi} = (a b 0)$

$V = \begin{pmatrix} X & ? & ? \\ ? & X & X \\ 0 & X & X \end{pmatrix}$ or $\begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$

**high-scale**

$m_{ei} = (A 0 0)$

$m_{vi} = (a 0 0)$

$V = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$
Example: the vectorlike part of $U_l$ is not in $U_{\nu c}$

<table>
<thead>
<tr>
<th></th>
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<th>high-scale</th>
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<td>$U_l$</td>
<td>$1+1+1$</td>
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</tr>
<tr>
<td>$U_{e^c}$</td>
<td>$1+1+1$</td>
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<td>$U_{\nu^c}$</td>
<td>$1+\bar{1}+1$</td>
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<td>$V = \begin{pmatrix} X &amp; ? &amp; ? \ ? &amp; X &amp; X \ 0 &amp; X &amp; X \end{pmatrix}$ or $\begin{pmatrix} ? &amp; ? &amp; X \ X &amp; X &amp; ? \ X &amp; X &amp; 0 \end{pmatrix}$</td>
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Example: the vectorlike part of $U_I$ is not in $U_{\nu c}$

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<td>$m_N = \begin{pmatrix} X \ X &amp; X \ X \end{pmatrix}$</td>
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<tr>
<td>$m_\nu = \begin{pmatrix} X &amp; X \ X &amp; X \end{pmatrix}$</td>
<td>$M = \begin{pmatrix} X \ X &amp; X \ X \end{pmatrix}$</td>
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</table>

$\det = 0$
Equivalence in the symmetric limit: $M_0$ singular

- Can still compare high-scale and low-scale predictions by taking

$$\lim_{\phi \to 0} m^T_N(\phi)M^{-1}(\phi)m_N(\phi)$$

(can exist or not, be finite or not)
Equivalence in the symmetric limit: $M_0$ singular

- **Necessary** condition: $U_l$ does not contain the chiral part of $U_{vc}$

- Example:

  \[
  U_l = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ \end{pmatrix} \quad m_{ei} = (A \ 0 \ 0) \quad m_{ei} = (A \ 0 \ 0)
  \]

  \[
  U_{ec} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ \end{pmatrix} \quad m_{vi} = (a \ 0 \ 0) \quad m_{vi} = (a \ 0 \ 0)
  \]

  \[
  U_{vc} = \begin{pmatrix} 1 & \text{real} \\ \end{pmatrix} \quad U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \\ \end{pmatrix} \quad U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \\ \end{pmatrix}
  \]
Equivalence in the symmetric limit: $M_0$ singular

- **Necessary** condition: $U_I$ does not contain the chiral part of $U^\nu_c$

- Example:
  
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<tr>
<td>$U^\nu_c = 1$ + real</td>
<td>$U = \begin{pmatrix} ? &amp; ? &amp; X \ X &amp; X &amp; ? \ X &amp; X &amp; 0 \end{pmatrix}$</td>
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Equivalence in the symmetric limit: $M_0$ singular

- **Necessary** condition: $U_l$ does not contain the chiral part of $U_{vc}$

- Example:

  \[
  U_l = 1+1+1 \quad \Rightarrow \quad m_{ei} = (A \ 0 \ 0)
  \]

  \[
  U_{ec} = 1+1+1 \quad \Rightarrow \quad m_{vi} = (a \ 0 \ 0)
  \]

  \[
  U_{vc} = 1 \quad \Rightarrow \quad U = \begin{pmatrix}
  ? & ? & X \\
  X & X & ? \\
  X & X & 0
  \end{pmatrix}
  \]

  \[
  U_{
u c} = \text{real} \quad \Rightarrow \quad U = \begin{pmatrix}
  X & X & ? \\
  X & X & X \\
  X & X & X
  \end{pmatrix}
  \]

  \[
  m_E = \begin{pmatrix}
  X & X \\
  X & X
  \end{pmatrix}
  \quad m_N = \begin{pmatrix}
  X & X \\
  X & X
  \end{pmatrix}
  \quad m_E = \begin{pmatrix}
  X & X \\
  X & X
  \end{pmatrix}
  \]

  \[
  m_{\nu} = \begin{pmatrix}
  X \\
  X
  \end{pmatrix}
  \quad M = \begin{pmatrix}
  X & X \\
  X & X
  \end{pmatrix}
  \quad m_{\nu} = \begin{pmatrix}
  X & X \\
  X & X
  \end{pmatrix}
  \]
Equivalence in the symmetric limit: $M_0$ singular

- **Necessary** condition: $U_l$ does not contain the chiral part of $U_{vc}$

- Example:

  **low-scale**

  $U_l = 1+1+1$

  $m_{ei} = (A\ 0\ 0)$

  $U_{e^c} = 1+1+1$

  $m_{vi} = (a\ 0\ 0)$

  $U_{\nu^c} = 1^+ \text{ real}$

  \[
  U = \begin{pmatrix}
  ? & ? & X \\
  X & X & ? \\
  X & X & 0 \\
  \end{pmatrix}
  \]

  $m_{E} = \begin{pmatrix}
  \ \\
  X & X \\
  \end{pmatrix}$

  $m_{\nu} = \begin{pmatrix}
  X \\
  \end{pmatrix}$

  $m_{N} = \begin{pmatrix}
  \ \\
  X & X \\
  \end{pmatrix}$

  $M = \begin{pmatrix}
  \ \\
  X & X \\
  \end{pmatrix}$

  $m_{E} = \begin{pmatrix}
  \ \\
  X & X \\
  \end{pmatrix}$

  $m_{\nu} = \begin{pmatrix}
  \ \\
  \end{pmatrix}$

  **high-scale**

  $U_{e^c} = 1+1+1$

  $m_{vi} = (a\ 0\ 0)$

  $U_{\nu^c} = 1^+ \text{ real}$

  \[
  U = \begin{pmatrix}
  X & X & ? \\
  X & X & X \\
  X & X & X \\
  \end{pmatrix}
  \]

  single RH neutrino dominance
Equivalence in the symmetric limit: $M_0$ singular

- **Necessary** condition: $U_l$ does not contain the chiral part of $U_{vc}$

- Example:
  
  **low-scale**
  
  $U_l = 1+1+1$
  
  $m_{ei} = (A \ 0 \ 0)$
  
  $U_{e^c} = 1+1+1$
  
  $m_{vi} = (a \ 0 \ 0)$
  
  $U_{\nu^c} = \begin{cases} 1 & \text{real} \\ \end{cases}$

  $$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$$

  $$m_E = \begin{pmatrix} X & X \\ X & X \end{pmatrix}$$

  $$m_\nu = \begin{pmatrix} X \\ \end{pmatrix}$$

  **high-scale**

  $m_{ei} = (A \ 0 \ 0)$

  $m_{vi} = (a \ 0 \ 0)$

  $$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$$

  $$m_E = \begin{pmatrix} X & X \end{pmatrix}$$

  $$m_\nu = \begin{pmatrix} X & X & X \end{pmatrix}$$

  single RH neutrino dominance

  $\det = 0$
Q3: can lepton flavour be accounted for by a flavour symmetry constraining a see-saw lagrangian close to its symmetric limit?
• If $U_{vc}$ vectorlike and the vectorlike part of $U_l$ is contained in $U_{vc}$: yes, at the same conditions as in the low-scale analysis.

• If instead the low- and high-scale analyses are not equivalent, predictive (non-unconstrained) cases corresponding to NH can be found.
Conclusions

• The complete set of lepton flavour predictions of any flavour group and representation in the symmetric limit has been found at low scale. The predictions only depend on the type, dimension, and equivalence of the irrep components.

• At low-scale: the symmetric limit is close to what observed only if neutrinos are unconstrained (anarchical) or inverted hierarchical.

• The measurement of the neutrino mass hierarchy allows to tell whether flavour symmetry breaking plays a leading order role in understanding lepton flavour observables, at low scale.

• In the high-scale case: the results do not change, except when the low- and high-scale analyses are not equivalent. The conditions for equivalence have been found when $M \neq 0$. A normal hierarchy for the neutrinos can be obtained.

• If the present hint for normal hierarchy was confirmed, a predictive symmetric limit can be close to what observed only because the low- and high-scale actions of the flavour symmetry are not equivalent. Otherwise, symmetry breaking effects necessary play a leading order role in determining lepton flavour observables.