Flavour symmetries in the symmetric limit (and the neutrino normal hierarchy)

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Reyimuaji, R 1801.10530, JHEP
Reyimuaji, R to appear

The content of this talk

Q1: can lepton flavour be accounted for by a flavour symmetry <u>constraining light neutrino Majorana masses</u> close to its symmetric limit?

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A1: yes, but only if neutrinos are inverted hierarchical or unconstrained (anarchical)

If NH is confirmed, symmetry breaking must play a leading role in the understanding of lepton flavour

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A2: not necessarily

Conditions for the equivalence in the case of see-saw I

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A3: yes, and neutrinos can be <u>normally hierarchical</u> if the high-scale and low-scale actions of the flavour symmetry are not equivalent

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vanishes for $\phi_k = 0$
"symmetric limit"

In our case

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$$g \in G_f : \quad \begin{cases} l_i & \to \quad U_l(g)_{ij}\, l_j \\ e_i^c & \to \quad U_{e^c}(g)_{ij}\, e_j^c \end{cases}$$

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Symmetric limit (low-scale)

$$\lambda^E_{ij} e^c_i l_j h \quad \rightarrow \quad \boxed{m^0_{E\,ij} e^c_i e_j}$$

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Symmetry breaking

$$m_E = \boxed{m^0_E} + m^1_E$$

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not invariant under G_f generated by ϕ

The symmetric limit provides an approximate description of lepton flavour

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e.g. $m_D^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_b \end{pmatrix}$ $m_U^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_t \end{pmatrix}$

$m_E^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix}$ $V_{\mathrm{CKM}}^0 = \mathbf{1}$

(θ_C undetermined)

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e.g. $G = A_4$

$$m_\nu^0 = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix} \qquad m_E^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

m_ν^1: H_1 invariant $\qquad$ m_E^1: H_2 invariant

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Neutrino masses

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PMNS matrix

$$\begin{pmatrix} X & X & 0 \\ X & X & X \\ X & X & X \end{pmatrix} \text{ or } \begin{pmatrix} X & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$$

($X \neq 0$ generic)

Q1: can lepton flavour be accounted for by a flavour symmetry constraining light neutrino Majorana masses close to its symmetric limit?

G_f U_l U_e leading, in the symmetric limit, to lepton masses and mixings in the above form

- A complete and concise classification is possible, as the predictions in the symmetric limit only depend on the structure of the decomposition of the representations in irreducible components (irreps) and in particular on their
  - **Type** (real, pseudoreal, complex)
  - **Dimension**
  - **Equivalence**
- Notation
  - "**n**": dimension n **complex** or **pseudoreal** irrep
  - "n": dimension n **real** irrep
  - "n, n', n"": dimension n **inequivalent** irreps

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U_l, U_{e^c} irreps	masses	ν hierarchy	PMNS zeros
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$\mathbf{1} \quad \mathbf{1} \quad \overline{\mathbf{1}}$ $\overline{\mathbf{1}} \quad r \not\supseteq \mathbf{1}, \overline{\mathbf{1}}$	$(A00)$ $(0aa)$	IH	none $(\mathbf{13})$
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- Only 6 cases
- Only d = 1 (abelian) irreps and no pseudoreal irreps on l (on e^c only if $m_{e,\mu} = 0$)
- Neutrinos are either inverted hierarchical or unconstrained (anarchical)
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1+1+1

- "1" = real one-dimensional: $f \rightarrow \pm f$
- 1+1+1: $U(g)_{ij} = \pm \mathbf{1}_{ij}$
- any m_ν is trivially invariant
- neutrino masses and mixing completely unconstrained
- (anarchy)

SU(5) and SO(10)

- SU(5): assume $U_{\bar{5}} = U_l$ and $U_{10} = U_{ec}$, require $(V_{CKM})_0 = \mathbf{1}$ or V_{12}
  - only unconstrained (anarchical) neutrinos are allowed
- SO(10): assume $U_l = U_{ec} = U_{16}$
  - no solutions

Features of the proof

- in 2 steps: masses first, then mixings
- no need to write down any mass matrix, texture: the flavour pattern is directly determined by the irrep decomposition
- in particular, the form of the PMNS matrix is given by

$$U = H_E P_E V D^{-1} P_\nu^{-1} H_\nu^{-1}$$

  - V commutes with U_l
  - D maximal rotation, if U_l contains conjugated complex irreps (Dirac substructure)
  - P permutations possibly needed to bring mass eigenvalues in standard ordering
  - H ambiguity in case of degeneracies (fixed by symmetry breaking effects)

Q2: is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?

[to appear]

Equivalence in the symmetric limit

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$$\mathcal{L}_{\text{high-scale}} = \lambda^E_{ij} e^c_i l_j h^* + \lambda^N_{ij} \nu^c_i l_j h + \frac{M_{ij}}{2} \nu^c_i \nu^c_j$$

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Given an invariant m_ν, is it always possible to find invariant m_N and M such that $m_\nu = - m_N^T M^{-1} m_N$? (converse is always true)

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- Necessary and sufficient conditions for LS to be equivalent to HS

  Require $M \neq 0$ in the symmetric limit

  1. U_{vc} vectorlike

  2. The vectorlike part of U_l is contained in U_{vc}

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- Is the low-scale version of a representation always equivalent to the high-scale version? **NO**

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Require $M \neq 0$ in the symmetric limit

1. U_{vc} vectorlike — real, or pairs of complex conjugated, or pairs of equivalent pseudoreal

2. The vectorlike part of U_I is contained in U_{vc}

Example: the vectorlike part of U_l is not in U_{vc}

	low-scale	high-scale
$U_l = 1+1+\mathbf{1}$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_e^c = 1+\mathbf{1}+\mathbf{1}$	$m_{vi} = (a\ b\ 0)$	$m_{vi} = (a\ 0\ 0)$
$U_v^c = \bar{1}+\mathbf{1}+\mathbf{1}$	$V = \begin{pmatrix} X & ? & ? \\ ? & X & X \\ 0 & X & X \end{pmatrix}$ or $\begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$V = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

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$U_l = \boxed{1+1}+\mathbf{1}$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_e^c = 1+\mathbf{1}+\mathbf{1}$	$m_{vi} = (a\ b\ 0)$	$m_{vi} = (a\ 0\ 0)$
$U_v^c = 1+\mathbf{1}+\mathbf{1}$	$V = \begin{pmatrix} X & ? & ? \\ ? & X & X \\ 0 & X & X \end{pmatrix}$ or $\begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$V = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

Example: the vectorlike part of U_l is not in U_{vc}

	low-scale	high-scale
$U_l = \boxed{1+1}+\mathbf{1}$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_e^c = 1+\mathbf{1}+\mathbf{1}$	$m_{vi} = (a\ b\ 0)$	$m_{vi} = (a\ 0\ 0)$
$U_\nu^c = 1+\mathbf{1}+\mathbf{1}$	$V = \begin{pmatrix} X & ? & ? \\ ? & X & X \\ 0 & X & X \end{pmatrix}$ or $\begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$V = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

$$m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix} \qquad m_N = \begin{pmatrix} X & & \\ & X & X \end{pmatrix} \qquad m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} & & \\ & X & X \\ & X & X \end{pmatrix}} \qquad M = \begin{pmatrix} & X & \\ X & & \\ & & X \end{pmatrix} \qquad \boxed{m_\nu = \begin{pmatrix} & & \\ & \boxed{\begin{matrix} X & X \\ X & X \end{matrix}} \end{pmatrix}}$$

det = 0

Equivalence in the symmetric limit: M_0 singular

- Can still compare high-scale and low-scale predictions by taking

$$\lim_{\phi \to 0} m_N^T(\phi) M^{-1}(\phi) m_N(\phi)$$

(can exist or not, be finite or not)

Equivalence in the symmetric limit: M_0 singular

- **Necessary** condition: U_l does not contain the chiral part of U_{vc}
- Example:

	low-scale	**high-scale**
$U_l = \mathbf{1}+\mathbf{1}+1$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_e^c = \bar{\mathbf{1}}+\bar{\mathbf{1}}+\bar{\mathbf{1}}$	$m_{\nu i} = (a\ 0\ 0)$	$m_{\nu i} = (a\ 0\ 0)$
$U_\nu^c = \bar{\mathbf{1}}+$ real	$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

Equivalence in the symmetric limit: M_0 singular

- **Necessary** condition: U_l does not contain the chiral part of U_{vc}
- Example:

	low-scale	**high-scale**
$U_l = \mathbf{1}+\mathbf{1}+1$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_e^c = \bar{\mathbf{1}}+\bar{\mathbf{1}}+\bar{\mathbf{1}}$	$m_{\nu i} = (a\ 0\ 0)$	$m_{\nu i} = (a\ 0\ 0)$
$U_\nu^c = \boxed{\bar{\mathbf{1}}} + \text{real}$	$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

Equivalence in the symmetric limit: M_0 singular

- **Necessary** condition: U_l does not contain the chiral part of U_{vc}
- Example:

$U_l = \mathbf{1}+\mathbf{1}+1$

$U_e^c = \bar{\mathbf{1}}+\bar{\mathbf{1}}+\bar{\mathbf{1}}$

$U_v^c = \boxed{\bar{\mathbf{1}}} + \text{real}$

low-scale

$m_{ei} = (A\ 0\ 0)$

$m_{vi} = (a\ 0\ 0)$

$$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$$

$$m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} X & & \\ & & \\ & & \end{pmatrix}}$$

high-scale

$m_{ei} = (A\ 0\ 0)$

$m_{vi} = (a\ 0\ 0)$

$$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$$

$$m_N = \begin{pmatrix} & & \\ & X & X \end{pmatrix} \qquad m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$M = \begin{pmatrix} X & X & \\ X & X & \\ & & \end{pmatrix} \qquad \boxed{m_\nu = \begin{pmatrix} & & \\ & X & X \\ & X & X \end{pmatrix}}$$

Equivalence in the symmetric limit: M_0 singular

- **Necessary** condition: U_l does not contain the chiral part of U_{vc}
- Example:

$U_l = \mathbf{1}+\mathbf{1}+1$

$U_e^c = \bar{\mathbf{1}}+\bar{\mathbf{1}}+\bar{\mathbf{1}}$

$U_\nu^c = \boxed{\bar{\mathbf{1}}}+$ real

low-scale

$m_{ei} = (A\ 0\ 0)$

$m_{\nu i} = (a\ 0\ 0)$

$$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$$

$$m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} X & & \\ & & \\ & & \end{pmatrix}}$$

high-scale

$m_{ei} = (A\ 0\ 0)$

$m_{\nu i} = (a\ 0\ 0)$

$$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$$

$$m_N = \begin{pmatrix} & & \\ & X & X \end{pmatrix} \quad m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$M = \begin{pmatrix} X & X & \\ X & X & \\ & & \end{pmatrix} \quad \boxed{m_\nu = \begin{pmatrix} & & \\ & X & X \\ & X & X \end{pmatrix}}$$

single RH neutrino dominance

Equivalence in the symmetric limit: M_0 singular

- **Necessary** condition: U_l does not contain the chiral part of U_{vc}
- Example:

$U_l = \mathbf{1}+\mathbf{1}+1$

$U_e^c = \bar{\mathbf{1}}+\bar{\mathbf{1}}+\bar{\mathbf{1}}$

$U_\nu^c = \boxed{\bar{\mathbf{1}}}+$ real

low-scale

$m_{ei} = (A\ 0\ 0)$

$m_{\nu i} = (a\ 0\ 0)$

$$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$$

$$m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} X & & \\ & & \\ & & \end{pmatrix}}$$

high-scale

$m_{ei} = (A\ 0\ 0)$

$m_{\nu i} = (a\ 0\ 0)$

$$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$$

$$m_N = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$M = \begin{pmatrix} X & X & \\ X & X & \\ & & \end{pmatrix}$$

single RH neutrino dominance

$$m_E = \begin{pmatrix} & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} & & \\ & \boxed{\begin{matrix} X & X \\ X & X \end{matrix}} \end{pmatrix}}$$

det = 0

Q3: can lepton flavour be accounted for by a flavour symmetry constraining a see-saw lagrangian close to its symmetric limit?

- If U_{vc} vectorlike and the vectorlike part of U_l is contained in U_{vc}: yes, at the same conditions as in the low-scale analysis

- If instead the low- and high-scale analyses are not equivalent, predictive (non-unconstrained) cases corresponding to NH can be found

Conclusions

- The **complete** set of lepton flavour **predictions** of any flavour group and representation **in the symmetric limit** has been found at low scale. The predictions only depend on the type, dimension, and equivalence of the irrep components.

- At **low-scale**: the symmetric limit is close to what observed only if neutrinos are **unconstrained** (anarchical) **or inverted hierarchical**.

- **The measurement of the neutrino mass hierarchy allows to tell whether flavour symmetry breaking plays a leading order role in understanding lepton flavour observables, at low scale.**

- In the **high-scale** case: the results do not change, except when the low- and high-scale analyses are **not equivalent**. The conditions for equivalence have been found when M≠0. A **normal hierarchy** for the neutrinos can be obtained.

- If the present hint for normal hierarchy was confirmed, **a predictive symmetric limit can be close to what observed only because the low- and high-scale actions of the flavour symmetry are not equivalent**. Otherwise, symmetry breaking effects necessary play a leading order role in determining lepton flavour observables.