# Recent Progress in inclusive $b \to c$ semileptonic transitions

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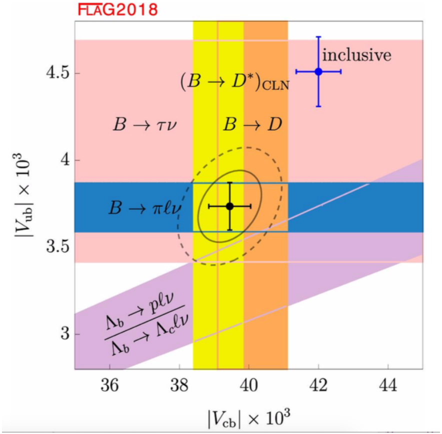
# Contents

# Introduction

Charged Current Semileptonics are under scrutiny:

- Tensions between inclusive and exclusive determinations of $V_{xb}$
- Tensions between semitauonic and semimuonic (exclusive) decays



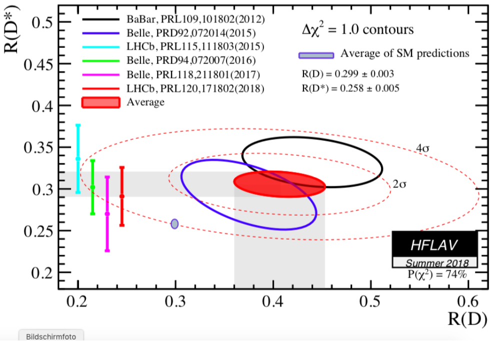

Exclusive $V_{cb}$ has shifted recently

- CLN form factor parametrization is too simple
- More sophisticated BGL parametriztion

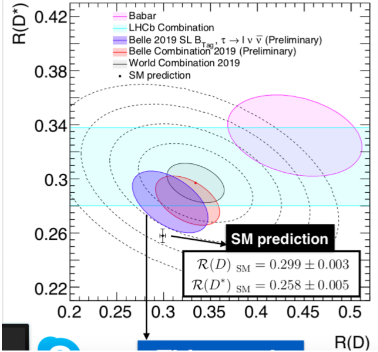
$|V_{cb}|$ = $(42.5 \pm 0.3 \pm 0.7 \pm 0.6) \times 10^{-3}$ **Exclusive $|V_{cb}|$ (BGL)**

$|V_{cb}|$ = $(38.4 \pm 0.2 \pm 0.6 \pm 0.6) \times 10^{-3}$ **Exclusive $|V_{cb}|$ (CLN)**

## Tension in exclusive Semi-tauonics:




New BELLE 2019



Present talk: Focus on $b \to c \ell \bar{\nu}$

- $V_{cb}$ is one of the best known CKM matrix elements
- Inclusive determination:
  - Based on the Heavy Quark Expansion (HQE)
  - Close to the 1% theoretical uncertainty
- Exclusive determination
  - Based on Lattice determinations of Form Factors
  - Simulations with finite quark masses
  - Not yet at the 1% level of uncertainties

Recent data indicate that there is no $V_{cb}$ problem

# Part I

## Alternative $V_{cb}$ Determination

ThM, K. K. Vos: arXiv:1802.09409

M. Fael, ThM, K. K. Vos: arXiv:1812.07472

# Inclusive $V_{cb}$ Determination

- Standard tool: Heavy Quark Expansion
- Structure of the expansion (@ tree):

$$
\begin{aligned}
d\Gamma \;=\;& d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4 \\
& + d\Gamma_5 \left( a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right) \\
& + \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4
\end{aligned}
$$

- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

- $\Gamma_0$ is the decay of a free quark (“Parton Model”)
- $\Gamma_1$ vanishes due to Heavy Quark Symmetries
- $\Gamma_2$ is expressed in terms of two parameters

$$
\begin{aligned}
2M_H\mu_\pi^2 &= -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle \\
2M_H\mu_G^2 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle
\end{aligned}
$$

$\mu_\pi$: Kinetic energy and $\mu_G$: Chromomagnetic moment

- $\Gamma_3$ two more parameters

$$
\begin{aligned}
2M_H\rho_D^3 &= -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle \\
2M_H\rho_{LS}^3 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle
\end{aligned}
$$

$\rho_D$: Darwin Term and $\rho_{LS}$: Spin-Orbit Term

- $\Gamma_4$ and $\Gamma_5$ have been computed Bigi, Uraltsev, Turczyk, TM, ...

# Inclusive $V_{cb}$ Determination

- Based on the HQE for the inclusive rates and for moments of spectra
- (Cut) moments of the charged lepton energy, hadronic energy and hadronic invariant mass spectra
- Extract the HQE parameters from this data
- Obtain $V_{cb}$ from the total semileptonic rate

Problem: Number of HQE parameters in higher orders!

- 4 up to $1/m^3$
- 13 up to $1/m^4$ (tree level)
- 31 up to order $1/m^5$ (tree level)
- Factorial Proliferation

# Reparametrization in HQE (Dugan, Golden, Grinstein, Chen, Luke, Manohar...

Start from the operator:

$$R(q) = \int d^4x \, e^{iqx} \, T[\bar{Q}(x)\Gamma q(x) \; \bar{q}(0)\Gamma^\dagger Q(0)]$$

and replace $Q(x) = \exp(-im(v \cdot x))Q_v(x)$

$$R(S) = \int d^4x \, e^{-iSx} \, T[\bar{Q}_v(x)\Gamma q(x) \; \bar{q}(0)\Gamma^\dagger Q_v(0)]$$

with $S = mv - q$.

These expressions are independent of $v$!

## Perform the OPE $\longrightarrow$ HQE

$$\begin{aligned} R(S) &= \sum_{n=0}^{\infty} \left[ C^{(n)}_{\mu_1 \cdots \mu_n}(S) \right]_{\alpha\beta} \bar{Q}_{v,\alpha}(iD_{\mu_1} \cdots iD_{\mu_n}) Q_{v,\beta} \\ &= \sum_{n=0}^{\infty} C^{(n)}_{\mu_1 \cdots \mu_n}(S) \otimes \bar{Q}_v(iD_{\mu_1} \cdots iD_{\mu_n}) Q_v \end{aligned}$$

These expressions are still invariant under reparametrization of $v$: (as long as the sum is not truncated)

$$\delta_{\rm RP}\, v_\mu = \delta v_\mu \quad \text{with} \quad v \cdot \delta v = 0$$

$$\delta_{\rm RP}\, iD_\mu = -m \delta v_\mu$$

$$\delta_{\rm RP}\, Q_v(x) = im(x \cdot \delta v) Q_v(x) \quad \text{in particular} \quad \delta_{\rm RP}\, Q_v(0) = 0 \, .$$

The RP connects different orders in $1/m$, which yields the master relation between the coefficients $n = 0, 1, 2, ...$

$$\boxed{\delta_{\rm RP} C^{(n)}_{\mu_1 \cdots \mu_n} \;=\; m\,\delta v^\alpha \left( C^{(n+1)}_{\alpha\mu_1\cdots\mu_n} + C^{(n+1)}_{\mu_1\alpha\mu_2\cdots\mu_n} + \cdots + C^{(n+1)}_{\mu_1\cdots\mu_n\alpha} \right)}$$

Use these coefficients, integrate over phase space, get a total rate $\Gamma = \mathrm{Im}\langle B|R|B\rangle = \mathrm{Im}\langle R\rangle$
The coeffcients of the OPE will depend only on $v$

$$R \;=\; \sum_{n=0}^{\infty} c^{(n)}_{\mu_1\cdots\mu_n}(v) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

and satisfy the master relation between different orders in the HQE

# Making use of RPI ...

- RPI is a consequence of Lorentz invariance of QCD
- RPI is an exact symmetry:
  the relations must hold to all order in $\alpha_s$
- Resummation of towers of terms from different orders
- For Lorentz invariant observables:
  - The master relations are identical for all observables
  - "Rigid" relations between coefficients
  - Reduction of HQE parameters due to RPI

## How does this happen? A Toy example without gluons

Look at the partonic result for the rate

$$R(p) = R(p^2) = R((mv+k)^2) = R(m^2 + 2m(vk) + k^2)$$
$$= R(m^2) + R'(m^2)(2m(vk)+k^2) + \frac{1}{2}R''(m^2)(2m(vk)+k^2)^2$$
$$= R(m^2)$$

if there are no gluons: Equation of motion:

$$(2m(vk)+k^2) \to 2m(iv\partial) + (i\partial)^2 \to 0$$

All the fully symmetrized (zero gluon) contributions are contained in the partonic result

## HQE parameters (for the total rate) to $O(1/m^4)$

$$
\begin{aligned}
2m_H\mu_3 &= \langle H(p)|\bar{Q}_v Q_v|H(p)\rangle = \langle \bar{Q}_v Q_v\rangle \\
2m_H\mu_G &= \langle \bar{Q}_v (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu}) Q_v\rangle \\
2m_H\rho_D &= \langle \bar{Q}_v \left[(iD^\mu), \left[\left((ivD) + \frac{(iD)^2}{2m}\right), (iD_\mu)\right]\right] Q_v\rangle \\
2m_H r_G^4 &= \langle \bar{Q}_v [(iD_\mu), (iD_\nu)]\,[(iD^\mu), (iD^\nu)]\, Q_v\rangle \\
2m_H r_E^4 &= \langle \bar{Q}_v [(ivD), (iD_\mu)]\,[(ivD), (iD^\mu)]\, Q_v\rangle \\
2m_H s_B^4 &= \langle \bar{Q}_v [(iD_\mu), (iD_\alpha)]\,[(iD^\mu), (iD_\beta)]\,(-i\sigma^{\alpha\beta}) Q_v\rangle \\
2m_H s_E^4 &= \langle \bar{Q}_v [(ivD), (iD_\alpha)]\,[(ivD), (iD_\beta)]\,(-i\sigma^{\alpha\beta}) Q_v\rangle \\
2m_H s_{qB}^4 &= \langle \bar{Q}_v [iD_\mu, [iD^\mu, [iD_\alpha, iD_\beta]]]\,(-i\sigma^{\alpha\beta}) Q_v\rangle
\end{aligned}
$$

- Less that had been identified before
- Depend on the quark mass
- Are expressed NOT in terms of $iD_\perp$, rather "full" derivatives
- Can be expressed in terms of full QCD operators via

$$ivD\, Q_v \to \left( ivD + \frac{(iD)^2}{m} \right) Q_v = \frac{1}{2m}((iD)^2 - m^2)Q$$

Thus

$$\begin{aligned}
2m_H \mu_3 &= \langle \bar{Q} Q \rangle \\
2m_H \mu_G &= \langle \bar{Q} (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu}) Q \rangle \\
2m_H \rho_D &= \frac{1}{2m} \langle \bar{Q} \left[ (iD^\mu) \, , \, \left[ (iD)^2 \, , \, (iD_\mu) \right] \right] Q \rangle \\
&\quad \ldots .
\end{aligned}$$

## Example 1: $b \to s\gamma$ ($O_7$ contribution only)

$$\Gamma_{b\to s\gamma} = \frac{\lambda^2 m_b^3}{4\pi}\left[\mu_3 - \frac{2}{m_b^2}\mu_G^2 - \frac{10\rho_D^3}{3m_b^3} - \frac{1}{3m_b^4}\left(4r_G^4 + 4r_E^4 + \frac{1}{4}s_{qB}^4 - 4s_E^4\right) + O(1/m_b^5)\right]$$

## Example 2: $b \to c\ell\bar{\nu}$ ($\rho = m_c^2/m_b^2$)

$$\begin{aligned}
\frac{\Gamma}{\Gamma_0} &= \mu_3\, z(\rho) - 2\,\frac{\mu_G^2}{m_b^2}(\rho-1)^4 + d(\rho)\,\frac{\tilde{\rho}_D^3}{m_b^3} + \frac{2}{3}(-1+\rho)^3(1+5\rho)\frac{s_B^4}{m_b^4} \\
&- \frac{8}{9}\frac{r_E^4}{m_b^4}\left(2 + 9\rho^2 - 20\rho^3 + 9\rho^4 + 6\log\rho\right) \\
&+ \frac{4}{9}\frac{r_G^4}{m_b^4}\left(16 - 21\rho + 9\rho^2 - 7\rho^3 + 3\rho^4 + 12\log\rho\right) \\
&+ \frac{1}{9}\frac{s_E^4}{m_b^4}\left(50 - 72\rho + 40\rho^3 - 18\rho^4 + 24\log\rho\right) \\
&+ \frac{1}{36}\frac{s_{qB}^4}{m_b^4}\left(-25 + 48\rho - 36\rho^2 + 16\rho^3 - 3\rho^4 - 12\log\rho\right) + \mathcal{O}(1/m_b^5)
\end{aligned}$$

## Differential Rates and Moments

Define generalized moments with weight function $w$:

$$\langle M[w]\rangle = \int \frac{d^4q}{(2\pi)^4}\widetilde{dk}\,\widetilde{dk'}\, w(v,k,k')\langle R(s)\rangle L(k,k')(2\pi)^4\delta^4(q-k-k')$$

which have an OPE in term of the “operator kernel”

$$M[w] = \sum_{n=0}^{\infty} a^{(n)}_{\mu_1\cdots\mu_n} \otimes \bar{b}_v(iD_{\mu_1}\cdots iD_{\mu_n})b_v$$

If the weight function is RPI (i.e. is independent of $v$)

$$\delta_{\mathrm{RP}} w(v,k,k') = 0$$

the coefficients $a^{(n)}$ of the operator kernel satisfy the same “rigid” relations as the total rates. Thus they depend on the same reduced set of HQE parameters.

For the case of $b \to c$ semileptonics, the weight function

$$w(v, k, k') = \delta(q^2 - (k + k')^2)$$

satisfies this: The leptonic invariant mass spectrum will depend on the reduced set of HQE parameters
This thus also holds for moments:

$$\frac{1}{\Gamma_0} \int d\hat{q}^2 (\hat{q}^2)^n \frac{d\Gamma}{d\hat{q}^2}$$

and for cut moments, as long as the cut is Lorentz invariant, e.g.

$$\frac{1}{\Gamma_0} \int_{\hat{q}^2_{\mathrm{cut}}} d\hat{q}^2 (\hat{q}^2)^n \frac{d\Gamma}{d\hat{q}^2}$$

# In the massless limit:

$$Q_1 = \frac{3}{10}\mu_3 - \frac{7}{5}\frac{\mu_G^2}{m_b^2} + \frac{\tilde\rho_D^3}{m_b^3}(19 + 8\log\rho) - \frac{r_E^4}{m_b^4}\left(\frac{1292}{45} + \frac{40}{3}\log\rho\right) - \frac{s_B^4}{m_b^4}(8 + 2\log\rho)$$
$$+ \frac{13}{120}\frac{s_{qB}^4}{m_b^4} + \frac{s_E^4}{m_b^4}\left(\frac{63}{5} + 4\log\rho\right) + \frac{r_G^4}{m_b^4}\left(\frac{827}{45} + \frac{22}{3}\log\rho\right), \tag{4.10}$$

$$Q_2 = \frac{2}{15}\mu_3 - \frac{16}{15}\frac{\mu_G^2}{m_b^2} + \frac{\tilde\rho_D^3}{m_b^3}\left(\frac{358}{15} + 8\log\rho\right) - \frac{r_E^4}{m_b^4}\left(\frac{2888}{45} + \frac{64}{3}\log\rho\right) - \frac{s_B^4}{m_b^4}\left(\frac{259}{15} + 4\log\rho\right)$$
$$+ \frac{s_{qB}^4}{m_b^4}\left(\frac{251}{180} + \frac{1}{3}\log\rho\right) + \frac{s_E^4}{m_b^4}\left(\frac{908}{45} + \frac{16}{3}\log\rho\right) + \frac{r_G^4}{m_b^4}\left(\frac{1373}{45} + \frac{28}{3}\log\rho\right), \tag{4.11}$$

etc.

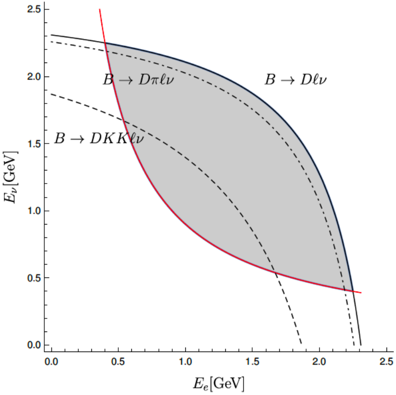
NB.: In the massless limit one need to include also four quark operators, and

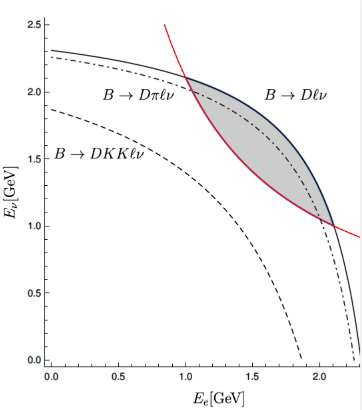
$\log\rho = \log(\mu^2/m_b^2)$, the $\mu$ dependence is compensated by the four quark operators

We have computed the full expression (for finite $\rho$) for all $Q_n$, as well as for the spectrum $d\Gamma/(dq^2)$.

# Cut Moments

Implementing a $q^2$ cut:




The expression for the cut moments are available!

# Conclusion on Part I

- $V_{cb}$ may be determined with a smaller number of independent HQE parameters
- Strategy is the same as before, but based on a different set of observables
- Perspective for a data-driven analysis up to $1/m_b^4$

# Part II

## Preliminary Results on $\alpha_s \Lambda_{\rm QCD}^3 / m_b^3$

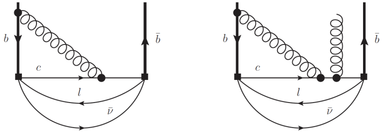
ThM, A. A. Pivovarov: SI-HEP-2018-36, arXiv:1903.xxxxx

## Status of the $b \to c$ semileptonic HQE Calculation

$$\Gamma = \Gamma_0 \left( C_0(\rho) \left[ 1 + \frac{\mu_\pi^2}{2m_b^2} \right] + C_G(\rho) \frac{\mu_G^2}{2m_b^2} + C_{rD}(\rho) \frac{\rho_D^3}{6m_b^3} + \cdots \right)$$

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the $\mu_\pi^2/m_b^2$ and $\mu_G^2/m_b^2$ is known
- QCD inspired modelling for the HQE matrix elements
- In the pipeline:
  - $\mathcal{O}(\alpha_s)$ for the $\rho_D/m_b^3$ and $\rho_{LS}/m_b^3$
  - Relations among the coefficients from RPI

# Some technical remarks ...




$$\tilde{\mathcal{T}} = C_0 \mathcal{O}_0 + C_v \frac{\mathcal{O}_v}{m_b} + C_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + C_G \frac{\mathcal{O}_G}{2m_b^2} + C_D \frac{\mathcal{O}_D}{2m_b^3}$$

$$\mathcal{O}_0 = \bar{h}_v h_v \qquad \mathcal{O}_v = \bar{h}_v v\pi h_v \qquad \mathcal{O}_\pi = \bar{h}_v \pi_\perp^2 h_v$$

$$\mathcal{O}_G = \bar{h}_v \frac{1}{2} [\gamma_\mu, \gamma_\nu] \pi_\perp^\mu \pi_\perp^\nu h_v \qquad \mathcal{O}_D = \bar{h}_v [\pi_\perp^\mu, [\pi_\perp^\mu, \pi v]] h_v$$

- Compute the three-loop Feynman Diagrams
- Perform the renormalization

... a few subtleties ...

(a) Rewrite

$$\bar{b} \not{v} b = \mathcal{O}_0 + \tilde{C}_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + \tilde{C}_G \frac{\mathcal{O}_G}{2m_b^2} + \tilde{C}_D \frac{\mathcal{O}_D}{2m_b^3} + O(\Lambda^4_{QCD}/m_b^4)$$

(b) Renormalization:

- Heavy Quark Fields (on-shell renomalization)
- Charm mass in the $\overline{\rm MS}$ scheme
- Renormalization of $\rho_D$ in $\overline{\rm MS}$
- Mixing with time-ordered products like

$$(-i) \int d^4x \, \mathrm{T}[\mathcal{L}_{\rm HQET}(x) \mathcal{O}_\pi]$$

All poles cancel ... (I still have to check this!)

# Result (preliminary!)

We have the analytic result for $C_{rD}$

Numerically we have ($m_c^2/m_b^2 = 0.07$, $\alpha_s(m_b) = 0.2$)

$$\begin{aligned} C_{rD} &= -57.1588 + \frac{\alpha_s}{4\pi}(-56.5941 C_A + 446.793 C_F) \\ &= -57.1588 + \frac{\alpha_s}{4\pi}(425.942) \\ &= -57.1588(1 - \frac{\alpha_s}{4\pi} 7.4519 \ldots) \\ &= -57.1588(1 - 0.12) \end{aligned}$$

- $\rho_D = \rho_D(m_b)$
- Leading order has a sizeable coefficient
- QCD corrections have the expected size.
- Impact on $V_{cb}$ will be small but visible

# Conclusion on Part II

- $\alpha_s \Lambda_{\rm QCD}^3 / m_b^3$ is almost completed for the total rate
- We can also compute moments of kinematic distributions
- We will not have the full phase space distributions, so we cannot implement an electron energy cut
- Can we use RPI to obtain the coefficient of $\rho_{\rm LS}$?

# Overall Conclusions

Inclusive $V_{cb}$ is getting more and more precise:

- The problem exclusive vs inclusive seems to disappear
- Subtle problems will arise once we want to increase the precision further:
  - Convergence of the HQE
  - Duality Violations
  - $\alpha_s^3$ for the partonic rate
  - ...