Axion Like Particles

- originally - the Axion propose as a solution to the strong CP problem
- appear in many BSM scenarios
- portal to dark matter and/or dark sector
- if very light can be a dark matter candidate
- predicted by string theory
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well motivated BSM scenario
Axion Like Particles

pseudo-scalar and pNGB

\[ \mathcal{L}_{\text{eff}} = - \frac{4\pi\alpha_s c_g}{\Lambda} a G_{\mu\nu} \tilde{G}_{\mu\nu} + \frac{c_\gamma}{4\Lambda} a F_{\mu\nu} \tilde{F}_{\mu\nu} \]

\[ \Lambda \gg m_a \]
Axion Like Particles

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\[ \Lambda \gg m_a \]

\[ c_\gamma = 1, c_g = 0 \]

superconducting
radiofrequency cavities
Axion Like Particles

pseudo-scalar and pNGB

\[ \mathcal{L}_{\text{eff}} = -4\pi\alpha_s c_g \frac{\mathcal{L}}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} + \frac{c_\gamma}{4\Lambda} a F^{\mu\nu} \tilde{F}_{\mu\nu} \]

\[ \Lambda \gg m_a \]

\( c_\gamma = 1, c_g = 0 \)

superconducting radiofrequency cavities

\( c_g \neq 0 \) or \( c_\gamma \neq 0 \)

photon beam experiments (PrimEx, GlueX) and hadronic decays
Probing ALPs and the Axiverse with Superconducting Radiofrequency Cavities
The Idea

probing off-shell ALPs via non-linear QED in a cavity

\[ \propto (F_{\mu \nu} \tilde{F}_{\mu \nu})^2 \propto (E \cdot B)^2 \]

non-linear Maxwell equations
The Idea
probing off-shell ALPs via non-linear QED in a cavity

\[ \propto (\tilde{F}_{\mu\nu} F^{\mu\nu})^2 \propto (E \cdot B)^2 \]
non-linear Maxwell equations

\[ \omega_s = \pm \omega_1 \pm \omega_1 \pm \omega_2 \]
pump
measure
The Idea
probing off-shell ALPs via non-linear QED in a cavity

\[ \propto (F^{\mu\nu} \tilde{F}_{\mu\nu})^2 \propto (E \cdot B)^2 \]

non-linear Maxwell equations

advantages:

* probes large range of masses - broad band
* does not rely on ALP been dark matter
The Euler Heisenberg effect

non-linear QED

\[ E \ll m_e \]

\[ \propto c_1(F^{\mu\nu}F_{\mu\nu})^2 + c_2(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 \]

never been measured below the electron mass!
measured at high energies
(light by light scattering)

Hiesenberg Euler, 1936
Schwinger, 1951
ATLAS, 2017
The Euler Heisenberg effect

non-linear QED

\[ E \ll m_e \]

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ALP vs EH

\[ \frac{c_\gamma/\Lambda}{m_a} \gtrsim \mathcal{O}(1) \times \frac{\alpha}{m_e^2} \sim \frac{10^{-10} \text{ GeV}^{-1}}{10^{-6} \text{ eV}} \]

comparable to the current limit on ALPs (by CAST)

Evans and Rafelski, 1810.06717
The Euler Heisenberg effect

\[ E \ll m_e \]

non-linear QED

\[ \propto c_1(F^{\mu\nu}F_{\mu\nu})^2 + c_2(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 \]

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comparable to the current limit on ALPs (by CAST)

Evans and Rafelski, 1810.06717

improve current bounds

sensitivity to EH
Detecting the EH effect by Superconducting radiofrequency cavities

non-linear QED in the SM

\[ E \ll m_e \]

\[ \propto c_1 (F^{\mu\nu} F_{\mu\nu})^2 + c_2 (F^{\mu\nu} \tilde{F}_{\mu\nu})^2 \]

Hiesenberg Euler, 1936
Schwinger, 1951

Brodin, Marklund, Stenflo, PRL, 2001
Eriksson, Brodin, Marklund, Stenflo, PRA, 2004

\( \omega_s = 2\omega_1 - \omega_2 \)

\( \omega_s > \omega_1, \omega_2 \)
Sensitivity to ALPs
Sensitivity to ALPs

the number of signal photons

\[ N_s = \frac{1}{2\omega_s} \int d^3x |E_a(x)|^2 = \frac{Q_s^2 V E_0^6 c^4}{2\omega_s \Lambda^4} \left\{ \begin{array}{l} \frac{K_0^2}{\omega_s^4} \\
\frac{K_\infty^2}{m_a^4} \end{array} \right\} \]

quality factor  cavity volume  pump field strength

\[ m_a \ll \omega_s \]

\[ m_a \gg \omega_s \]

electric field of the induced ALP current

modes overlap
Sensitivity to ALPs

the number of signal photons

\[ N_s = \frac{1}{2\omega_s} \int d^3x \left| E_a(x) \right|^2 = \frac{Q_s^2 V E_0^6 c^4}{2\omega_s \Lambda^4} \left\{ \frac{K_0^2}{\omega_s^4} \right\} m_a \ll \omega_s \]

signal-to-noise ratio (SNR)  
(Dicke radiometer equation)

\[ \text{SNR} \approx \frac{N_s}{N_{th}} \frac{1}{2LQ_s} \sqrt{\frac{t}{B}} \]

quality factor  
cavity volume  
pump field strength  

electric field of the induced ALP current  

\[ \frac{K_2^2}{\omega_s^4} \]

\[ m \gg \omega_s \]

\[ c \gamma \Lambda \]

\[ m_a \ll \omega_s \]

\[ K_{\infty}^2/m_a^4 \]  

modes overlap  

\[ \frac{Q_s^2 V E_0^6 c^4}{2\omega_s \Lambda^4} \]

\[ K_0^2/\omega_s^4 \]

\[ m \gg \omega_s \]

\[ \frac{K_2^2}{\omega_s^4} \]

\[ m_a \ll \omega_s \]

\[ \frac{Q_s^2 V E_0^6 c^4}{2\omega_s \Lambda^4} \]

\[ m \gg \omega_s \]

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\[ \frac{K_2^2}{\omega_s^4} \]

\[ m_a \ll \omega_s \]

\[ \frac{Q_s^2 V E_0^6 c^4}{2\omega_s \Lambda^4} \]

\[ m \gg \omega_s \]

\[ \frac{K_2^2}{\omega_s^4} \]
\[ \frac{c_\gamma}{\Lambda} = \begin{cases} \left( \frac{4TL}{Q_sV_E^6K_0^2} \sqrt{\frac{B}{t}} \text{SNR} \right)^{1/4} \omega_s & m_a \ll \omega_s \\ \left( \frac{4TL}{Q_sV_E^6K_\infty^2} \sqrt{\frac{B}{t}} \text{SNR} \right)^{1/4} m_a & m_a \gg \omega_s \end{cases} \]

\[ \text{SNR} = 5 \]
Sensitivity to ALPs

$$\frac{c_\gamma}{\Lambda} = \left\{ \begin{array}{ll}
\left( \frac{4TL}{Q_s V E_0^6 K_0^2} \sqrt{\frac{B}{t} \text{SNR}} \right)^{1/4} & \omega_s \quad m_a \ll \omega_s \\
\left( \frac{4TL}{Q_s V E_0^6 K_\infty^2} \sqrt{\frac{B}{t} \text{SNR}} \right)^{1/4} & m_a \quad m_a \gg \omega_s
\end{array} \right.$$  

SNR = 5

$$a = 0.5 \text{ m}, \quad d = 1.56 \text{ m}, \quad V = 1.23 \text{ m}^3$$

$$\omega_1 = \text{TE}_{011}, \quad \omega_2 = \text{TM}_{010}, \quad \omega_s = \text{TM}_{020}$$

$$\omega_s/(2\pi) = 527 \text{ Hz}$$

$$K_0 = 0.4, \quad K_\infty = 0.18$$

$$E_0 = 45 \text{ MV/m} \quad T = 1.5 \text{ K}$$
Sensitivity to ALPs

\[ \frac{c_\gamma}{\Lambda} = \begin{cases} 
\left( \frac{4TL}{Q_sVE_0^6K_0^2} \sqrt{\frac{B}{t} \text{SNR}} \right)^{1/4} & \omega_s \ m_a \ll \omega_s \\
\left( \frac{4TL}{Q_sVE_0^6K_\infty^2} \sqrt{\frac{B}{t} \text{SNR}} \right)^{1/4} & m_a \ m_a \gg \omega_s 
\end{cases} \]

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\[ a = 0.5 \text{ m}, \ d = 1.56 \text{ m}, \ V = 1.23 \text{ m}^3 \]
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Sensitivity to ALPs

\[
\frac{c_\gamma}{\Lambda} = \begin{cases} 
\left( \frac{4TL}{Q_s V E^6 K^2_0} \frac{1}{\sqrt{t}} \text{SNR} \right)^{1/4} & \omega_s \ll m_a, \\
\left( \frac{4TL}{Q_s V E^6 K^2_\infty} \frac{1}{\sqrt{t}} \text{SNR} \right)^{1/4} & m_a \gg \omega_s
\end{cases}
\]

SNR = 5

\[
a = 0.5 \text{ m}, \quad d = 1.56 \text{ m}, \quad V = 1.23 \text{ m}^3
\]

\[
\omega_1 = T E_{011}, \quad \omega_2 = T M_{010}, \quad \omega_s = T M_{020}
\]

\[
\omega_s / (2\pi) = 527 \text{ Hz}
\]

\[
K_0 = 0.4, \quad K_\infty = 0.18
\]

\[
E_0 = 45 \text{ MV/m}, \quad T = 1.5 \text{ K}
\]

20 days run to detect EH!
ALPs at the MeV to the GeV scale
$\mathcal{L}_{\text{eff}} = - \frac{4\pi\alpha_s c_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} + \frac{c_\gamma}{4\Lambda} a F^{\mu\nu} \tilde{F}_{\mu\nu}$

$c_g \neq 0$ or $c_\gamma \neq 0$

- probing at photon beam (Primakoff like) experiments
- estimate of hadronic decay rates
ALPs at Primakoff like experiments

photon beam
5-10 GeV

target $N$: $p$, $C$, $Si$, $Pb$
ALPs at Primakoff like experiments

- Photon beam: 5-10 GeV
- Target $N$: $p$, $C$, $Si$, $Pb$
- SM final state: prompt decay
ALPs at Primakoff like experiments

 Photon beam 5-10 GeV

 Target $N$: $p$, C, Si, Pb

 SM final state prompt decay

$$\gamma \quad \gamma^* \quad a$$

$$\frac{c_\gamma}{4\Lambda} a F_{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\gamma \quad V^{\prime*} \quad a$$

$$\frac{4\pi\alpha_s c_g}{\Lambda} a G_{\mu\nu} \tilde{G}_{\mu\nu}$$
ALP photons coupling

\[ \frac{c_\gamma}{4\Lambda} a F^{\mu\nu} \tilde{F}_{\mu\nu} \]
ALP photons coupling

\[ \gamma \rightarrow a \]

\[ \gamma^* \]

\[ N \rightarrow aN \]

\[ \frac{c_\gamma}{4\Lambda} a F^{\mu\nu} \tilde{F}_{\mu\nu} \]

\[ \frac{d\sigma_{\gamma N \to aN}^{\text{elastic}}}{dt} = \alpha Z_N^2 F_N^2(t) \Gamma_{a \rightarrow \gamma \gamma} \mathcal{H}(m_N, m_a, s, t) \]

- target charge
- form factor
- kinematical function
Primakoff production of ALPs and pion/eta are similar

$$\frac{d\sigma_{\gamma N \rightarrow aN}^{\text{elastic}}}{dt} = \frac{\alpha Z_{\gamma}^2 F_{\gamma}^2(t) \Gamma_{a \rightarrow \gamma \gamma} \mathcal{H}(m_N, m_a, s, t)}{\Gamma_{P \rightarrow \gamma \gamma} \mathcal{H}(m_N, m_p, s, t)}$$

Data-driven signal normalisation
(canceled form factor and flux dependence)
ALP photons coupling
**ALP photons coupling**

- **FASER**
- **ALP photons coupling**
- **dumps**
- **beam**
- **SeaQuest**
- **Belle-II**
- **PrimEx**

The extracted decay width combined for the two targets was extracted by fitting the data to the theoretical model. The fit results for individual contributions from the di-lepton decay process through the hadron exchange, for individual contributions from the di-hadron decay process, and for individual contributions from the pion photoproduction from nuclei at forward angles. The cross section of this process can be extracted accurately using a full theoretical description based on the Glauber model. The systematic uncertainties were verified by two groups within the PrimEx collaboration. The quoted total systematic uncertainty (2.1%) is the quadratic sum of all the estimated uncertainties in this experiment [9].

The angular resolutions of the four processes mentioned above folded with the experimental results with the theoretical cross sections and the phase angle, respectively. The cross sections and the phase angle, respectively, also used to check the model dependence of the extracted total photon flux for each tagger energy bin, the number of atoms in the target, the acceptance of the experimental setup and the inelasticities of the detectors. The uncertainty reached in the photon flux measurement, as determined through model dependence and the parameters inside of the model, contributed coherently, as well as to the nucleus; the incident photon; the theoretical predictions at the level of 1.5% and will be estimated to be 0.3%.

The cross sections for these well-known processes agree with the theoretical predictions at the level of 1.5% and will be estimated to be 0.3%.

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ALP photons coupling

![Graph showing ALP photons coupling with various experimental data points and projections. The graph includes data from PrimEx, LEP, Belle-II, SeaQuest, and SHiP, among others. There are annotations indicating on-tape PrimEx data and a projection for GlueX.](image-url)
ALP gluons coupling

\[ \gamma \rightarrow V^{'\ast} \rightarrow a \]

\[ p \rightarrow V^\ast \rightarrow p \]

\[ -\frac{4\pi\alpha_s c_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} \]
$\gamma \rightarrow a$\hspace{1cm} p

$\frac{4\pi\alpha_s c_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu}$

Fig. 3 [1701.08123]

di-photon final state
ALP gluons coupling

\[ \gamma \rightarrow \gamma V^* \rightarrow V^* a \]

\[ p \rightarrow p \]

\[ \frac{4\pi\alpha_s c_g}{\Lambda} \alpha G^{\mu\nu} \tilde{G}_{\mu\nu} \]

what if ALP can decay hadronically?
(above 3 pion mass)
ALP gluons coupling

\[- \frac{4\pi \alpha_s c_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} \]

How to estimate hadronic rates for ALPs with QCD scale mass?
How to estimate hadronic rates for ALPs with QCD scale mass?

\[ m_a \lesssim \text{GeV} \quad \text{???} \quad m_a \gtrsim 2 \text{ GeV} \]

chiral PT \quad \text{pQCD}
How to estimate hadronic rates for ALPs with QCD scale mass?

\[ m_a \lesssim \text{GeV} \quad \quad \quad \quad m_a \gtrsim 2 \text{ GeV} \]

chiral PT

?????

pQCD

use data!!

\[ -\frac{4\pi \alpha_s c_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} \]
How to estimate hadronic rates for ALPs with QCD scale mass?

\[ \frac{4\pi\alpha_s c_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} \]

- \( m_a \lesssim \text{GeV} \)
- \( m_a \gtrsim 2 \text{GeV} \)

chiral PT

pQCD

use data!!

\( e^+ e^- \rightarrow \text{hadrons} \)

information on specific \( U(3)_{\text{flavor}} \) combinations
$-\frac{4\pi\alpha_s c g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu}$

How to estimate hadronic rates for ALPs with QCD scale mass?

$m_a \lesssim \text{GeV}$

chiral PT

$m_a \gtrsim 2 \text{ GeV}$

pQCD

use data!!

$e^+ e^- \rightarrow \text{hadrons}$

information on specific $U(3)_{\text{flavor}}$ combinations

directly deduce the hadronic rates of vectors
Figure 3. Decay branching fractions for the (top left) $A'$, (top right) $B-L$, (middle left) $B$, and (middle right) protophobic models. The branching fractions of the $B$ boson decaying into specific hadronic final states are shown in Fig. 10. (bottom) Ratio of the branching fractions to leptons for $B-L$, $B$, and the protophobic model relative to the $A'$.
ALP gluons coupling

\[- \frac{4\pi \alpha_s c_g}{\Lambda} a G^{\mu \nu} \tilde{G}_{\mu \nu} \]

ALPs hadronic rates?

\[e^+ e^- \rightarrow V_1^* \rightarrow V_2 P\]
ALP gluons coupling

\[
-\frac{4\pi\alpha_s c_g}{\Lambda} \, a \, G^{\mu\nu} \tilde{G}_{\mu\nu}
\]

ALPs hadronic rates?

\[ e^+ e^- \rightarrow V_1^* \rightarrow V_2 P \]

\[
\mathcal{A}(V_1 \rightarrow V_2 P) = \epsilon_{\mu\nu\alpha\beta} \epsilon_{1}^{\mu} \epsilon_{2}^{\nu} P^{\alpha} P^{\beta} \mathcal{F}(p_1^2, p_2^2, q^2) \times \frac{3g^2}{4\pi^2 f_\pi} \langle V_1 V_2 P \rangle
\]

one Lorentz structure  
modified VMD
**ALP gluons coupling**

\[ -\frac{4\pi\alpha_s c g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} \]

\[ e^+ e^- \rightarrow V_1^* \rightarrow V_2 P \]

**ALPs hadronic rates?**

\[ \mathcal{A}(V_1 \rightarrow V_2 P) = \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu_1 \epsilon^\rho_{\nu'} p_1^\alpha p_2^\beta \mathcal{F} \left(p_1^2, p_2^2, q^2\right) \times \frac{3g^2}{4\pi^2 f}\langle V_1 V_2 P \rangle \]

one Lorentz structure

modified VMD

---

**from data**

\[ \mathcal{F}(m) = \begin{cases} 
1 & \text{interpolation for } m < 1.4 \text{ GeV} \\
\left(\frac{m}{\beta_m}\right)^4 & \text{for } 1.4 \leq m \leq 2 \text{ GeV} \\
\end{cases} \]

for \( m > 2 \text{ GeV} \)
ALP gluons coupling

\[ -\frac{4\pi\alpha_s c_g}{\Lambda} a \, G^{\mu\nu} \tilde{G}_{\mu\nu} \]

\[ e^+ e^- \rightarrow V_1^* \rightarrow V_2 P \]

\[ \mathcal{A}(V_1 \rightarrow V_2 P) = \epsilon_{\mu\nu\rho\sigma} \epsilon_1^{\mu*} p_1^\alpha p_2^\beta \mathcal{F}(p_1^2, p_2^2, q^2) \times \frac{3g^2}{4\pi^2 f_p} \langle V_1 V_2 P \rangle \]

one Lorentz structure
modified VMD

\[ \mathcal{F}(m) = \begin{cases} 
1 & \text{for } m < 1.4 \text{ GeV} \\
\left( \frac{\beta_\mathcal{F}}{m} \right)^4 & \text{for } 1.4 \leq m \leq 2 \text{ GeV} \\
& \text{for } m > 2 \text{ GeV} 
\end{cases} \]

from data

ALPs hadronic rates?
$\frac{4\pi\alpha_sc_g}{\Lambda} a \, G^{\mu\nu} \tilde{G}_{\mu\nu}$

replace $P$ by $P$-ALP mixing

$\eta_c$ – cross check
ALP gluons coupling

\[ -\frac{4\pi\alpha_sc_g}{\Lambda} a G^{\mu\nu} \tilde{G}_{\mu\nu} \]

\[ \Lambda \]

\[ a \]

\[ m_{\eta} \]

\[ m_{\eta'} \]

\[ b \rightarrow s a \text{ (inclusive)} \]

\[ m_{\eta\pi\pi} \text{ & } m_{KK^*} \text{ windows} \]

\[ \nu \text{-CAL} \]

\[ E137 \]

\[ \text{gray constraints depend on UV completion} \]

\[ \text{results shown assume } \mathcal{U} \mathcal{V} \approx \log \frac{\Lambda^2_{\text{UV}}}{m_t^2} \pm \mathcal{O}(1) \Rightarrow 1 \]
Outlook

- Superconducting radiofrequency cavities can be the most powerful to probe ALPs in the lab.

- The Euler Heisenberg effect (QED light by light scattering) can be probed, for the first time, in the same experimental setup.

- On tape PrimEx data can improve the sensitive to ALP with QCD mass scale, future GlueX data will improve it by order of magnitude.

- ALP hadronic rates for, mass around the GeV, can be estimated using data-driven method.
Backups
Overlap

![Graph showing relative field strength versus $\rho/\alpha$.](image)

- $E_{3,z}$
- $J_{0,z}\cos \omega \nu t$
- $J_{0,z}\sin \omega \nu t$
- $J_{\infty,z}\omega \nu$
Cavity vs LSW

\[
\frac{N_s}{N_i} \mid_{\text{LSW}} \sim \left( \frac{c_\gamma}{\Lambda} \right)^4 B_{\text{prod.}}^2 B_{\text{det.}}^2 L^4
\]

\[
\frac{N_s}{N_i} \mid_{\text{cavity}} \sim Q_s^2 \left( \frac{c_\gamma}{\Lambda} \right)^4 E_0^4 L^4
\]

\[
\text{SNR} = \frac{P_s}{T} \sqrt{\frac{t}{B}} \approx \frac{N_s}{N_{\text{th}}} \frac{1}{2LQ_s} \sqrt{\frac{t}{B}}
\]
Disentangling EH and ALPs

Proof of concept with rectangular cavity

pump 3 modes:

\[ E_p = r_1E_1 + r_1' E_1' + r_2E_2 \]

\[ \text{TE}_{221}/\text{TM}_{221}/\text{TM}_{121} \]

signal mode:

\[ \text{TM}_{163} \]

matching condition

\[ \omega_s = 2\omega_1 - \omega_2 \]

\[ K_\infty = 0.047r_2(r_1^2 - 0.18r_1^2) \]

\[ K_{EH} = 0.059r_2(r_1^2 - 8.24r_1^2) \]
\[ \mathcal{H}(m_N, m_a, s, t) \equiv 128\pi \frac{m_N^4}{m_a^3} \frac{m_a^2 t(m_N^2 + s) - m_a^4 m_N^2 - t((s - m_N^2)^2 + st)}{t^2(s - m_N^2)^2(t - 4m_N^2)^2} \]