The Z-Penguin in Generic Extensions of the Standard Model

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Based on [1903.05116]
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Descriptions of new physics

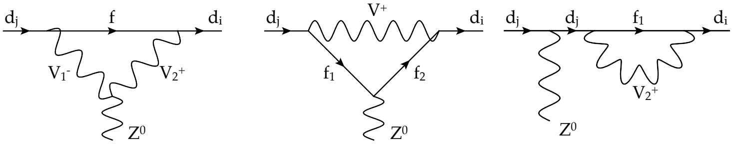
- Effective theories:
  - good separation of scales
  - parameterise heavy new physics
    (except for huge number of independent operators)
  - only if momenta are not too large
- Explicit models:
  - correlate observables – low energy $\leftrightarrow$ high p_T
  - falsify validity of effective theory
  - light weakly coupled new physics
- Generic models / Simplified models
  - cover larger set of allowed model space
  - correlate low energy $\leftrightarrow$ high p_T?
  - often neither renormalisable nor unitary

This talk

- Goal: Constrain generic model to achieve
  - perturbative unitarity
  - renormalisibilty
- For the example of a FCNC Z-Penguin
- Define generic Lagrangian
- Renormalisation for extra charged vectors
- Extensions to arbitrary fermions/scalars/vectors

Toy example: extra vectors

Consider tower of vectors V and $d_j d_i Z$ Green's function:



We only need cubic $\psi - \psi - V$ and $V - V - V$ interactions:

$$\mathcal{L}_3^V = \sum_{f_1 f_2 v_1 L/R} g^{L/R}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2}$$

$$+\frac{i}{6} \sum_{v_1 v_2 v_3} g^{abc}_{v_1 v_2 v_3} \Big(V^a_{v_1,\mu} V^b_{v_2,\nu} \partial^{[\mu} V^{c,\nu]}_{v_3}$$

$$+ V^c_{v_3,\mu} V^a_{v_1,\nu} \partial^{[\mu} V^{b,\nu]}_{v_2} + V^b_{v_2,\mu} V^c_{v_3,\nu} \partial^{[\mu} V^{a,\nu]}_{v_1} \Big).$$

Couplings in the Standard Model

$$\mathcal{L}_3^V = \sum_{f_1 f_2 v_1 L/R} g^{L/R}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2} + \frac{i}{6} \sum_{v_1 v_2 v_3} g^{abc}_{v_1 v_2 v_3} \Big(V^a_{v_1,\mu} V^b_{v_2,\nu} \partial^{[\mu} V^{c,\nu]}_{v_3} + \dots \Big).$$

In SM we would need the following couplings:

- $g^L_{W^+\bar{u}_j d_k} = \frac{e}{s_w \sqrt{2}} V_{jk}, \quad y^L_{G^+\bar{u}_j d_k} = \frac{m_{uj}}{M_W} \frac{e}{s_w \sqrt{2}} V_{jk}$
- $g^L_{Z\bar{f}_j f_k} = \frac{2e}{s_{2w}} \left(T_3^f - Q_f s_w^2 \right) \delta_{jk}, \quad g^R_{Z\bar{f}_j f_k} = -\frac{2e}{s_{2w}} Q_f s_w^2 \delta_{jk}$
- $g_{ZW^+W^-} = \frac{e}{t_w}, \; g_{ZW^+G^-} = -t_w^2 \frac{e}{t_w}, g_{ZG^+G^-} = \left(1 - \frac{1}{2c_W^2}\right) \frac{e}{t_w}$

General generic Lagrangian

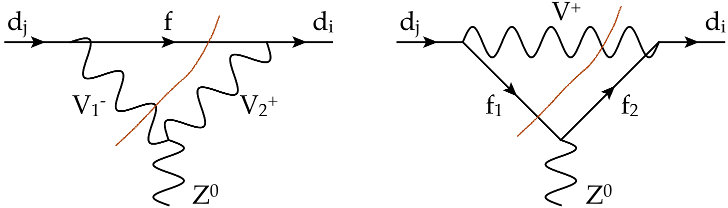
Incorporating Goldstones we arrive at:

$$\mathcal{L}_3 = \sum_{f_1 f_2 v_1 L/R} y^{L/R}_{s_1 \bar{f}_1 f_2} h_{s_1} \bar{\psi}_{f_1} P_{L/R} \psi_{f_2} + \frac{1}{2} \sum_{v_1 v_2 s_1} g_{v_1 v_2 s_1} V_{v_1,\mu} V^{\mu}_{v_2} h_{s_1}$$
$$- \frac{i}{2} \sum_{v_1 s_1 s_2} g_{v_1 s_1 s_2} V^{\mu}_{v_1} \Big(h_{s_1} \partial_\mu h_{s_2} - \left(\partial_\mu h_{s_1} \right) h_{s_2} \Big) + \mathcal{L}^V_3 \,.$$

- h extra physical scalars (Goldstones $h \to \phi$)
- Add R_ξ gauge-fixing
- Adding $SU(3) \times U(1) \to$ higher order corrections
- Using Lagrangian will give divergent results

Finite FCNC Z-Penguin at one-loop?

- Perturbative Unitary ↔ massive vectors from SSB
  [Llewellyn Smith ’73; Cornwall et.al. 73/74]
- Need correct high-energy behaviour in loops:
  - Gauge-structure from Slavnov-Taylor (STIs)
  - Traditionally used in high-energy scattering ("Goldstone-boson Equivalence Theorem")
  - UV behaviour controls renormalization properties



Remnants of gauge symmetry

- Massive vector bosons originate from a spontaneously broken gauge symmetry
- Fix the gauge for massive vector ($\sigma_{V^\pm} = \pm i$, $\sigma_V = 1$)

$$\mathcal{L}_{\text{fix}} = -\sum_v (2\xi_v)^{-1} F_{\bar{v}} F_v \,, \qquad F_v = \partial_\mu V_v^\mu - \sigma_v \xi_v M_v \phi_v \,,$$

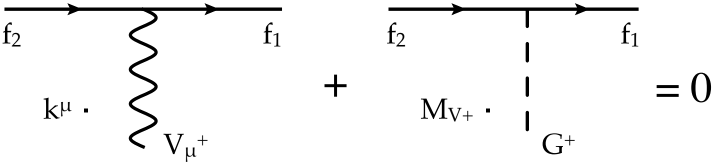
- BRST invariant field combination $s(\dots)_{\text{ph}} = 0$
- STIs from $s\langle T\{\bar{u}_v(\dots)_{\text{ph}}\}\rangle = 0$ at required order:

$$\langle T\Big\{k^\mu \underline{V_v^\mu} - i\sigma_{\bar{v}} M_v \underline{\phi_v}\Big\}(\dots)_{\text{ph}}\rangle \,,$$

- E.g. for $(\dots)_{\text{ph}} = \bar{f}_1 f_2$ we have

$$y^{L/R}_{\phi_1 \bar{f}_1 f_2} = -i\sigma_{v_1} \frac{1}{M_{v_1}} \Big(m_{f_1} g^{L/R}_{v_1 \bar{f}_1 f_2} - g^{R/L}_{v_1 \bar{f}_1 f_2} m_{f_2} \Big)$$

Elimination of gauge boson couplings



- From $(VV)_{\rm ph}, (Vh)_{\rm ph}, (hh)_{\rm ph}$ we obtain 3-point STIs:

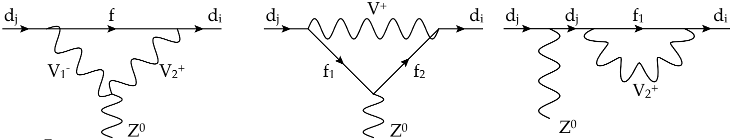
$$g_{v_1\phi_2\phi_3} = \sigma_{v_2}\sigma_{v_3}\,\frac{M_{v_2}^2+M_{v_3}^2-M_{v_1}^2}{2\,M_{v_2}M_{v_3}}\,g_{v_1v_2v_3}\,, \qquad g_{v_1\phi_2 s_1} = -i\sigma_{v_2}\,\frac{1}{2\,M_{v_2}}\,g_{v_1v_2s_1}\,,$$

$$g_{v_1v_2\phi_3} = -i\sigma_{v_3}\,\frac{M_{v_1}^2-M_{v_2}^2}{M_{v_3}}\,g_{v_1v_2v_3}\,, \qquad g_{\phi_1 s_1 s_2} = i\sigma_{v_1}\,\frac{M_{s_1}^2-M_{s_2}^2}{M_{v_1}}\,g_{v_1s_1s_2}\,,$$

$$g_{\phi_1\phi_2 s_1} = -\sigma_{v_1}\sigma_{v_2}\,\frac{M_{s_1}^2}{2\,M_{v_1}M_{v_2}}\,g_{v_1v_2s_1}\,, \qquad g_{\phi_1\phi_2\phi_3} = 0\,.$$

- Allows us to eliminate all Goldstone couplings

Results in terms of physical parameters



$$\sum_{f_1 f_2 v_1}\left[\tilde{k}^L_{f_1 f_2 v_1}\Big(\tilde{C}_0\big(m_{f_1}, m_{f_2}, M_{v_1}\big) - \tfrac{1}{2}\Big) + k^L_{f_1 f_2 v_1} C_0\big(m_{f_1}, m_{f_2}, M_{v_1}\big) + k'^L_{f_1 f_2 v_1}\right]$$

$$+ \sum_{f_1 v_1 v_2}\left[\tilde{k}^L_{f_1 v_1 v_2}\Big(\tilde{C}_0\big(m_{f_1}, M_{v_1}, M_{v_2}\big) + \tfrac{1}{2}\Big) + k^L_{f_1 v_1 v_2} C_0\big(m_{f_1}, M_{v_1}, M_{v_2}\big) + k'^L_{f_1 v_1 v_2}\right]$$

The divergent loop functions $\tilde{C}_0$ are multiplied with:

$$\tilde{k}^L_{f_1 f_2 v_1} = \left(g^L_{Z\bar{f}_2 f_1} + \frac{m_{f_1} m_{f_2}}{2M^2_{v_1}} g^R_{Z\bar{f}_2 f_1}\right) g^L_{\bar{v}_1 \bar{d}_i f_2} g^L_{v_1 \bar{f}_1 d_j},$$

$$\tilde{k}^L_{f_1 v_1 v_2} = -\left(3 + \frac{m^2_{f_1}(M^2_{v_1} + M^2_{v_2} - M^2_Z)}{4M^2_{v_1} M^2_{v_2}}\right) g_{Z v_1 \bar{v}_2} g^L_{\bar{v}_1 \bar{d}_i f_1} g^L_{v_2 \bar{f}_1 d_j}$$

$$- \frac{1}{2}\left(1 + \frac{m^2_{f_1}}{2M^2_{v_1}}\right)\left(g^L_{Z\bar{d}_i d_i} g^L_{v_1 \bar{d}_i f_1} g^L_{\bar{v}_1 \bar{f}_1 d_j} + g^L_{v_1 \bar{d}_i f_1} g^L_{\bar{v}_1 \bar{f}_1 d_j} g^L_{Z\bar{d}_j d_j}\right)\delta_{v_1 v_2},$$

Consider SM fermions and extra vectors

Derive STIs for $f - f - V - V$ function:

- Relations between products of trilinear couplings

$$\sum_{v_3} g^{L/R}_{v_3 \bar{f}_1 f_2} g_{v_1 v_2 \bar{v}_3} = \sum_{f_3} \left(g^{L/R}_{v_1 \bar{f}_1 f_3} g^{L/R}_{v_2 \bar{f}_3 f_2} - g^{L/R}_{v_2 \bar{f}_1 f_3} g^{L/R}_{v_1 \bar{f}_3 f_2} \right)$$

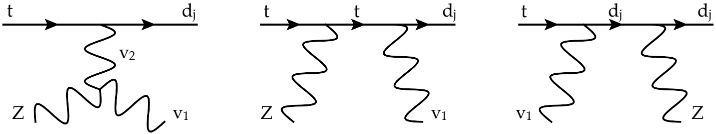
- $v_1 \to W_1^+, v_2 \to W_2^-, f_1 \to d_i, f_2 \to d_j$ and $g_{Z\bar{d}_i d_j} = 0$:

$$0 = \sum_{f_3} g^L_{W_2^- \bar{s} f_3} g^L_{W_1^+ \bar{f}_3 d} \quad \text{CKM unitarity}$$

- We still obtain a divergence proportional to

$$\sum_{v_1, v_2} \left(\frac{1}{2M^2_{v_1}} (g^R_{Z\bar{t}t} - g^L_{Z\bar{d}d}) \delta_{v_1 v_2} - \frac{(M^2_{v_1} + M^2_{v_2} - M^2_Z)}{4 M^2_{v_1} M^2_{v_2}} g_{Z v_1 \bar{v}_2} \right) g^L_{\bar{v}_1 \bar{d}_i t} g^L_{v_2 \bar{t} d_j}$$

Two additional STIs:



Setting $v_3 = Z$, $f_2 = d_j$ there are two additional STIs:

$$g^L_{Z\bar{t}t} g^L_{v_1^+\bar{t}d_j} = g^L_{v_1^+\bar{t}d_j} g^L_{Z\bar{d}_j d_j} + \sum_{v_2} g_{Z v_1^+ v_2^-} g^L_{v_2^+\bar{t}d_j}$$

$$g^R_{Z\bar{t}t} g^L_{v_1^+\bar{t}d_j} = \tfrac{1}{2} g^L_{v_1^+\bar{t}d_j}\left(g^L_{Z\bar{t}t} + g^L_{Z\bar{d}_j d_j}\right) + \sum_{v_2} \frac{M^2_{v_1} - M^2_Z}{2M^2_{v_2}} g_{Z v_1^+ v_2^-} g^L_{v_2^+\bar{t}d_j}$$

Which can be used to eliminate $g^{L/R}_{Z\bar{t}t}$ from the expression

Results for extra vectors

The resulting expression comprises less parameters

$$\hat{C}^L_{d_j d_i Z} = \sum_{v_1 v_2} f_V(m_t, M_{v_1}, M_{v_2}) g_{Z v_2^+ v_1^-} g^L_{v_1^+ \bar{t} d_j} g^L_{v_2^- \bar{d}_i t}$$

and a finite loop function

$$\begin{aligned} f_V(m_i, m_j, m_k) = {} & m_i^2 C_0\left(m_i, m_k, m_k\right) - \frac{m_i^2\left(m_j^2 + m_k^2 - M_Z^2\right)}{4 m_j^2 m_k^2} \\ & + \frac{m_i^2\left(-3m_j^2 + m_k^2 - M_Z^2\right) + 4m_k^2\left(m_j^2 - m_k^2 + M_Z^2\right)}{4 m_j^2 m_k^2} m_i^2 C_0\left(m_i, m_i, m_k\right) \\ & + \frac{-M_Z^2\left(3m_j^2 + 4m_k^2\right) - 13 m_j^2 m_k^2 + 3m_j^4 + 4m_k^4}{4 m_j^2 m_k^2} m_i^2 C_0\left(m_i, m_j, m_k\right). \end{aligned}$$

SM couplings: $g^L_{W^+ \bar{u}_j d_k} = \dfrac{e}{s_w \sqrt{2}} V_{jk}$ and $g_{ZW^+W^-} = \dfrac{e}{t_w}$.

Including extra scalars

$$
\begin{aligned}
\hat{C}^L_{d_j d_i Z} = & \sum_{s_1 s_2} f_S(m_t, M_{s_1}, M_{s_2}) y^L_{s_2^+ \bar{t} d_j} \left(\delta_{s_1 s_2} y^R_{s_2^- \bar{d}_i t} \left(g^L_{Z \bar{d}_j d_j} - g^L_{Z \bar{t} t} \right) + g_{Z s_1^+ s_2^-} y^R_{s_1^- \bar{d}_i t} \right) \\
& + \sum_{v_1 v_2} f_V(m_t, M_{v_1}, M_{v_2}) g_{Z v_2^+ v_1^-} g^L_{v_1^+ \bar{t} d_j} g^L_{v_2^- \bar{d}_i t} \\
& + \sum_{s_1 v_1} f_{VS}(m_t, M_{s_1}, M_{v_1}) y^L_{s_1^+ \bar{t} d_j} g^L_{v_1^- \bar{d}_i t} g_{Z v_1^+ s_1^-} \\
& + \sum_{s_1 v_1} f_{VS'}(m_t, M_{s_1}, M_{v_1}) y^R_{s_1^- \bar{d}_i t} g^L_{v_1^+ \bar{t} d_j} g_{Z v_1^- s_1^+} \, .
\end{aligned}
$$

- $f_{S,VS,VS'}$ are again functions of C_0 and provide results for LR-Models.
- E.g. for one charged Higgs we reproduce 2HDM type II results in Literature by specifying:

$$
g_{Z h^- h^+} = -e \frac{c_{2w}}{2 s_w c_w} \, , \quad y^L_{h^+ \bar{t} d_i} = \frac{m_t}{t_\beta} \frac{V_{td} e}{\sqrt{2} s_w M_W} \, .
$$

Generic fermions, vectors and scalars

- Renormalisation procedure works also for the most generic Lagrangian
- Simplified formulas given for charged particles in [1903.05116]
- Checked against results in the literature
  - general MSSM reproduce Literature (but explicitly finite)
  - Vector-like-quarks reproduce SMEFT logs
- Method also works for neutral particles

Generic fermions, vectors and scalars

$$\begin{aligned}
\hat{C}^L_{d_j d_i Z} = & \sum_{f_1 f_2 v_1} g^L_{Z\bar{f}_2 f_1} g^L_{v_1 \bar{f}_1 d_j} g^L_{\bar{v}_1 \bar{d}_i f_2} F_V\left(m_{f_1}, m_{f_2}, M_{v_1}\right) \\
& + \sum_{f_1 f_2 v_1} g^R_{Z\bar{f}_2 f_1} g^L_{v_1 \bar{f}_1 d_j} g^L_{\bar{v}_1 \bar{d}_i f_2} F_{V'}\left(m_{f_1}, m_{f_2}, M_{v_1}\right) \\
& + \sum_{f_1 v_1 v_2} g_{Z v_2 \bar{v}_1} g^L_{v_1 \bar{f}_1 d_j} g^L_{\bar{v}_2 \bar{d}_i f_1} F_{V''}\left(m_{f'}, m_{f_1}, M_{v_1}, M_{v_2}\right) \\
& + \sum_{f_1 f_2 s_1} g^L_{Z\bar{f}_2 f_1} y^L_{s_1 \bar{f}_1 d_j} y^R_{\bar{s}_1 \bar{d}_i f_2} F_S\left(m_{f_1}, m_{f_2}, M_{s_1}\right) \\
& + \sum_{f_1 s_1 s_2} \left(g_{Z s_2 \bar{s}_1} + \delta_{s_1 s_2} g^L_{Z\bar{d}_j d_j}\right) y^L_{s_1 \bar{f}_1 d_j} y^R_{\bar{s}_2 \bar{d}_i f_1} F_{S'}\left(m_{f_1}, M_{s_1}, M_{s_2}\right) \\
& + \sum_{f_1 f_2 s_1} g^R_{Z\bar{f}_2 f_1} y^L_{s_1 \bar{f}_1 d_j} y^R_{\bar{s}_1 \bar{d}_i f_2} F_{S''}\left(m_{f_1}, m_{f_2}, M_{s_1}\right) \\
& + \sum_{f_1 s_1 v_1} g_{Z v_1 \bar{s}_1} y^L_{s_1 \bar{f}_1 d_j} g^L_{\bar{v}_1 \bar{d}_i f_1} F_{SV}\left(m_{f_1}, M_{s_1}, M_{v_1}\right) \\
& + \sum_{f_1 s_1 v_1} g_{Z \bar{v}_1 s_1} g^L_{v_1 \bar{f}_1 d_j} y^R_{\bar{s}_1 \bar{d}_i f_1} F_{SV'}\left(m_{f_1}, M_{s_1}, M_{v_1}\right) ,
\end{aligned}$$

Penguins come with boxes

- Results for $\Delta F = 2$ boxes known [Senjanovic et.al.]
- Both $\Delta F = 2$ and $\Delta F = 1$ boxes given for our most general fermion-scalar-vector interactions [1903.05116]
- Results are finite by power counting.
- See also [1904.05890] [Arnan, Crivellin, Fedele, Mescia] for boxes from fermion-scalar interactions.

Outlook

- Agrees with explicit calculations in the SM, MSSM, LR-Model, multi Higgs models & vector-like quarks
- For heavy new physics reproduces the logs of an EFT calculation
- Provides a Framework for
  - Simplified unitary models
  - Studying unitarisation of effective theories
  - Studying weakly coupled light physics
- Could be extended to $SU(3) \times SU(2) \times U(1)$ SMEFT matching
- Extend to different operators and develop code for numerical evaluation [Fady Bishara, Joachim Brod, MG, Ulserik Moldanazarova]