# Asympotic Safety of $f(\mathrm{Ric})$ Gravity

Yannick Kluth

8 July 2019

University of Sussex

UNIVERSITY OF SUSSEX

# This Talk ...

Motivation

Asymptotic Safety

Functional Renormalisation Group

f($\mathrm{Ric}$) Gravity

Summary

# Motivation

## Quantum Gravity

- General Relativity is tested to very high precisions but can't be end of story: Black holes, consistency with QM, ...
- Quantum Field Theory describes all forces except for gravitation to very high precision
- What do we know about quantum gravity?
    - Gravity is perturbatively non-renormalisable
    - Inifnite number of counterterms
    - Only predictive as low energy effective field theory

$$\mathcal{S} = \int \mathrm{d}^d x \sqrt{g} \left[ \lambda_0 + \lambda_1 R + \lambda_{2,1} R^2 + \lambda_{2,2} R_{\mu\nu} R^{\mu\nu} + \lambda_{2,3} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \lambda_{2,4} \Box R + \dots \right]$$

- How to determine open coupling constants?

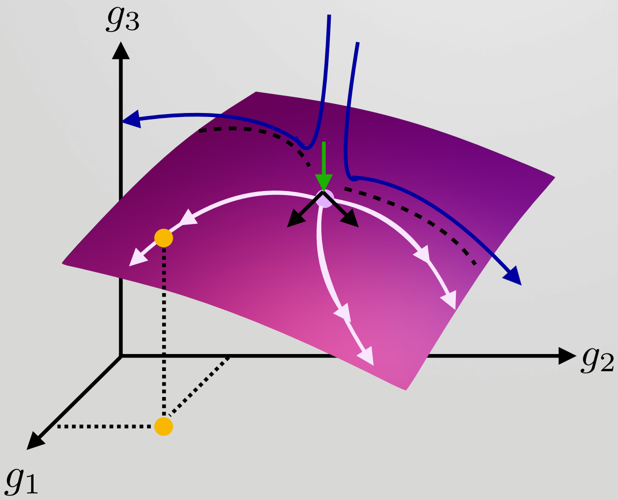
# Asymptotic Safety

# UV Fixed Points

- Theory should be valid up to arbitrary high energies
- Couplings are energy dependent
  - ➤ Avoid Landau poles by requiring scale invariance in the UV!
- Dimensionless couplings $\tilde{g}_i(k) = k^{-d_i} g_i(k)$ should stop running in the UV, $\tilde{\beta}_i(k) \xrightarrow{k \to \infty} 0$
  - ➤ UV Fixed Point!
- Only specific RG trajectories flow into UV fixed point

[Weinberg, 1979]

## RG Flow




Graphic taken from [arXiv:1810.07615]

- Schematic RG flow with three couplings
- Trajectories flow towards IR
- All trajectories which flow into fixed point are on UV critical surface (purple)
- What is the dimensionality of the UV critical surface?

## Stability Matrix

Expand beta functions around fixed point $\tilde{g}^*$:

$$\tilde{\beta}_i = \left.\frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\right|_* (\tilde{g}_j - \tilde{g}_j^*) + \mathcal{O}(\tilde{g}_j - \tilde{g}_j^*)^2$$

This is solved by

$$\tilde{g}_i = \tilde{g}_i^* + \sum_n C_n k^{\lambda_n} (\underline{v}_n)_i$$

with

$$\left.\frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\right|_* (\underline{v}_n)_j = \lambda_n (\underline{v}_n)_i$$

The $C_n$ are now the open parameters of the theory

- $\lambda_i > 0$ : Eigenmode diverges in the UV $\rightarrow$ $C_i = 0$ (Irrelevant direction)
- $\lambda_i < 0$ : Eigenmode vanishes in the UV $\rightarrow$ $C_i =$ unknown (Relevant direction)

➤ **Numer of relevant directions is given by number of negative eigenvalues**

## Gaussian Scaling

Relation between massive and massless couplings:

$$\tilde{g}_i = k^{-d_i} g_i$$
$$\Rightarrow \tilde{\beta}_i = -d_i \tilde{g}_i + k^{-d_i} \beta_i$$

Stability matrix:

$$\left.\frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\right|_* = -d_i \delta_{ij} + \text{quantum corrections}$$

At Gaussian fixed point ($\tilde{g}_i = 0$):

$$\lambda_i = -d_i$$

At Gaussian fixed point the number of relevant directions is given by number of couplings with positive mass dimension. What about interacting fixed points?

# Functional Renormalisation Group

# Wetterich Equation

- A priori no reason to believe that perturbation theory works for an interacting UV fixed point
  - ➤ Use non-perturbative Functional Renormalisation Group (FRG) [arXiv:1710.05815]

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[ \left( \partial_t \mathcal{R}_k \right) \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \right]$$

  - $\mathcal{R}_k$ is a $k$-dependent mass term (IR regulator)
  - $\Gamma_k$ is effective average action.
    - (Scale dependent effective action)
- Solve FRG approximately by making an ansatz for the form of $\Gamma_k$
  - ➤ Extract beta functions

## Gravitational Truncations

Easiest truncation given by $f(R)$ action

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g}\, f(R) = \int \mathrm{d}^d x \sqrt{g} \left[\lambda_0 + \lambda_1 R + \lambda_2 R^2 + \lambda_3 R^3 + \dots\right]$$

- This gives three relevant directions [arXiv:1410.4815, arXiv:0805.2909]

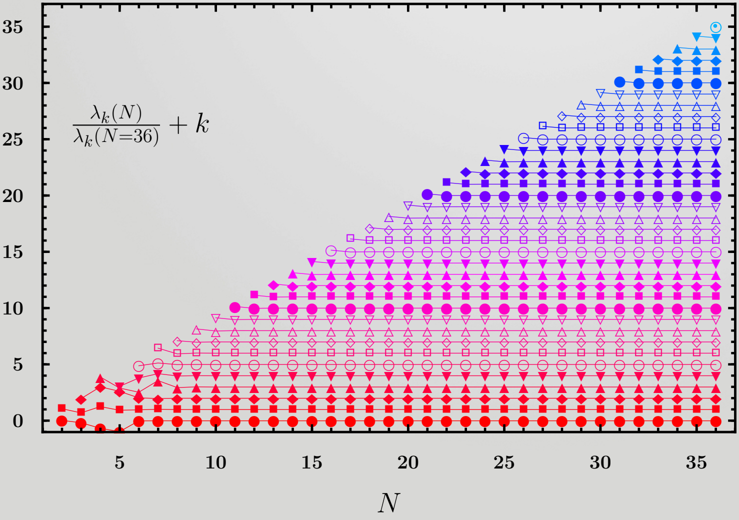
Here, we consider

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g}\, f(\mathrm{Ric}) = \int \mathrm{d}^d x \sqrt{g} \left[\lambda_0 + \lambda_1 R^{\mu}{}_{\mu} + \lambda_2 R^{\mu}{}_{\nu} R^{\nu}{}_{\mu} + \lambda_3 R^{\mu}{}_{\nu} R^{\nu}{}_{\rho} R^{\rho}{}_{\mu} + \dots\right]$$

truncated to include $N$ different operators.

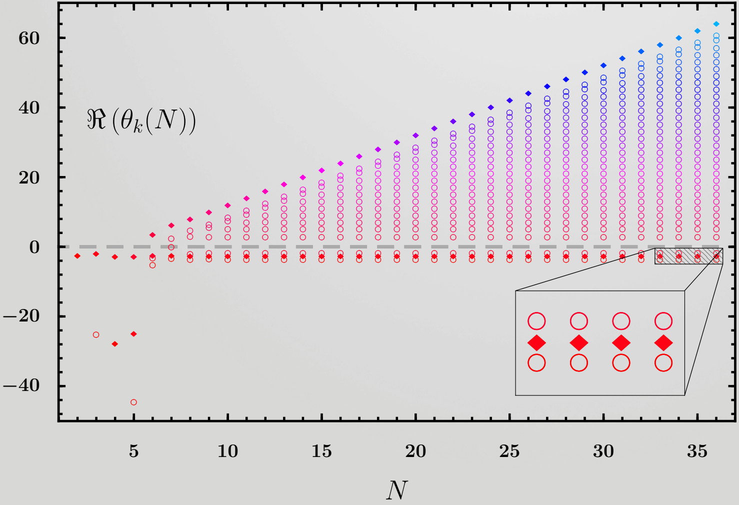
# f(Ric) Gravity

# Couplings




- Convergence of fixed point couplings $\lambda_k(N)$ with $N$ being the number of included operators
- High orders show strong convergence
- Unstable below $N = 6$

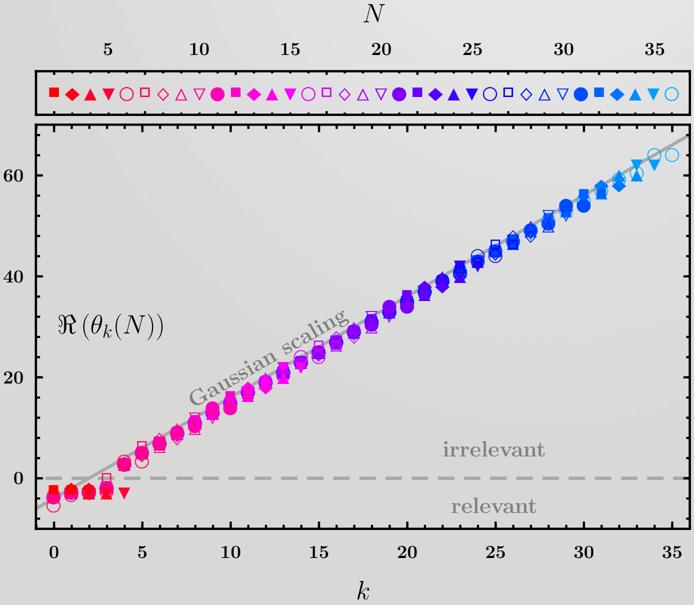
# Eigenvalues




- Eigenvalues $\theta_k(N)$ at different orders $N$
- Convergence similar to couplings
- Four relevant directions
- Gaussian scaling?

Circles ⇔ Real eigenvalues // Diamonds ⇔ Complex conjugate eigenvalues

# Gaussian Scaling




- All eigenvalues up to $N = 36$ are shown (different symbols for different orders)
- Eigenvalues close to Gaussian scaling

# Summary

## Summary & Outlook

- Make Quantum Gravity asymptotically safe by introducing UV fixed point
  - Number of open parameters given by number of negative eigenvalues
- Using FRG, stable fixed points can be found in $f(\mathrm{Ric})$ truncation
  - Eigenvalues follow Gaussian scaling with small deviations
  - 4 relevant directions
- What happens in other truncations?
- What happens if more operators are included?

Any Questions?