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Parton Branching at Amplitude Level

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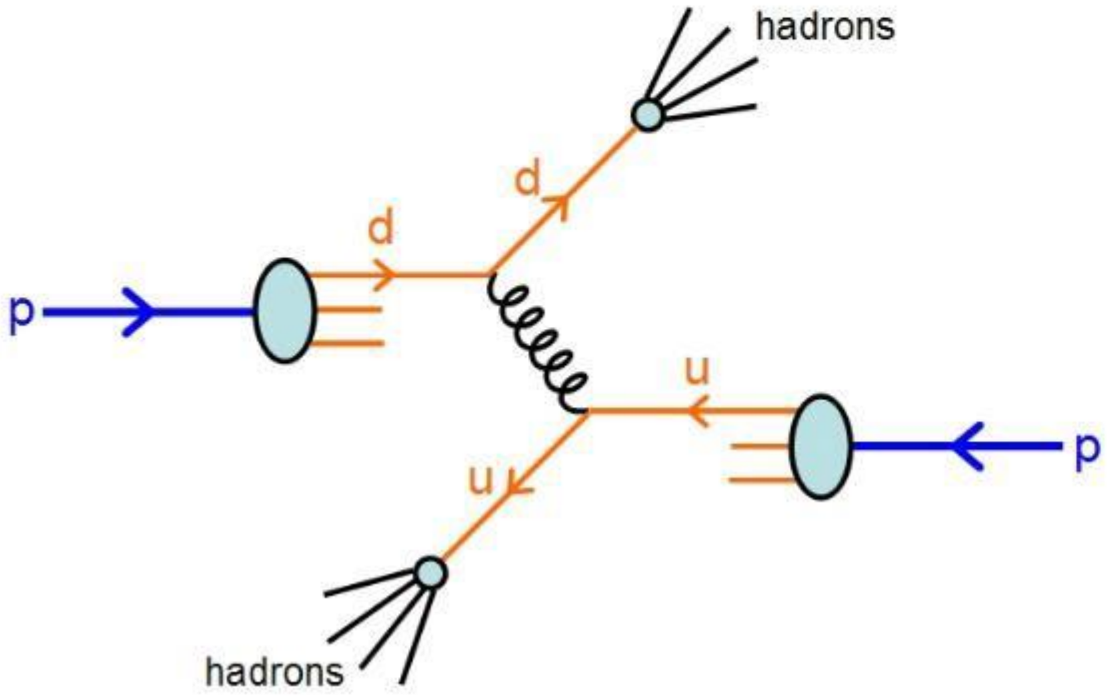
Background and motivation

What we want to compute:

$$A \propto \left\langle \text{stuff; out} \,\middle|\, P_1, P_2; \text{in} \right\rangle$$

Challenges:

1. Hadronic physics in non-perturbative.
2. Loads of divergences.
3. $\alpha_s \in [0.1, 1]$ at useful scales: radiative corrections from all orders contribute.



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What we want to compute:

$$A \propto \left\langle \text{stuff; out} \,\middle|\, P_1, P_2; \text{in} \right\rangle$$

Methods:

- Regge Theory: ${}^{s}/_{t} \gg 1$
- Other analytic booststraps.

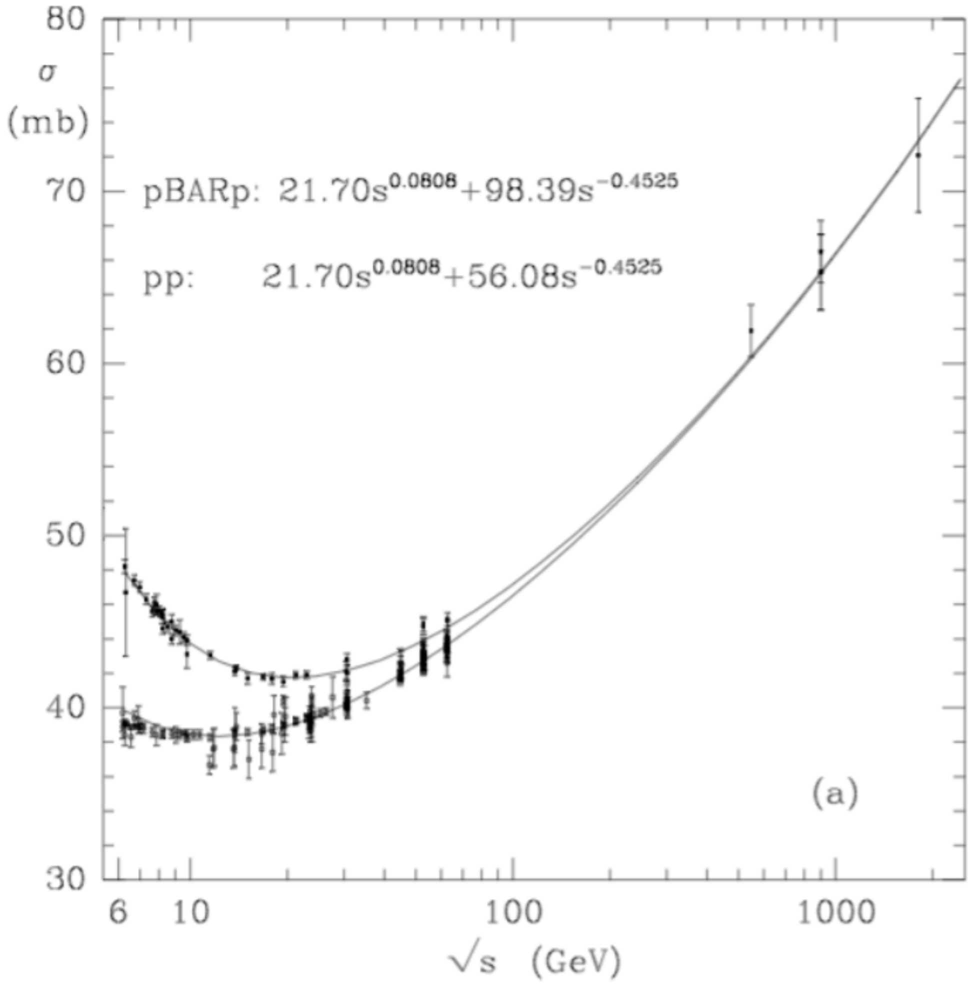

Figure: Donnachie and Landshoff (1992)

Background and motivation

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$$A \propto \left\langle \text{stuff; out} \,\middle|\, P_1, P_2; \text{in} \right\rangle$$

Methods:

- Perurbative QCD.
- Long scale – short scale factorisation

$$A \propto \left\langle \mathrm{q}_1 \ldots \mathrm{q}_\mathrm{n}; \text{out} \,\middle|\, p_1, p_2; \text{in} \right\rangle \otimes \mathcal{H}$$

$$\frac{d\sigma}{dQ^2\,dy} = \sum_{a,b} \int_0^1 d\xi_A\, d\xi_B\, f_{a/A}(\xi_A, Q^2)\, H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q^2\right) f_{b/B}(\xi_B, Q^2).$$

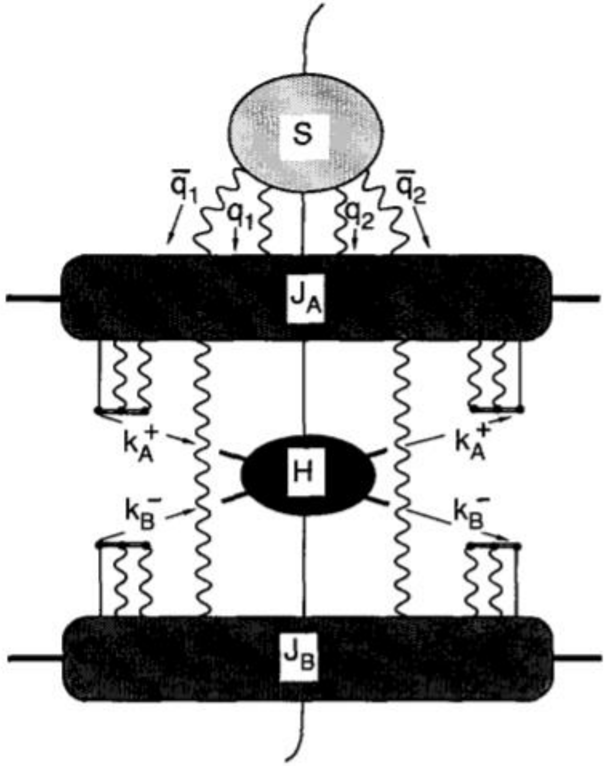


Figure: Collins, Soper and Stermn 1988

Background and motivation

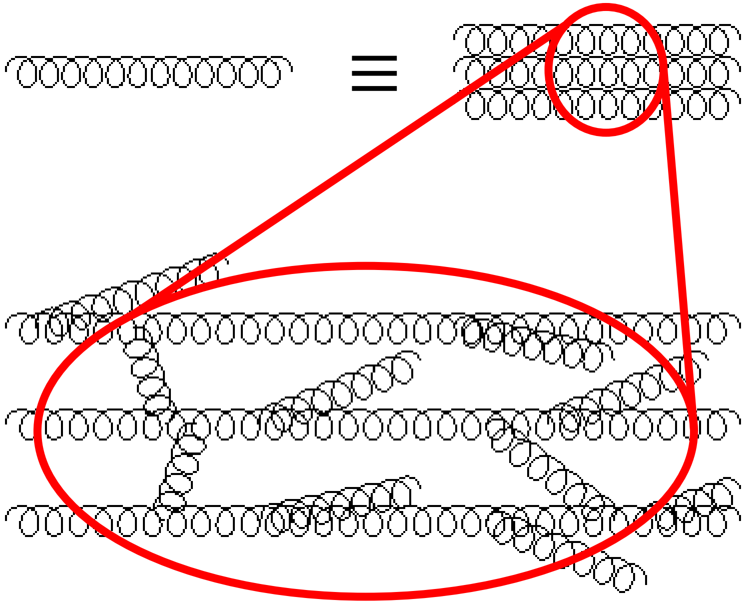
What we want to compute:

$$A \propto \cancel{\langle \text{stuff; out} | P_1, P_2; \text{in} \rangle}$$

$$\langle \mathrm{q}_1 \dots \mathrm{q}_\mathrm{n}; \text{out} | p_1, p_2; \text{in} \rangle \otimes \mathcal{H}$$

Problems:

- Infra-red divergences – degenerate final states.
- Large logarithms of large scales.



Background and motivation

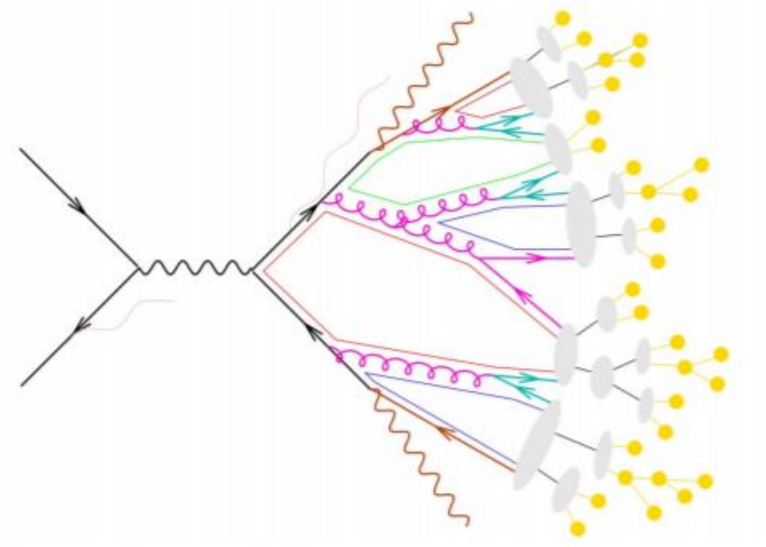
What we want to compute:

$$A \propto \cancel{\left\langle \text{stuff; out} \middle| P_1, P_2; \text{in} \right\rangle}$$

$$\left\langle \mathrm{q}_1 \ldots \mathrm{q}_\mathrm{n}; \text{out} \middle| p_1, p_2; \text{in} \right\rangle \otimes \mathcal{H}$$

Solutions

- Analytic resummations and parton showers.
- Soft-collinear effective theory.



SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
	mass-modes $(q_m,\ A_m^\mu)$	$p_m^\mu \sim Q(\lambda, \lambda, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

Background and motivation

What we want to compute:

$$A \propto \cancel{\left\langle \text{stuff; out} \middle| P_1, P_2; \text{in} \right\rangle}$$

$$\left\langle \mathrm{q}_1 \ldots \mathrm{q}_n; \text{out} \middle| p_1, p_2; \text{in} \right\rangle \otimes \mathcal{H}$$

Problems:

- SCET and traditional resummations are not all purpose. They're observable dependent.
- Parton showers aren't rigorous.

New tool box:

$$\mathcal{M}^{c_1,\ldots,c_n}_{s_1,\ldots,s_n} = \langle c_1, \ldots, c_n; s_1, \ldots, s_n | \mathcal{M} \rangle$$

$$|\mathcal{M}_{n+1}\rangle = \mathbf{D}_{n+1} |\mathcal{M}_n\rangle$$

$$\mathbf{D}_n \sim \mathbf{T}_i \otimes \mathbf{S}^{s_n} = T_i^{c_n} |c_n; s_n\rangle$$

$$\sum_{i \in n} \mathbf{T}_i \otimes \boldsymbol{O} |\mathcal{M}_n\rangle = 0$$

Basic construction

$$\langle \mathcal{M} | \mathbf{D}_1^\dagger \mathbf{D}_1 | \mathcal{M} \rangle = \langle \mathcal{M} | \mathbf{SoftCol} | \mathcal{M} \rangle + \langle \mathcal{M} | \mathbf{WideSoft} | \mathcal{M} \rangle + \langle \mathcal{M} | \mathbf{HardCol} | \mathcal{M} \rangle + \langle \mathcal{M} | \mathbf{Remainder} | \mathcal{M} \rangle$$

$\langle \mathcal{M} | \mathbf{Eikonal} | \mathcal{M} \rangle = \langle \mathcal{M} | \mathbf{SoftCol} | \mathcal{M} \rangle + \langle \mathcal{M} | \mathbf{WideSoft} | \mathcal{M} \rangle$,

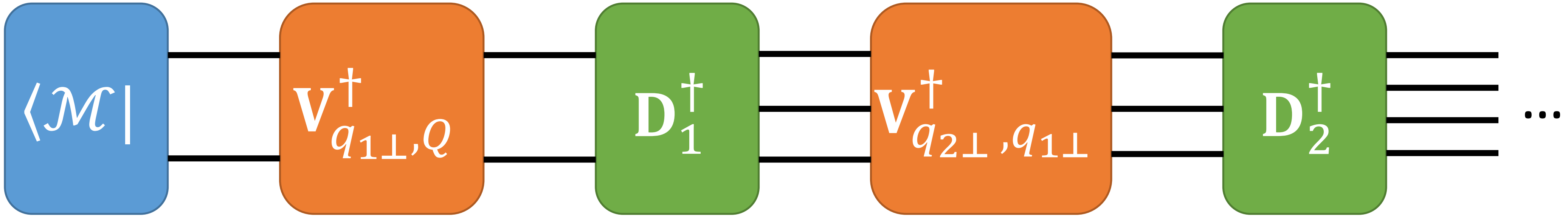
$\langle \mathcal{M} | \mathbf{SplittingFunctions} | \mathcal{M} \rangle = \langle \mathcal{M} | \mathbf{SoftCol} | \mathcal{M} \rangle + \langle \mathcal{M} | \mathbf{HardCol} | \mathcal{M} \rangle$,

we ignore $\langle \mathcal{M} | \mathbf{Remainder} | \mathcal{M} \rangle$ as not it's logarithmically enhanced.

So $\langle \mathcal{M} | \mathbf{D}_1^\dagger \mathbf{D}_1 | \mathcal{M} \rangle = \langle \mathcal{M} | \mathbf{Eikonal} | \mathcal{M} \rangle + \langle \mathcal{M} | \mathbf{SplittingFunctions} | \mathcal{M} \rangle - \langle \mathcal{M} | \mathbf{SoftCol} | \mathcal{M} \rangle$

i.e. we can't just interleave soft and collinear emissions, else we double count the soft-collinear region.

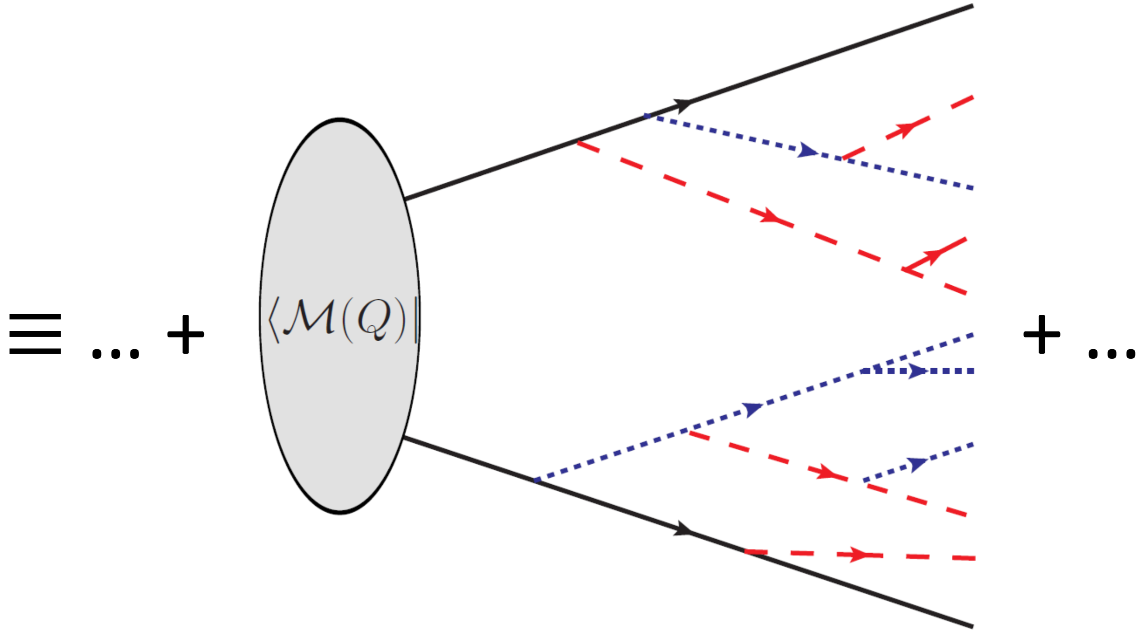
The algorithm: arXiv:1905.08686



$\langle\mathcal{M}|$ is a conjugate amplitude.

$\mathbf{V}^\dagger_{b,a}$ is an amplitude level Sudakov factor (i.e. it has the complete colour structure of exponentiated one loop soft and collinear exchanges).

$\mathbf{D}^\dagger_i$ is an amplitude level operator that emits a single parton, i.



The algorithm: arXiv:1905.08686

$$\mathrm{Tr}\left[\mathbf{V}_{\mu,q_{1\perp}} \, \mathbf{D}_1 \, \mathbf{V}_{q_{1\perp},Q} \, |\mathcal{M}\rangle\langle\mathcal{M}| \, \mathbf{V}^\dagger_{q_{1\perp},Q} \, \mathbf{D}_1^\dagger \, \mathbf{V}^\dagger_{\mu,q_{1\perp}} \right] = \mathrm{d}\sigma_1$$

$$\begin{aligned} \Sigma(\mu) &= \int \sum_n \mathrm{d}\sigma_n \, u_n(q_1, \ldots, q_n), \\ &= \int \sum_n \left(\prod_{i=1}^{n} \mathrm{d}\Pi_i \right) \mathrm{Tr}\, \mathbf{A}_n(\mu; \{p\}_n) \, u_n(q_1, \ldots, q_n), \end{aligned}$$

The algorithm (extra details)

$$d\sigma_0 = \mathrm{Tr}\left(\mathbf{V}_{\mu,Q}\mathbf{H}(Q;\{p\})\mathbf{V}^\dagger_{\mu,Q}\right) = \mathrm{Tr}\,\mathbf{A}_0(\mu;\{p\}),$$

$$d\sigma_1 = \int \prod_{i=1}^{n_H+1} d^4p_i\,\mathrm{Tr}\left(\mathbf{V}_{\mu,q_{1\perp}}\mathbf{D}_1\mathbf{V}_{q_{1\perp},Q}\mathbf{H}(Q;\{p\})\mathbf{V}^\dagger_{q_{1\perp},Q}\mathbf{D}_1^\dagger\mathbf{V}^\dagger_{\mu,q_{1\perp}}\right) d\Pi_1$$

$$= \mathrm{Tr}\,\mathbf{A}_1(\mu;\{\tilde{p}\}\cup q_1)\,d\Pi_1,$$

$$d\sigma_n = \mathrm{Tr}\,\mathbf{A}_n(\mu;\{p\}_n)\prod_{i=1}^{n} d\Pi_i,$$

$$\ldots\mathbf{D}_i\mathcal{O}\mathbf{D}_i^\dagger\ldots = \ldots\mathbf{S}_i\mathcal{O}\mathbf{S}_i^\dagger\ldots + \ldots\mathbf{C}_i\mathcal{O}\mathbf{C}_i^\dagger\ldots.$$

$$\mathbf{S}_i = \sum_j \left(\frac{q_{i\perp}^{(j\vec{m})}}{2\tilde{p}_j\cdot q_i}\mathbb{T}_j^g \otimes \left(\tilde{p}_j\cdot\epsilon_+^*(q_i)\mathbb{S}^{1_i} + \tilde{p}_j\cdot\epsilon_-^*(q_i)\mathbb{S}^{-1_i}\right)\right)\mathfrak{R}_{ij}^{\mathrm{soft}}(\{p\},\{\tilde{p}\},q_i),$$

$$\mathbf{C}_i = \sum_j \frac{q_{i\perp}^{(j\vec{n})}}{2\sqrt{z_i}}\Delta_{ij}\,\overline{\mathbf{P}}_{ij}\,\mathfrak{R}_{ij}^{\mathrm{coll}}(\{p\},\{\tilde{p}\},q_i),$$

where

$$\mathbf{A}_n(q_\perp;\{\tilde{p}\}_{n-1}\cup q_n) = \int\prod_{i=1}^{n_H+n} d^4p_i\mathbf{V}_{q_\perp,q_{n\perp}}\mathbf{D}_n\mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1})\mathbf{D}_n^\dagger\mathbf{V}^\dagger_{q_\perp,q_{n\perp}}\Theta(q_\perp\le q_{n\perp}). \tag{2.2}$$

The algorithm (extra details)

$$\mathbf{P}_{ij} = \delta_{s_j,\frac{1}{2}}\delta_j^{\text{final}}\Bigg(\sqrt{\frac{\mathcal{P}_{qq}}{2\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{\langle q_i\tilde{p}_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{2\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{[\tilde{p}_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i})$$

$$+\sqrt{\frac{\mathcal{P}_{gq}}{2\mathcal{C}_{\text{F}}(2-2z_i+z_i^2)}}\frac{1}{\langle\tilde{p}_j q_i\rangle}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) + \sqrt{\frac{(1-z_i)^2\mathcal{P}_{gq}}{2\mathcal{C}_{\text{F}}(2-2z_i+z_i^2)}}\frac{1}{[q_i\tilde{p}_j]}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i})\Bigg)$$

$$+\delta_{s_j,\frac{1}{2}}\delta_j^{\text{initial}}\sqrt{\frac{1}{z_i}}\Bigg(\sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i})$$

$$+\sqrt{\frac{(1-z_i)^2\mathcal{P}_{qg}}{n_f\mathcal{C}_F(1-2z_i(1-z_i))}}\frac{1}{[p_j q_i]}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i})$$

$$+\sqrt{\frac{z_i^2\mathcal{P}_{qg}}{n_f\mathcal{C}_F(1-2z_i(1-z_i))}}\frac{1}{\langle q_i p_j\rangle}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i})\Bigg)$$

$$+\cdots$$

These are the terms that generate emissions from a positive helicity electron.

The algorithm (extra details)

$$
\begin{aligned}
\mathbf{P}_{ij} = {} & \delta_{s_j,\frac{1}{2}}\delta_j^{\text{final}}\Bigg(\sqrt{\frac{\mathcal{P}_{qq}}{2\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{\langle q_i\tilde{p}_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{2\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{[\tilde{p}_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{\mathcal{P}_{gq}}{2\mathcal{C}_{\text{F}}(2-2z_i+z_i^2)}}\frac{1}{\langle\tilde{p}_j q_i\rangle}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) + \sqrt{\frac{(1-z_i)^2\mathcal{P}_{gq}}{2\mathcal{C}_{\text{F}}(2-2z_i+z_i^2)}}\frac{1}{[q_i\tilde{p}_j]}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i})\Bigg) \\
& + \delta_{s_j,-\frac{1}{2}}\delta_j^{\text{final}}\Bigg(\sqrt{\frac{\mathcal{P}_{qq}}{2\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{[\tilde{p}_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{2\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{\langle q_i\tilde{p}_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{\mathcal{P}_{gq}}{2\mathcal{C}_{\text{F}}(2-2z_i+z_i^2)}}\frac{1}{[q_i\tilde{p}_j]}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) + \sqrt{\frac{(1-z_i)^2\mathcal{P}_{gq}}{2\mathcal{C}_{\text{F}}(2-2z_i+z_i^2)}}\frac{1}{\langle\tilde{p}_j q_i\rangle}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i})\Bigg) \\
& + \delta_{s_j,1}\delta_j^{\text{final}}\Bigg(\sqrt{\frac{(1-z_i)^2\mathcal{P}_{qg}}{2T_{\text{R}}(1-2z_i(1-z_i))}}\frac{1}{[\tilde{p}_j q_i]}(\mathbb{W}^{ij}-\mathbb{1})(\mathbb{T}_j^g\otimes\mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{+\frac{1}{2}_i}) \\
& + \sqrt{\frac{z_i^2\mathcal{P}_{qg}}{2T_{\text{R}}(1-2z_i(1-z_i))}}\frac{1}{[\tilde{p}_j q_i]}(\mathbb{W}^{ij}-\mathbb{1})(\mathbb{T}_j^g\otimes\mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) \\
& + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{\langle q_i\tilde{p}_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^4\mathcal{P}_{gg}}{2\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{[q_i\tilde{p}_j]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{2\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{[\tilde{p}_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{P}_j^1\mathbb{S}^{+1_i})\Bigg) \\
& + \delta_{s_j,-1}\delta_j^{\text{final}}\Bigg(\sqrt{\frac{(1-z_i)^2\mathcal{P}_{qg}}{2T_{\text{R}}(1-2z_i(1-z_i))}}\frac{1}{\langle q_i\tilde{p}_j\rangle}(\mathbb{W}^{ij}-\mathbb{1})(\mathbb{T}_j^g\otimes\mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) \\
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& + \delta_{s_j,\frac{1}{2}}\delta_j^{\text{initial}}\sqrt{\frac{1}{z_i}}\Bigg(\sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) + \sqrt{\frac{z_i^2\mathcal{P}_{qq}}{\mathcal{C}_{\text{F}}(1+z_i^2)}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{(1-z_i)^2\mathcal{P}_{qg}}{n_f\mathcal{C}_F(1-2z_i(1-z_i))}}\frac{1}{[p_j q_i]}\mathbb{W}^{ij}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) \\
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& + \delta_{s_j,1}\delta_j^{\text{initial}}\sqrt{\frac{1}{z_i}}\Bigg(\sqrt{\frac{2n_f\mathcal{P}_{gq}}{T_{\text{R}}(2-2z_i+z_i^2)}}\frac{1}{\langle p_j q_i\rangle}(\mathbb{T}_j^g\otimes\mathbb{P}_j^2\mathbb{S}^{+\frac{1}{2}_i}) \\
& + \sqrt{\frac{2n_f(1-z_i)^2\mathcal{P}_{gq}}{T_{\text{R}}(2-2z_i+z_i^2)}}\frac{1}{[q_i p_j]}(\mathbb{T}_j^g\otimes\mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) + \sqrt{\frac{\mathcal{P}_{gg}}{\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^4\mathcal{P}_{gg}}{\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{[q_i p_j]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{\langle q_i p_j\rangle}(\mathbb{T}_j^g\otimes\mathbb{P}_j^1\mathbb{S}^{-1_i})\Bigg) \\
& + \delta_{s_j,-1}\delta_j^{\text{initial}}\sqrt{\frac{1}{z_i}}\Bigg(\sqrt{\frac{2n_f\mathcal{P}_{gq}}{T_{\text{R}}(2-2z_i+z_i^2)}}\frac{1}{[q_i p_j]}(\mathbb{T}_j^g\otimes\mathbb{P}_j^2\mathbb{S}^{-\frac{1}{2}_i}) \\
& + \sqrt{\frac{2n_f(1-z_i)^2\mathcal{P}_{gq}}{T_{\text{R}}(2-2z_i+z_i^2)}}\frac{1}{\langle p_j q_i\rangle}(\mathbb{T}_j^g\otimes\mathbb{P}_j^1\mathbb{P}_j^2\mathbb{S}^{+\frac{1}{2}_i}) + \sqrt{\frac{\mathcal{P}_{gg}}{\mathcal{C}_{\text{A}}(1-z_i+z_i^2)^2}}\frac{1}{[p_j q_i]}(\mathbb{T}_j^g\otimes\mathbb{S}^{-1_i}) \\
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\end{aligned}
\tag{A.1}
$$

What we have done so far

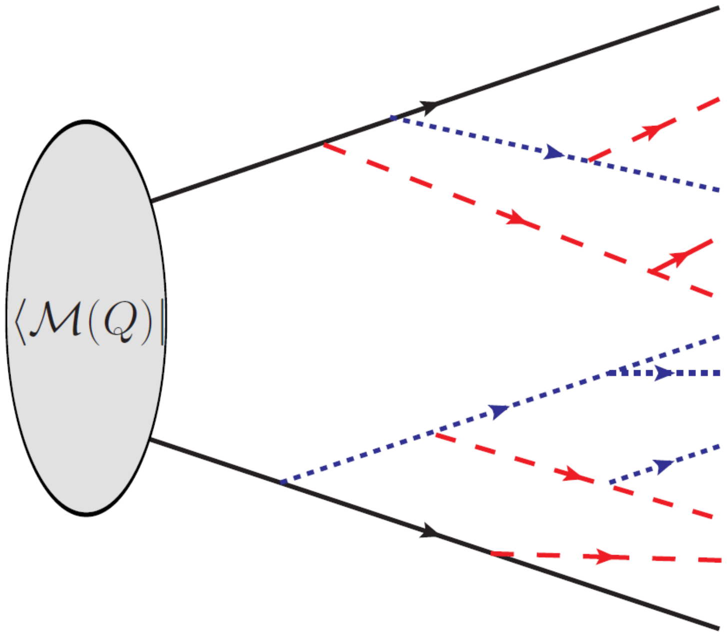
In [arXiv:1905.08686] you will find:

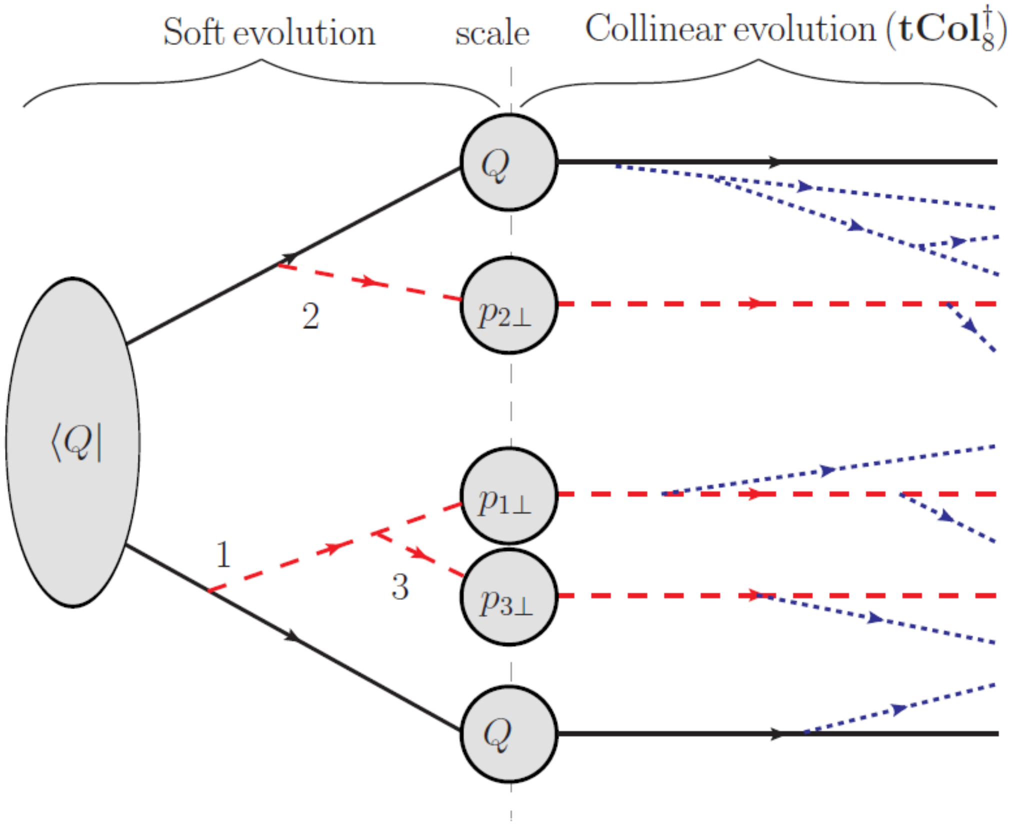
- Alternative formations of the algorithm.
- Examples of analytic resummations calculated using the algorithm. DGLAP, BMS, thrust, …
- Discussions on the spin correlations and pole/colour structures in the algorithm.
- Full definitions of all operators involved, which are often unwieldy.
- Derivation of QCD factorisation theorems.
- Links to SCET can be found in previous papers.

What is in the works?

- Re-derivation of and constraints on current angular ordered and dipole evolution.
- Analysis of non-Global structures.
- Derivation of super-leading contributions.
- Formal derivation of the algorithm using amplitudes and RG flow. Starting with YM (i.e. turn off quarks), then maybe N=4 SYM as half way house then full QCD derivation.

Factorisation



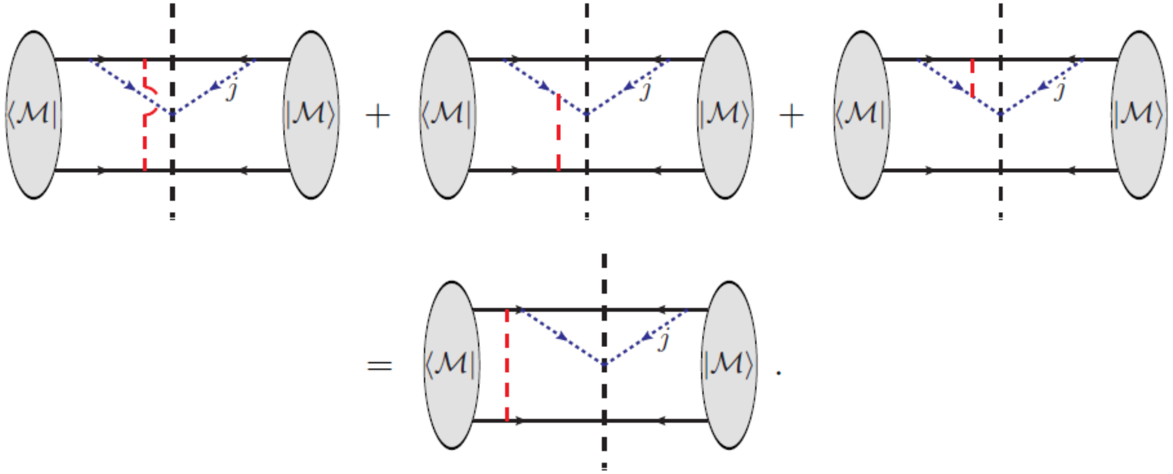


Factorisation

$$\left[\mathbf{D}_i - \overline{\mathbf{C}}_i, \overline{\mathbf{C}}_j\right] \simeq 0, \qquad \left[\mathbf{V}_{a,b}(\mathbf{V}^{\mathrm{col}}_{a,b})^{-1}, \overline{\mathbf{C}}_j\right] \simeq 0,$$

$$\left[\mathbf{V}_{a,b}(\mathbf{V}^{\mathrm{col}}_{a,b})^{-1}, \mathbf{V}^{\mathrm{col}}_{c,d}\right] \simeq 0, \qquad \left[\mathbf{D}_i - \overline{\mathbf{C}}_i, \mathbf{V}^{\mathrm{col}}_{a,b}\right] \simeq 0.$$

The equality only holds when considering only the real part of these diagrams. The soft loop also generates imaginary parts; Coulomb/Glauber exchanges.



Conclusions

- We've explored the theoretical basis for an algorithm for parton evolution at amplitude level.
- This work will be used to inform future work on the CVolver code.
- Independent of CVolver, development of this algorithm has opened a number of future avenues for research: i.e.
  - Rederiving current algorithms for parton showers to try and evaluate their accuracy.
  - Re-formulating our algorithm as evolution equations might allow us to make a direct link to SCET.