Stefano Profumo

Santa Cruz Institute for Particle Physics
University of California, Santa Cruz

Cosmology and Dark Matter

Lecture 1

IX NExT PhD Workshop
The Cosener's House, Abingdon, UK
July 8-11th, 2019

✓ PhD **Theoretical Particle Physics** (2004)

International School for Advanced Studies (SISSA-ISAS), Trieste, Italy

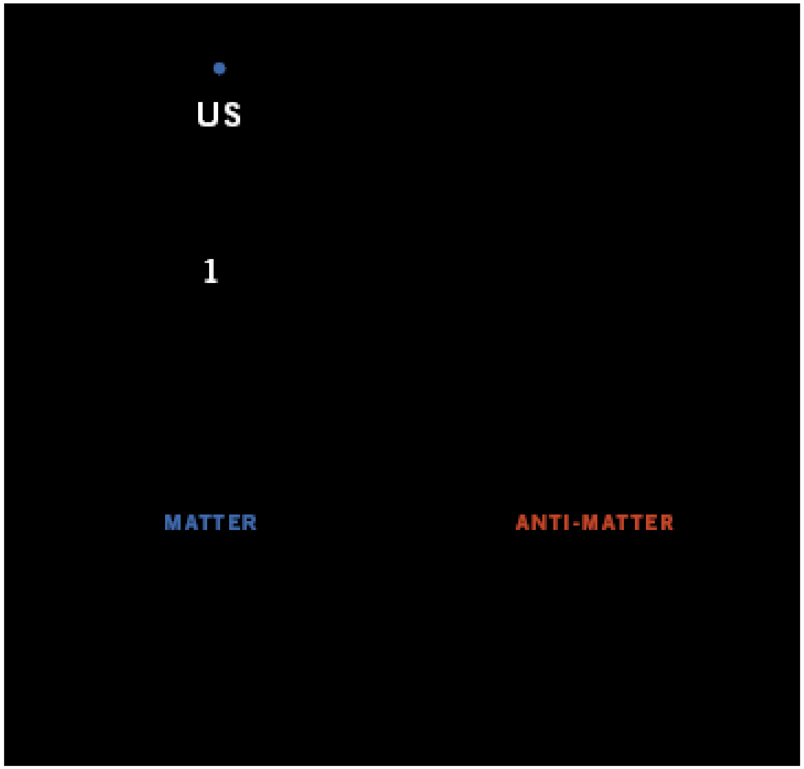
✓ Postdoc, FSU and California Institute of Technology (2005-2007)

Theoretical Astrophysics and Particle Physics

✓ Joined **UCSC Physics** Faculty (Assistant Professor, 2007-2011,
Associate Professor, July 2011-2015
Full Professor, July 2015-)

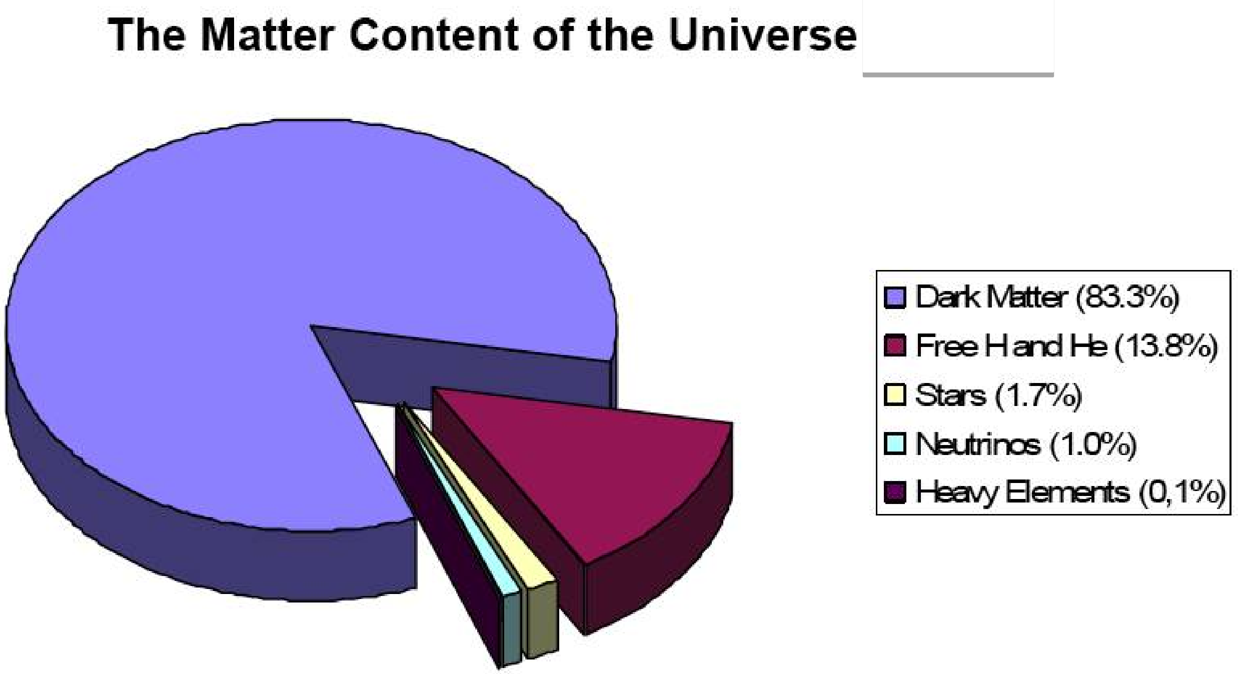
✓ Director of UCSC Physics **Graduate Studies** (2012-)

✓ **SCIPP Deputy Director** for **Theory** (July 2011-)



1. What is the origin of the tiny excess of matter over anti-matter?

2. What is the fundamental particle physics nature of Dark Matter?



Please come **introduce yourselves**!

If you are ever on the **US West Coast** please let me know!

Never **underestimate** the importance of **networking** in science!

Feel free to **interrupt** me during the lectures!

(Rare occurrence, but I do occasionally make mistakes…)

"R2D2 is known for having been wrong in the past"

Lectures **plan**:

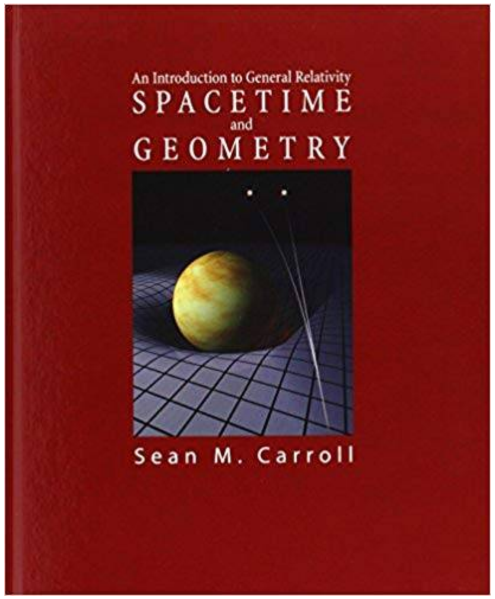
- the **metric** of a homogeneous and isotropic **universe**
- **stuff** in the universe and how to measure it
- the relation between **density** and **destiny** of the universe

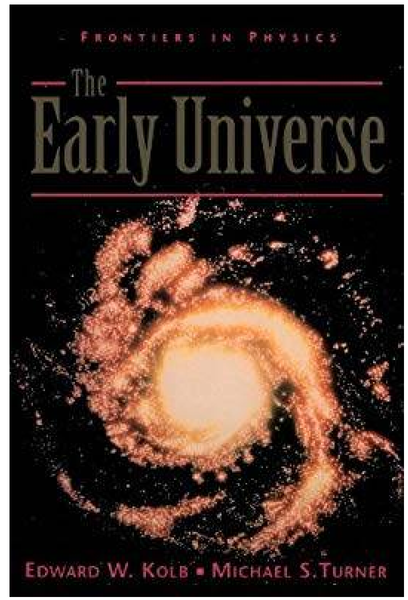
- **thermodynamics** of the early universe
- thermal **history** of the universe

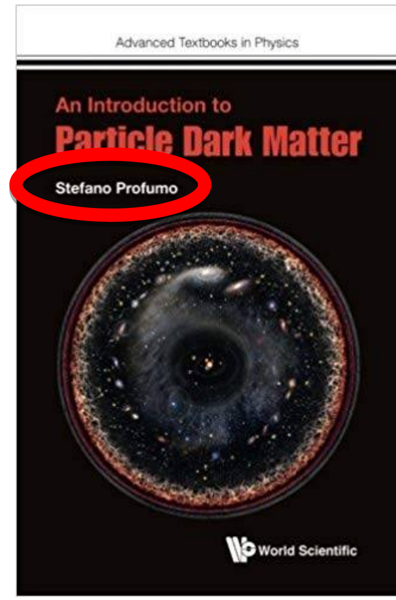
- hot, warm, and cold **relics**
- dark matter **kinetic** decoupling and **structure** formation

- (**WIMP**) dark matter **detection**
- **axions**, **sterile neutrinos**, and searches

References:







21. BIG-BANG COSMOLOGY

Revised September 2011 by K.A. Olive (University of Minnesota) and J.A. Peacock (University of Edinburgh).

21.1. Introduction to Standard Big-Bang Model

26. Dark Matter

Revised August 2018 by L. Baudis (Zürich U.) and S. Profumo (University of California, Santa Cruz).

26.1 The case for dark matter

Contemporary cosmology assumes
"**Copernican Principle**":
the universe is (pretty much) the **same** everywhere

(are we in a totally **random place** in the universe?
...or are we the **center** of the universe?)

The Copernican Principle is clearly **false** locally...
but quite true on **large scales**:
galaxy surveys, X-ray, γ-ray backgrounds, CMB background

Mathematically: Copernican Principle is related to **properties** of a **manifold**: **isotropy** and **homogeneity**

Isotropy: take a **specific point**: from there, space looks the **same** in **every direction**

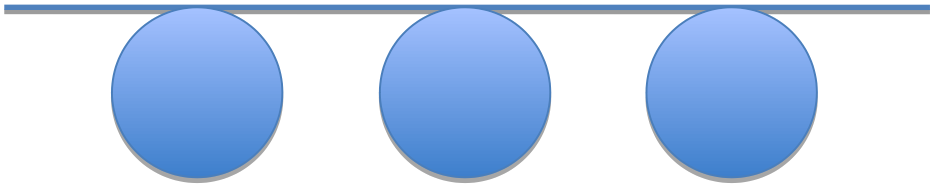
(**CMB** really tells us about isotropy!)

Homogeneity: the **metric** is the **same** throughout space (given any two points p, q on a manifold M, there is an isometry which takes p into q)

Notice that there is **no necessary relationship** between isotropy and homogeneity!
(in other words ***one does NOT imply the other*** *and vice versa)*

Example 3D manifold that is **homogeneous everywhere** but **nowhere isotropic**:

RxS^2



Example of a 2D manifold **isotropic** around a point but **not homogeneous**: cone (around its vertex)

…however: if a manifold is **isotropic everywhere** then it is **homogeneous**!

Also: if it is **homogeneous**, and **isotropic around at least one point**, then **isotropic everywhere**

Modern **cosmology** assumes isotropy and homogeneity ***in space***.

However, the observation that distance galaxies are receding indicates that the **universe is changing in time**!

In **differential geometry** terms: the universe can be **foliated** into **homogeneous** and **isotropic space-like slices**

$$\boldsymbol{M = R \times \Sigma}$$

if Σ is a 3D homogeneous and isotropic manifold, differential geometry demands it must be **maximally symmetric,**
and provides us with the **general form** for the **metric**

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}(u)\mathrm{d}u^i\mathrm{d}u^j$$

time-like coordinate

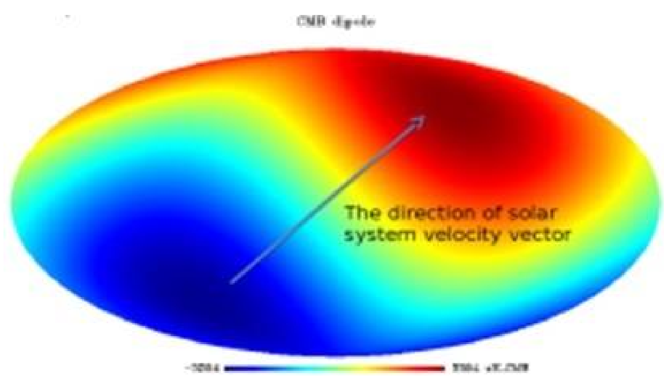
scale factor (only dep. on t)

maximally-symmetric 3D metric on Σ

coordinates on Σ

This choice of metric, such that $dtdu^i$ are absent and there's a universal $a(t)$: "**comoving**" coordinates

Only a **comoving observer** (observer at fixed *u* coordinates) will see the universe as **isotropic**
(**we are not** quite **comoving** – in fact we see a CMB dipole anisotropy…)



In differential geometry, requiring the **metric** to satisfy the general condition for **maximal symmetry**…

$$R_{\rho\sigma\mu\nu} = a^{-2}(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\nu}g_{\sigma\mu})$$

…**forces** the **metric** to have the form

$$ds^2 = -dt^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right]$$

Notice that the metric is left **unchanged** by simultaneously

$$\left\{\begin{array}{lcl} k & \rightarrow & \frac{k}{|k|} \\ r & \rightarrow & \sqrt{|k|}\,r \\ a & \rightarrow & \frac{a}{\sqrt{|k|}} \end{array}\right.$$

…and thus it **only** depends on ***k/|k|***,
leaving three distinct possibilities: ***k = -1, 0, 1***

- **k=0** "**flat**". For good reasons…!

$$d\sigma^2 = dr^2 + r^2 d\Omega^2 = dx^2 + dy^2 + dz^2$$

metric on **space-like** slice

- **k=+1** "**closed**" universe:

$$r = \sin\chi$$

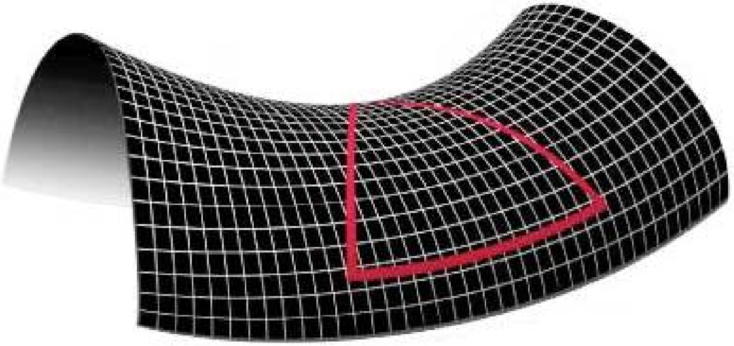
$$d\sigma^2 = d\chi^2 + \sin^2\chi\, d\Omega^2$$

spacelike slices are spheres!

- **k=-1** "**open**" universe

$$d\sigma^2 = d\psi^2 + \sinh^2\psi\, d\Omega^2$$

$$r = \sinh\psi$$



OK, not let's put "**stuff**" in the universe and see what ***a(t)*** does via the super powers of **General Relativity**!!

Zero-th order assumption: "stuff" is **isotropic** (everywhere) in its rest frame ("**perfect fluid**")

The **energy-momentum tensor** of a perfect fluid reads

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ij}p & \\ 0 & & & \end{pmatrix}$$

...we will need the **trace** of this in Einstein's equations ("mostly +" metric)

$$T = T^{\mu}{}_{\mu} = -\rho + 3p$$

in GR (and in field theory) Noether's theorem tells us that **conservation of 4-momentum** is equivalent to the vanishing of a (covariant) **derivative** of the **energy-momentum tensor**

$$
\begin{aligned}
0 &= \nabla_\mu T^\mu{}_0 = \partial_\mu T^\mu{}_0 + \Gamma^\mu_{\mu 0} T^0{}_0 - \Gamma^\lambda_{\mu 0} T^\mu{}_\lambda \\
&= -\partial_0 \rho - 3\frac{\dot{a}}{a}(\rho + p) \, .
\end{aligned}
$$

little to do further unless we know something about the relation between **density** and **pressure** ("equation of state")

Luckily, for relevant fluids in the early (and late) universe, ***p=wρ***

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \qquad\qquad \rho \propto a^{-3(1+w)}$$

$p = w\rho$

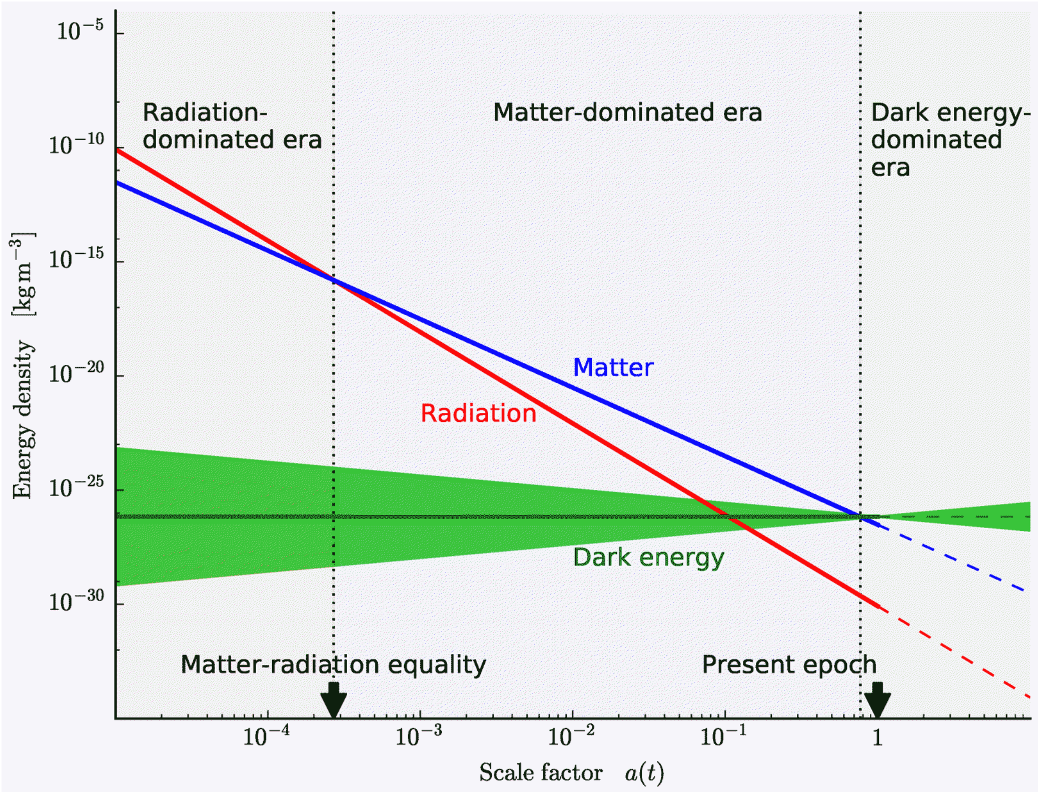
$$\rho \propto a^{-3(1+w)}$$

Matter: defined as a p=0 fluid, $\rho \sim a^{-3}$

Radiation: w=1/3 (see later why) , $\rho \sim a^{-4}$

Cosmological **constant**: w=-1, $\rho \sim a^{0}$

the scaling with ***a(t)***, and **measurements** today, imply **which fluid** dominated the energy density, and **when**



See how this is done!
Let's **solve** for *a(t)*!
Use **Einstein's equations**

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

$$\mu\nu = 00 \qquad -3\frac{\ddot{a}}{a} = 4\pi G(\rho + 3p)$$

$$\mu\nu = ij \qquad \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G(\rho - p)$$

$$\left.\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \end{aligned}\right\} \text{Friedmann's Equations}$$

Hubble parameter
(or Hubble rate)

$$H = \frac{\dot{a}}{a}$$

…to **solve** Friedmann's equations, we need boundary conditions!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

In addition to **Hubble rate today** (H_0), two other key parameters:

deceleration parameter: $$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

density parameter: $$\Omega = \frac{8\pi G}{3H^2} = \frac{\rho}{\rho_{\text{crit}}} \qquad \rho_{\text{crit}} = \frac{3H^2}{8\pi G}$$

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G} \qquad \Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_{\text{crit}}} \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

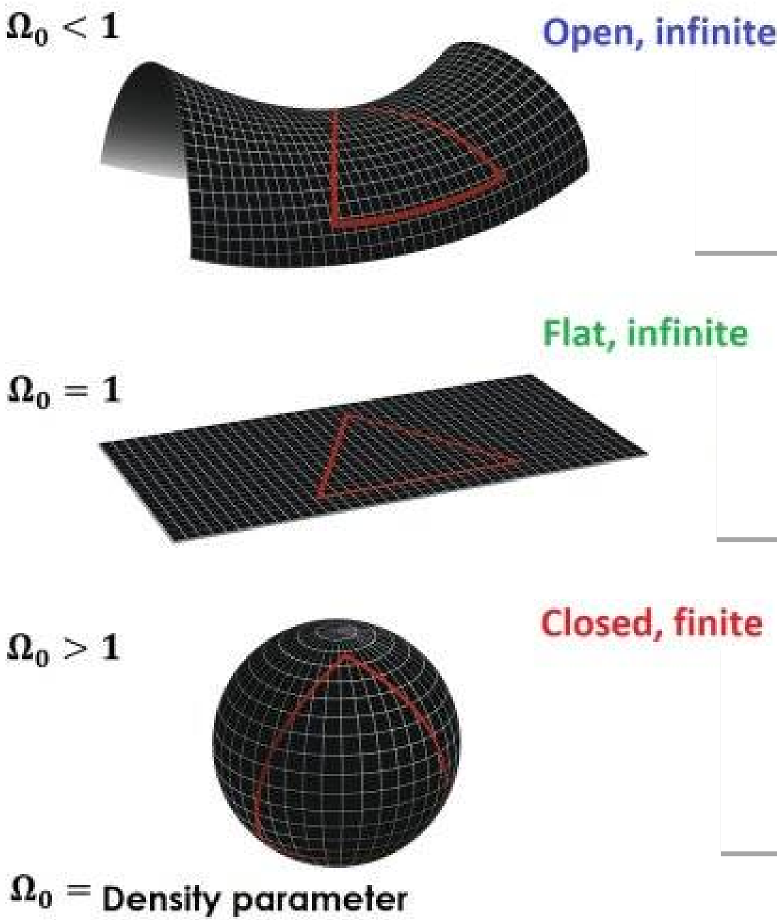
divide by H^2, use definition of Ω...

$$\Omega - 1 = \frac{k}{H^2 a^2}$$

thus the **sign** of ***k*** is determined by whether $\boldsymbol{\Omega}$ is bigger or smaller than 1 (i.e. ρ bigger or smaller than ρ_{crit})

$$\begin{array}{ccccccc}
\rho < \rho_{\text{crit}} & \leftrightarrow & \Omega < 1 & \leftrightarrow & k = -1 & \leftrightarrow & \text{open} \\
\rho = \rho_{\text{crit}} & \leftrightarrow & \Omega = 1 & \leftrightarrow & k = 0 & \leftrightarrow & \text{flat} \\
\rho > \rho_{\text{crit}} & \leftrightarrow & \Omega > 1 & \leftrightarrow & k = +1 & \leftrightarrow & \text{closed}
\end{array}$$

$$
\begin{array}{ccccccc}
\rho < \rho_{\rm crit} & \leftrightarrow & \Omega < 1 & \leftrightarrow & k = -1 & \leftrightarrow & \text{open} \\
\rho = \rho_{\rm crit} & \leftrightarrow & \Omega = 1 & \leftrightarrow & k = 0 & \leftrightarrow & \text{flat} \\
\rho > \rho_{\rm crit} & \leftrightarrow & \Omega > 1 & \leftrightarrow & k = +1 & \leftrightarrow & \text{closed}
\end{array}
$$



let's understand the right-hand **bubbles**!

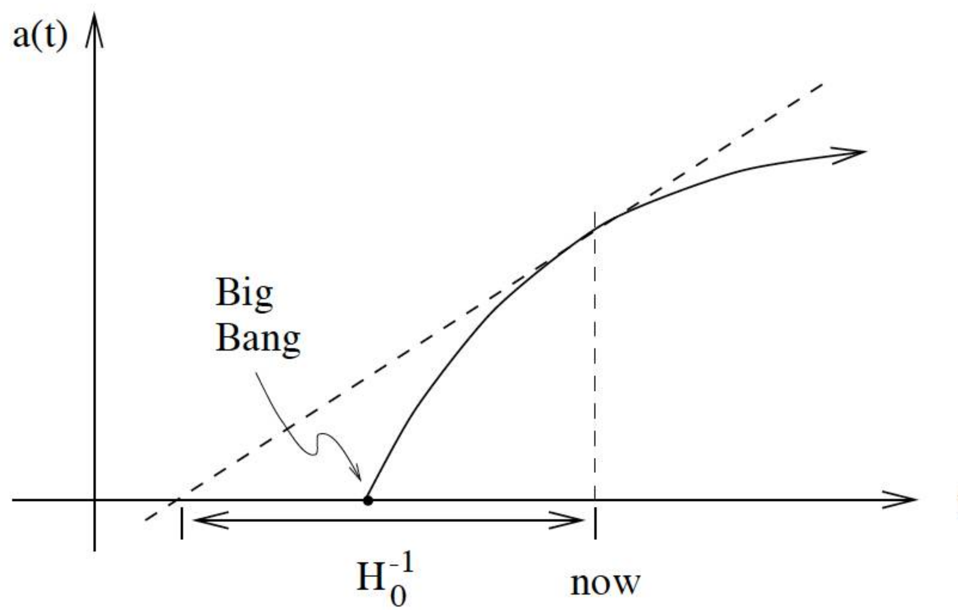
The **past** and **future** of the universe are **predetermined**!
Assume that the universe is filled with fluids with **positive** energy density, and **non-negative** pressure (thus no cosmological constant!)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

tells us that the universe is always **decelerating**! (expected)

(also tells us that to accelerate it, $p<-\rho/3$, or $w<-1/3$)

at some point in the past, thus, *a(t)* ✉ *0* (**Big Bang**)



the **future** evolution will depend on the values of *k*: (1) open or flat

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \text{for} \quad k \leq 0$$

R.H.S.>0 implies that ***da/dt > 0* at all times**

(we know from Hubble that *da/dt>0* now and it **can't change sign**!):

...but then open and flat universes

(with positive energy densities) **expand forever**!

for **closed** universes, k=+1, we have to work a bit harder…

$$\frac{d}{dt}(\rho a^3) = a^3\left(\dot\rho + 3\rho\frac{\dot a}{a}\right) = -3pa^2\dot a \ .$$

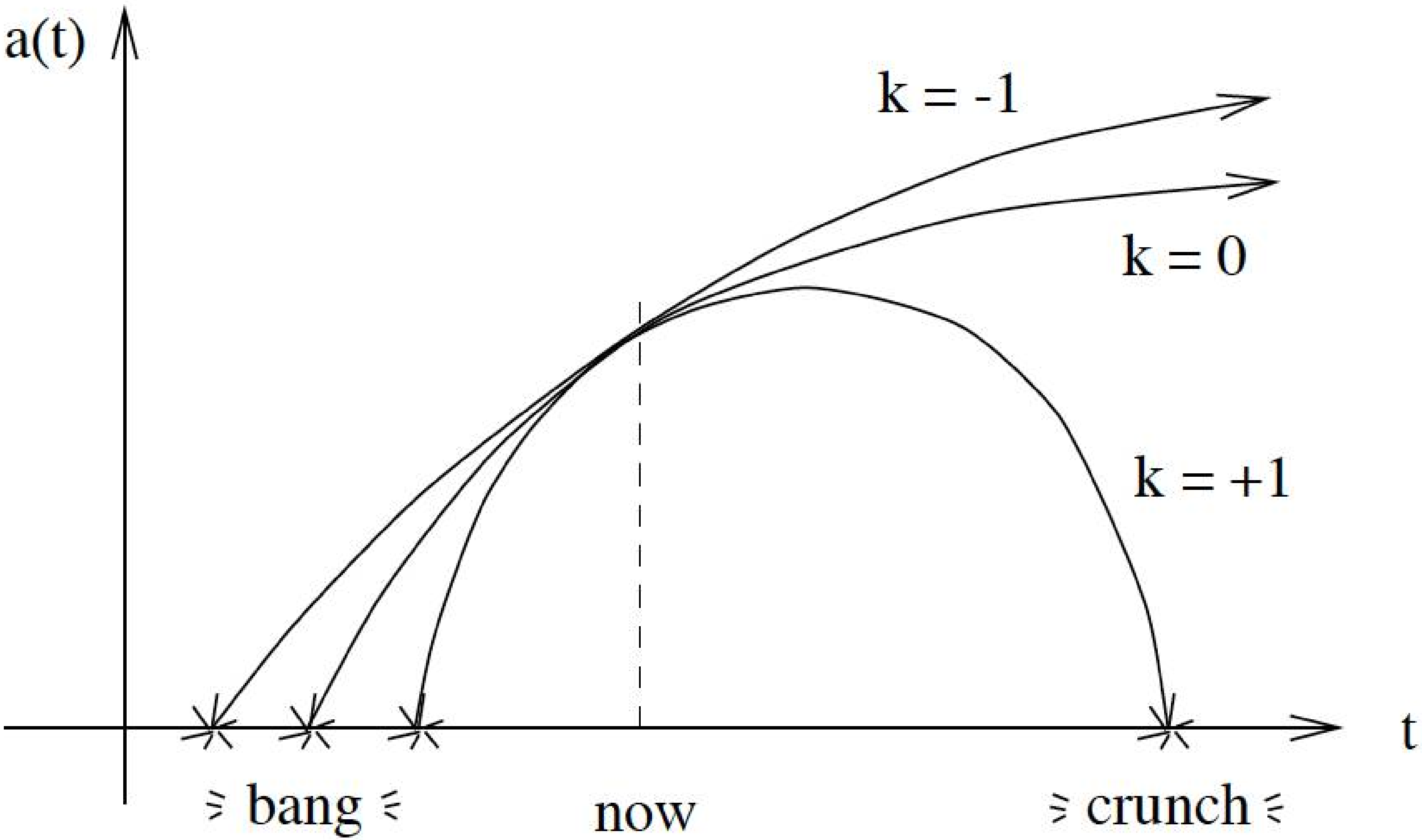
$$\frac{d}{dt}(\rho a^3) \leq 0 \ .$$

thus for *a* ✉ infinity, ρ *a*^{2} must go to **zero** for ρ *a*^{3} to **decrease**…
(also implies that if k=0, da/dt ✉ *0)*

$$\dot a^2 = \frac{8\pi G}{3}\rho a^2 - 1$$

…but the RHS **cannot be negative**!

…thus there is a **maximal** value for
the expansion factor, and then **recollapse**



a(t)
k = -1
k = 0
k = +1
t
bang
now
crunch

how do we **measure** cosmological **parameters**?

define a **luminosity** distance as $$d_L^2 = \frac{L}{4\pi F}$$

where *F* is the measured **flux** and *L* the (absolute) source **luminosity**

$$F/L = 1/A(d) = 1/4\pi d^2$$

in an expanding universe, the distance is a "**comoving**" distance *r* times the scale factor *a*, $d=a_0 r$

define a **redshift** *z* as

$$\begin{aligned} z &= \frac{\lambda_0 - \lambda_1}{\lambda_1} \\ &= \frac{a_0}{a_1} - 1 \ . \end{aligned}$$

now, the **red-shifted** luminosity distance is

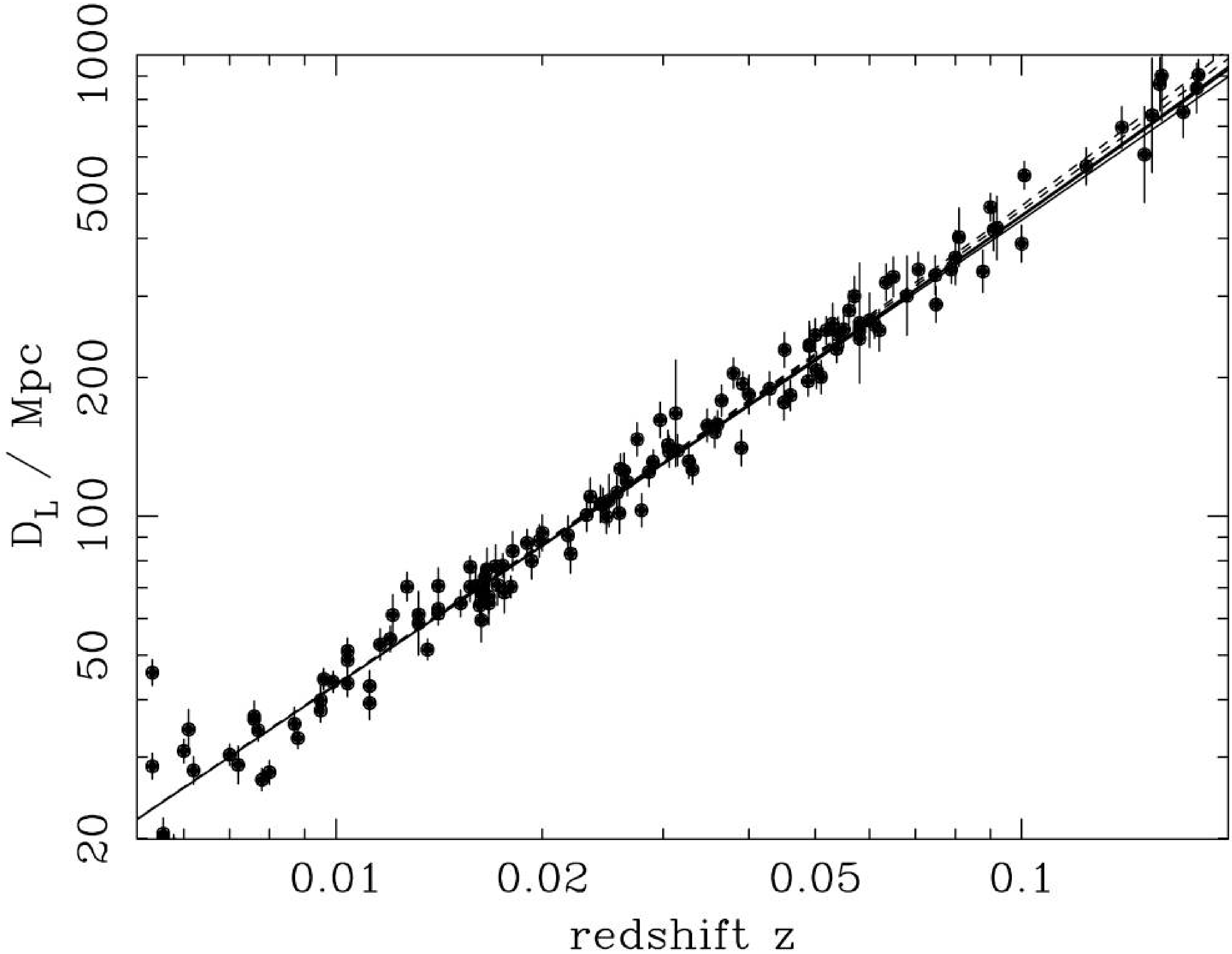
$$\frac{F}{L} = \frac{1}{4\pi a_0^2 r^2 (1+z)^2}$$

$$d_L = a_0 r (1+z) \ .$$

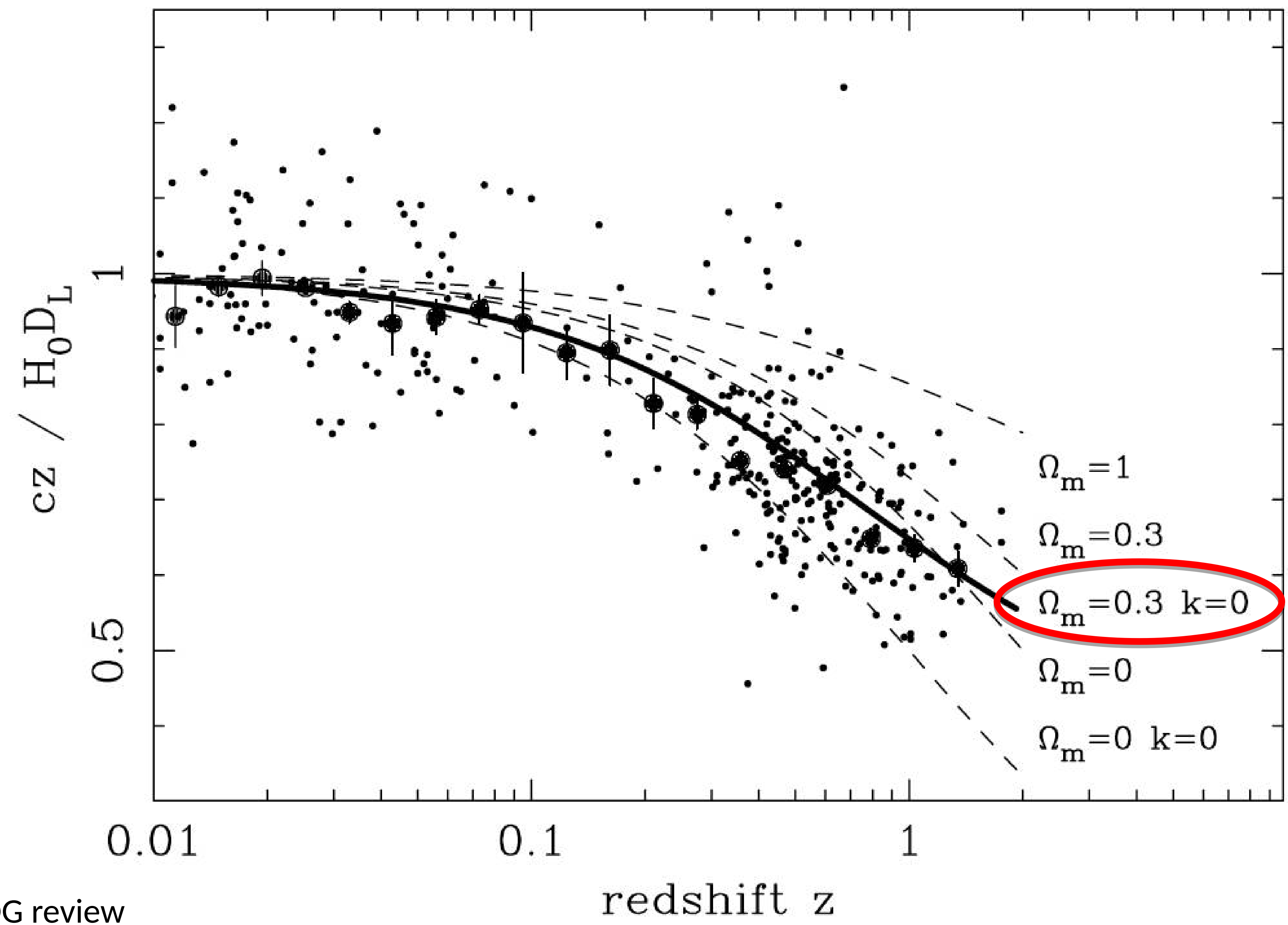
eliminating *r*, we can **recast *d_L*** in terms of ***H_0*** and ***q_0***

$$d_L = H_0^{-1}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right]$$

Recipe to discover what kind of FRW we live in (and to get Nobel prize): measure ***$d_L(z)$*** for objects of known luminosity (**standard candels**)!



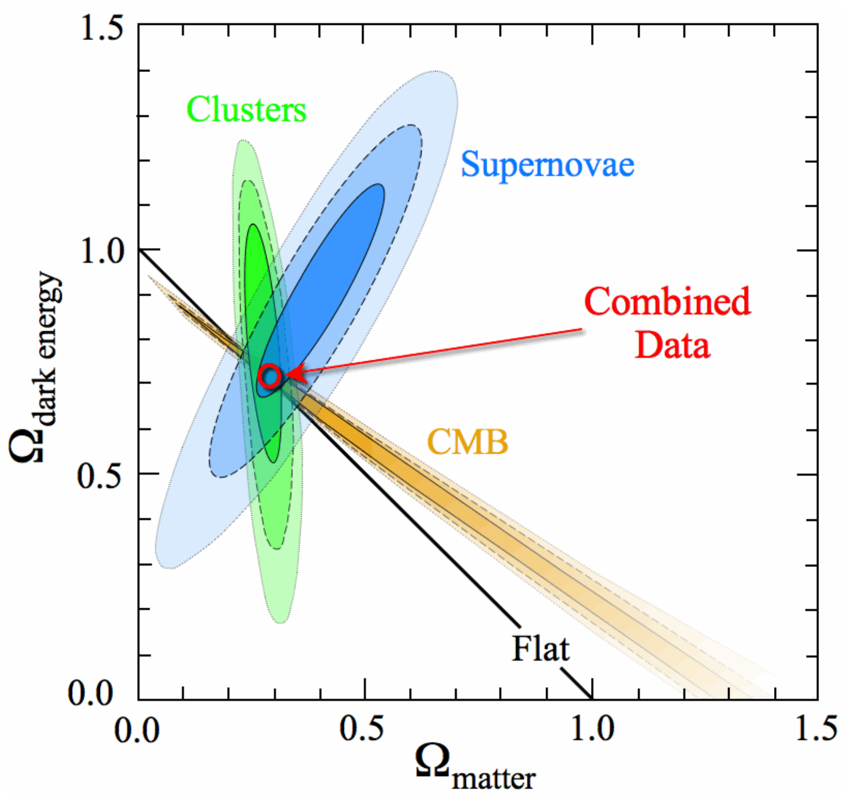
PDG review



cz / H_0D_L
1
0.5
0.01
0.1
1
redshift z
$\Omega_m=1$
$\Omega_m=0.3$
$\Omega_m=0.3$ k=0
$\Omega_m=0$
$\Omega_m=0$ k=0

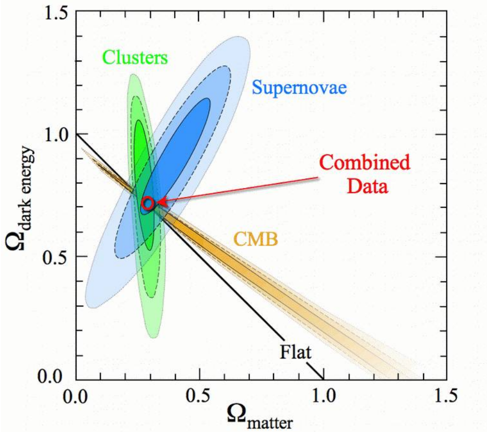
PDG review

key **parameters** for the **geometry** of the universe are **measured** in a variety of additional ways…

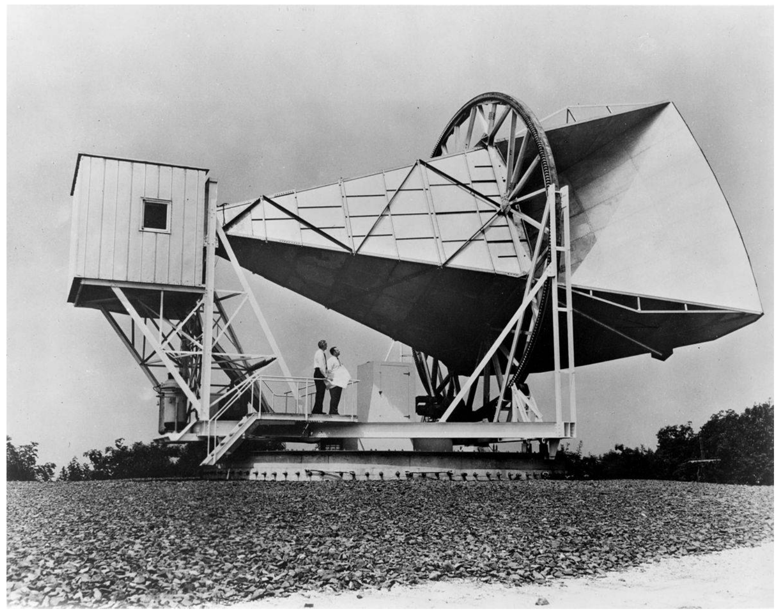


Executive Summary:

data indicate that universe today is compatible with being **flat**, with ~30% pressureless matter, 70% cosmological-constant-like perfect fluid, negligible relativistic matter



fundamental tool to pinpoint cosmological parameters is the **cosmic microwave background** – relic photons from the early universe

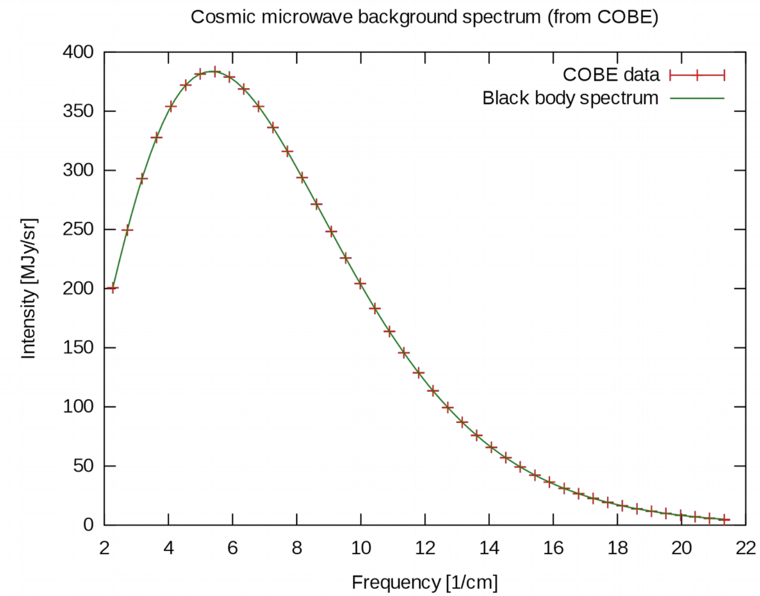


the Holmdel Horn Antenna (Penzias, Wilson, 1964)

key observation: universe filled with a gas of photons with a **black-body spectrum** at a temperature of 2.7K, or **$2.4\text{x}10^{-4}$ eV** in natural units

the observed **specific intensity**
(ergs/cm^2/sec/sr/Hz)

$$I_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT_0} - 1}$$



of course, the photons are **NOT** in thermal equilibrium any more, but their **momentum distribution** indicates that they once were…

this, and the fact that eventually, as $a \to 0$ densities diverge, tells us that **once** the universe contained species in **thermal equilibrium**…

let's explore the **thermodynamics** of the universe

$$(\hbar = c = 1 = k_B = 1)$$

$$f(\vec{p}) = \left[\exp\left(\frac{E-\mu}{T}\right) \pm 1\right]^{-1} \qquad E(p) = (p^2 + m^2)^{1/2}$$

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3p,$$

$$\rho = \frac{g}{(2\pi)^3} \int f(\vec{p})\, E(\vec{p})\, d^3p$$

$$P = \frac{g}{(2\pi)^3} \int f(\vec{p})\, \frac{|\vec{p}|^2}{3E}\, d^3p$$

If **expansion timescale** of the Universe is **long** compared with the timescales for **reactions** that maintain thermal equilibrium, then fluids are in thermal equilibrium with **adiabatic** changes, meaning the **entropy** per comoving volume will be constant.

Second law of thermodynamics

(per comoving volume):

$$T dS = d(\rho V) + P dV = d[(\rho + P)V] - V dP.$$

$$dP = (\rho + P)(dT/T)$$

$$dS = \frac{1}{T} d[(\rho + P)V] - (\rho + P)V \frac{dT}{T^2} = d\left[\frac{(\rho + P)V}{T} + \text{constant}\right]$$

$$S = a^3(\rho + P)/T.$$

$$s \equiv S/V = (\rho + p)/T \propto a^{-3}$$

entropy *density*

An important **limit** we will use later on: $T \gg m$ and $T \gg \mu$

$$\rho = \begin{cases} (\pi^2/30)gT^4 & \text{bosons} \\ (7/8)(\pi^2/30)gT^4 & \text{fermions} \end{cases}$$

$$n = \begin{cases} [\zeta(3)/\pi^2]gT^3 & \text{bosons} \\ (3/4)[\zeta(3)/\pi^2]gT^3 & \text{fermions} \end{cases}$$

$$\rho = \frac{g}{(2\pi)^3}\int f(\vec{p})\,E(\vec{p})\,d^3p,$$

$$P = \frac{g}{(2\pi)^3}\int f(\vec{p})\,\frac{|\vec{p}|^2}{3E}\,d^3p,$$

$$P = \frac{1}{3}\rho$$

…also **entropy** density ($s=(\rho+P)/T$) scales like T^3, and ***a ~ 1/T***

$$\rho = \begin{cases} (\pi^2/30)gT^4 & \text{bosons} \\ (7/8)(\pi^2/30)gT^4 & \text{fermions} \end{cases}$$

if the universe is filled with **multiple relativistic** species, the **total** radiation and pressure density can be cast as

$$\rho_R = (\pi^2/30)g_*T^4, \qquad P_R = \rho_R/3,$$

$$g_* \equiv \sum_{i=\text{bosons}} g_i(T_i/T)^4 + \frac{7}{8}\sum_{i=\text{fermions}} g_i(T_i/T)^4$$

T_i are **different** if the species are not in statistical ("**kinetic**") equilibrium!

Quick **shortcut** that will be useful later on: assume **flat** universe (k=0),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$H^2 = \frac{8\pi G}{3}\rho$$

$$M_P = \sqrt{\frac{1}{8\pi G}}$$

$$\rho_{\rm rad} = \frac{\pi^2}{30} g_* T^4, \quad g_* \simeq 106.75$$

$$H = \sqrt{\frac{\pi^2 g_*}{3 \cdot 30}} \frac{T^2}{M_P} \simeq 3.4 \frac{T^2}{M_P}$$

similarly for **entropy** density

$$s = (2\pi^2/45) g_{*s} T^3$$

$$g_{*s} \equiv \sum_{i=\text{bosons}} g_i (T_i/T)^3 + (7/8) \sum_{i=\text{fermions}} g_i (T_i/T)^3$$

…so for an adiabatic universe, $g_{*s} T^3 a^3$ is a **constant**!

…back to radiation energy density and ***g_*(T)***

$$\rho_R = (\pi^2/30) g_* T^4$$

$$g_* \equiv \sum_{i=\mathrm{bosons}} g_i (T_i/T)^4 + \frac{7}{8} \sum_{i=\mathrm{fermions}} g_i (T_i/T)^4$$

…***g_*(T)*** is very useful in model building!

E.g. if there exist new "**dark**" degrees of freedom (e.g. light mediators) they should contribute to the **effective number of degrees of freedom**, which is something we measure!

First, let’s calculate the effective number of relativistic degrees of freedom in the **late universe**

Two species: **photons** and **neutrinos** and
Issue: they have **different temperatures!**

As we will see shortly, neutrinos decouple around a temperature of **1 MeV**

After that, **electrons and positrons** annihilate, and “heat up*” the photons (but not that neutrinos), so need to correct for the **mismatch** between electron and neutrino temperatures

*not true, they just slow down the temperature drop with *1/a*

Now using **conservation** of **entropy**,
and the fact that entropy scales like gT^3,
assuming instantaneous decays ($a_0=a_1$)
(0=after, 1=before)

$$\left(\frac{g_0}{g_1}\right)^{\frac{1}{3}} = \frac{T_1}{T_0}, \qquad \frac{T_\nu}{T_\gamma} = \left(\frac{2}{2+2\times 7/8+2\times 7/8}\right)^{\frac{1}{3}} = \left(\frac{4}{11}\right)^{\frac{1}{3}}.$$

e^- e^+ (71%)

the contribution of neutrinos to the radiation energy density can be then **parameterized** by the effective number of **neutrinos species**, times this correction for the neutrino vs photon temperature

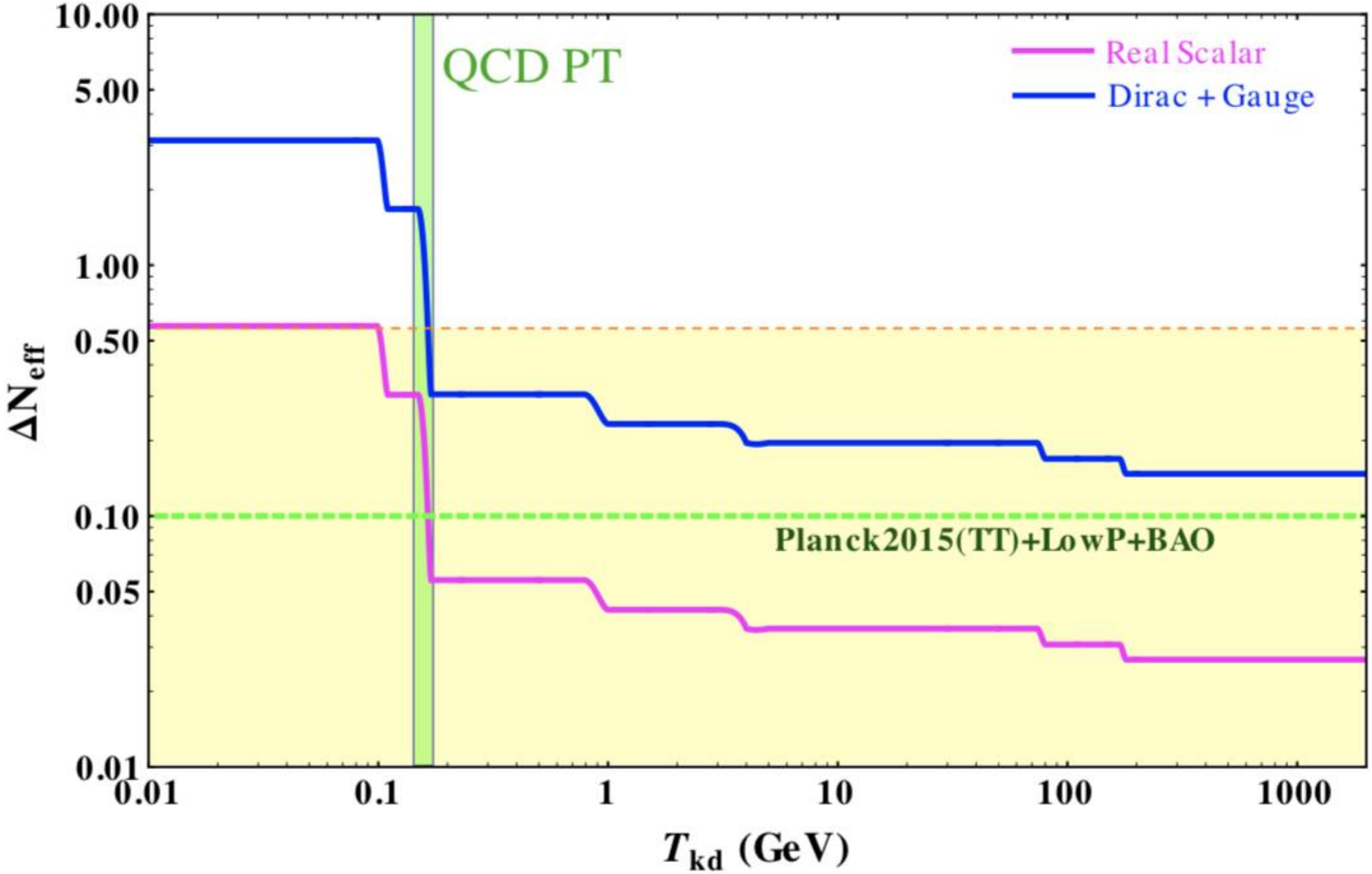
$$1+\frac{7}{8}N_\nu\left(\frac{4}{11}\right)^{\frac{4}{3}}$$

plugging in the numbers, g_* today should be 3.36, equivalent to ***N_ν ~ 3***

…in fact the SM prediction is N_ν~3.046 since neutrinos are **not completely decoupled** at electron-positron annihilation

N_ν impacts several "**late-universe**" observables:

- **Big Bang Nucleosynthesis** $N_\nu = 3.14\,^{+0.70}_{-0.65}$ at 68%
- **Baryon Acoustic Oscillations** (SN+WMAP) $N_\nu = 4.34\,^{+0.88}_{-0.86}$ at 68%
- **CMB** (Planck) $N_\nu = 3.15 \pm 0.23.$



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Stefano Profumo

Santa Cruz Institute for Particle Physics

University of California, Santa Cruz

Cosmology and Dark Matter

Lecture 2

IX NExT PhD Workshop

The Cosener's House, Abingdon, UK

July 8-11th, 2019

Second important **limit** we will use later on: $m \gg T$

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3p,$$

$$\rho = \frac{g}{(2\pi)^3} \int f(\vec{p})\, E(\vec{p})\, d^3p$$

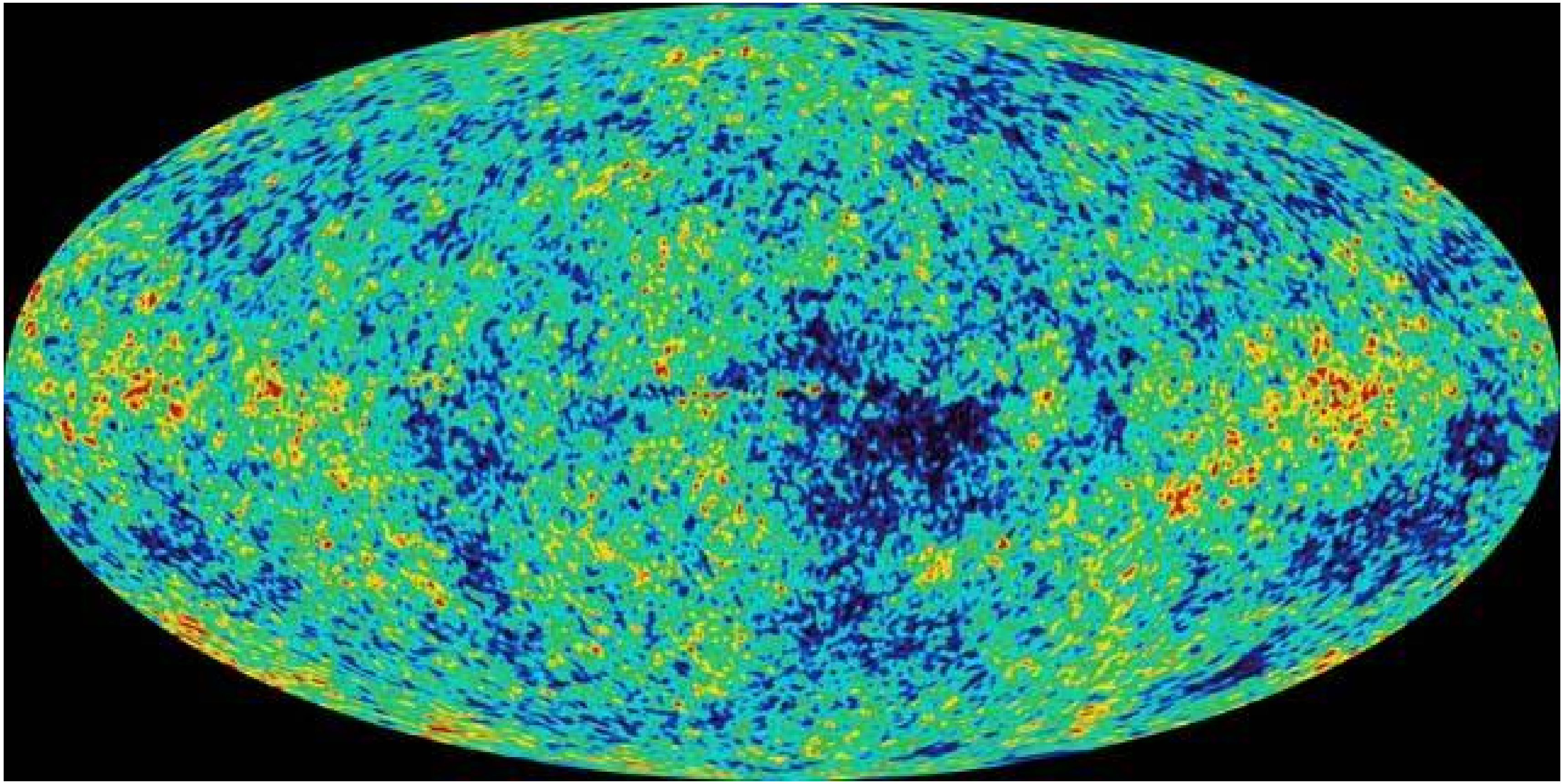
$$P = \frac{g}{(2\pi)^3} \int f(\vec{p})\, \frac{|\vec{p}|^2}{3E}\, d^3p$$

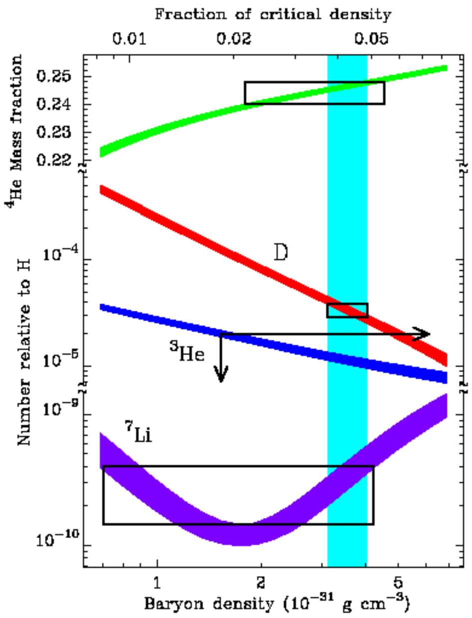
$$f(\vec{p}) = \left[\exp\left(\frac{E-\mu}{T}\right) \pm 1\right]^{-1}$$

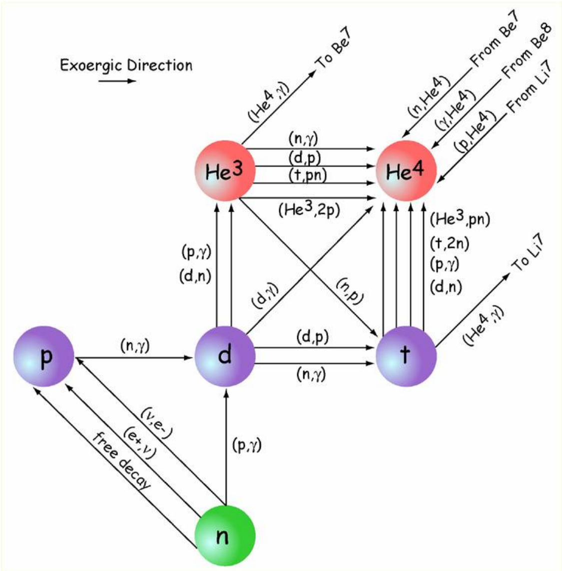
$$n = g\left(\frac{mT}{2\pi}\right)^{2/3} e^{-(m-\mu)/T}, \qquad \rho = mn, \qquad P = nT \ll \rho.$$

The Universe's **Thermodynamics** provides a successful framework for the **origin of species** in the early universe:

thermal decoupling

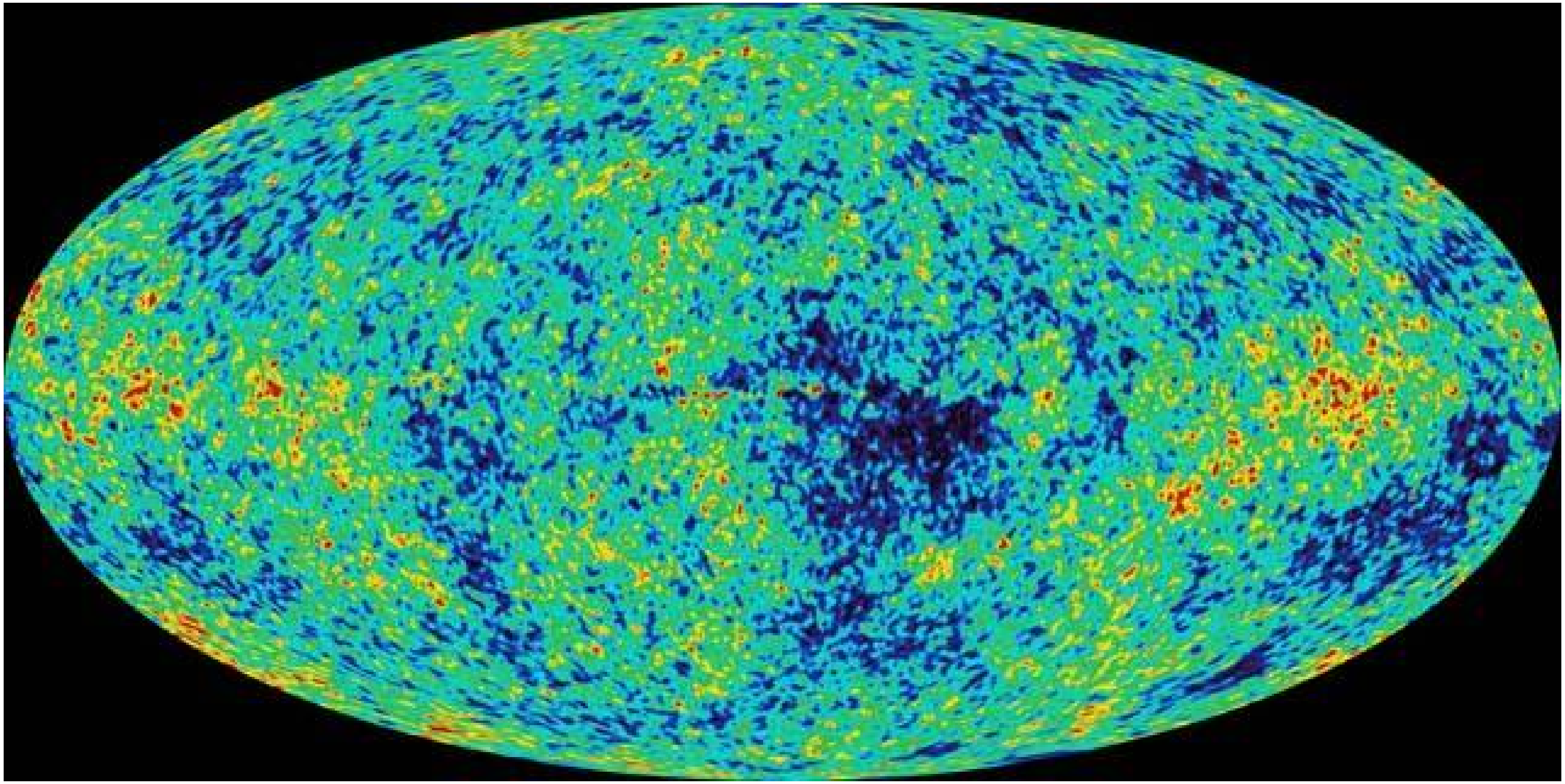






A successful ***synergy*** of ***statistical mechanics***,
general relativity, and of ***nuclear and particle physics***
making **predictions** testable to exquisite accuracy
with ***astronomical*** observations!

Event	time t
Inflation	10^{-34} s (?)
Baryogenesis	?
EW phase transition	20 ps
QCD phase transition	20 μs
Dark matter freeze-out	?
Neutrino decoupling	1 s
Electron-positron annihilation	6 s
Big Bang nucleosynthesis	3 min
Matter-radiation equality	60 kyr
Recombination	260–380 kyr
Photon decoupling	380 kyr
Reionization	100–400 Myr
Dark energy-matter equality	9 Gyr
Present	13.8 Gyr



CMB decouples because free electrons bind with protons to form Hydrogen atoms (**recombination**)
when temperatures are around **13.6 eV**

n_e, n_p, and n_H

$$e^- + p \leftrightarrow H \qquad \mu_p + \mu_e = \mu_H$$

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right)$$

$$n_H = g_h \left(\frac{m_H T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_p + \mu_e - m_H}{T}\right) = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

$$B \equiv m_p + m_e - m_H = 13.6 \text{ eV} \qquad m_p \simeq m_H$$

$$n_H = g_h \left(\frac{m_H T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_p + \mu_e - m_H}{T}\right) = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

$$g_p = g_e = 2 \qquad g_H = 4 \qquad n_p = n_e$$

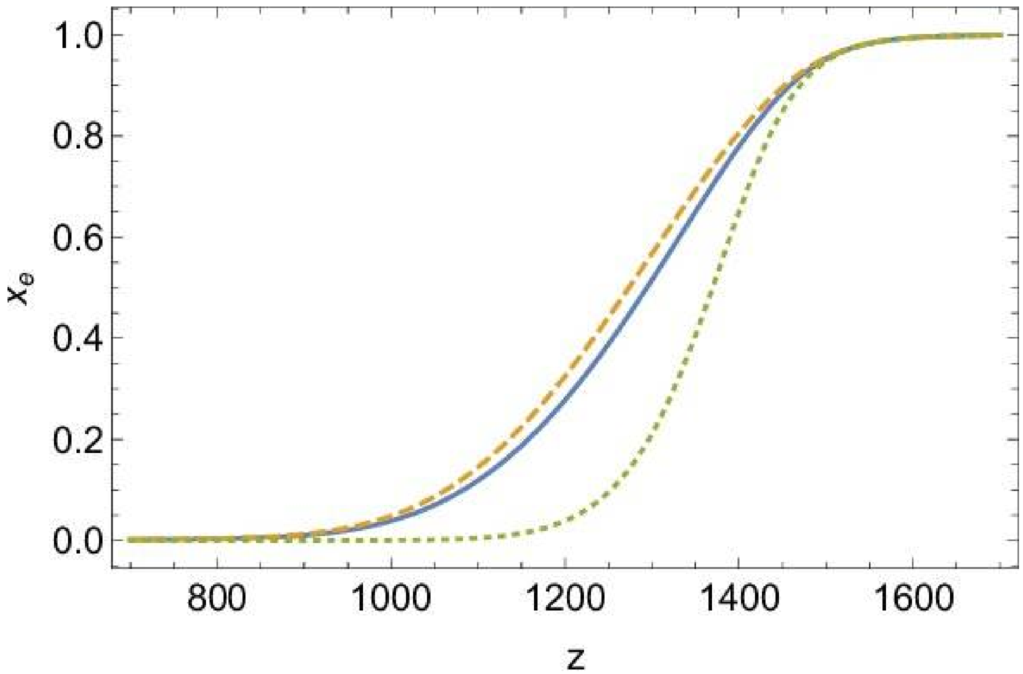
ionization **fraction**
(fraction of free electrons)

$$X_e \equiv n_p/n_H \qquad n_H = (1 - X_e) n_p$$

$$\frac{(1 - X_e^{\mathrm{eq}})}{(X_e^{\mathrm{eq}})^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}} \eta \left(\frac{T}{m_e}\right)^{-3/2} e^{B/T}$$

$$T(z) = T_0(1 + z)$$

Saha's equation and its solution



$$\frac{(1 - X_e^{\rm eq})}{(X_e^{\rm eq})^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}}\eta\left(\frac{T}{m_e}\right)^{-3/2} e^{B/T}$$

Saha's equation describes recombination when the **reaction** leading to bound hydrogen atoms is in **equilibrium**

in reality, n_e=n_p are **dropping** very fast, so the process ends when it goes **out of equilibrium**

Let's investigate the process of "**freeze-out**" from the thermal bath, and actually calculate when **photons** decouple from **matter**

Key **idea** of thermal decoupling:
if the **reaction** keeping a species in equilibrium
is **faster** than the **expansion rate** of the universe,
the reaction is in **statistical equilibrium**;
if it's **slower**, the species **decouples** ("freeze-out")

$$\Gamma \ll H(T) \qquad\qquad \Gamma(T_{\mathrm{t.o.}}) \sim H(T_{\mathrm{t.o.}})$$

the **reaction rate** (from definition of cross section!)

$$\Gamma = n \cdot \sigma \cdot v$$

when do **CMB** photons decouple?

key process: **Thompson** scattering off of **free electrons** (X_e)

free electron number density are fixed by
(1) electron-positron **asymmetry** and (2) **recombination** (i.e. X_e)

$$\Gamma = n_e \sigma_T v_e$$

(we will see later v_e is not too small)

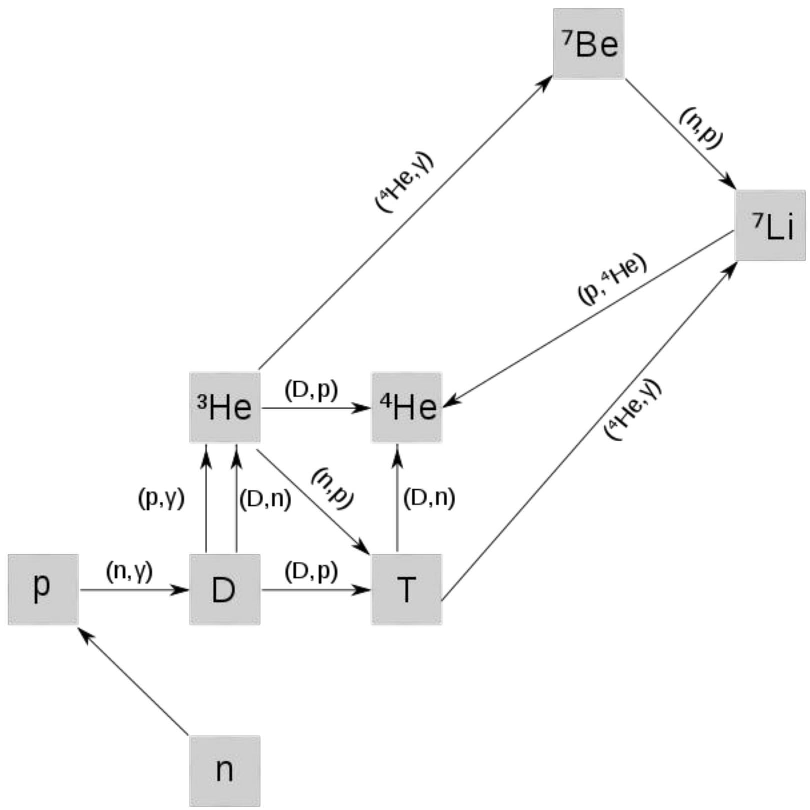
$$\sigma_T = 6.625 \times 10^{-25} \text{ cm}^2$$

$$H \simeq \frac{T^2}{M_P} \sim X_e \left(\frac{n_e}{s}\right) T^3 \sigma_T \simeq 0.01 \cdot 10^{-10} \cdot (6 \times 10^{-25}) \cdot 2.5 \times 10^{27} \cdot T^3$$

$$T \simeq \frac{3 \times 10^{-11} \text{ GeV}}{X_e} \simeq 0.3 \text{ eV}$$

redshifted to today, this **agrees** with the CMB black body spectrum!

Another (earlier) thermal **decoupling** process:
Big Bang **Nucleosynthesis**



$$\begin{aligned}
p + n &\longrightarrow {}^2\mathrm{H} + \gamma \\
p + {}^2\mathrm{H} &\longrightarrow {}^3\mathrm{He} + \gamma \\
{}^2\mathrm{H} + {}^2\mathrm{H} &\longrightarrow {}^3\mathrm{He} + n \\
{}^2\mathrm{H} + {}^2\mathrm{H} &\longrightarrow {}^3\mathrm{H} + p \\
{}^3\mathrm{He} + {}^2\mathrm{H} &\longrightarrow {}^4\mathrm{He} + p \\
{}^3\mathrm{H} + {}^2\mathrm{H} &\longrightarrow {}^4\mathrm{He} + n
\end{aligned}$$

will focus on one key prediction: **Helium4** mass fraction **~ 25%**

Early BBN: neutron decoupling $T \gg \text{MeV}$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e$$

$$\mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\nu_e + n \leftrightarrow p + e^-$$

$$\frac{n}{p} \equiv \frac{n_n}{n_p} = \exp\left[-Q/T + (\mu_e - \mu_\nu)/T\right]$$

$$Q \equiv m_n - m_p = 1.293 \text{ MeV}$$

$$\Gamma_{pe\leftrightarrow\nu n} = \begin{cases} \tau_n^{-1}(T/m_e)^3 e^{-Q/T} & T \ll Q, m_e \\ \frac{7\pi^4\tau_n^{-1}}{30\lambda_0}\left(\frac{T}{m_e}\right)^5 \simeq G_F^2 T^5 & T \gg Q, m_e \end{cases}$$

$$T \gtrsim m_e,\ (\Gamma/H) \simeq (T/0.8\,\mathrm{MeV})^3$$

$$\left(\frac{n}{p}\right)_{\mathrm{freezeout}} = e^{-Q/T_f} \simeq \frac{1}{6}$$

- all neutrons “find” protons and (via *D*) **form ^4He** nuclei
- 1 in 5 neutrons **decay** ✉ *n/p~1/7*

$$X_4 = \frac{4n_{^4He}}{n_N} = \frac{4(n_n/2)}{n_n + n_p} = \frac{2(n/p)}{1+(n/p)} \simeq 0.25$$

Go further up in **temperature** and back in **time**!

(Standard Model) neutrino freeze-out (**hot** thermal relic)

language definition: **hot** = relativistic at $T_{f.o}$

cold = $v<c=1$. (actually not by much, typically!)

key reaction: neutrino-antineutrino (back-)conversion to electron-positron pairs

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

$$n(T_\nu) \cdot \sigma(T_\nu) = H(T_\nu) \qquad \sigma \sim G_F^2 T_\nu^2$$

suppose this is a hot relic… ***$n \sim T_\nu^3$***

$$T_\nu^3 G_F^2 T_\nu^2 = T_\nu^2 / M_P$$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3} \text{ GeV} \sim 1 \text{ MeV}$$

happy about **two** things in particular:

1. **hot** relic assumption works! $T_\nu \gg m_\nu$

2. **Fermi** effective theory OK! $T_\nu \ll m_W$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \mathrm{GeV} \sim 1\ \mathrm{MeV}$$

now, how do we calculate the **relic** thermal **abundance** of this prototypical hot relic?

Introduce ***Y=n/s*** (ratio of number and entropy **density**, $V=a^3$)

If universe is iso-entropic (i.e. adiabatic), ***s x a³=S*** is conserved

Y ~ n a³ is thus ~ **comoving number density**, and (without entropy injection)

$$Y_{\rm today} = Y_{\rm freeze-out} = Y(T_\nu)$$

$$Y_{\rm freeze-out} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$n_{\text{today}} = s_{\text{today}} \times Y_{\text{today}} = s_{\text{today}} \times Y_{\text{freeze-out}}$$

$$\rho_{\nu,\text{today}} = m_\nu \times Y_{\text{freeze-out}} \times s_{\text{today}}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\text{crit}}} h^2 \simeq \frac{m_\nu}{91.5 \text{ eV}}$$

Cowsik-Mc-Clelland limit

That was **fun**! Let's see if it works for something else…

Try **proton-antiproton** freeze-out:
what's the **relic** matter **abundance** in a baryon-symmetric Universe?

$$\sigma \sim \Lambda_{\rm QCD}^{-2}.$$

$n\ \sigma\ = H$ ✉ $T^3\ \Lambda^{-2} = T^2/M_P$ ✉ $T = \Lambda^2/M_P$

doesn't quite work, we're way **outside**
the regime of validity for **hot relics**, since $T <<<<<<< m_p$ …

Need to work out the case of **cold relics**, which looks nastier by eye

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$

Here's the trick: **freeze-out** condition gives

$$n_{\mathrm{f.o.}} \sim \frac{T_{\mathrm{f.o.}}^2}{M_P \cdot \sigma}$$

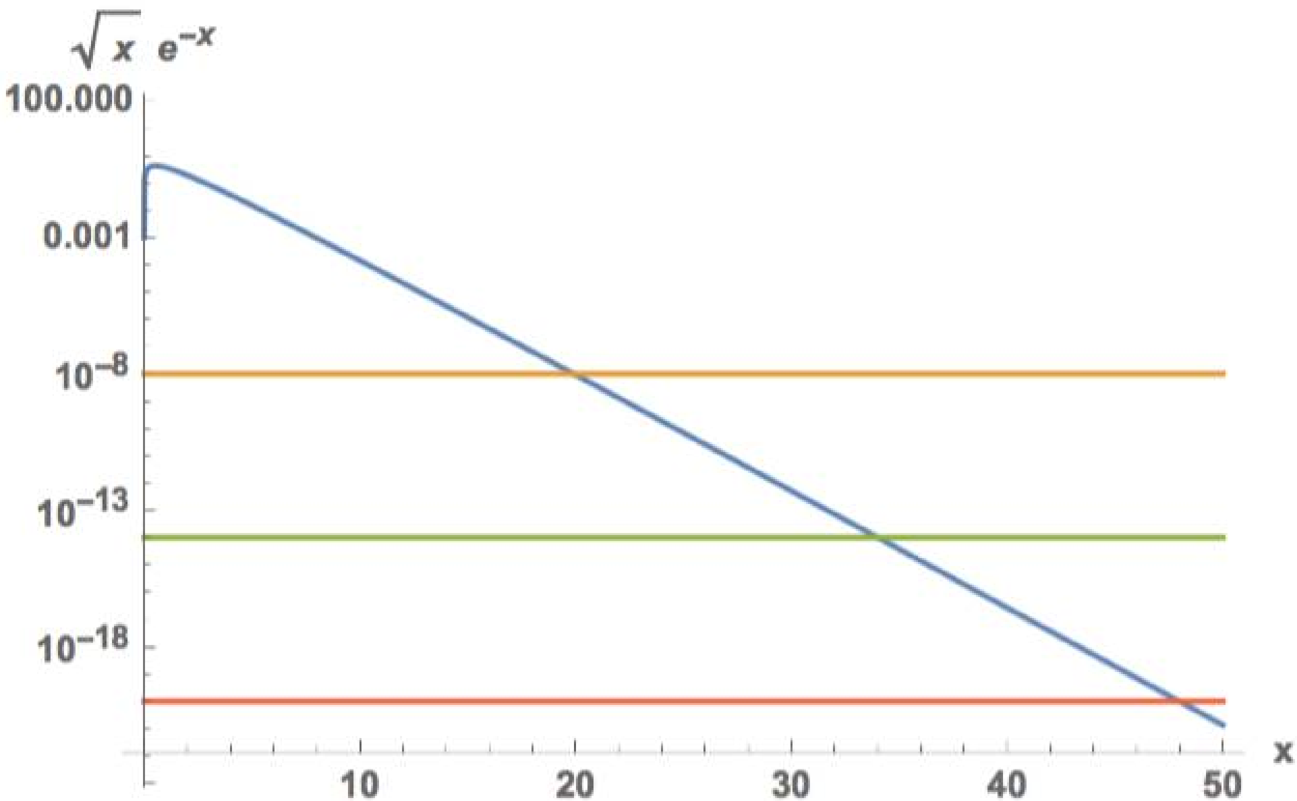
now define $m_\chi / T \equiv x$ (cold relic: **x>>1**)

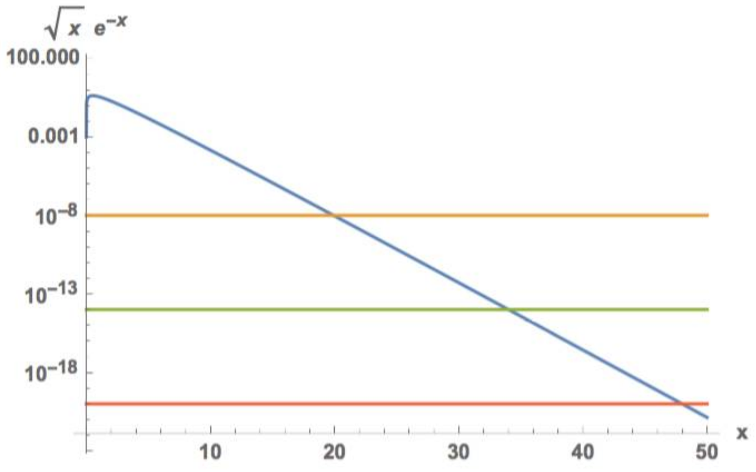
Freeze-out condition (x) now reads

$$\frac{m_\chi^3}{x^{3/2}} e^{-x} = \frac{m_\chi^2}{x^2 \cdot M_P \cdot \sigma}$$

…so we gotta **solve** $\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$





$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$

$$\sigma \sim G_F^2 m_\chi^2$$

Take e.g. a "**weakly interacting massive particle**"

$$m_\chi \sim 10^2 \text{ GeV.}$$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \sim \frac{1}{10^2 \cdot 10^{18} \cdot 10^{-6}} \sim 10^{-14}.$$

$x = m_\chi / T \sim 35$

proton-antiproton: (1 GeV 10^{18} GeV 10^{-2} GeV^{-2}) ~ 10^{-16}

$x = m_p / T \sim 37$

Off to calculating the **thermal relic density**

$$\Omega_\chi = \frac{m_\chi \cdot n_\chi(T=T_0)}{\rho_c} = \frac{m_\chi T_0^3}{\rho_c}\frac{n_0}{T_0^3}$$

iso-entropic universe aT ~const $\qquad \frac{n_0}{T_0^3} \simeq \frac{n_{\text{f.o.}}}{T_{\text{f.o.}}^3}$

$$\Omega_\chi = \frac{m_\chi T_0^3}{\rho_c}\frac{n_{\text{f.o.}}}{T_{\text{f.o.}}^3} = \frac{T_0^3}{\rho_c} x_{\text{f.o.}} \left(\frac{n_{\text{f.o.}}}{T_{\text{f.o.}}^2}\right) = \left(\frac{T_0^3}{\rho_c M_P}\right)\frac{x_{\text{f.o.}}}{\sigma}$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\text{f.o.}}}{20}\left(\frac{10^{-8}\ \text{GeV}^{-2}}{\sigma}\right)$$

Notice we neglected relative **velocity**…
What is the velocity of a cold relic at freeze-out?

$$\frac{3}{2}T = \frac{1}{2}mv^2$$

…just use **equipartition** theorem… $v=(3/x)^{1/2}$ ~ 0.3

Now, back to **relic density**:

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\mathrm{f.o.}}}{20}\left(\frac{10^{-8}\ \mathrm{GeV}^{-2}}{\sigma}\right)$$

$$\sigma_{\mathrm{EW}} \sim G_F^2 T_{\mathrm{f.o.}}^2 \sim G_F^2\left(\frac{E_{\mathrm{EW}}}{20}\right)^2 \sim 10^{-8}\ \mathrm{GeV}^{-2},$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20}\left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

Is this **unique** to **WIMPs**? **No**.

$$\sigma \sim \frac{g^4}{m_\chi^2}$$

"**WIMPless**" miracle... what did we use?

$$m_\chi \cdot \sigma \cdot M_P \gg 1$$

$$\sigma \sim 10^{-8}\ {\rm GeV}^{-2}$$

Substitute and find that ***m_χ >> 0.1 eV*** !

In practice various **constraints** on light thermal relics from structure formation, relativistic degrees of freedom at BBN, CMB... ***m_χ > MeV***

What is the **range** of **masses** expected for cold relics?

Cross section cannot be arbitrarily large: **unitarity** limit

$$\sigma \lesssim \frac{4\pi}{m_\chi^2}$$

$$\frac{\Omega_\chi}{0.2} \gtrsim 10^{-8}\ \mathrm{GeV}^{-2} \cdot \frac{m_\chi^2}{4\pi}$$

$$\left(\frac{m_\chi}{120\ \mathrm{TeV}}\right)^2 \lesssim 1$$

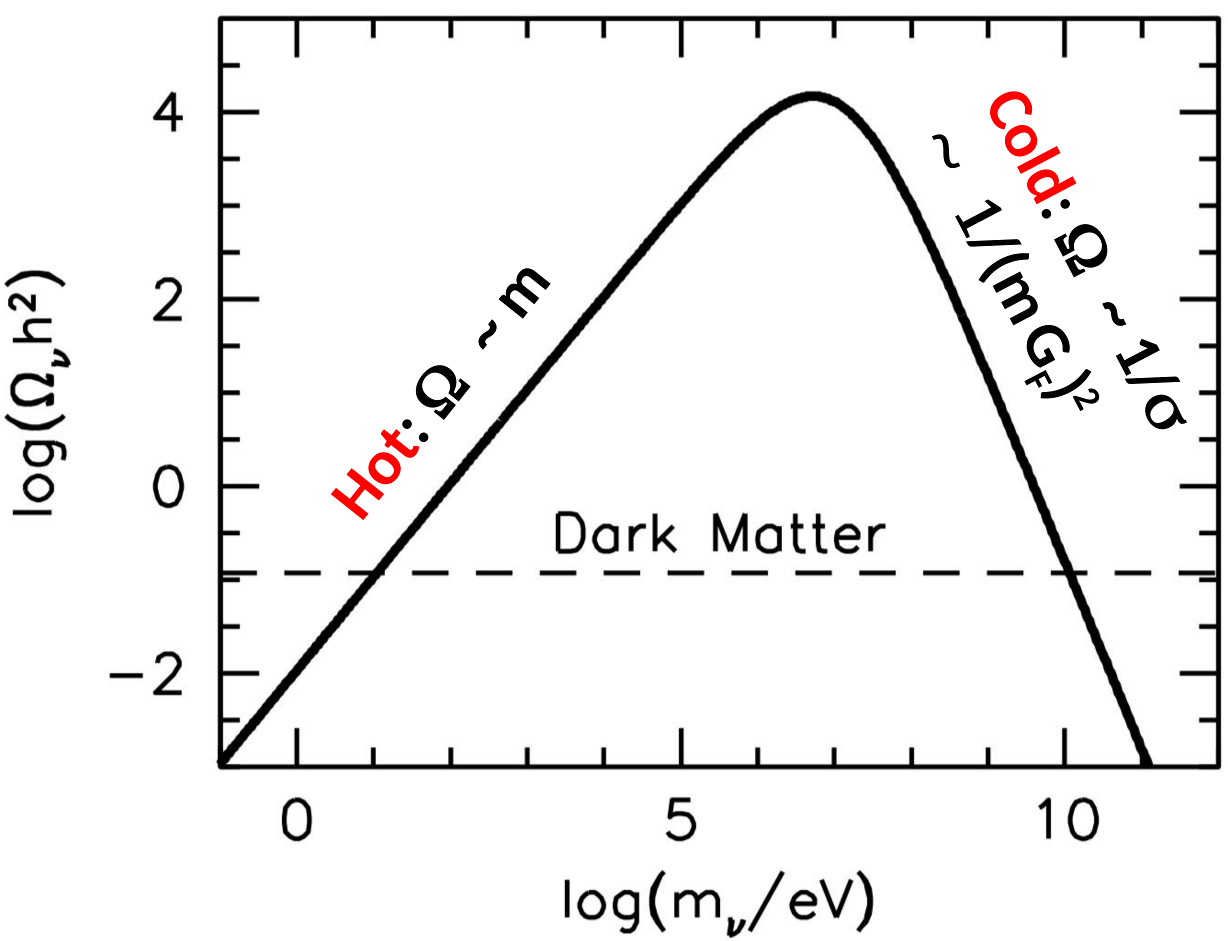
What is the **range** of **masses** expected for cold relics?

If you have a **WIMP**, defined by a cross section $\sigma \sim G_F^2\, m_\chi^2$

$$\Omega_\chi h^2 \sim 0.1 \frac{10^{-8}\ \mathrm{GeV}^{-2}}{G_F^2\, m_\chi^2} \sim 0.1 \left(\frac{10\ \mathrm{GeV}}{m_\chi}\right)^2$$

"**Lee-Weinberg**" limit

WIMP's **thermal** relic density



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SANTA CRUZ INSTITUTE FOR PARTICLE PHYSICS

UC SANTA CRUZ

Stefano Profumo

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Cosmology and Dark Matter

Lecture 3

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Key ideas from last lecture

- the **metric** of a homogeneous and isotropic **universe**
- **stuff** in the universe and how to measure it
- the relation between **density** and **destiny** of the universe
- **thermodynamics** of the early universe

- (re-)**combination**: exact formula for **free electron fraction** in eq.
- photons **decouple** when free electrons are gone, around 0.3 eV
- BBN: key is thermal **decoupling of neutrons**, when $T \sim m_n - m_p$, fixes n/p
- ^{4}He **mass fraction** ~only depends on n/p
- happens at 0.8..0.1 MeV, t~few seconds to minutes old

Key ideas from last lecture

- **neutrino decoupling**: hot thermal relic,

  freezes out at T~1 MeV ~ $(G_F^2 M_P)^{-1/3}$

- present **number density**: define Y=n/s; $Y_{dec}=Y_{today}$,

  so $n_{today}=Y_{decoupling} \times s_{today}$

- present energy density: $\rho_\nu = n_{today} * m_\nu$ (generic for **hot** thermal relics)

- **Cold** relics: $\Omega \sim 1/\sigma$, with $\sigma \sim 10^{-8}$ GeV^{-2} ~ σ_{EW} (**WIMP**(—less) miracle!)
- **Relic** protons and antiprotons ~ 10^{-10} matter abundance

Discussion so far OK for a **qualitative** assessment of **relic density**

State of the art much more sophisticated: Solve **Boltzmann equation**

$$\hat{L}[f] = \hat{C}[f]$$

$$\hat{L}_{\rm NR} = \frac{\rm d}{{\rm d}t} + \frac{{\rm d}\vec{x}}{{\rm d}t}\vec{\nabla}_x + \frac{{\rm d}\vec{v}}{{\rm d}t}\vec{\nabla}_v$$

$$\hat{L}_{\rm cov} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma}\, p^\beta\, p^\gamma \frac{\partial}{\partial p^\alpha}$$

Looks ugly, but for the **FRW** metric **phase-space** density simplifies…

$$f(\vec{x}, \vec{p}, t) \rightarrow f(|\vec{p}|, t) \qquad f(E, t)$$

$$\hat{L}[f] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|^2\, \frac{\partial f}{\partial E}$$

Now, what we are interested in are **number** densities, which in terms of **phase-space** densities are simply…

$$n(t) = \sum_{\text{spin}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f(E,t)$$

…**integrate** the Liouville operator over **momentum space** and get

$$\int L[f] \cdot g \frac{\mathrm{d}^3 p}{(2\pi)^3} = \frac{\mathrm{d}n}{\mathrm{d}t} + 3H \cdot n,$$

Back to **Boltzmann** equation, suppose a **2-to-2** reaction, with 3, 4 **in eq.**

$$1 + 2 \leftrightarrow 3 + 4$$

Consider the **collision** factor, and again integrate over **momenta**…

$$g_1 \int \hat{C}[f_1] \frac{\mathrm{d}^3 p}{(2\pi)^3} = -\langle \sigma \cdot v_{\mathrm{M\o l}} \rangle \left(n_1 n_2 - n_1^{\mathrm{eq}} n_2^{\mathrm{eq}} \right)$$

…where the **cross section**

$$\sigma = \sum_f \sigma_{12 \to f}$$

$$g_1 \int \hat{C}[f_1] \frac{\mathrm{d}^3 p}{(2\pi)^3} = -\langle \sigma \cdot v_{\mathrm{M\o l}} \rangle \left(n_1 n_2 - n_1^{\mathrm{eq}} n_2^{\mathrm{eq}} \right)$$

let’s understand the rest of the equation:

$$v_{\mathrm{M\o l}} \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 \, E_2}$$

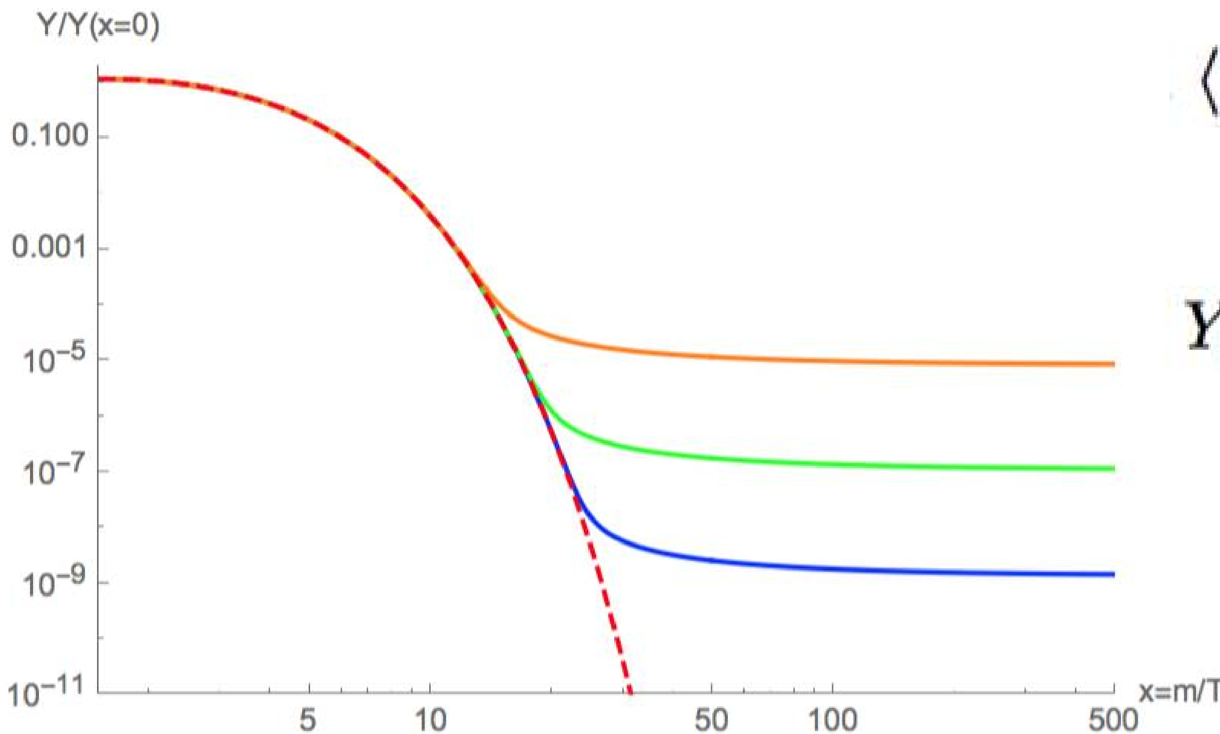
$$\langle \sigma \cdot v_{\mathrm{M\o l}} \rangle = \frac{\int \sigma \cdot v_{\mathrm{M\o l}} \, e^{-E_1/T} e^{-E_2/T} \, \mathrm{d}^3 p_1 \, \mathrm{d}^3 p_2}{\int e^{-E_1/T} e^{-E_2/T} \, \mathrm{d}^3 p_1 \, \mathrm{d}^3 p_2}$$

Final version of
Boltzmann Eq.

$$\dot{n} + 3Hn = \langle \sigma v \rangle \left(n_{\mathrm{eq}}^2 - n^2 \right)$$

$$\dot{n} + 3Hn = \langle \sigma v \rangle \left(n_{\rm eq}^2 - n^2 \right)$$

$$\frac{dY(x)}{dx} = -\frac{xs\langle \sigma v \rangle}{H(m)} \left(Y(x)^2 - Y_{\rm eq}^2(x) \right)$$



$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 x^{-n}$$

$$Y_{\rm today} \simeq \frac{n+1}{\lambda} x_{\rm f.o.}^{n+1}$$

$$\lambda = \frac{\langle \sigma v \rangle_0 s_0}{H(m)}$$

There exist important "**exceptions**" to this standard story:

1. **Resonances**

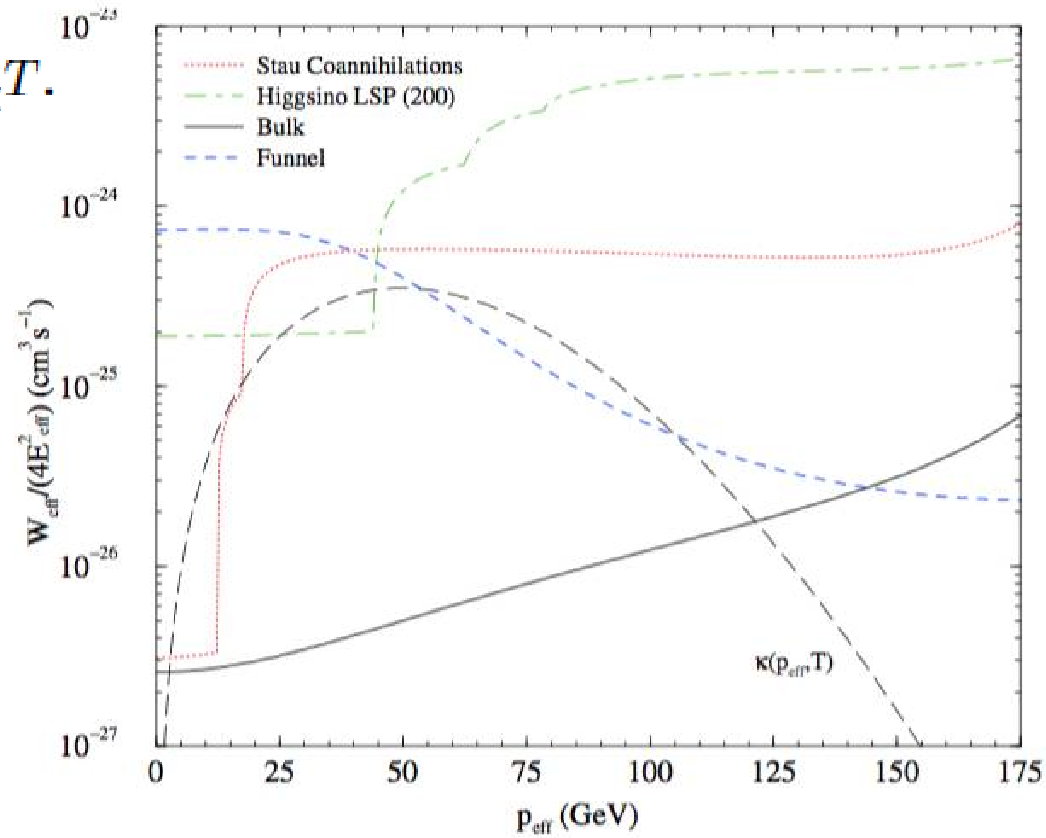
$$\langle s \rangle \simeq 4m_\chi^2 + 6m_\chi T .$$

2. **Thresholds**

3. **Co-annihilation**

$$\langle \sigma v \rangle \;\to\; \langle \sigma_{\rm eff} v \rangle = \frac{\sum_{i<j=1}^{N} \sigma_{ij} \exp\left(-\frac{\Delta m_i + \Delta m_j}{T}\right)}{\sum_{i=1}^{N} g_i \exp\left(-\frac{\Delta m_i}{T}\right)} .$$

Affects what the **pair-annihilation** rate **today** is compared to what it was at **freeze-out**!



$$\langle \sigma_{\rm eff} v \rangle = \int_0^\infty dp_{\rm eff} \frac{W_{\rm eff}(p_{\rm eff})}{4E_{\rm eff}^2} \kappa(p_{\rm eff}, T) \qquad E_{\rm eff}^2 = \sqrt{p_{\rm eff}^2 + m^2} .$$

So far we looked into what happens if we fiddle with the left hand side of

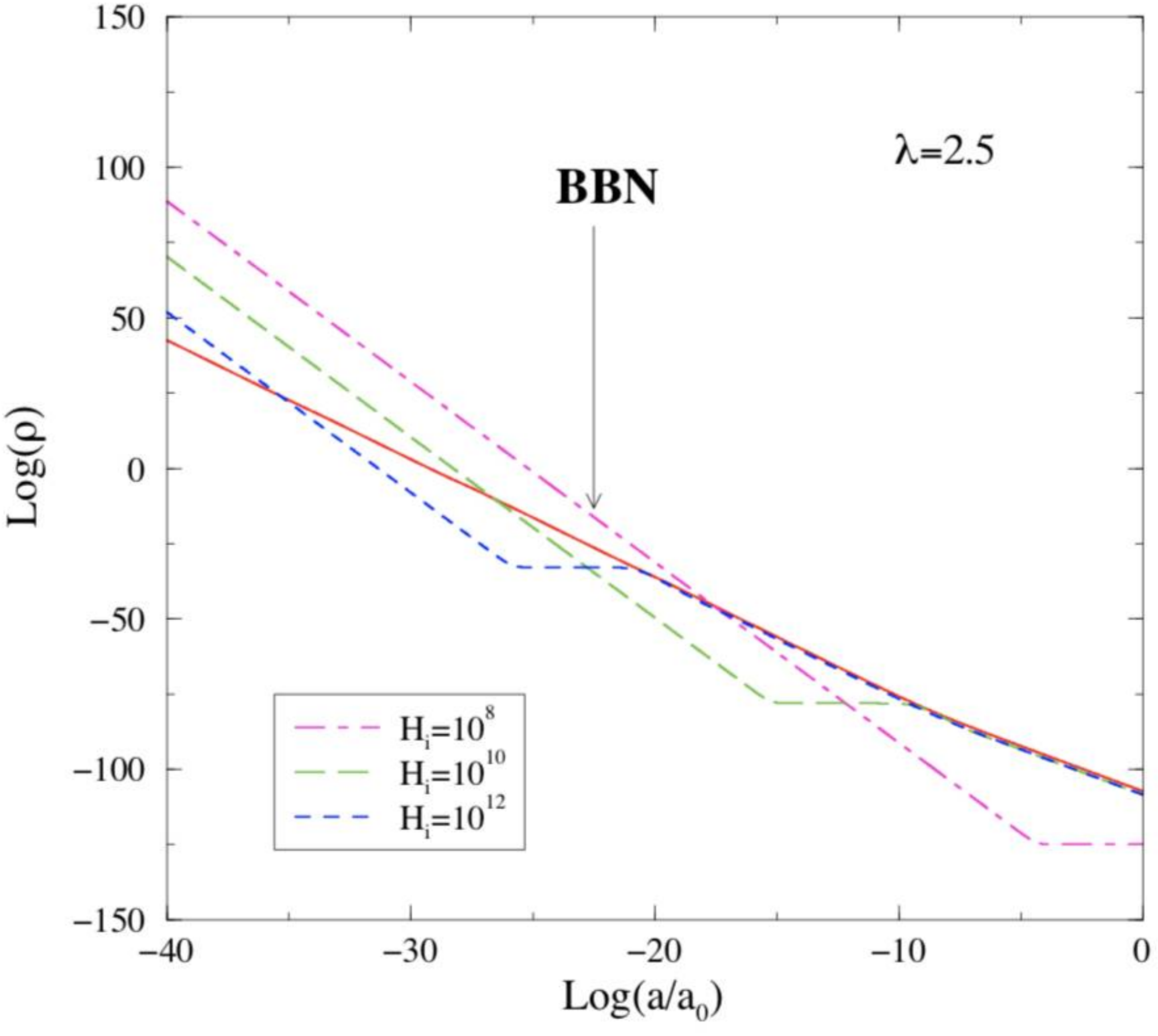
$$\Gamma = n \cdot \sigma \sim H$$

Consider a "**Quintessence**" dark energy model – homogeneous real scalar field

$$\rho_\phi = \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)^2 + V(\phi)$$

$$P_\phi = \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)^2 - V(\phi)$$

$$w = P_\phi/\rho_\phi \qquad \rho_\phi \sim a^{-3(1+w)} \qquad \rho \sim a^{-6}$$



Profumo and Ullio, many years ago

$$H \sim \frac{T^2}{M_P}\frac{T}{T_{\rm KRE}} \qquad (T \gtrsim T_{\rm KRE})$$

$$n_{\rm f.o.}\langle\sigma\, v\rangle \sim \frac{T^2}{M_P}\frac{T}{T_{\rm KRE}}.$$

$$\frac{n_{\rm f.o.}}{T_{\rm f.o.}^2} \sim \frac{1}{M_P\,\langle\sigma\, v\rangle}\frac{T_{\rm f.o.}}{T_{\rm KRE}}. \qquad \Omega_\chi^{\rm quint} = \frac{T_0^3}{M_P\cdot\rho_c}x_{\rm f.o.}\left(\frac{n_{\rm f.o.}}{T_{\rm f.o.}^2}\right)$$

$$\frac{\Omega_\chi^{\rm quint}}{\Omega_\chi^{\rm standard}} \sim \frac{T_{\rm f.o.}}{T_{\rm KRE}} \lesssim \frac{m_\chi}{20}\frac{1}{T_{\rm BBN}} \sim 10^4\frac{m_\chi}{100\ {\rm GeV}}.$$

After **chemical** decoupling (number density freezes out),

DM can still be in **kinetic** equilibrium

(i.e. its **velocity** distribution is in equilibrium)

generically, this is the case, since for **cold** relics

$$\chi\chi \leftrightarrow f f \qquad \rightarrow \quad \Gamma = n_{\mathrm{non-rel}} \cdot \sigma$$
$$\chi f \leftrightarrow \chi f \qquad \rightarrow \quad \Gamma = n_{\mathrm{rel}} \cdot \sigma$$

Think of a **prototypical WIMP**:

$$\sigma_{\chi f \leftrightarrow \chi f} \sim G_F^2 T^2$$

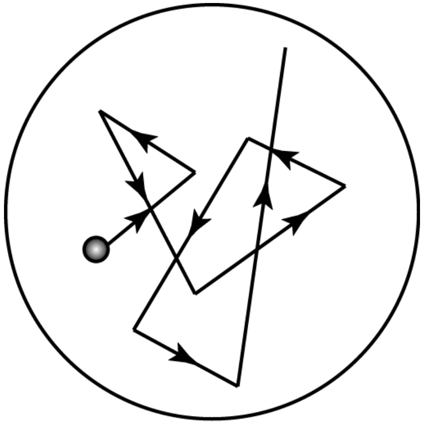
Problem: every collision has a **momentum transfer** $\delta p \sim T$

...but we need to keep the (cold) DM momentum in equilibrium, i.e.

$$\frac{p^2}{2m_\chi} \sim T \qquad p \sim \sqrt{m_\chi T}$$

so ***δp << p***, we need a bunch of kicks!

However, **subtlety**: kicks are in **random directions**!



$$N = \left(\frac{p}{\delta p}\right)^2 \sim \frac{m_\chi T}{T^2} = \frac{m_\chi}{T} \gg x_{\rm f.o.} \gtrsim 20$$

Let's estimate a typical WIMP **kinetic decoupling temperature**

$$n_{\rm rel} \cdot \sigma_{\chi f \leftrightarrow \chi f} \left(\frac{\delta p}{p}\right)^2 \sim T^3 \cdot G_F^2 T^2 \cdot \frac{T}{m_\chi} \sim H \sim \frac{T^2}{M_P}.$$

$$T_{\rm kd} \sim \left(\frac{m_\chi}{M_P \cdot G_F^2}\right)^{1/4} \sim 30 \text{ MeV} \left(\frac{m_\chi}{100 \text{ GeV}}\right)^{1/4}$$

What does this implies for **structure formation**?

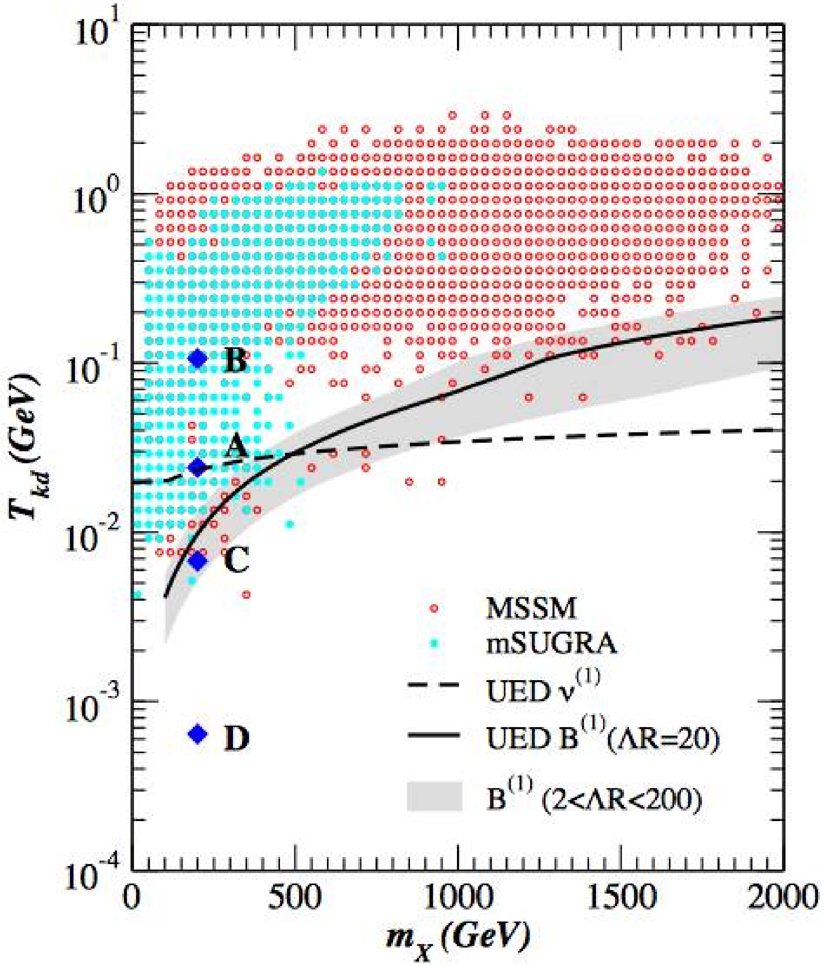
$$M_{\rm ao} \sim \frac{4\pi}{3}\left(\frac{1}{H(T_{\rm kd})}\right)^3 \rho_{\rm DM}(T_{\rm kd}) \sim 30\ M_\oplus \left(\frac{10\ {\rm MeV}}{T_{\rm kd}}\right)^3$$

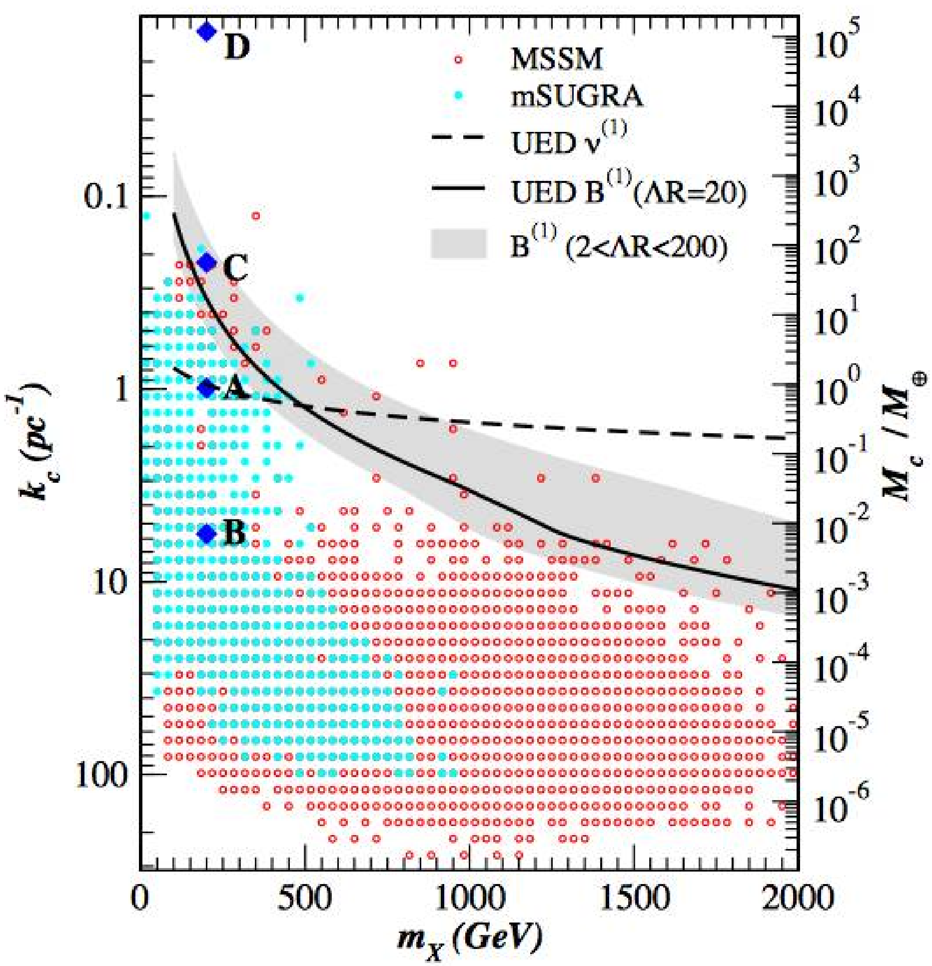
$$M_\oplus \simeq 3\times 10^{-6} M_\odot$$

First structures that collapse are these tiny **minihalos**
(maybe some survive today?)

Structures then **merge** into bigger and bigger halos
(**bottom-up** structure formation)

Notice that the kinetic decoupling/cutoff scale **varies** significantly even for a selected particle dark matter scenario!
e.g. for **SUSY, UED**





What happens instead for **hot relics**?

They decouple when **$T >> m_\nu$**

Structures can only collapse when **$T \sim m_\nu$**

(i.e. when things slow down enough for gravitational collapse!)

Structures are cutoff to the **horizon size** at that temperature

$$d_\nu \sim H^{-1}(T \sim m_\nu) \qquad d_\nu \sim \frac{M_P}{m_\nu^2}$$

$$d_\nu \sim \frac{M_P}{m_\nu^2}$$

$$M_{\text{cutoff, hot}} \sim \left(\frac{1}{H(T = m_\nu)}\right)^3 \rho_\nu(T = m_\nu) \sim \left(\frac{M_P}{m_\nu^2}\right)^3 m_\nu \cdot m_\nu^3 = \frac{M_P^3}{m_\nu^2}$$

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\ M_\odot \left(\frac{m_\nu}{30\ \text{eV}}\right)^{-2} \sim 10^{12}\ M_\odot \left(\frac{m_\nu}{1\ \text{keV}}\right)^{-2}$$

How does this compare with **observations**?

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\ M_\odot \left(\frac{m_\nu}{30\ \mathrm{eV}}\right)^{-2} \sim 10^{12}\ M_\odot \left(\frac{m_\nu}{1\ \mathrm{keV}}\right)^{-2}$$

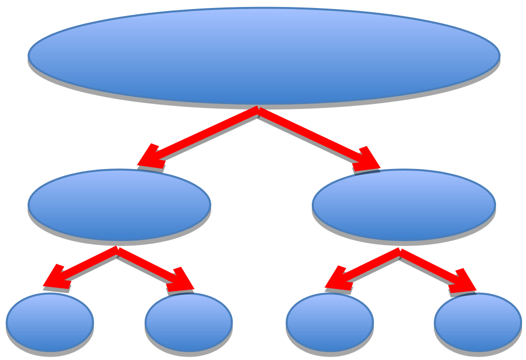
Observational **constraints** give

$$M_{\mathrm{cutoff}} \ll M_{\mathrm{Ly}-\alpha} \simeq 10^{10}\ M_\odot$$

So at best dark matter can be **keV** scale, if produced thermally

Structure formation looks strikingly different

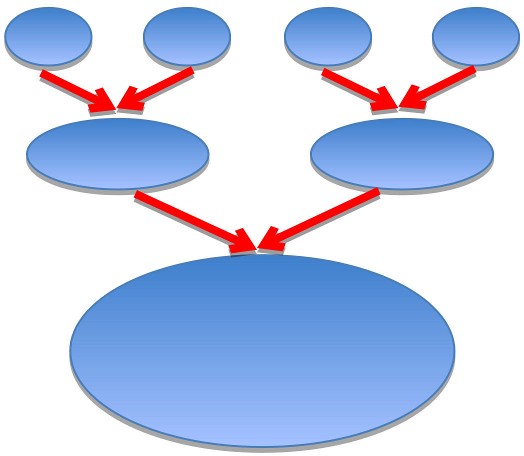
for hot and cold dark matter



Hot Dark Matter

Top-Down

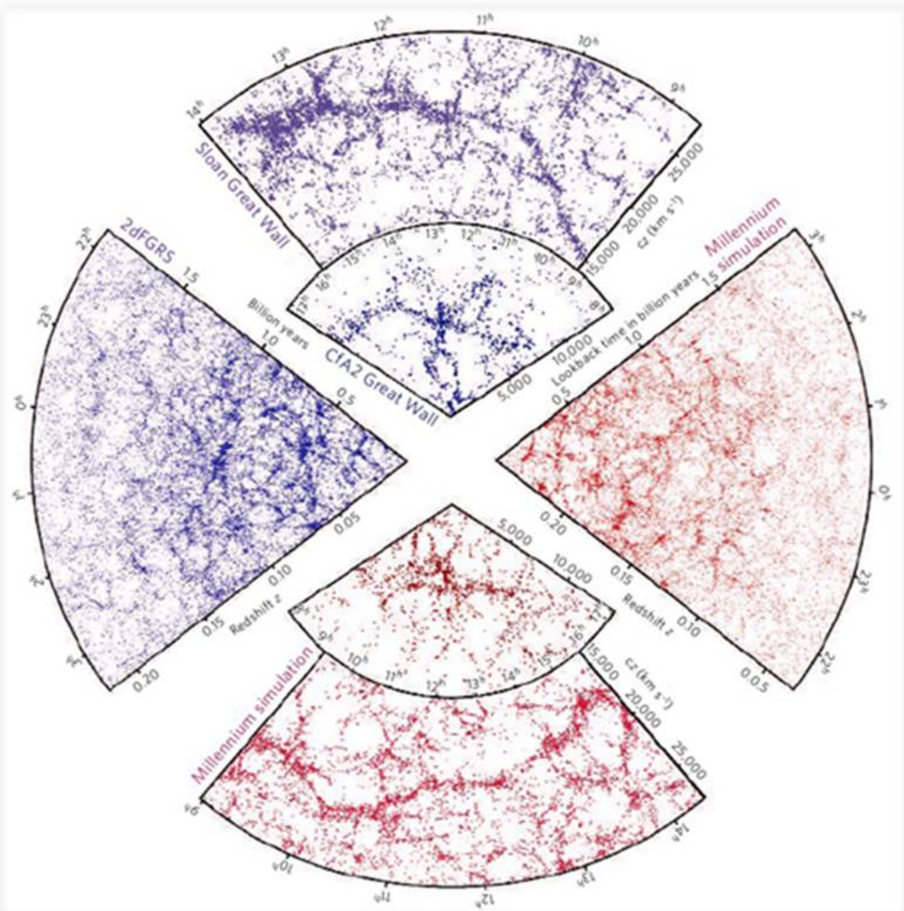
[doesn’t work!]



Cold Dark Matter

Bottom-Up

[Yeah!]



1980's: Davis, Efstathiou, Frenk and White show that simulations of structure formation in a universe with **cold dark matter** match observed structure incredibly well!!

…so something we **know** quite well about **Dark Matter** is its **abundance**

DM average density in "**astro**" units…

$$\bar{\rho}_{\rm DM} \sim 10^{10}\ \frac{M_\odot}{\rm Mpc^3}$$

clusters… 10^5 denser!

in "**particle physics**" units…

$$\bar{\rho}_{\rm DM} \sim 10^{-6}\ \frac{\rm GeV}{\rm cm^3}$$

galaxies… 10^6 denser!

Knowledge of the dark matter average **density**
is a powerful **model-building** tool

Models that **predict** the “right” **amount** of dark matter get kudos

Dark Matter “**cosmogony**” well-motivated guideline to model building

prototypical example: dark matter
as a ***thermal relic***… as we saw before,
but this is **NOT the only possible** story!

Stefano Profumo

Santa Cruz Institute for Particle Physics
University of California, Santa Cruz

Cosmology and Dark Matter

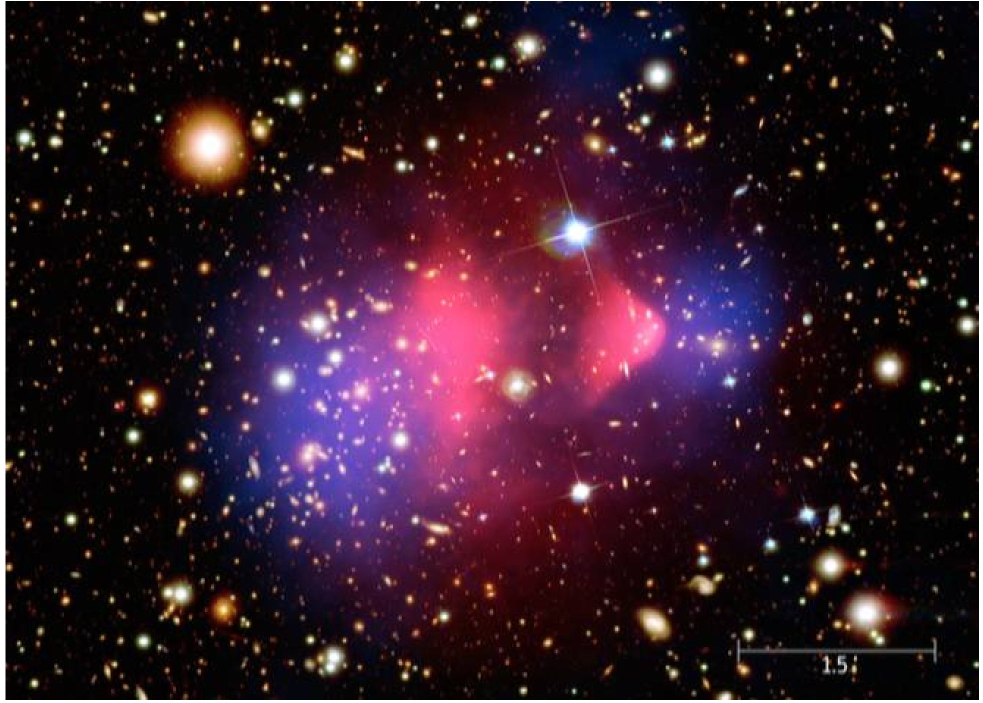
Lecture 4

IX NExT PhD Workshop
The Cosener's House, Abingdon, UK
July 8-11th, 2019

What else do we know about the **microscopic** nature of dark matter from its **macroscopic** features?

- "**Dark**" …but detailed constraints on electric charge of dark matter are model-dependent… Milli-charge allowed… Phenomenologically: DM is nearly **dissipationless** (maybe not entirely though, see dark photons, dark disks…)

- **Collisionless**… really? Let's calculate the relevant constraints!

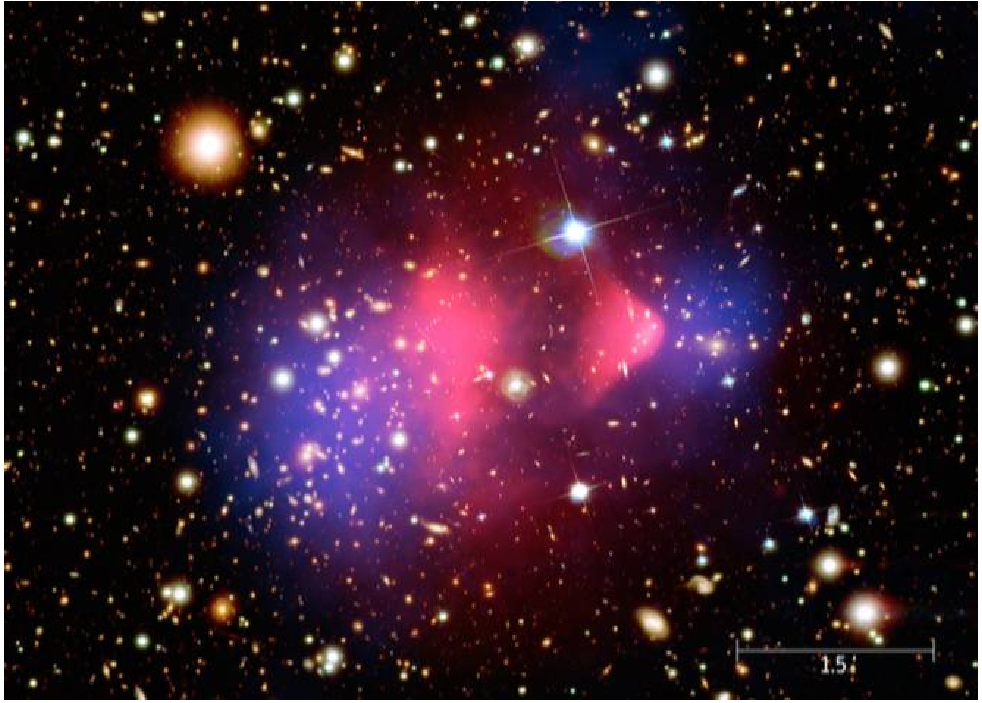


mean free path λ larger than cluster size, ~ 1 Mpc

cluster **density**: ρ ~ 1 GeV/cm^3, thus…

$\lambda = 1/(\sigma\ (\rho/m)) > 1$ Mpc ✉ σ /m < 1 Mpc / 1 GeV/cm^3

✉ ***σ /m < 1 cm^2/g***, or ***1 barn/GeV***



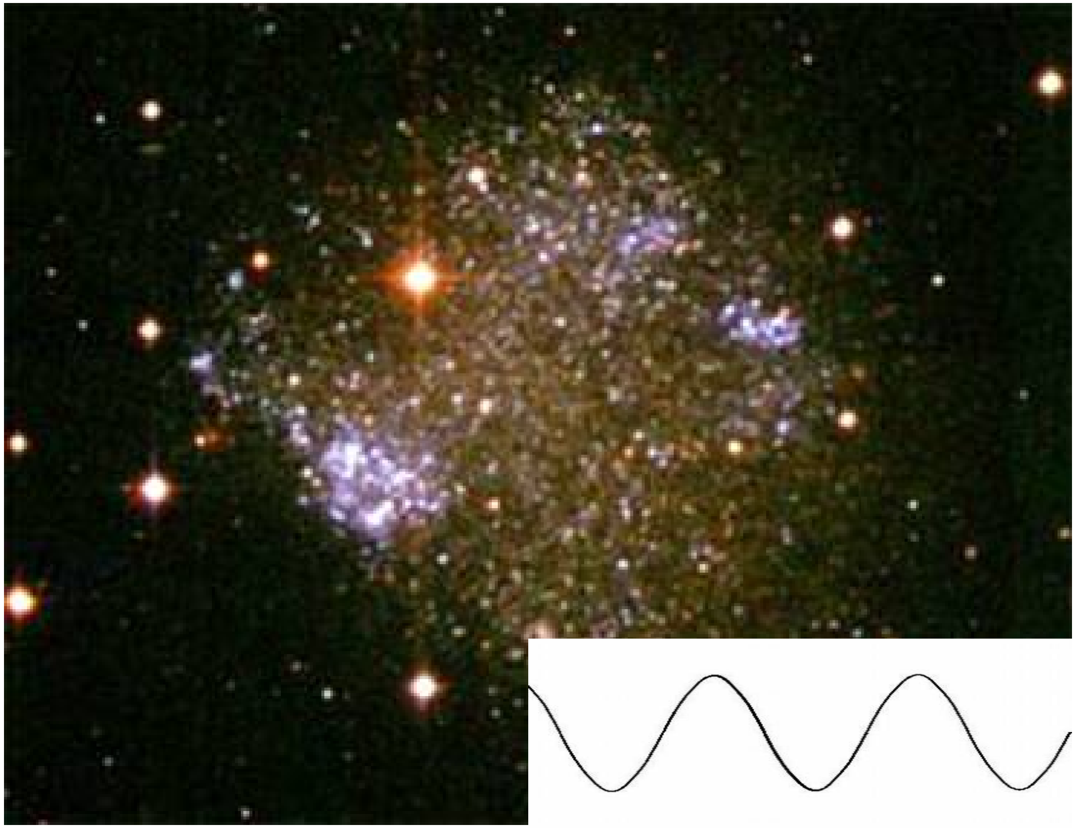
1 barn/GeV… which is **strong interaction**-size…

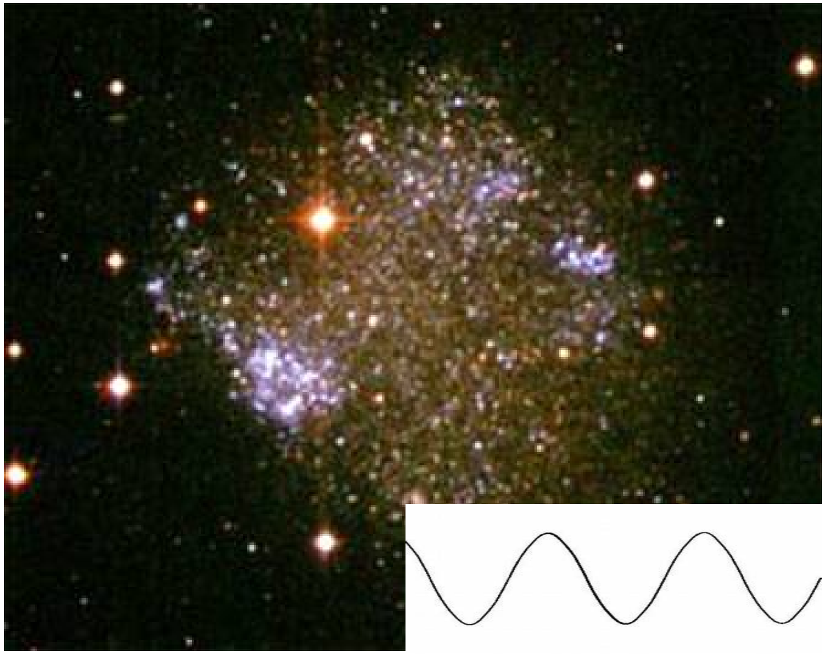
is this **small**?

Also, if cross section is **slightly smaller**, no **visible effect**…
if cross section **slightly larger**, **disaster**…

Begs the question: is “collisional” **self-interacting** dark matter a “**natural**” possibility??

- **Classical**: needs to be confined (gravitationally bound) on scales at least as large as dSph… if de Broglie wavelength is larger, disaster strikes!





little exercise: consider $v \sim 100$ *km/s*, show that $\boldsymbol{\lambda = h/p}$ is

$$\lambda \sim 3 \text{ mm} \left(\frac{1 \text{ eV}}{m}\right)$$

which means that to have $\lambda \ll$ kpc ~ $3\text{x}10^{21}$ cm, **$m > 10^{-22}$ eV**

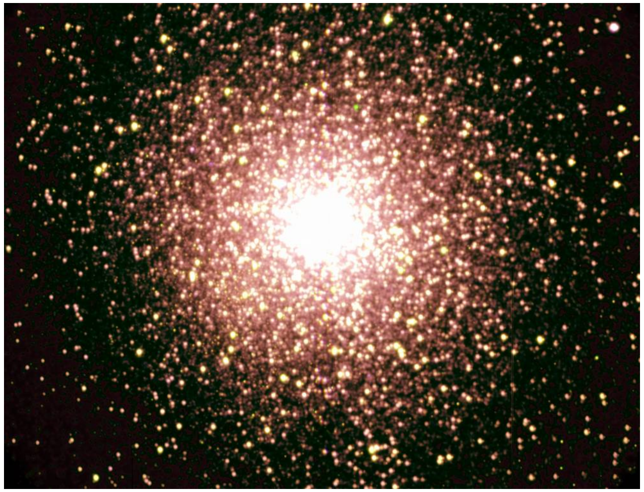
Much, much **better constraints** if the DM is a fermion – we know that the **phase space** density is bounded (Pauli blocking): $f = gh^{-3}$

Using **observed** density and velocity dispersion of dSph, **Tremaine-Gunn** limit (1979): observed phase space density cannot exceed upper bound! (Liouville theorem) Exercise!

$\sigma \sim 150$ km/s $\qquad$ $\rho \gtrsim 1$ GeV/cm^3

$$m^4 > \frac{\rho h^3}{[g(2\pi\sigma^2)^{3/2}]} \sim (25\ \mathrm{eV})^4.$$

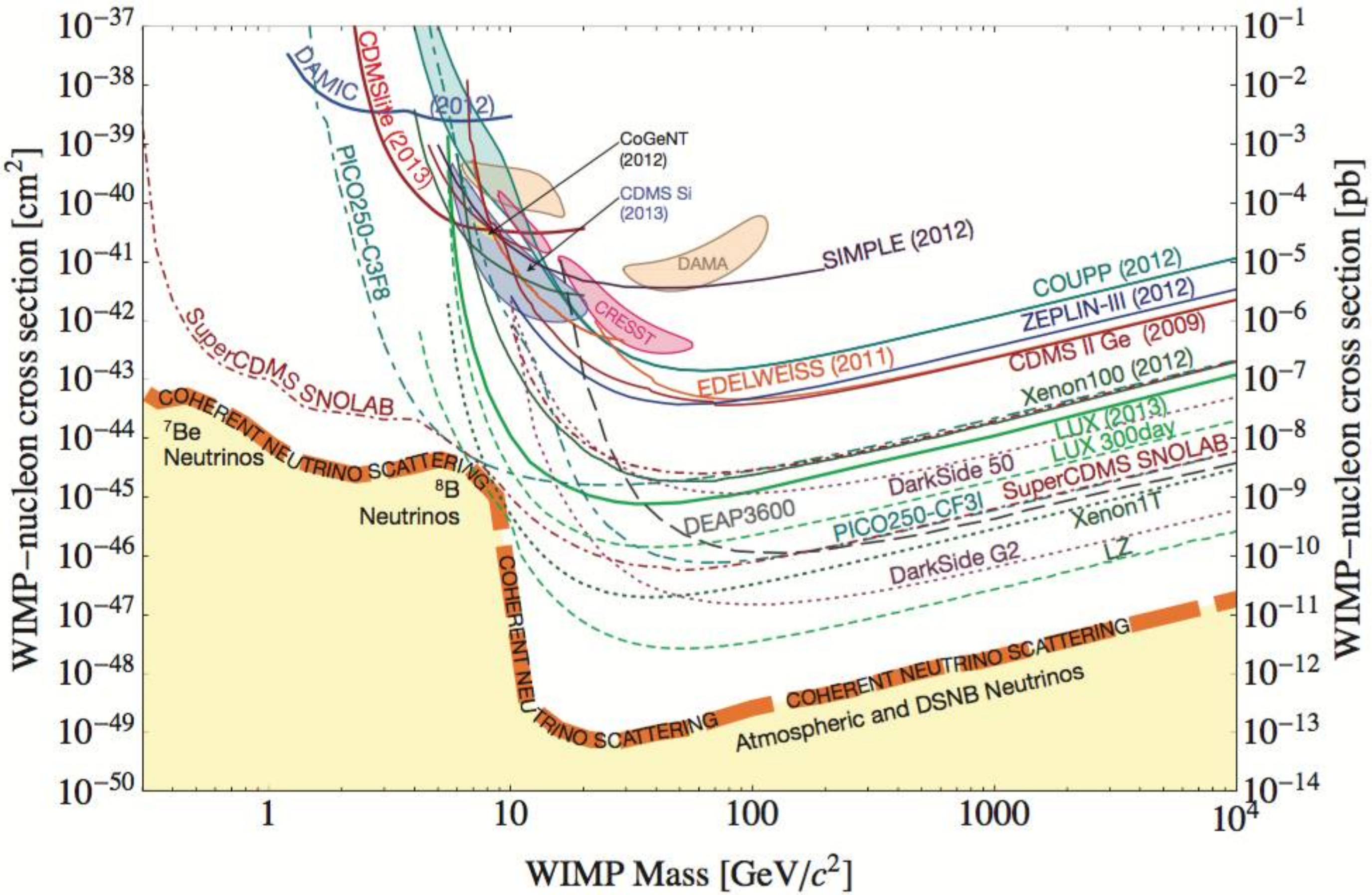
- **Fluid**: don't want to **disrupt** pretty (and old!) **clusters** of stars



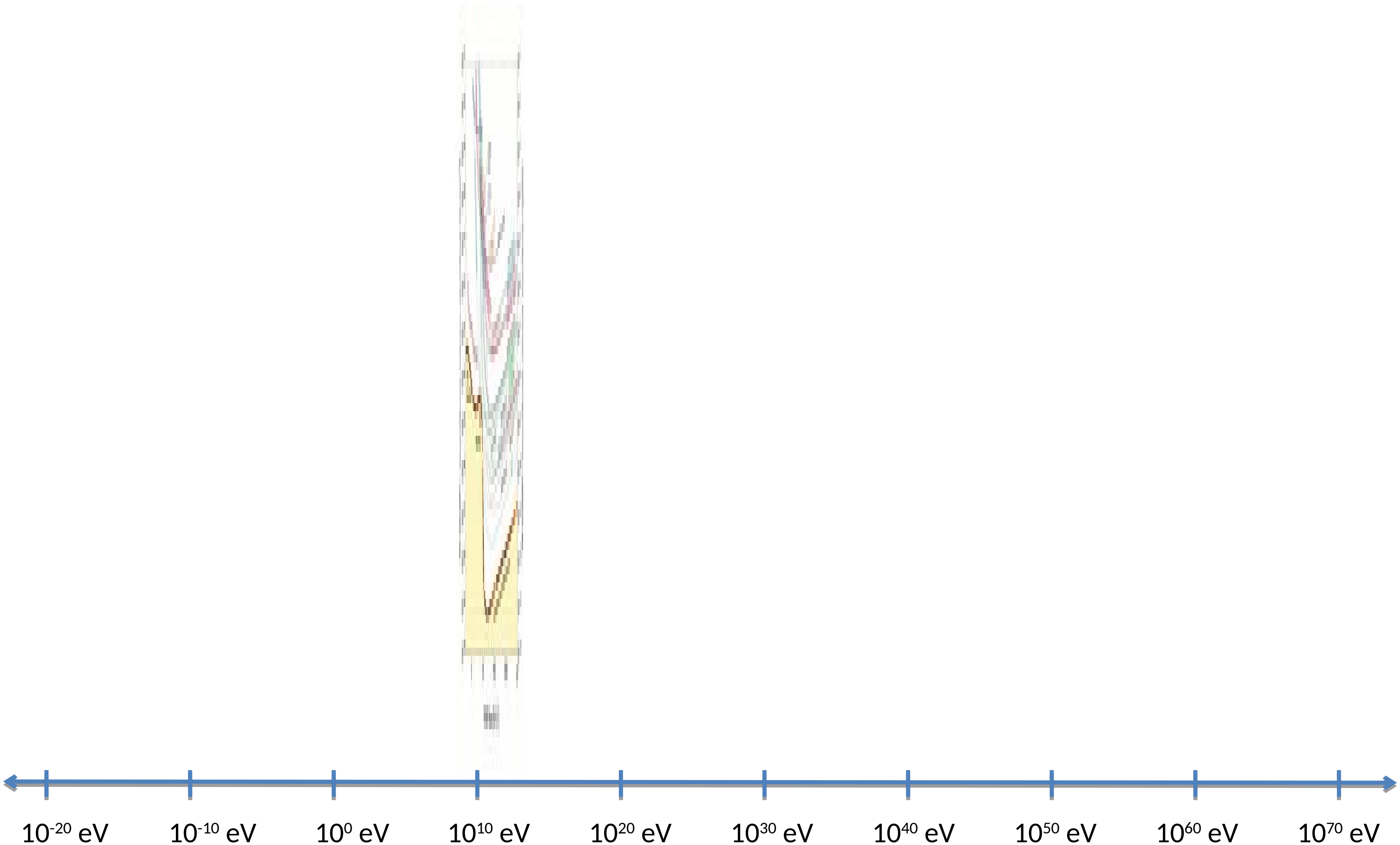
Neat exercise to estimate the **energy exchanged** by encounters of GC and BH, in the impulse approximation, demand that that energy be smaller than binding energy, get maximal mass for BH

Also constraints on **disk stability** ("heating")

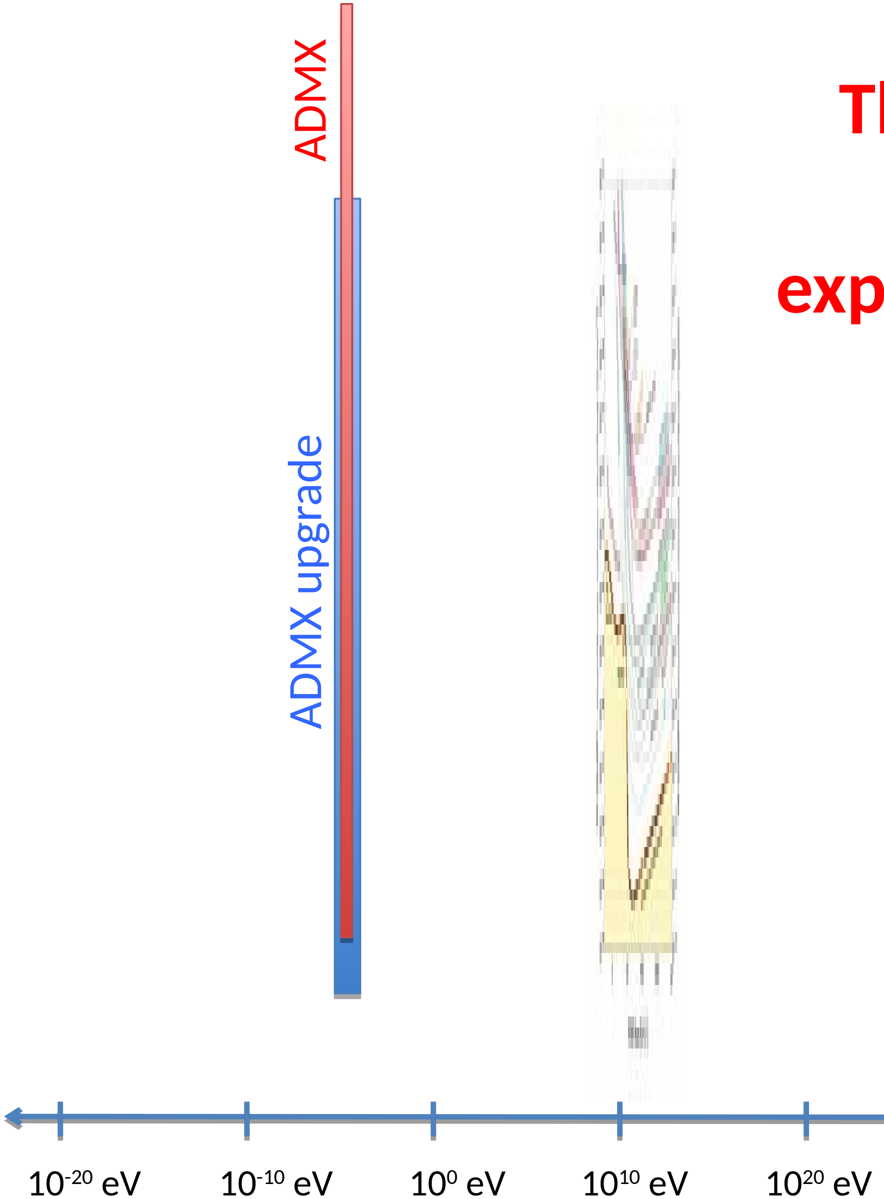
Bottom line: $m < 10^3$ solar masses ~ 10^{70} eV

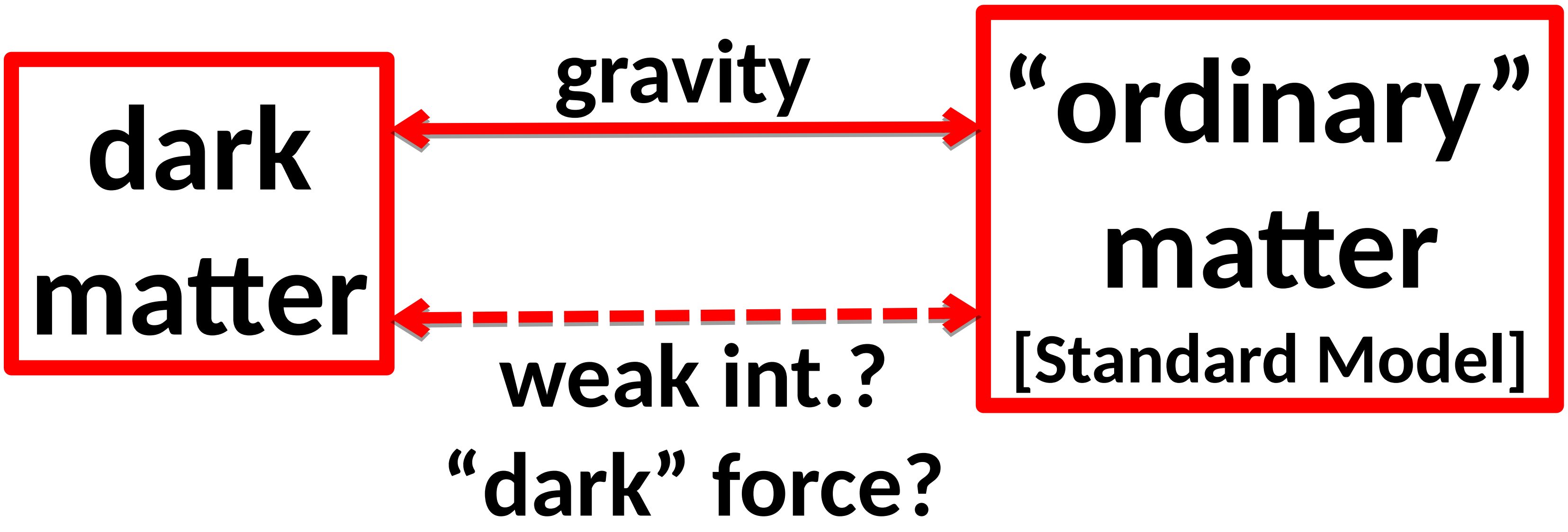


10^{-20} eV 10^{-10} eV 10^{0} eV 10^{10} eV 10^{20} eV 10^{30} eV 10^{40} eV 10^{50} eV 10^{60} eV 10^{70} eV

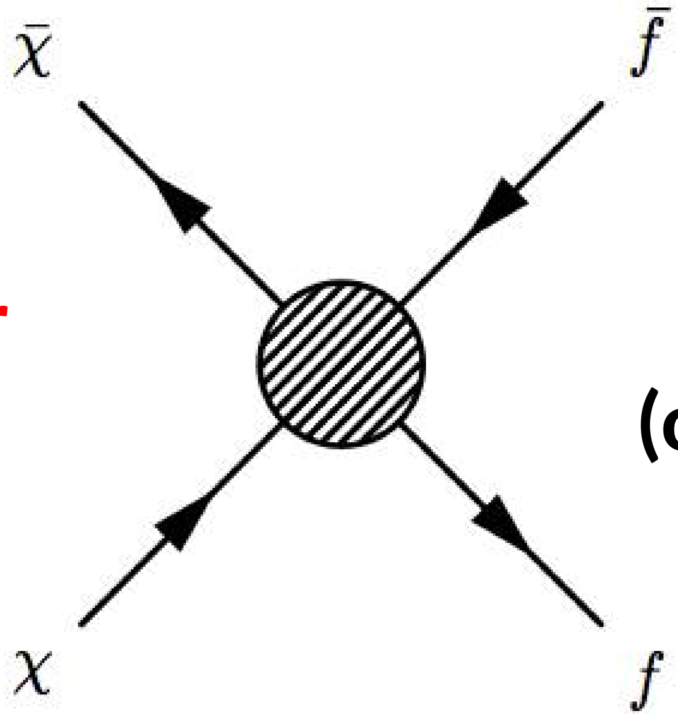

10^{-20} eV
10^{-10} eV
10^{0} eV
10^{10} eV
10^{20} eV
10^{30} eV
10^{40} eV
10^{50} eV
10^{60} eV
10^{70} eV

Theorists must keep an open attitude, and observers / **experimentalists** must cast a very wide net!



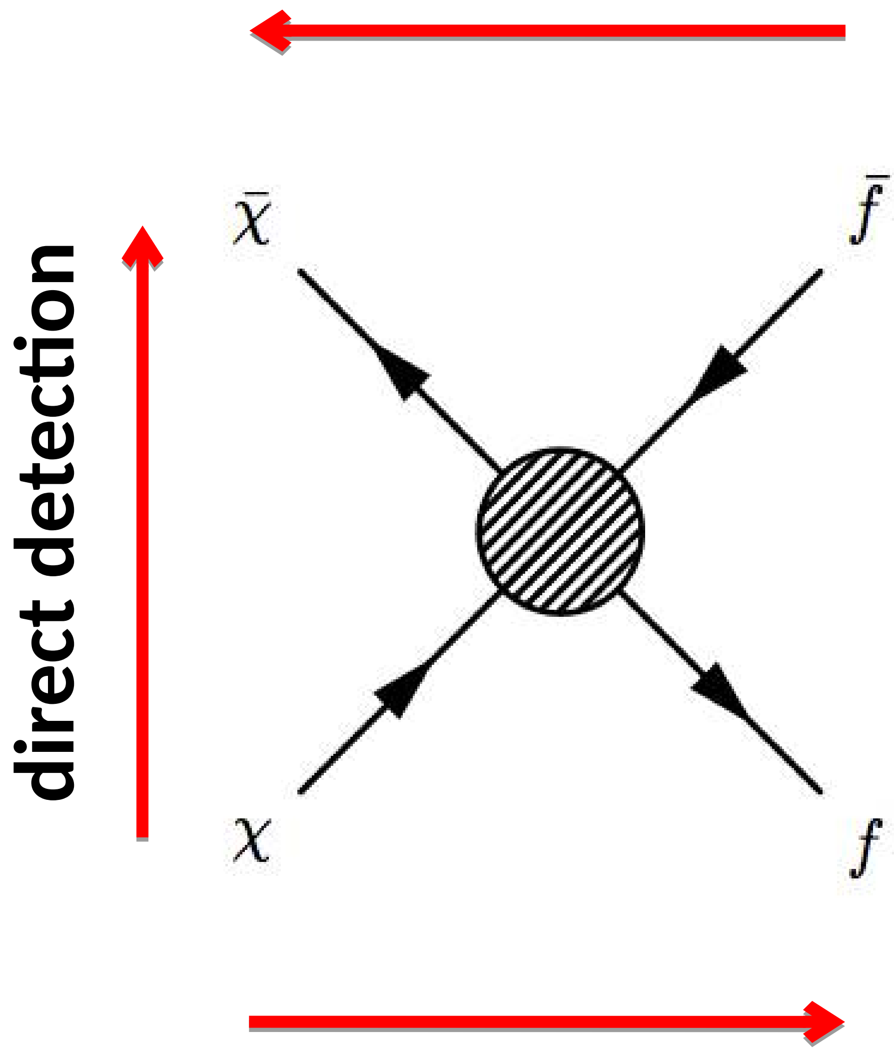

dark matter
gravity
weak int.?
"dark" force?
"ordinary" matter
[Standard Model]

Dark Matter
Particles

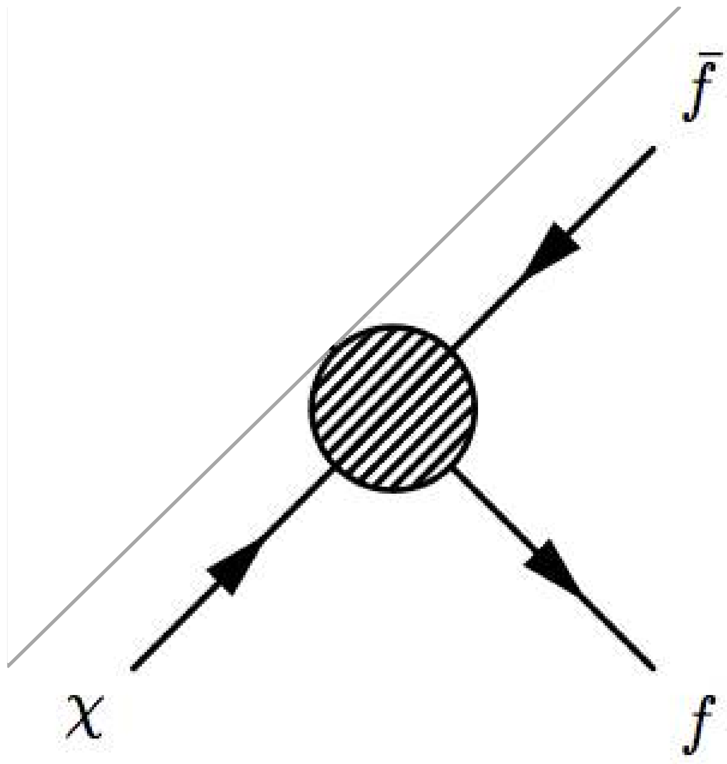


Standard Model
(ordinary) Particles

collider production



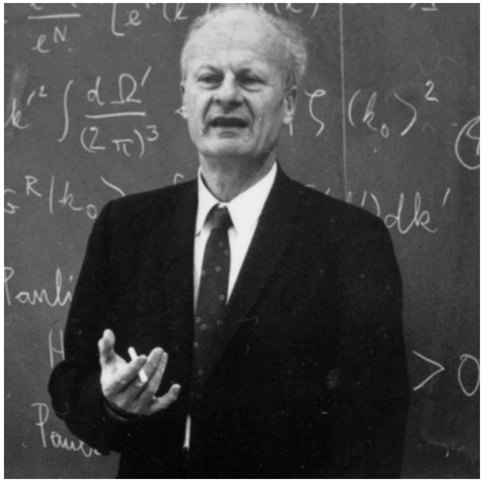
thermal equilibrium ?
[pair annihilation]



long-lived, but metastable

Consider **direct** detection

Detecting particles that interact **weakly** has always been known to be a **tough job**



H. Bethe

R. Peierls

After **estimating** in 1934 the **cross section** for $\bar{\nu}_e + p \to e^+ + n$

$$\sigma_{\bar{\nu}_e + p \to e^+ + n} \approx 10^{-43} \ (E_\nu/\mathrm{MeV})^2 \ \mathrm{cm}^2.$$

"It is therefore ***absolutely impossible*** to observe processes of this kind"

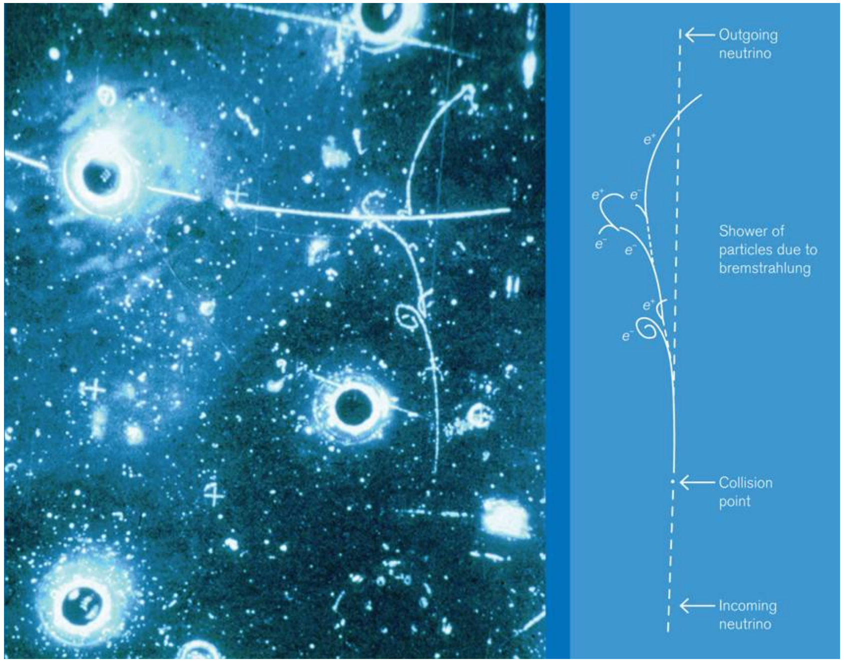
Bethe and Peierls were too **pessimistic**/**conservative**:

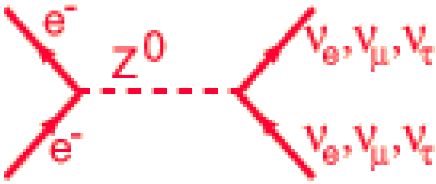
neutrinos were detected in 1953, abundantly in 1956

Inelastic process (maybe relevant for DM?)

$$\bar{\nu}_e + p \to e^+ + n \qquad\qquad \chi + X \to \chi' + Y,$$

Elastic neutrino scattering took **much longer** (Gargamelle 1973)





Let's use **WIMPs** again as **prototypical** DM particles

First, which **energies** and what **masses** are we talking about?

maximal recoil momentum for a DM particle
with velocity v is $\boldsymbol{2m_\chi v}$, so maximal energy to nucleons N is

$$E_{\max} = (2m_\chi v)^2/(2m_N)$$

Now, the **maximal velocity** a DM particle can have in the Galaxy is the **escape velocity** $v_{max} \sim 500\text{-}700\ km/s$

✉ ***E~ keV*** for GeV particles!

Plug in numbers for a detector with an energy threshold ~ keV...
minimal detectable **DM mass ~ GeV**

OK, now what about the **event rate**?

$$R = K\phi\sigma.$$

$$K \simeq 6.0 \times 10^{26}/A$$

$$\phi = v\rho_{\mathrm{DM}}/m_\chi$$

Plug in sensible **benchmark** values…

$$R = \frac{0.06\ \text{events}}{\text{kg day}}\left(\frac{100}{A}\right)\left(\frac{\sigma}{10^{-38}\ \text{cm}^2}\right)\left(\frac{\rho_{\mathrm{DM}}}{0.3\ \text{GeV}/\text{cm}^3}\right)\left(\frac{v}{200\ \text{km/s}}\right)$$

To have a detection need both enough **signal** events,

and enough **background** suppression

1. slowly decaying "primeval" nuclides (U, Th, ^{40}K),

   ab. 10^{-4}, half lives ~10^{9} yr

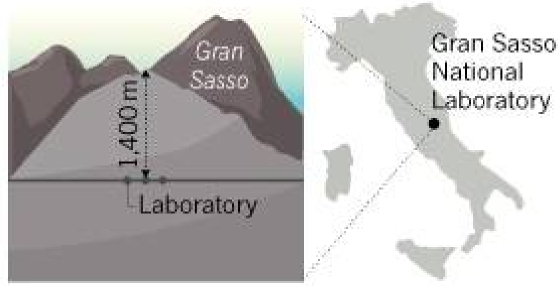
2. rare, fast decaying trace elements like tritium, ^{14}C:

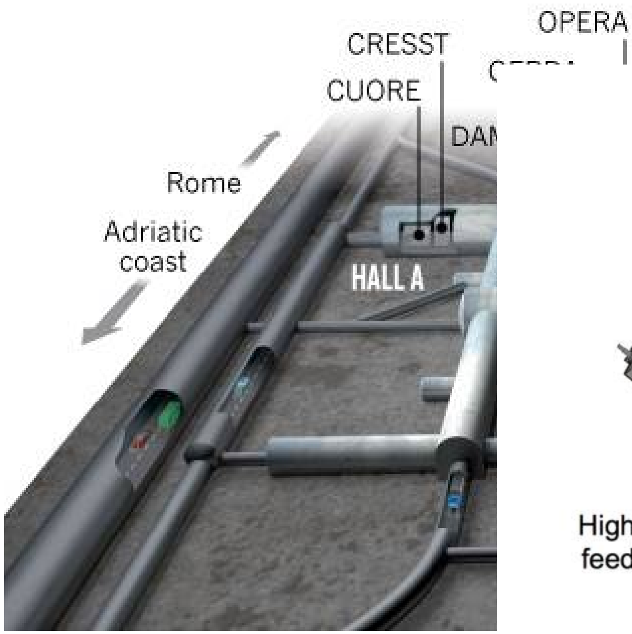
   ab 10^{-18} , half lives 10 yr

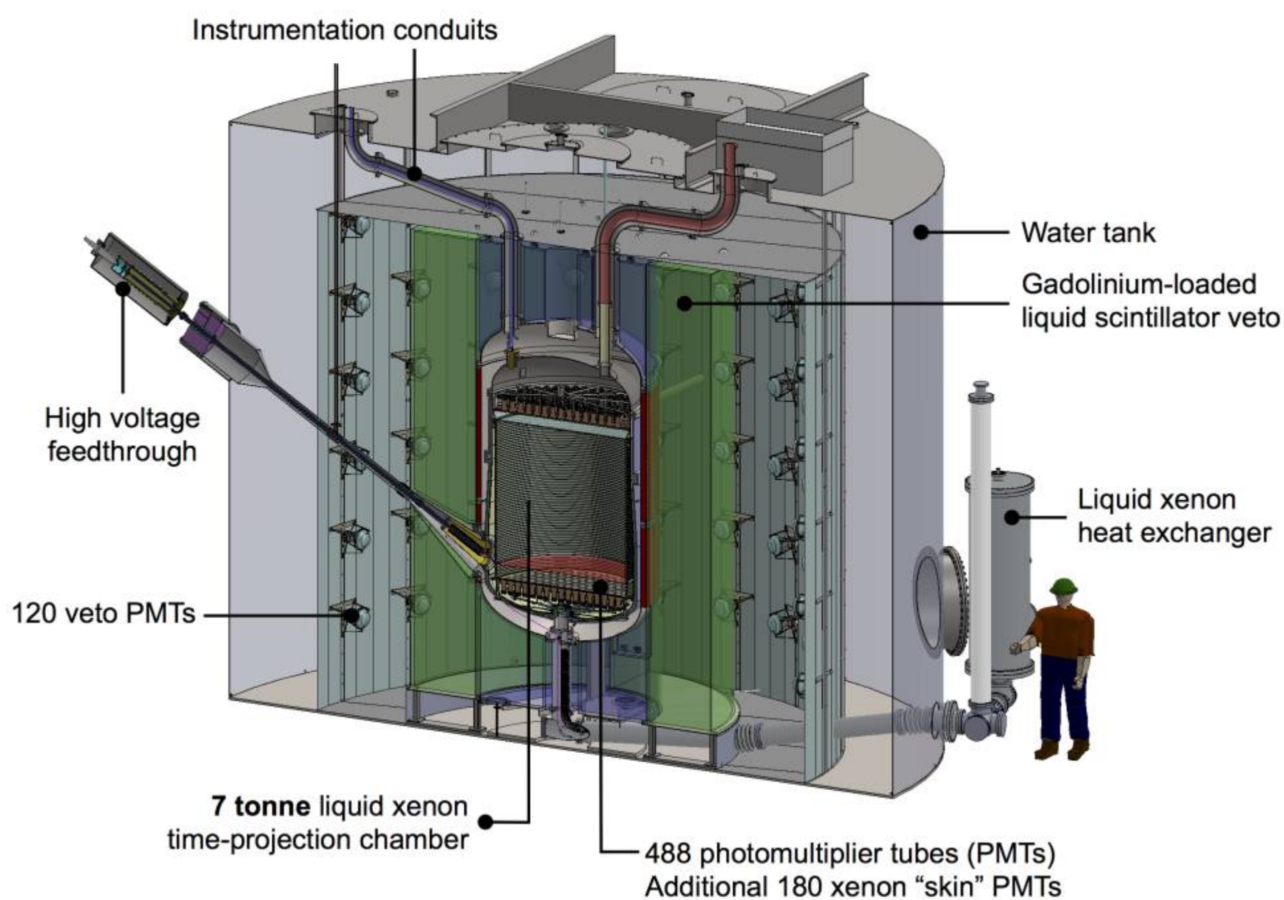
Big detectors, in **underground**, actively **shielded** environments…

THE A, B AND C OF GRAN SASSO

Experiments at the Gran Sasso National Laboratory are housed in and around three huge halls carved deep inside the mountain, where they are shielded from cosmic rays by 1,400 metres of rock.

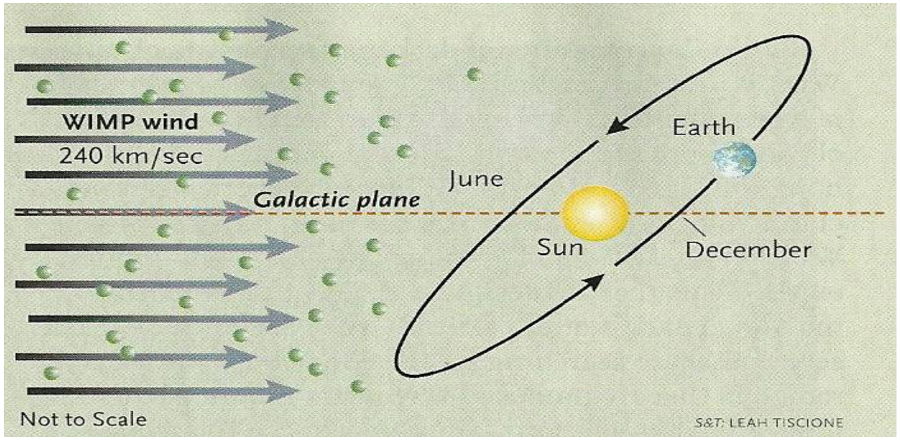






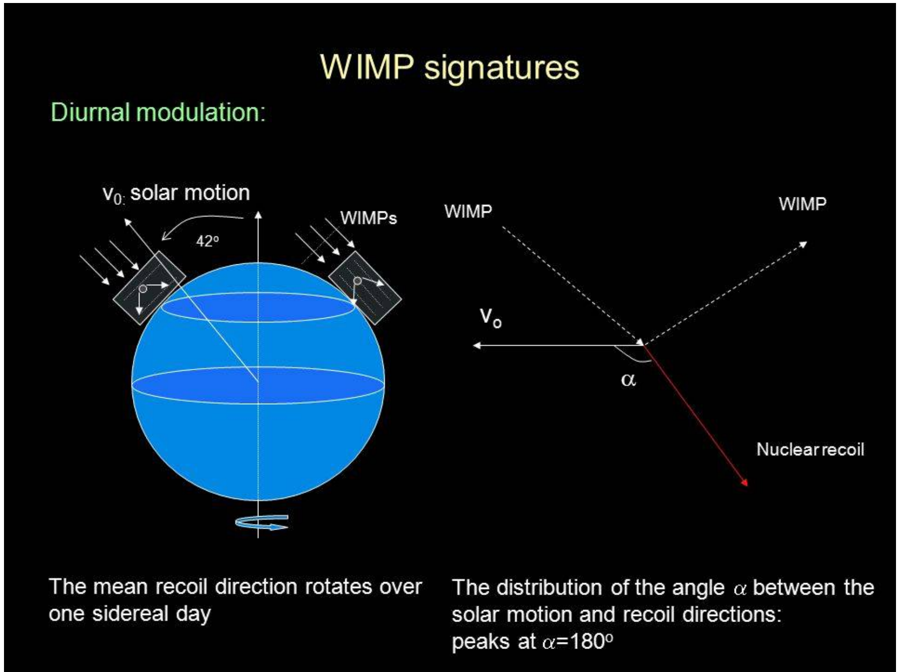
Handles on a DM **signal** versus radioactive **background**:

1. **Seasonal** modulation



2. **Diurnal** modulation

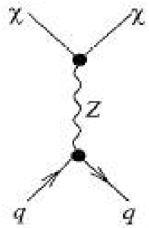
3. **Directional** information

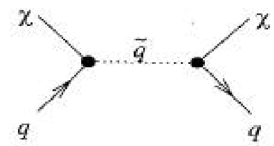


Given a **microscopic** theory of dark matter,
how does one get to the **DM-nucleus cross section**?

An interesting **multi-layered** problem in **effective field theory**!

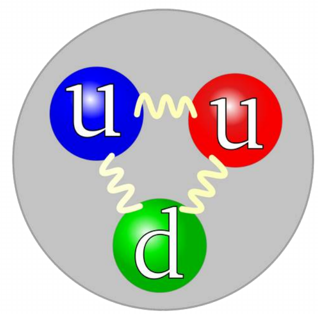
(1) Low-energy EFT **Dark Matter-quark**





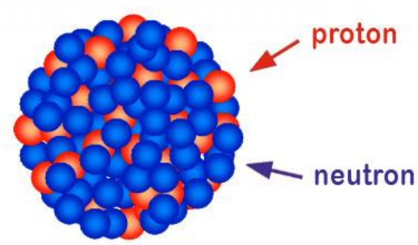
(2) Nucleon matrix elements

Dark Matter-nucleon



(3) Form factors

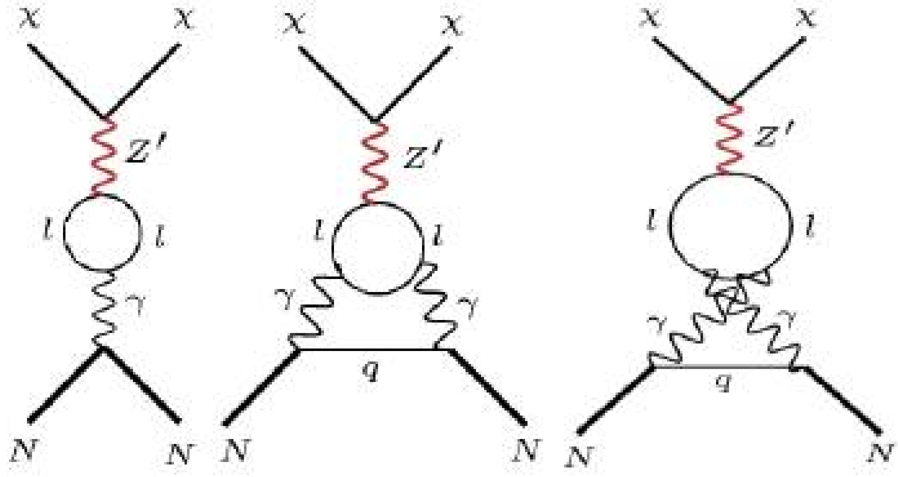
Dark Matter-nucleus

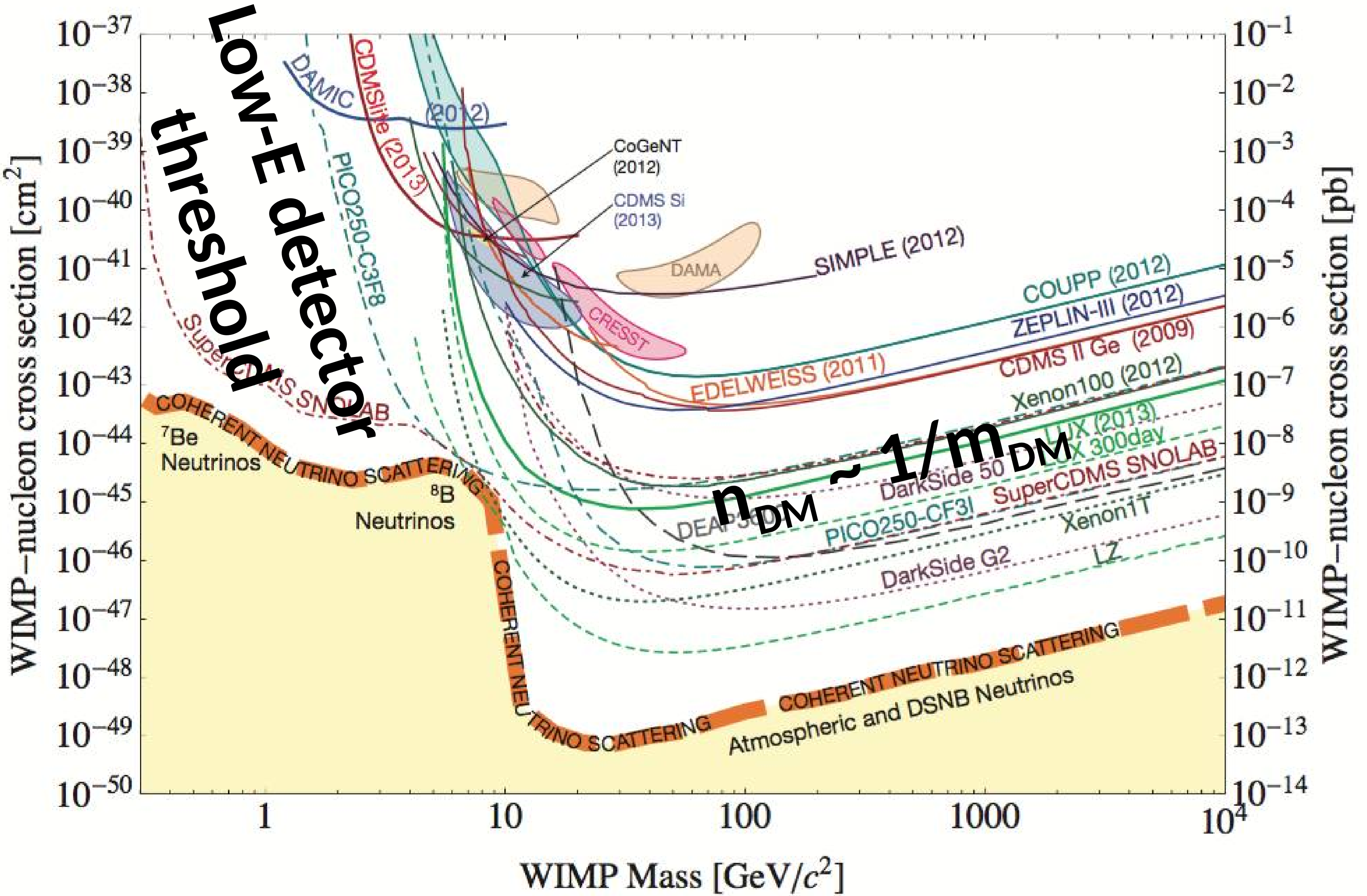


Sometimes life is simpler, e.g. if DM is (**milli**-electric-)**charged**

$$\sigma_N = \frac{16\pi\alpha^2\varepsilon^2 Z^2 \mu_N^2}{q^4}$$

Sometimes life is nastier, e.g. if DM is **lepto-philic**




Low-E detector threshold
$n_{DM} \sim 1/m_{DM}$
WIMP–nucleon cross section [cm^2]
WIMP–nucleon cross section [pb]
WIMP Mass [GeV/c^2]
DAMIC
CDMSlite (2013)
(2012)
PICO250-C3F8
CoGeNT (2012)
CDMS Si (2013)
DAMA
SIMPLE (2012)
CRESST
COUPP (2012)
ZEPLIN-III (2012)
CDMS II Ge (2009)
EDELWEISS (2011)
Xenon100 (2012)
LUX (2013)
LUX 300day
SuperCDMS SNOLAB
DarkSide 50
PICO250-CF3I
Xenon1T
DarkSide G2
LZ
COHERENT NEUTRINO SCATTERING
^{7}Be Neutrinos
^{8}B Neutrinos
Atmospheric and DSNB Neutrinos

Now off to **indirect** dark matter detection

Idea: use the **debris** of DM **pair-annihilation**
(likely large if thermal relic) or **decay**

$$\Gamma_{\mathrm{SM,\ ann}} \sim \left(\int_V \frac{\rho_{\mathrm{DM}}^2}{m_\chi^2} \mathrm{d}V \right) \times (\sigma v) \times (N_{\mathrm{SM,\ ann}}) \,,$$

What do we know about these **rates**?

σv from **thermal production** (with caveats!)

What about annihilation **final state**?

Very **model-dependent**

1. if DM belongs to an SU(2) **multiplet**, then well-defined combination of *ZZ*, *WW* final states…

2. In UED, DM is KK-1 mode of **hypercharge gauge boson**, thus

$$|M|^2 \propto |Y_f|^4 \qquad Y_{u_L} = 4/3 \quad Y_{e_R} = 2$$

3. Special "**selection rule**", e.g. helicity suppression for Marjorana fermion (analogous to charged pion decay)

$$|M|^2 \propto m_f^2$$

Annihilation (or decay) of DM can be **detected** or **constrained** in a variety of ways

Here's one possible **classification**:

1. **Very Indirect**: effects induced by dark matter on **astrophysical objects** or on **cosmological observations**

2. **Pretty Indirect**: probes that don't "trace back" to the annihilation event, as their trajectories are bent as the particles propagate: **charged cosmic rays**

3. **Not-so-indirect**: **neutrinos** and **gamma rays**, with the great added advantage of traveling in **straight lines**

Very indirect probes include e.g.

- **Solar Physics** (dark matter can affect the Sun's core temperature, the sound speed inside the Sun,…)
- **Neutron Star Capture**, possibly leading to the formation of black holes (notably e.g. in the context of asymmetric dark matter)
- **Supernova** and **Star** cooling
- **Protostars** (e.g. WIMP-fueled population-III stars)
- **Planets warming**
- **Big Bang Nucleosynthesis**, on the **cosmic microwave background**, on **reionization**, on **structure formation**…

Pretty Indirect Probes: **charged cosmic rays**

Good idea is to use **rare** cosmic rays, such as **anti-matter**

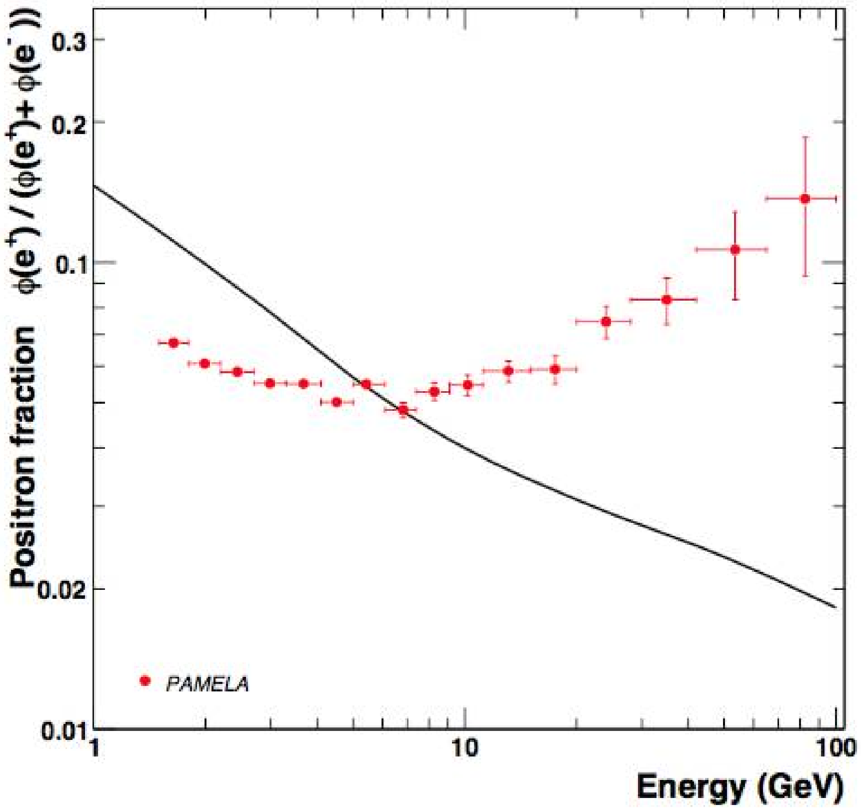
antiprotons, **positrons** relatively abundant
(mostly from inelastic processes CR p on ISM p)

Interesting probe: **antideuterons** (or even **anti-^{3}He** !!)

$$\bar{D}: \quad p + p \rightarrow p + p + \bar{p} + p + \bar{n} + n$$

large energy **threshold** (~17 GeV), so typically large momentum, while from DM produced at very low momentum! Select **low-energy antideuterons**

positrons (and in part antiprotons) have attracted attention because of "**anomalies**" reported by PAMELA, AMS-02, DAMPE



Very **hard** to explain the anomaly with **dark matter**!

- No excess **anitprotons** – must be "leptophilic" (possible but not generic)

- No observed **secondary radiation** from brems or IC

- Needed **pair-annihilation rate** very large for thermal production, leads to unseen gamma-ray or radio emission

$$\langle \sigma v \rangle \sim 10^{-24} \frac{\mathrm{cm}^3}{\mathrm{s}} \cdot \left(\frac{m_\chi}{100\ \mathrm{GeV}} \right)^{1.5}$$

Alternate explanation: nearby **point source** injecting a burst of **positrons** (a.k.a. Green's function, a.k.a. **PSR**)

$$\psi \propto Q \cdot \exp\left(-\left(\frac{r}{r_{\rm diff}}\right)^2\right)$$

Estimate **Age** and **Distance** of putative source

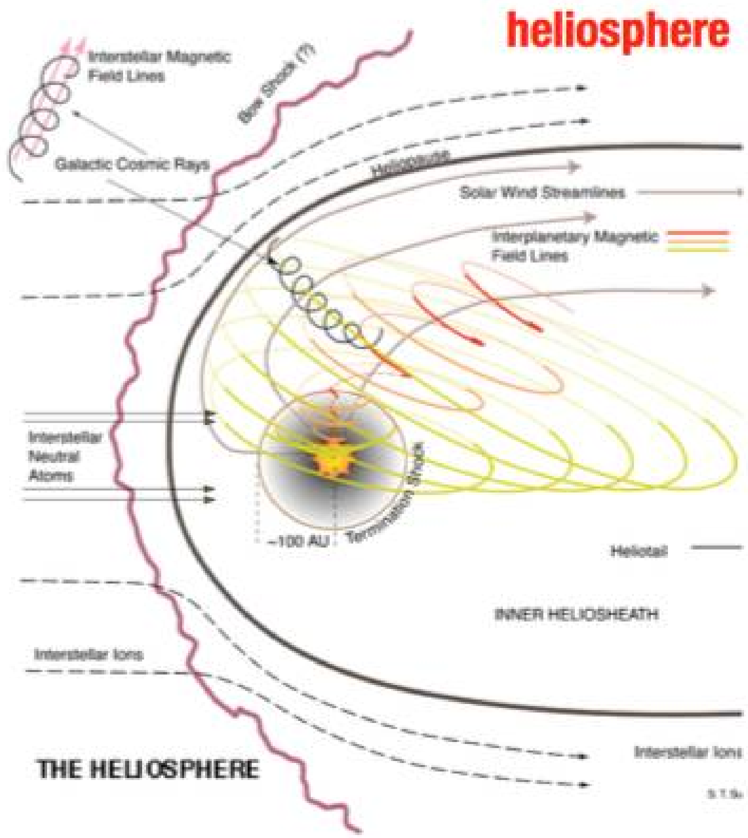
$$t_{\rm psr} \ll \tau_{\rm loss} = \frac{E}{b(E)}; \text{ for } E = 100 \text{ GeV}, \tau_{\rm loss} \sim \frac{100}{10^{-16} \cdot 100^2} \text{ s} \sim 10^{14} \text{ s} \sim 3 \text{ Myr}.$$

$$r_{\rm diff} \simeq \sqrt{D(E) \cdot t}.$$

$$\sqrt{D(E) \cdot t_{\rm psr}} \gg \text{ distance} \rightarrow \text{distance} \ll (3 \times 10^{28} \cdot 100^{0.7} \cdot 10^{14})^{1/2} \text{ cm} \sim 10^{22} \text{ cm} \sim 3 \text{ kpc}.$$

One possible way to **disentangle** PSR from DM: **anisotropy**

Complication: Larmor radius for **heliospheric** magnetic fields ***B~ nT***, is of the order of the **solar system size** (exercise)



Not-so-indirect DM detection: **neutrinos**!

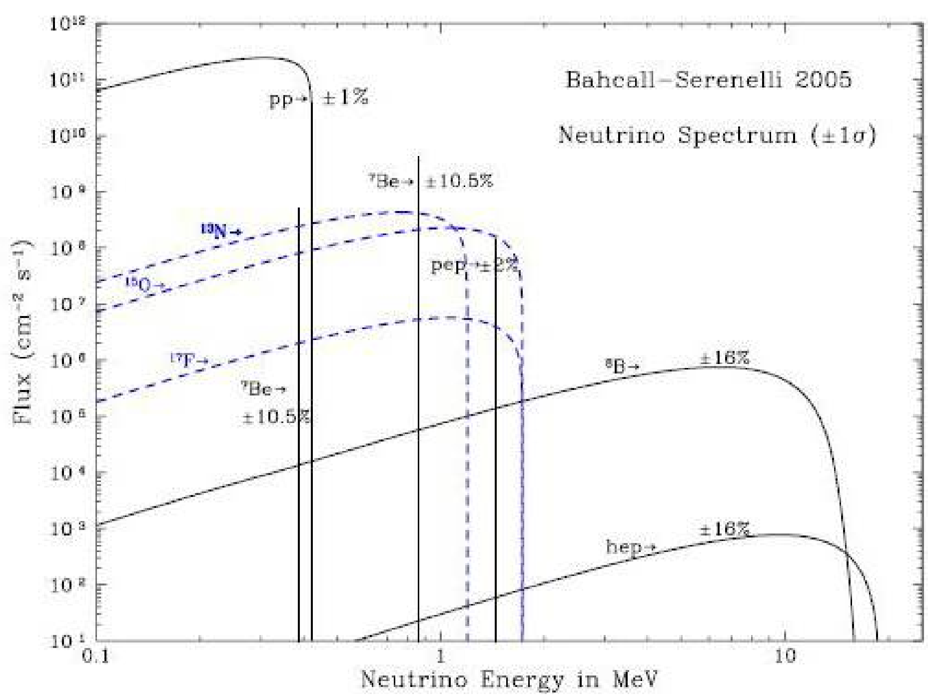
Only **two** observed astrophysical sources of neutrinos!

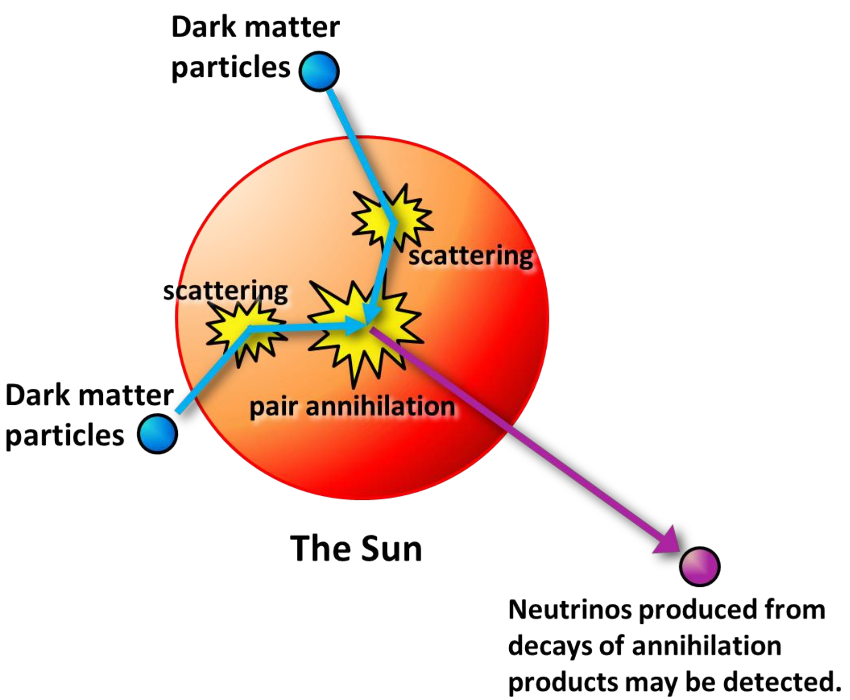
Hard (but not impossible) to detect particles

flip side: neutrinos have very **long mean free paths** in matter!

idea: DM can be **captured** in celestial bodies, **accrete** in sizable densities, start pair-annihilating

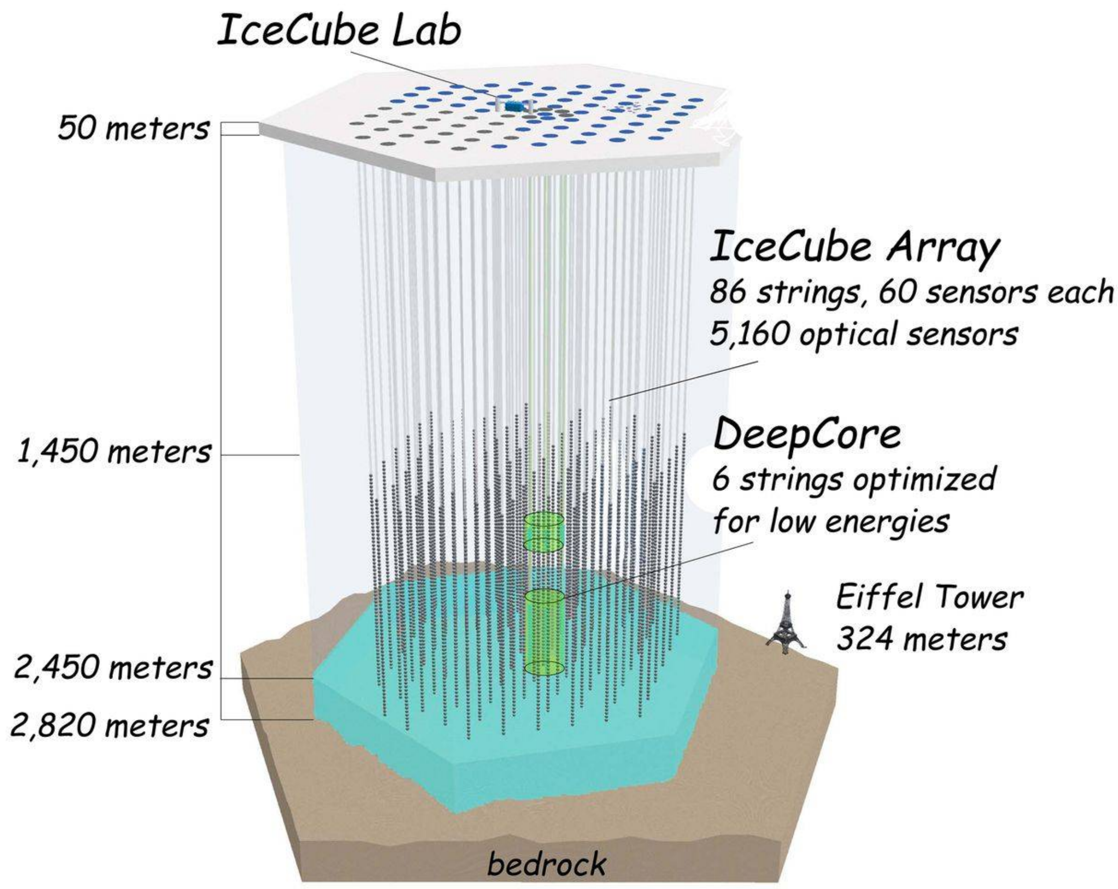
if the process of capture and annihilation is in **equilibrium**, large **fluxes** of neutrino can escape best target: **Sun**! Large, **nearby**, **low-E** neutrino emission



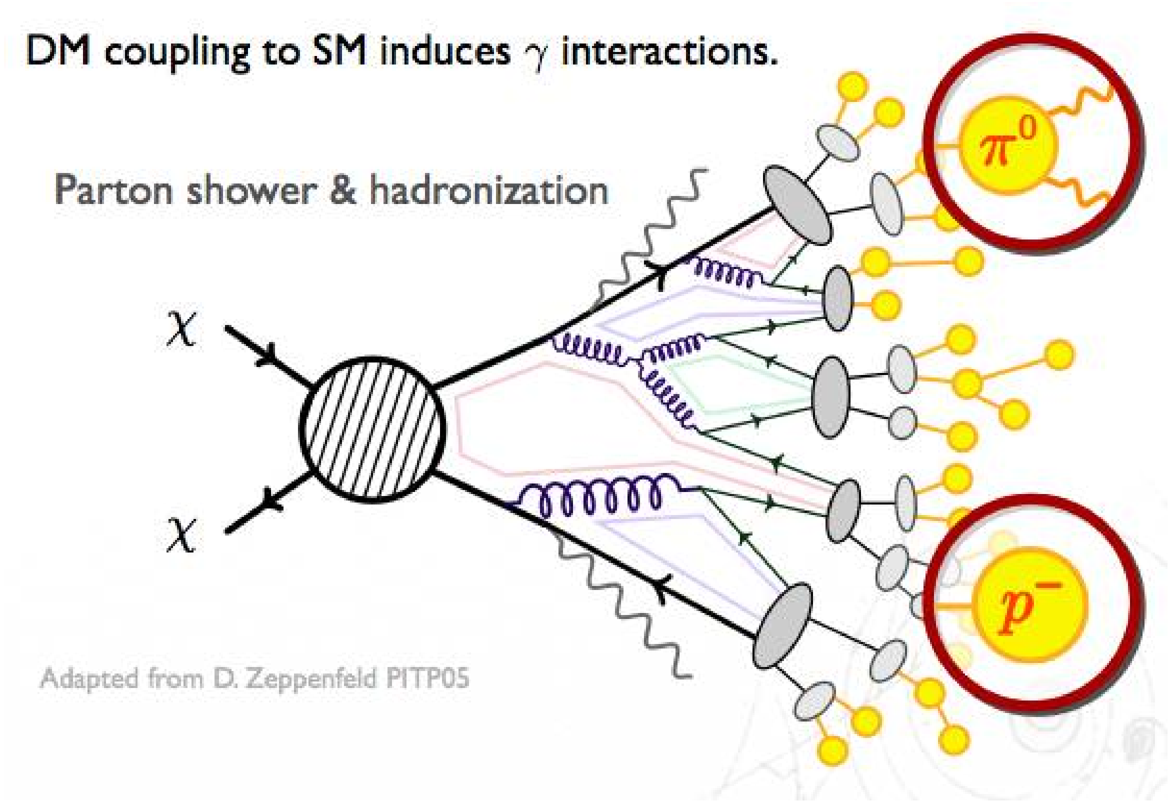


So far **no anomalous events** from Sun observed; **Earth** less promising

Opportunities with
lower-energy
threshold sub-detectors
DeepCore, **PINGU**



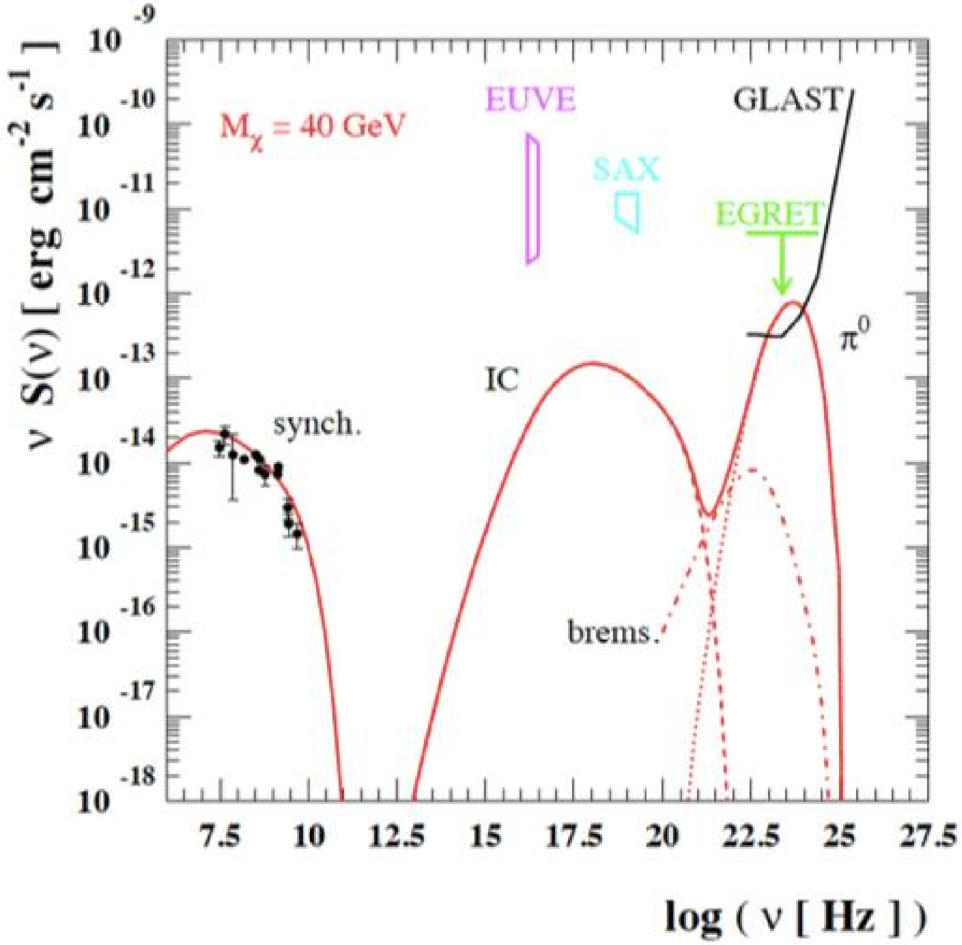
Light from **dark matter**!



Primary photons: prompt, or internal brems; just run Pythia (if you can!)

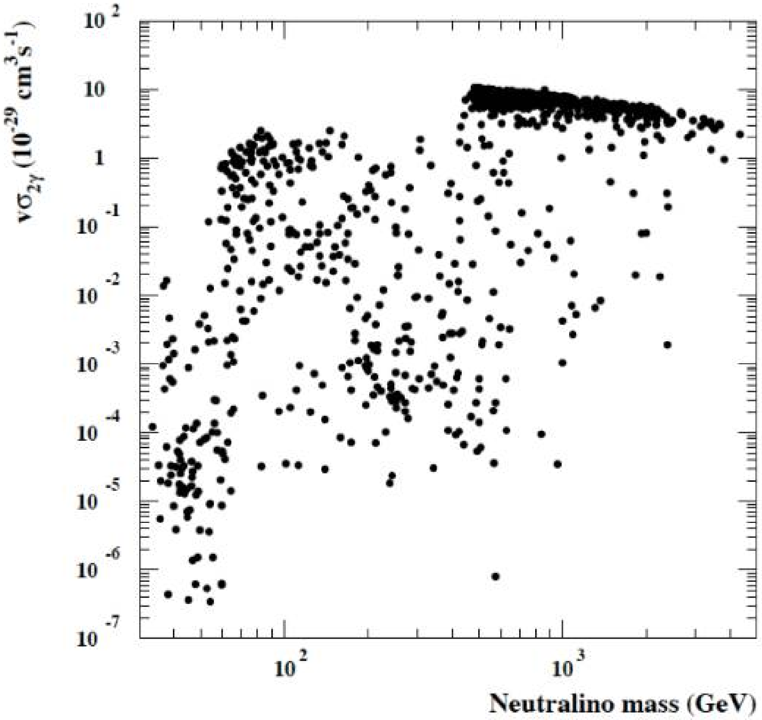
Secondary photons: Inverse Copmton, synchrotron

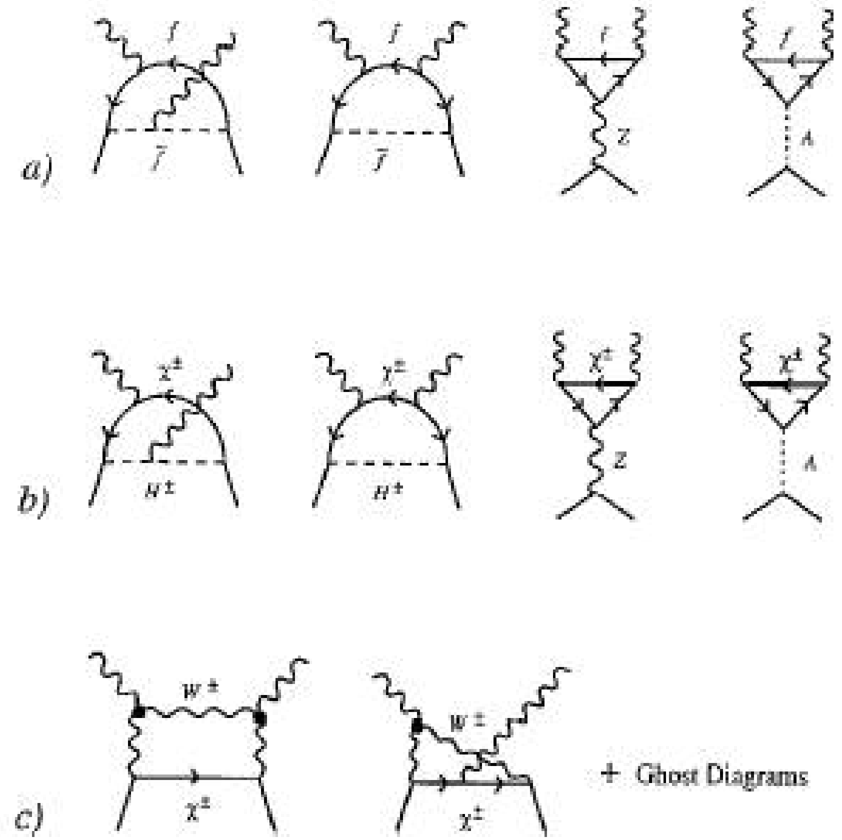
Overall **emission** looks like this, e.g. in a **cluster** of galaxies



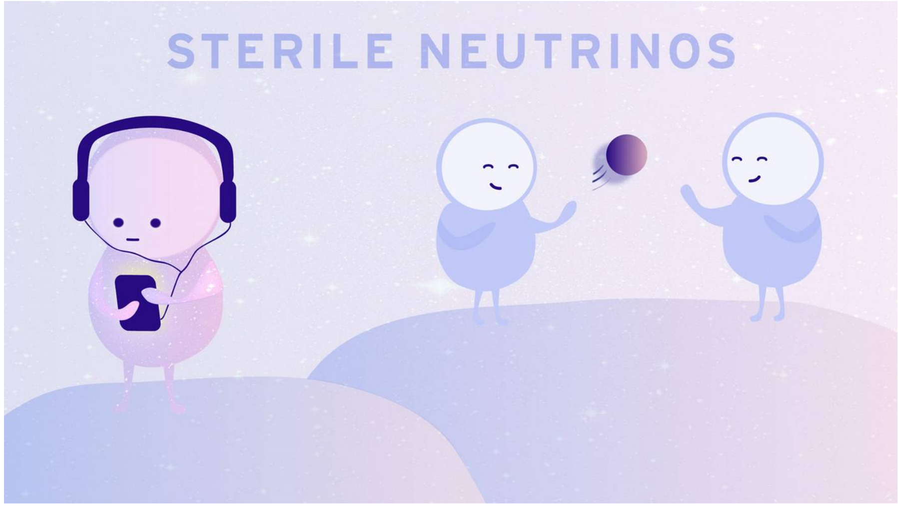
In addition, **monochromatic** photons $\chi\chi \to \gamma\gamma$

$$\frac{\langle\sigma v\rangle_{\gamma\gamma}}{\langle\sigma v\rangle_{\rm tot}} \sim \frac{\alpha^2}{16\pi^2}$$






AXION
EL VERDADERO ARRANCAGRASA
Antibacterial
AXION
EL VERDADERO ARRANCAGRASA
Limón
AXION
EL VERDADERO ARRANCAGRASA
BICARBONATO
AXION
EL VERDADERO ARRANCAGRASA
OXY
Plus
BICARBONATO
AXION
EL VERDADERO ARRANCAGRASA
Lima


STERILE NEUTRINOS

Axions and **ALP**s as dark matter candidates

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{j=1}^{n} \left[\bar{q}_j \gamma^\mu i D_\mu q_j - \left(m_j q^\dagger_{\mathrm{L}j} q_{\mathrm{R}j} + \mathrm{h.c.} \right) \right] + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

"theta" term innocuous **perturbatively** (total derivative), but entering pheno via **non-perturbative QCD effects**, producing large **neutron el. dipole moment**

$$d_n \simeq 5 \times 10^{-16} \bar{\theta} \ \ e \text{ cm} \qquad\qquad d_n < \text{few} \times 10^{-26} \ \ e \text{ cm}$$

PQ: promote θ to **dynamical** variable,
driven to zero by its own **classical potential**

Postulate a global (quasi-)**symmetry** of the theory (broken by non-perturbative effects) $U(1)_{PQ}$;
Symmetry spontaneously **broken** at a scale f_a.
Axion is the (pseudo-)Nambu-Goldstone boson associated with $U(1)_{PQ}$

Axion **mass** is $$m_a \sim \frac{\Lambda^2_{\rm QCD}}{f_a} \sim 0.6\,{\rm eV}\left(\frac{10^7\,{\rm GeV}}{f_a}\right)$$

QCD effects produce effective (slightly model-dependent) **couplings** to **fermions** and **photons**, which drive phenomenology

$$\mathcal{L}_{a\bar{f}f} = i g_f \frac{m_f}{(f_a/N)} a \bar{f}\gamma_5 f \qquad \mathcal{L}_{a\gamma\gamma} = -g_\gamma \frac{\alpha}{\pi}\frac{a}{f_a}\vec{E}\cdot\vec{B}.$$

Similar setup for axion-like particles (**ALP**s): new global U(1) symmetry spontaneously broken by a **hidden Higgs-type mechanism** at a scale v_h

Recast “Higgs” field as $$H_h(x) = \frac{1}{\sqrt{2}}\left(v_h + h_h(x)\right) e^{ia(x)/v_h}$$

The potential for the ALP field ***a(x)*** is flat, and depending on the model realization one generates **couplings** to SM particles

$$\mathcal{L}_{ALP} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{\alpha_s}{8\pi} C_{ag} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} - \frac{\alpha}{8\pi} C_{a\gamma} \frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} \frac{C_{af}}{f_a} \partial_\mu a \bar{f} \gamma^\mu \gamma^5 f$$

Because of coupling to SM particles,
axions **decay** to **two photons**,

$$\tau_{a\to\gamma\gamma} \sim \frac{16\pi^2}{\alpha^2}\frac{\Lambda^4_{\rm QCD}}{m_a^5} \simeq 10^{24}\,{\rm s}\left(\frac{1\,{\rm eV}}{m_a}\right)^5$$

To have a **sufficiently long-lived axion** we must demand

$$\tau_U \sim 10^{10}\times(\pi 10^7)\,{\rm s} \lesssim 10^{24}\,{\rm s}\left(\frac{1\,{\rm eV}}{m_a}\right)^5 \Rightarrow m_a \lesssim 25\,{\rm eV},\quad f_a \gtrsim 4\times 10^6\,{\rm GeV}$$

Axions can have dramatic **impact** on **stars**:
Compton-like and brems-like processes

$$\gamma + e \to a + e \qquad e + Z \to a + e + Z.$$

produce an **axion luminosity**, e.g. for the Sun, of

$$L_a \sim 6 \times 10^{-4} \left(\frac{m_a}{1\ \mathrm{eV}}\right)^2 L_\odot$$

Since solar luminosity is whatever it is, axion emission would require **enhanced nuclear energy production**, thus larger **neutrino** flux!
Limits are around 1 eV…

Axions would also **cool supernovae**, $L_a \sim 10^{59} \text{ergs/s} \left(\frac{m_a}{1 \text{ eV}}\right)^2$

$L_\nu \sim 10^{53}$ ergs/s

$L_a \gg L_\nu$ for $m_a \gg 10^{-3}$ eV.

If axions are **too massive**, they get **trapped** and they don't contribute to SN luminosity efficiently

$$10^{-3} \lesssim m_a/(1 \text{ eV}) \lesssim 2.$$

How can axions be **produced**? **Thermally**?

$$a+g \leftrightarrow \bar{q}+q \text{ or } g+g, \text{ or } a+q(\bar{q}) \leftrightarrow g+q(\bar{q})$$

$$\frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} \qquad \sigma_{q,g} \sim \frac{\alpha_s^3}{\pi^2 f_a^2}$$

$$\frac{\Gamma}{H} \sim 1 \quad \Rightarrow \quad N_c N_f T^3 \sigma_{q,g} \sim \frac{T^2}{M_P}$$

$$T_{\mathrm{th.ax.}} \simeq \text{few} \times 10^{11}\ \mathrm{GeV}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)$$

…but we know this doesn't work! **Hot DM** not good! Also, other constraints on axion mass… how about **non-thermal production**?

mis-alignment mechanism and axion strings

$$\Omega_{\rm mis} h^2 \simeq 0.4 \left(\frac{m_a}{10\ \mu\rm eV} \right)^{-1.18} \left(\frac{\bar\theta_1}{\pi} \right)^2$$

$$(\bar\theta_1)_{\rm RMS} \equiv \left(\int_{-\pi}^{\pi} {\rm d}\bar\theta_1 \frac{\bar\theta_1^2}{2\pi} \right)^{1/2} = \frac{\pi}{\sqrt{3}}$$

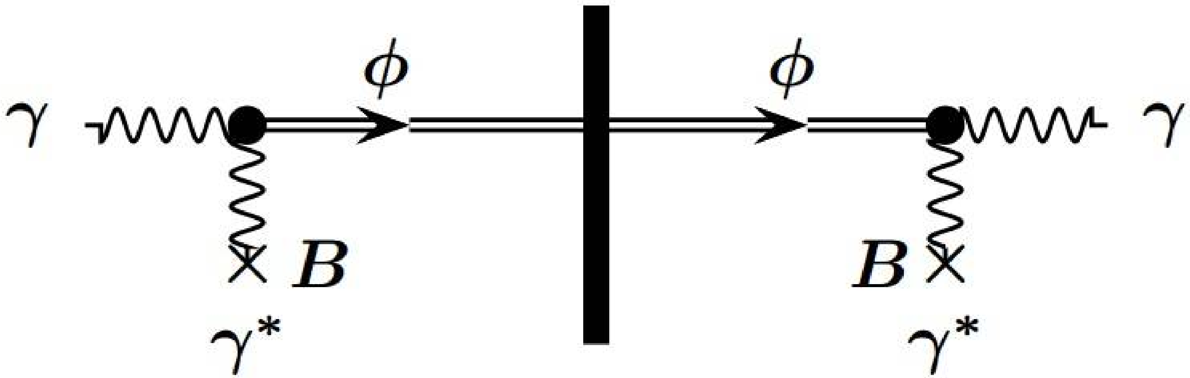
$$\Omega_{\rm mis,RMS} h^2 \sim 0.13 \left(\frac{m_a}{10\ \mu\rm eV} \right)^{-1.18}$$

$$\Omega_{\rm strings+domain\ walls} h^2 = (3.5 \pm 1.7) \left(\frac{m_a}{10\ \mu\rm eV} \right)^{-1.18}$$

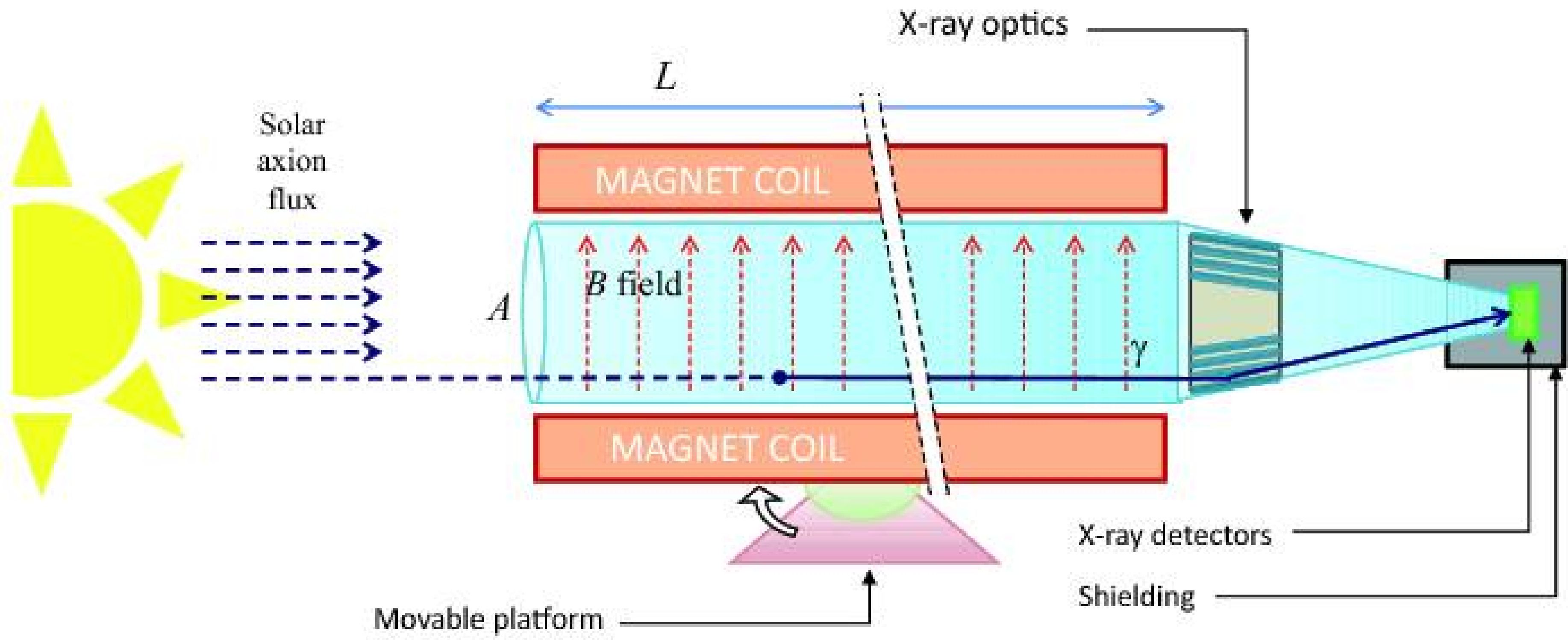
$$\Omega_{\rm strings+domain\ walls} h^2 \sim 0.4 \left(\frac{m_a}{10\ \mu\rm eV} \right)^{-1.18}$$

Axion laboratory searches based on
light-shining-through-wall experiments

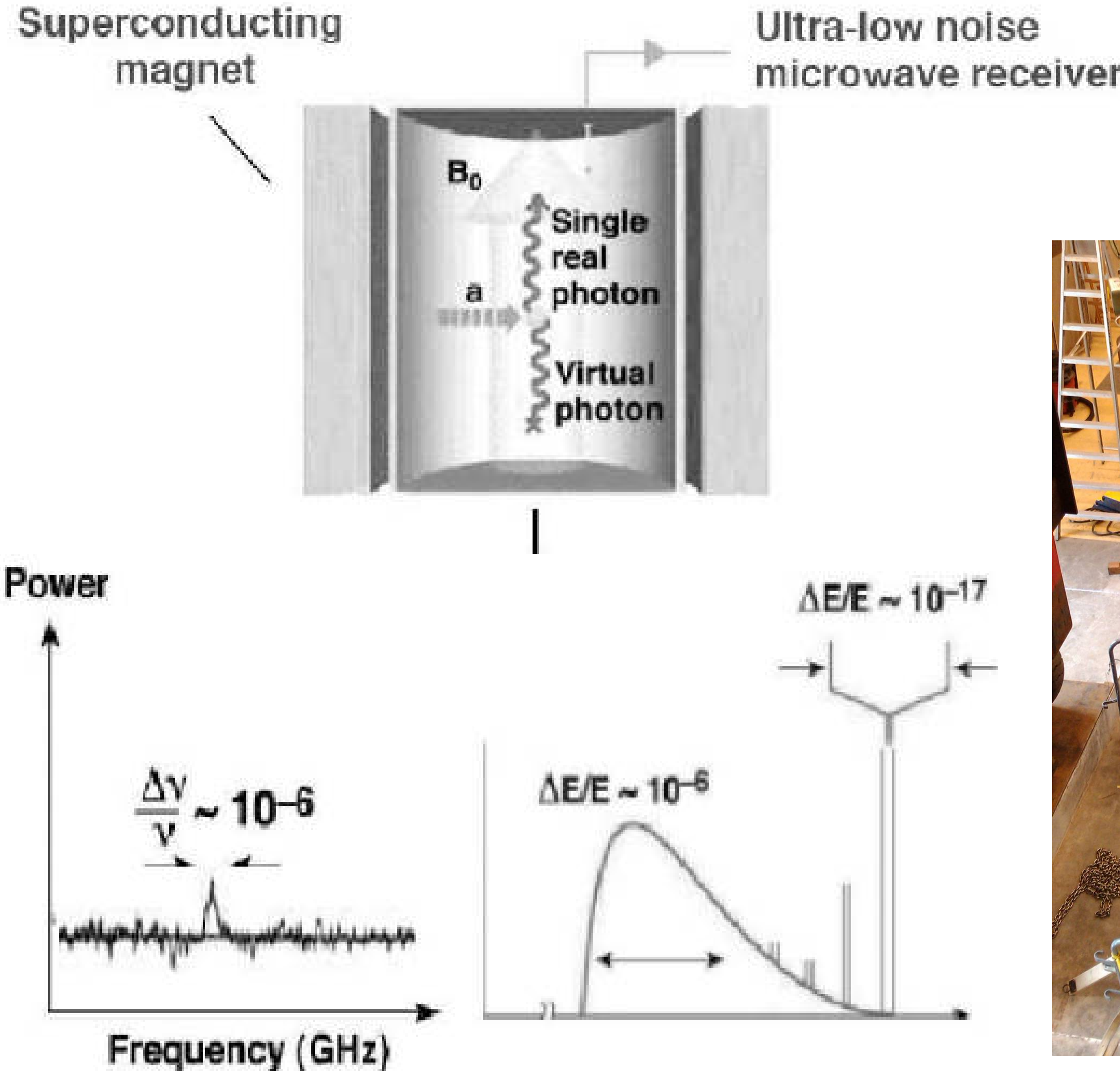
$$\gamma + Ze \leftrightarrow Ze + a$$

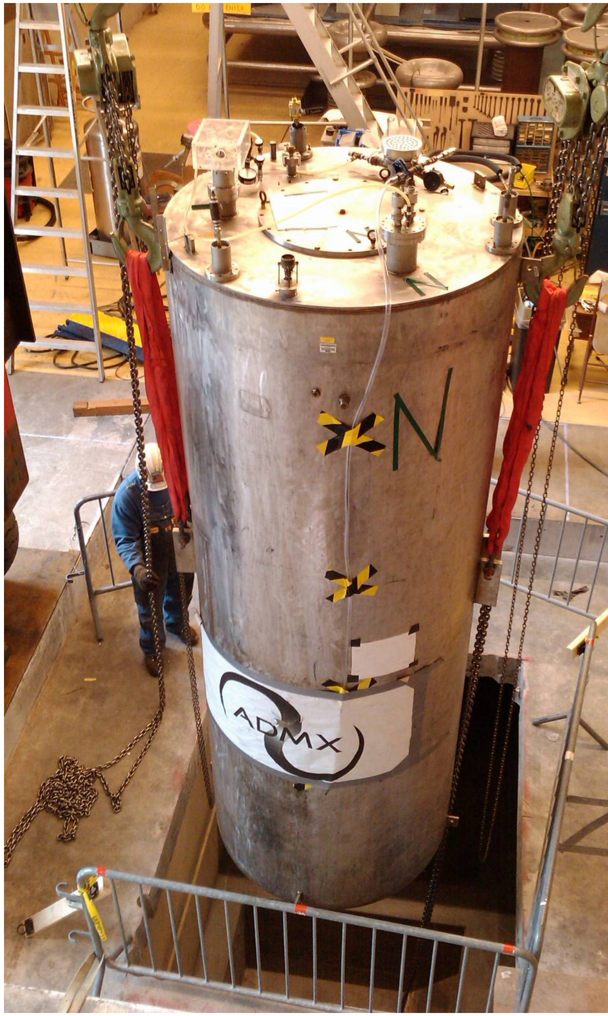


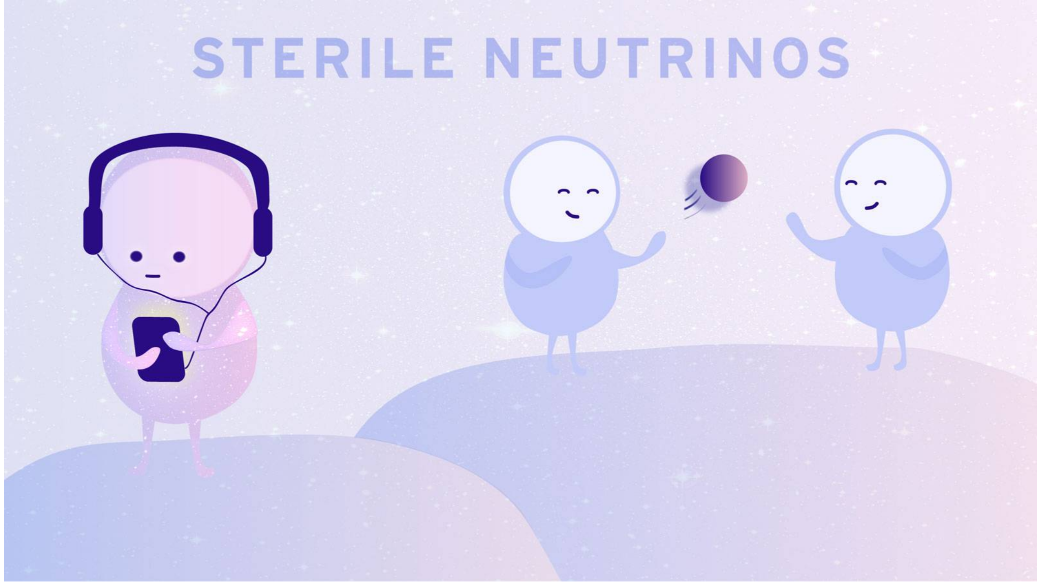
microwave cavities, and "**helioscopes**"



microwave cavities, and "**helioscopes**"


Superconducting magnet
Ultra-low noise microwave receiver
B_0
Single real photon
a
Virtual photon
Power
$\frac{\Delta\nu}{\nu} \sim 10^{-6}$
$\Delta E/E \sim 10^{-6}$
$\Delta E/E \sim 10^{-17}$
Frequency (GHz)


ADMX


STERILE NEUTRINOS

SM Neutrinos are strictly **massless**;
however, they are not observed to be!

Simplest addition: set of ***n*** singlet fermions ***N_a***, gauge singlets

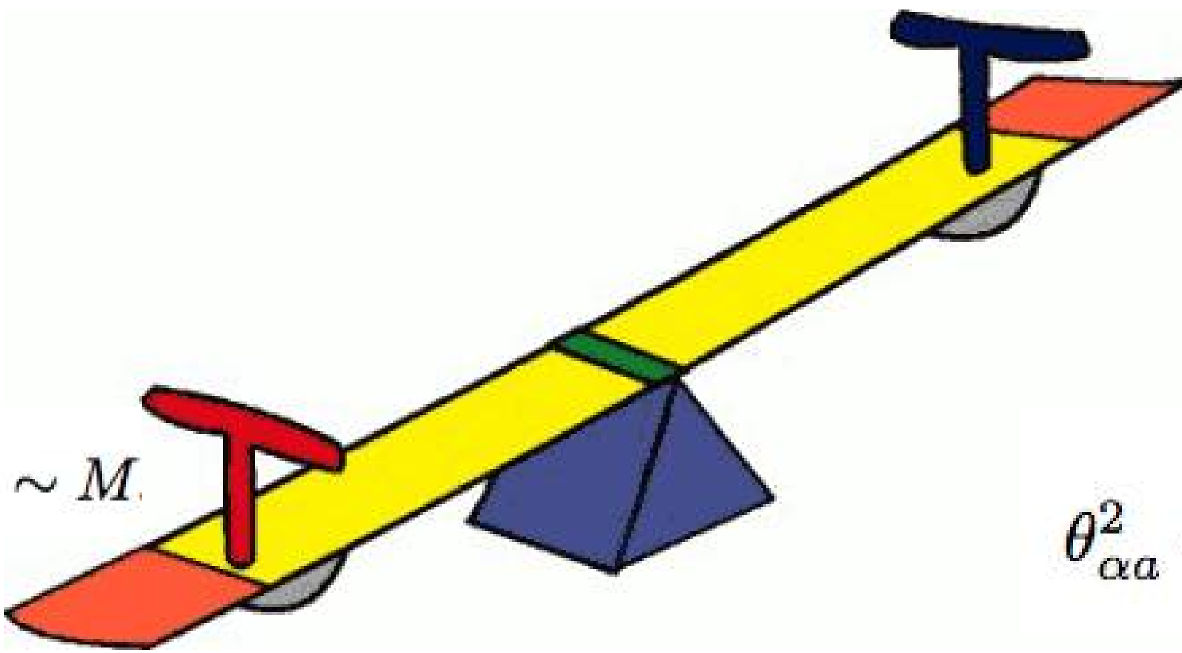
$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\bar{N}_a \partial\!\!\!/ N_a - y_{\alpha a} H^\dagger \bar{L}_\alpha N_a - \frac{M_a}{2} \bar{N}_a^c N_a$$

$$M^{(n+3)} = \begin{pmatrix} 0 & y_{\alpha a}\langle H \rangle \\ y_{\alpha a}\langle H \rangle & \mathrm{diag}(M_1, ..., M_n) \end{pmatrix}$$

If the following holds $y_{\alpha a}\langle H\rangle \sim yv \ll M_a \sim M$

“**See-saw**” mechanism!

$$M(\nu_{1,2,3}) \sim \frac{y^2 v^2}{M}$$



$$m(\nu_a) \sim M$$

$$\theta^2_{\alpha a} \sim \frac{y^2_{\alpha a} v^2}{M^2}$$

Sterile neutrinos mix via explicit (but possibly very small) **mixing** with ordinary neutrinos

…as such, they **decay** (into 3 SM neutrinos)

$$\Gamma \sim \theta^2 G_F^2 m^5 \sim \theta^2 \left(\frac{m}{\mathrm{keV}}\right)^5 \; 10^{-40}\;\mathrm{GeV} \Rightarrow \tau \sim 10^{16}\mathrm{s}\;\theta^{-2}\left(\frac{m}{\mathrm{keV}}\right)^{-5}$$

$$\theta^{-2}\left(\frac{m}{\mathrm{keV}}\right)^{-5} \gg 1$$

Being fermions, ***m > keV*** (e.g. Tremaine-Gunn)

How can sterile neutrinos be **produced**?

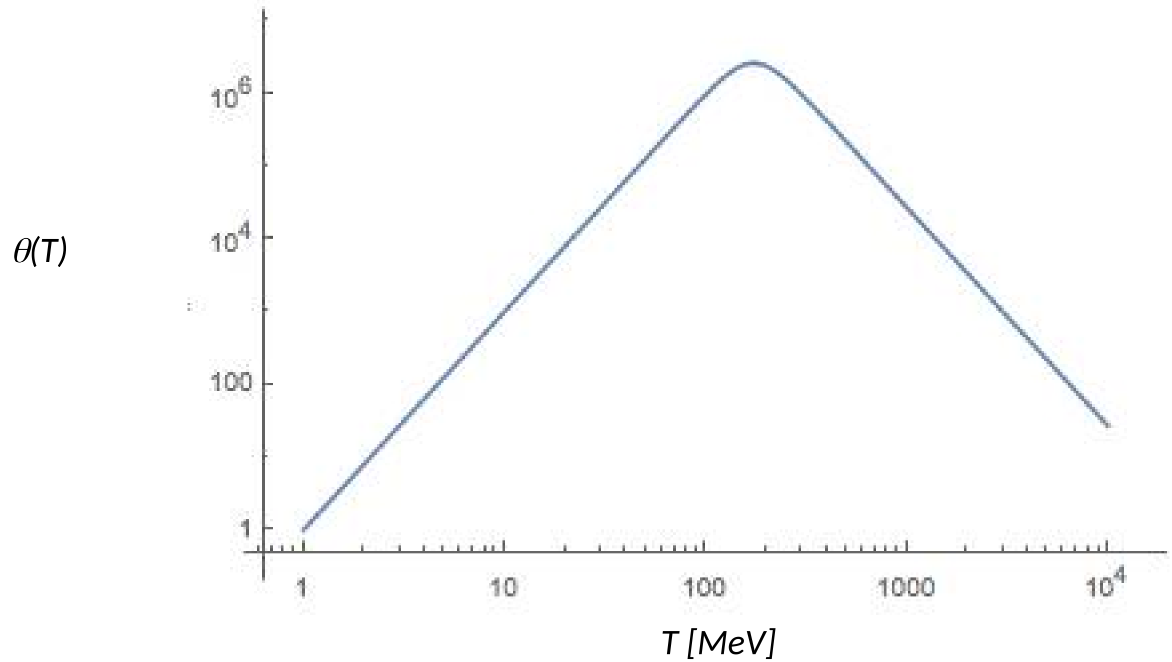
Basically, **freeze-in**: dump out-of-equilibrium sterile ν's through the universe history

$$\Gamma_{\nu_s} \sim (G_F^2 T^5)\theta^2(T)$$

Subtlety is **matter effects**, inducing ***T*-dependence** in the mixing angle

$$\theta \to \theta_M \simeq \frac{\theta}{1 + 2.4\left(\frac{T}{200\ \mathrm{MeV}}\right)^6 \left(\frac{1\ \mathrm{keV}}{m}\right)^2}$$

Sterile ν yield **Y=*n/s*** scales as production rate times Hubble time $t_H = M_P/T^2$



Maximal yield in **100-200 MeV** range ✉ QCD phase transition effects

$$\Omega_{\nu_s} h^2 \sim 0.1 \left(\frac{\theta^2}{3 \times 10^{-9}}\right) \left(\frac{m_s}{3\ \mathrm{keV}}\right)^{1.8}$$

(**Dodelson**-Widrow)

Additional important effect from Mikheyev-Smirnov-Wolfenstein effect with large **lepton asymmetries** (**Shi-Fuller** resonant production)

Other possibilities: **non-thermal production** from singlet scalar coupling

$$\frac{h_a}{2} S \bar{N}_a^c N_a$$

$$SH^\dagger H \quad \text{and/or} \quad S^2 H^\dagger H$$

$$\frac{n_N}{s} \sim \frac{n_S}{s} \tau \Gamma \sim \frac{M_P}{M_S^2} \frac{h^2}{16\pi} M_S$$

$$\Omega_N \sim 0.2 \left(\frac{h}{10^{-8}} \right)^3 \frac{\langle S \rangle}{m_S}$$

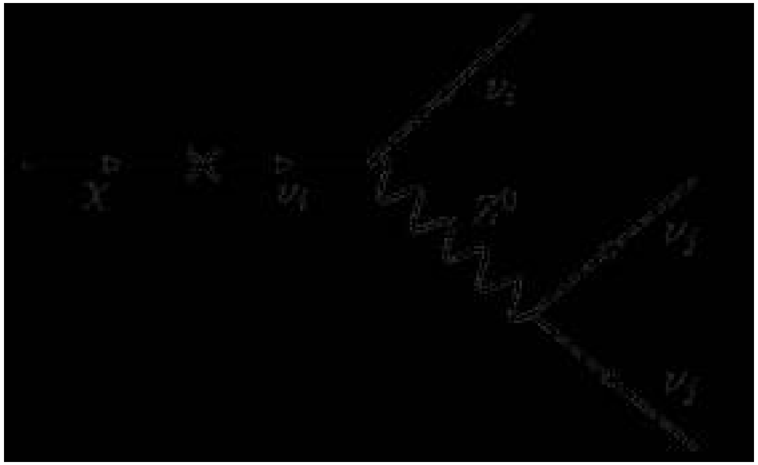
Sterile neutrino interesting from the standpoint of **structure formation** – remember

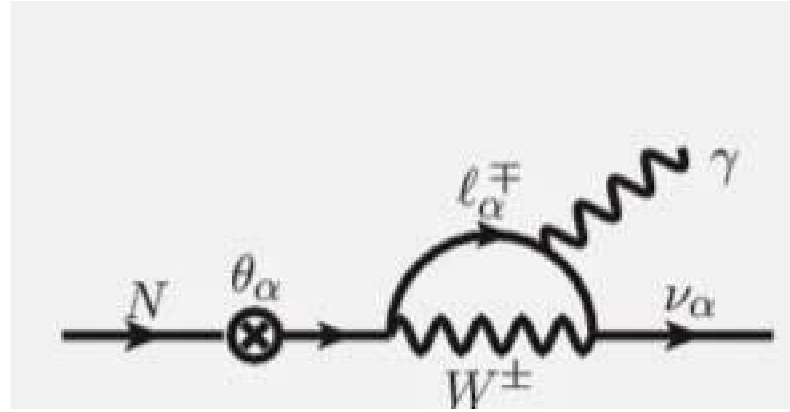
$$M_{\mathrm{cutoff,\ hot}} \sim \left(\frac{1}{H(T=m_\nu)}\right)^3 \rho_\nu(T=m_\nu) \sim \left(\frac{M_P}{m_\nu^2}\right)^3 m_\nu \cdot m_\nu^3 = \frac{M_P^3}{m_\nu^2}$$

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\, M_\odot \left(\frac{m_\nu}{30\ \mathrm{eV}}\right)^{-2} \sim 10^{12}\, M_\odot \left(\frac{m_\nu}{1\ \mathrm{keV}}\right)^{-2}$$

…and could explain high-velocity **pulsars**!

How would we **detect** sterile neutrino dark matter?





$$\Gamma_{\nu_s \to \gamma \nu_a} \approx \frac{\alpha}{16\pi^2} \theta^2 G_F^2 m^5$$

$$\phi_\gamma = \frac{\Gamma_{\gamma\nu}}{4\pi} \frac{E_\gamma}{m} \int_{fov} d\Omega \int_{\text{line of sight}} \frac{\rho_{\text{DM}}}{m} dr(\psi) = \frac{\Gamma_{\gamma\nu}}{8\pi m} J(\Delta\Omega, \psi)$$

few $\times 10^{18}$ GeV/cm^2

key background: diffuse **cosmic X-ray background**

$$\phi_{\mathrm{CXB}} \sim 9.2 \times 10^{-7} \left(\frac{E}{1\ \mathrm{keV}}\right)^{-0.4} \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{arcmin}^{-2} \quad \rightarrow \quad \sim 10^{-4}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$

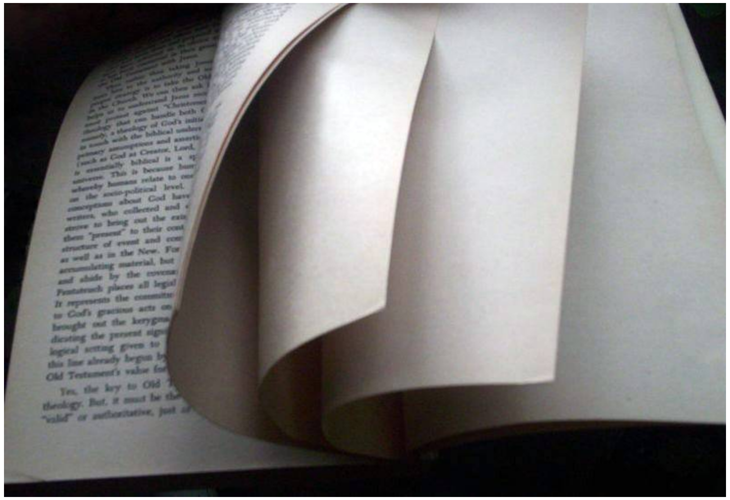
$$\phi_\gamma = \frac{\Gamma_{\gamma\nu}}{8\pi}\frac{J}{m} \sim 10^{-4}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} \left(\frac{\theta^2}{10^{-7}}\right)\left(\frac{m}{1\ \mathrm{keV}}\right)^4 \left(\frac{J}{10^{18}\ \mathrm{GeV/cm^2}}\right)$$

$$\left(\frac{\theta^2}{10^{-7}}\right)\left(\frac{m}{1\ \mathrm{keV}}\right)^4 \lesssim 1$$

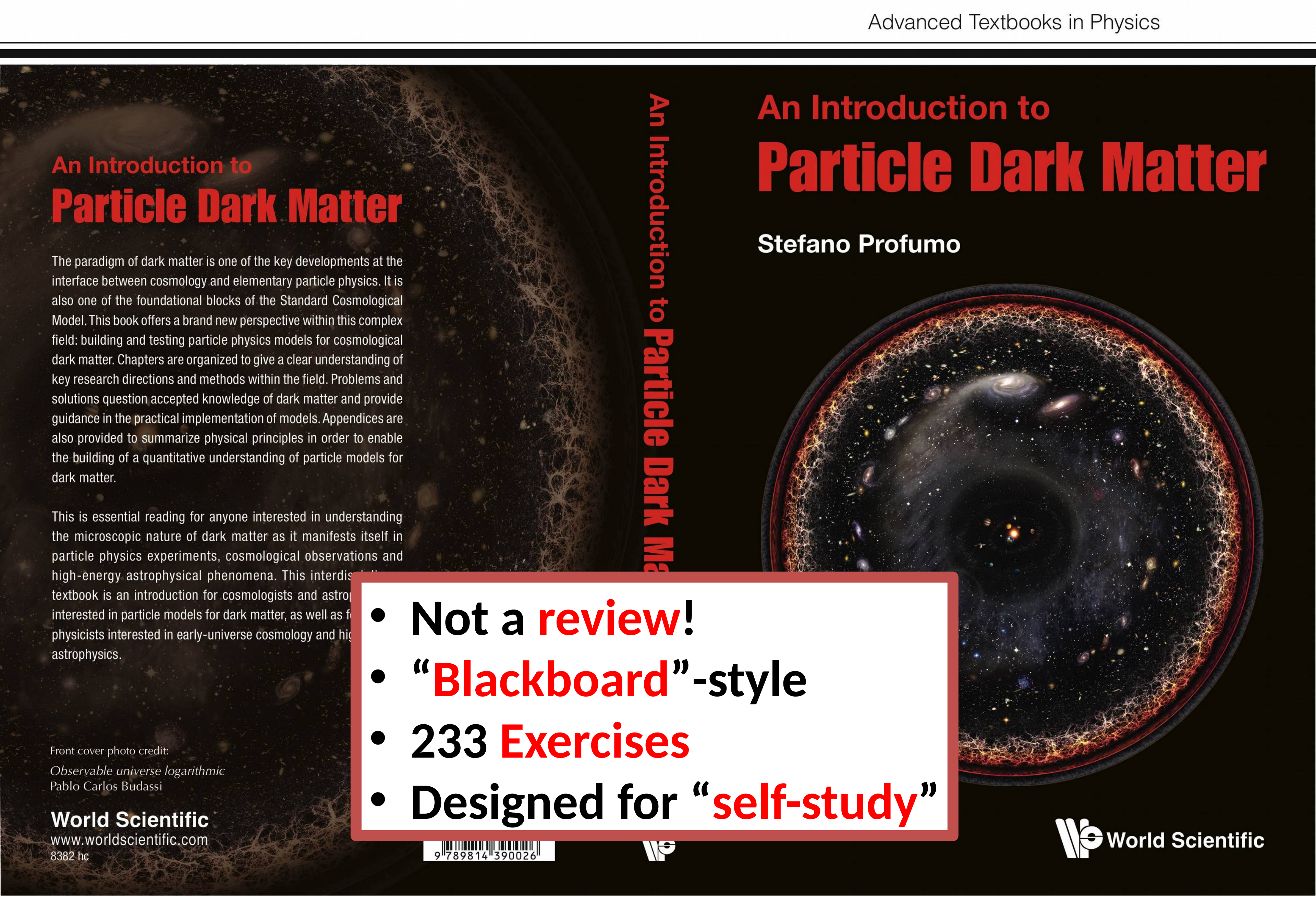
Have we **detected** it? **3.5 keV** line!

Many other interesting **possibilities**…

The pages on the **particle** nature of **dark matter** in the great book of physics are yet to be written – a field full of **opportunities**, which will reward creativity and critical thinking!



Make sure to **learn lessons** from recent **successes** and **failures**, and the **bag of tricks** that comes with those lessons!



- Not a **review**!
- “**Blackboard**”-style
- 233 **Exercises**
- Designed for “**self-study**”

$$H^2 = \frac{8\pi G_N}{3}\rho.$$

GR+SM: **energy density** in radiation

$$\rho \simeq \rho_{\rm rad} = \frac{\pi^2}{30} \cdot g \cdot T^4 \longrightarrow H \simeq T^2/M_P$$

now, the red-shifted luminosity distance is $\frac{F}{L} = \frac{1}{4\pi a_0^2 r^2 (1+z)^2}$

$$d_L = a_0 r (1+z) \ .$$

we want to get rid of the (unmeasurable) r, so consider a null geo

$$0 = ds^2 = -\mathrm{d}t^2 + \frac{a^2}{1-kr^2}\mathrm{d}r^2 \qquad \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^r \frac{dr}{(1-kr^2)^{1/2}}$$

expand the integral for "small" time intervals,

$$a(t_1) = a_0 + (\dot{a})_0(t_1 - t_0) + \frac{1}{2}(\ddot{a})_0(t_1 - t_0)^2 + \dots$$

find r…

$$r = a_0^{-1}\left[(t_0 - t_1) + \frac{1}{2}H_0(t_0 - t_1)^2 + \ldots\right]$$

…express in terms of redshift, using Hubble and the deceleration

$$\frac{1}{1+z} = 1 + H_0(t_1 - t_0) - \frac{1}{2}q_0H_0^2(t_1 - t_0)^2 + \ldots$$

…and finally plug into the luminosity distance formula:

$$d_L = H_0^{-1}\left[z + \frac{1}{2}(1 - q_0)z^2 + \ldots\right]$$

Recipe to discover what kind of FRW we live in (and to get Nobel p
measure dL(z) for objects of known luminosity (standard cande

in chemical equilibrium, $\mu_i + \mu_j = \mu_k + \mu_l$
$i + j \leftrightarrow k + l$

One pretty big open question – why is there more matter than antimatter? begs analyzing particle-antiparticle asymmetries.

ecause one can always radiate photons off of charged particles, the chemic

$$e^- + p \leftrightarrow e^- + p + \gamma$$

Also, because particles and antiparticles can pair-

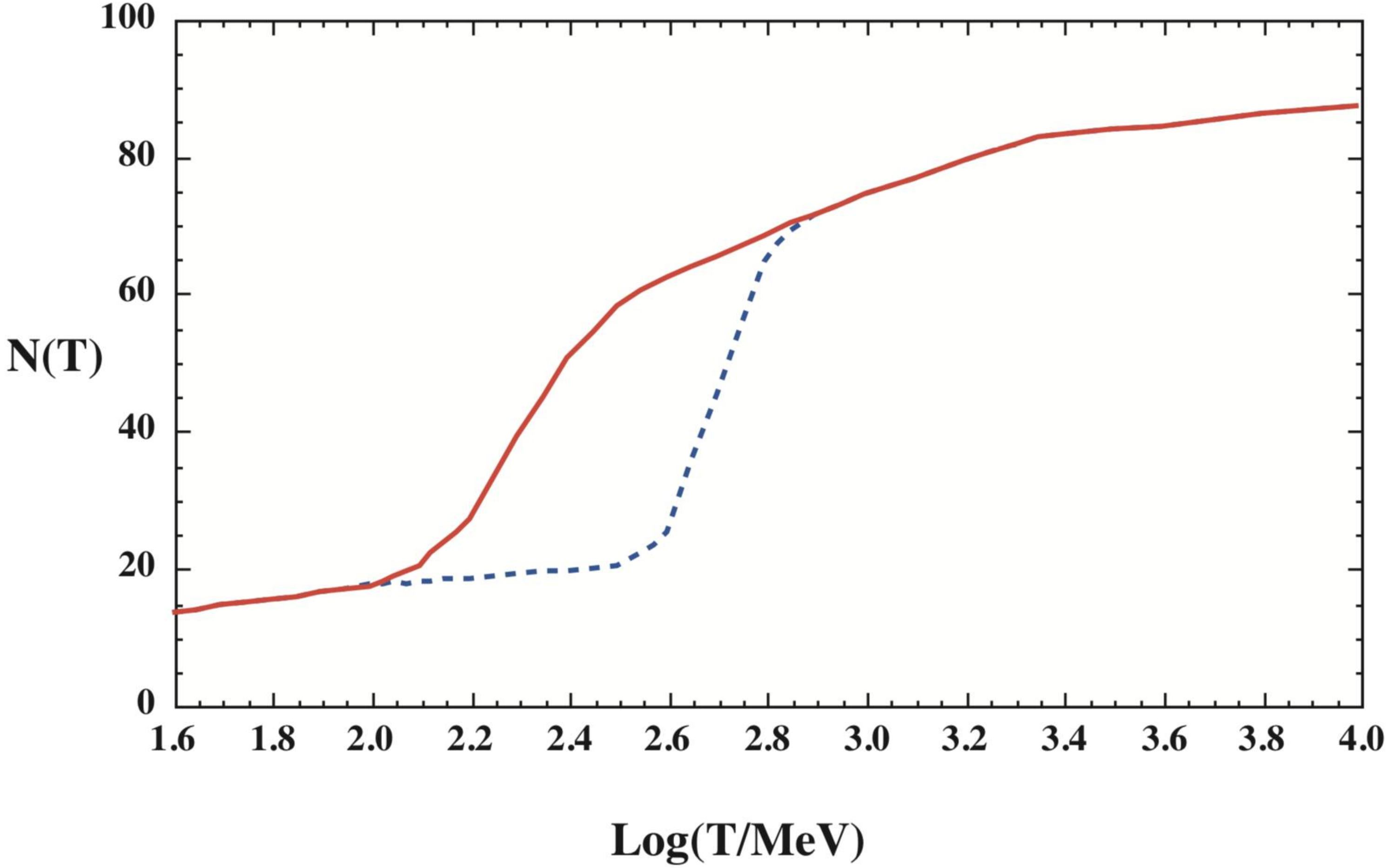
$$X^+X^- \leftrightarrow \gamma\gamma \qquad\qquad \mu_+ = -\mu_-$$

$$n_+ - n_- = \begin{cases} (gT^3/6\pi^2)\left[\pi^2(\mu/T) + (\mu/T)^3\right] & T \gg m \\ 2g(mT/2\pi)^{3/2}\sinh(\mu_+/T)e^{-m/T} & T \ll m \end{cases}$$

Looking back at the early universe…

Temperature	New Particles	$4N(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c{}^\dagger$	π's	69
$T_c < T < m_{\text{strange}}$	π's + $u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{\text{charm}}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\text{bottom}}$	$\tau^\pm$	303
$m_b < T < m_{\text{W,Z}}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\text{Higgs}}$	$W^\pm, Z$	381
$m_H < T < m_{\text{top}}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

Looking back at the early universe…



$$s = 1.80\, g_{*s} n_\gamma \qquad\qquad n_{\gamma 0} \simeq 411\,\mathrm{cm}^{-3}$$

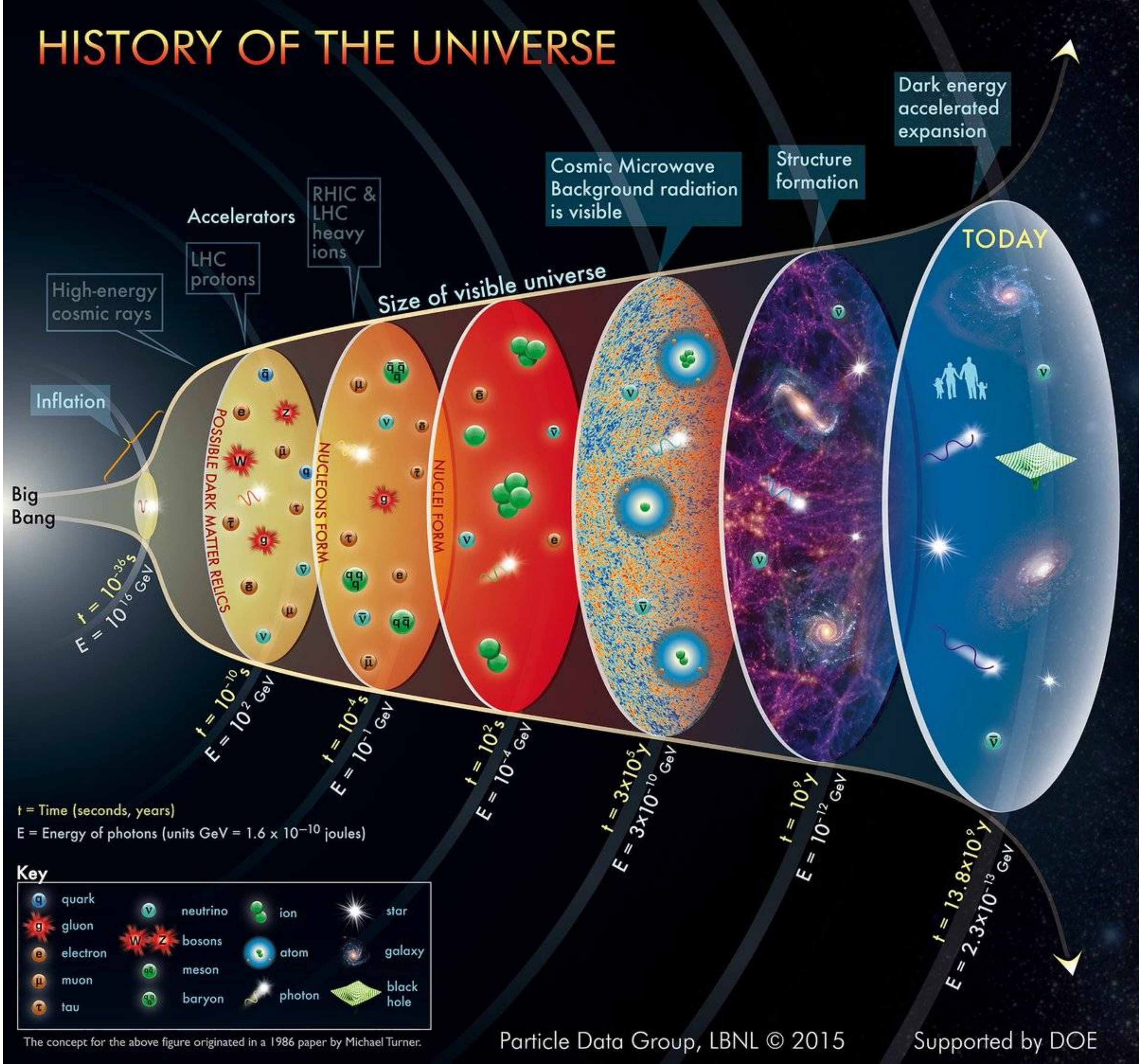
$$g_{*s} T^3 a^3 = \text{constant}$$

$$s_0 = 7.04\, n_\gamma \simeq 3000\,\mathrm{cm}^{-3}$$

$$n_B/s \equiv (n_b - n_{\bar{b}})/s$$

$$\eta \equiv (n_B/n_\gamma 0)_0 = 1.8\, g_{*s}(n_B/s)$$

$$\eta = 5.1 \times 10^{-10}(\Omega_b h^2/0.019)$$



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
Inflation
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
Big Bang
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
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(1) borrow **equilibrium number densities** from stat mech

$$n_{\rm rel} \sim T^3 \quad \text{for} \;\; m \ll T,$$
$$n_{\rm non-rel} \sim (mT)^{3/2} \exp\left(-\frac{m}{T}\right) \quad \text{for} \;\; m \gg T.$$

(2) borrow **Hubble rate** from general relativity
(FRW **solution** to Einstein's eq.)

$$H^2 = \frac{8\pi G_N}{3}\rho.$$

useful to know the **scale-factor** versus **time** dependence

for a **flat** universe, just use the scaling of density with scale factor, and Friedmann eq:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho_0 a^n$$

$$\frac{da}{dt} \propto a^{\frac{n+2}{2}}$$

$$a \sim t^{2/n}$$

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$$n = 3 \quad \text{(matter)} \quad a \sim t^{2/3}$$

$$n = 4 \quad \text{(radiation)} \quad a \sim t^{1/2}$$

$$n = 6 \quad \text{(kination)} \quad a \sim t^{1/3}$$

$$n = 0 \quad \text{(cosmological const.)} \quad a \sim \exp(c \cdot t)$$

- **k=0** "**flat**". For good reasons…!

$$d\sigma^2 = dr^2 + r^2 d\Omega^2 = dx^2 + dy^2 + dz^2$$

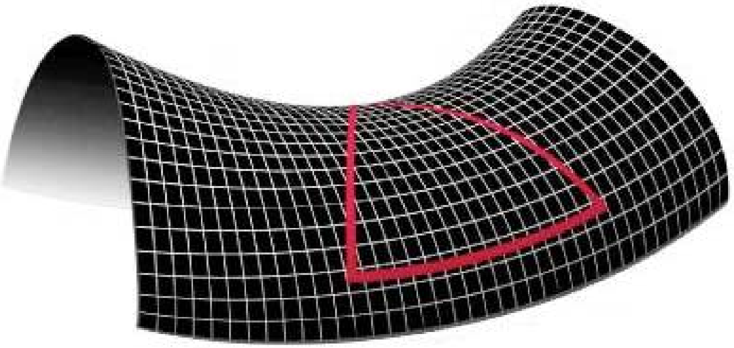
metric on **space-like** slice

- **k=+1** "**closed**" universe:

$$r = \sin\chi$$

$$d\sigma^2 = d\chi^2 + \sin^2\chi\, d\Omega^2$$

spacelike slices are spheres!



- **k=-1** "**open**" universe

$$r = \sinh\psi$$

$$d\sigma^2 = d\psi^2 + \sinh^2\psi\, d\Omega^2$$

for all cases, the **Ricci scalar** is

$$R = \frac{6}{a^2}(a\ddot{a} + \dot{a}^2 + k)$$

chemical potential: energy "cost" of changing **number** of particles

$$\mathrm{d}U = T\,\mathrm{d}S - P\,\mathrm{d}V + \sum_{i=1}^{n} \mu_i \mathrm{d}N_i \qquad \mu_i = \left(\frac{\partial U}{\partial N_i}\right)_{S,V,N_{j\neq i}}$$

Stories from the **thermal history** of the universe!

- (re-)**combination**: exact formula for **free electron fraction** in equilibrium
- photons **decouple** when free electrons are gone, around 0.3 eV, t~380 kyr

- BBN: key is thermal **decoupling of neutrons**, when $T \sim m_n - m_p$, fixes n/p
- ^4He **mass fraction** ~only depends on n/p
- happens at 0.8..0.1 MeV, t~few seconds to minutes old

- **neutrino decoupling**: hot thermal relic, freezes out at T~1 MeV ~ $(G_F^2 M_P)^{-1/3}$
- present **number density**: define Y=n/s; $Y_{dec} = Y_{today}$, so $n_{today} = Y_{decoupling} \times s_{today}$
- present energy density: $\rho_\nu = n_{today} * m_\nu$ (generic for **hot** thermal relics)

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