Ionization cooling provides the only practical solution to prepare high brightness beams necessary for a Neutrino Factory or Muon Collider because it is the only method fast enough to cool the beam on a timescale of the muon lifetime.

1. Motivation
- **Ionization Cooling**
  1. Energy loss by ionization ($dE/dx$) reduces both $p_x$ and $p_y$
  2. Heating from multiple scattering
  3. $p_z$ restored by RF cavities

$$\frac{dp_z}{d\tau} = \epsilon \frac{dE}{dx} + \frac{1}{2} \sigma_t \left( \frac{16}{3} \beta_1^2 + \frac{16}{3} \beta_2^2 \right)$$

→ Equilibrium emittance means cooling = heating

→ To maximize cooling, one must use a material with low-Z placed at a position where $\beta_1$ has a minimum.

1. Energy loss by ionization ($dE/dx$) reduces both $p_x$ and $p_y$.

2. Heating from multiple scattering.

3. $p_z$ restored by RF cavities.

4. Transverse phase-space volume reduction in MICE Step IV

1. **Ionization Cooling**

2. **MICE**
   - MICE is a key R&D towards Neutrino Factory and Muon Collider
   - Design, build, commission and operate a realistic section of a cooling channel
   - MICE collaboration: Bulgaria, China, Italy, Japan, Netherlands, Serbia, Switzerland, UK, USA: ~80 collaborators

3. **Goals**
   - Measure cooling performance in a variety of modes of operation and beam conditions
   - LHe or LHe absorber
   - 140–240 MeV/c central momentum
   - 3–10 mm input emittance
   - Achieve an absolute normalized transverse emittance measurement resolution of 0.1 %

4. **Transverse phase-space volume reduction in MICE Step IV**

1. Energy loss by ionization ($dE/dx$) reduces both $p_x$ and $p_y$.

2. Heating from multiple scattering.

3. $p_z$ restored by RF cavities.

**Transverse amplitude**

$$A_t = \epsilon \mu U \sum_{i=1}^{N} u_i \sim \epsilon \sigma_\mu \sigma_u$$

(1)

with $\mu$ the centred phase-space vector, i.e. the degree at which a particle is removed from the core distribution.

Assuming a Gaussian distribution, the 4D RMS ellipse encloses 99% of the beam particles. The volume of the corresponding contour, $V_n$, can be related to normalised emittance through

$$\epsilon_n = \frac{1}{\sqrt{2\pi\sigma_p^2}}$$

(2)

**Voronoi tessellation**

Tessellating the space allows to estimate local phase-space density for each region.

Cooling translates in an increased amount of small Voronoi regions.

**Alternative density estimators**

Many additional non-parametric density estimators exist and are under study to deal with beam nonlinearities.

- Optimal binning (here)
- k Nearest Neighbours
- LRD, DFTE, PBATDE, ...

5. Conclusions

- The emittance of a muon beam was measured particle-by-particle in MICE for the first time with its trackers.

- Cooling data has been taken for the LHe absorber and is being analysed right now.

- MICE is poised to observe transverse phase-space volume reduction in its current Step IV configuration.