Neutrinoless Double Beta Decay in Particle Physics

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Outline

$0\nu\beta\beta =$ Lepton Number Violation

- **Standard Interpretation:** light active Majorana neutrinos
  - General
  - Example: mass determination
- **Non-Standard Interpretations:** something else
  - heavier Majorana neutrinos
  - SUSY
  - RH currents
  - ...

Standard Interpretation

Neutrinoless Double Beta Decay is mediated by light, active and massive Majorana neutrinos (the ones which oscillate) and all other mechanisms potentially leading to $0\nu\beta\beta$ give negligible or no contribution

Why do we need neutrino mass and probe LNV?

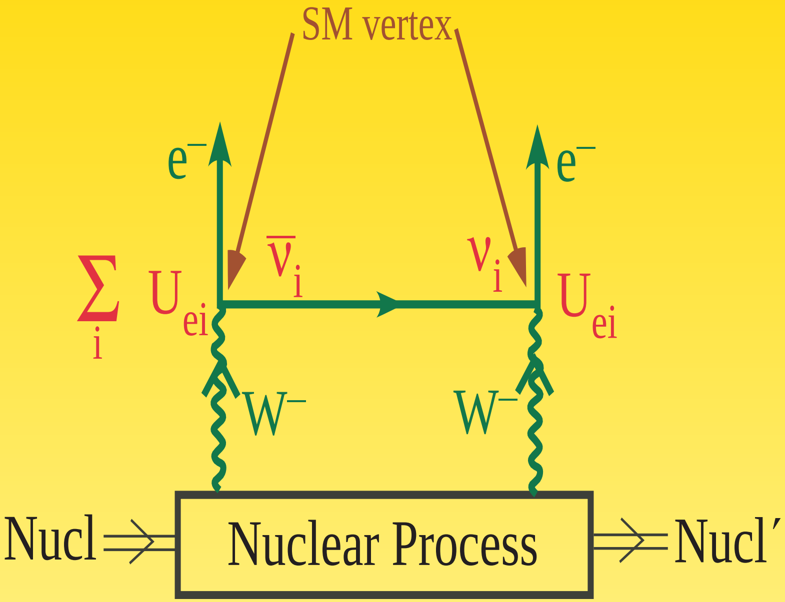
- in all seesaws (type I, II, III): neutrino mass inverse proportional to its origin
- GUTs: normal hierarchy...
- IH and QD neutrinos: special flavor symmetries required...
- QD: HDM, strong RG effects, leptogenesis,...
- mass hierarchy is moderate:

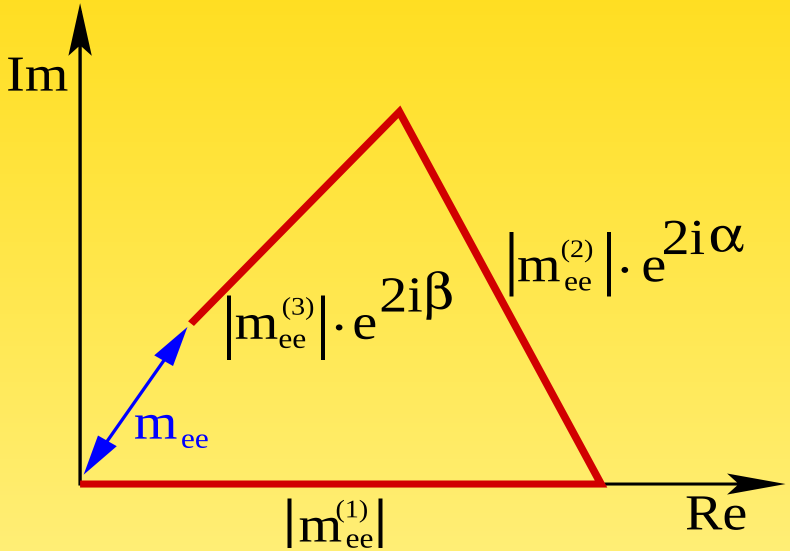
$$\mathrm{NH}: \quad \frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m_\odot^2}{\Delta m_\mathrm{A}^2}} \gtrsim \frac{1}{5} \simeq \sqrt{\frac{m_\mu}{m_\tau}} \simeq \sqrt{\frac{m_s}{m_b}} \simeq \sqrt{\sqrt{\frac{m_c}{m_t}}}$$

$\Rightarrow$ Neutrino masses are strange and can tell us a lot

$\Rightarrow$ Lepton Number Violation as important as Baryon Number Violation

$\Delta L \neq 0$: Neutrinoless Double Beta Decay





Amplitude proportional to coherent sum (“effective mass”):

$$|m_{ee}| \equiv \left|\sum U_{ei}^2\, m_i\right| = \left||U_{e1}|^2\, m_1 + |U_{e2}|^2\, m_2\, e^{2i\alpha} + |U_{e3}|^2\, m_3\, e^{2i\beta}\right|$$

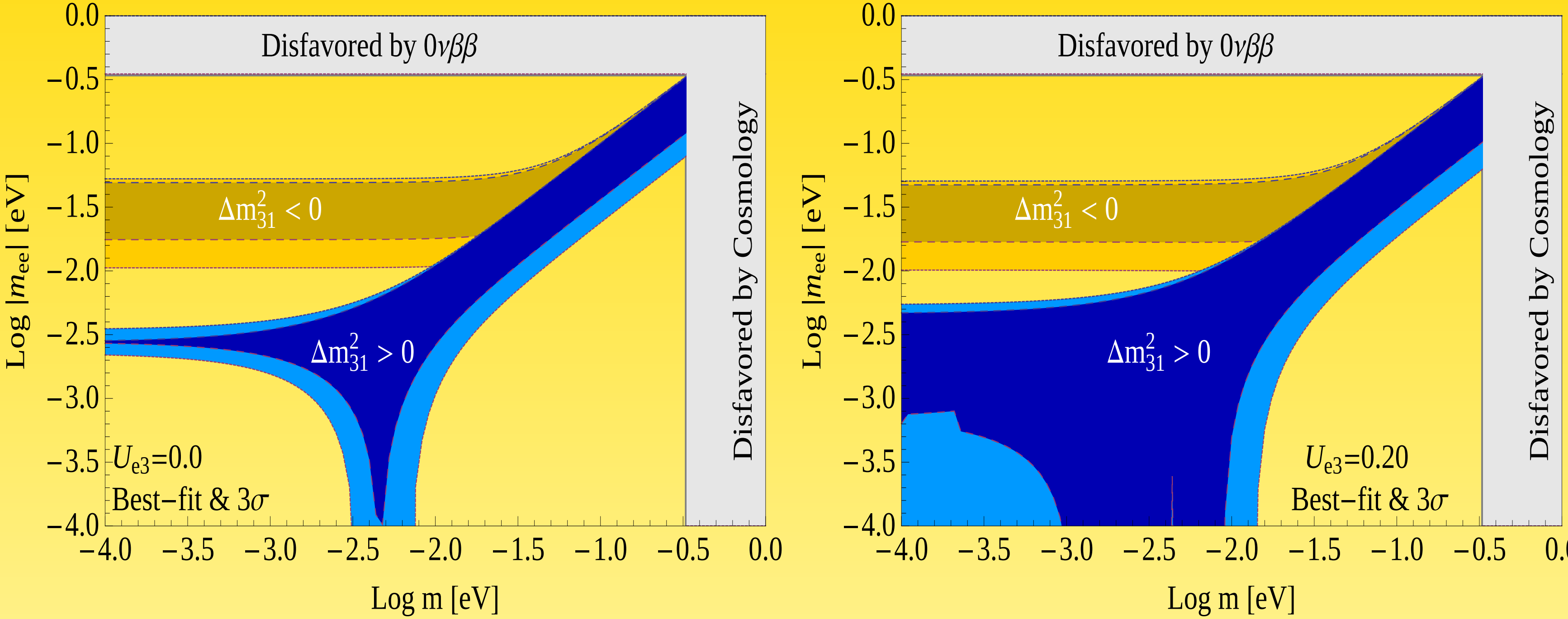
7 out of 9 parameters of m_ν:

$$|m_{ee}| = f\left(\theta_{12}, m_i, |U_{e3}|, \mathrm{sgn}(\Delta m^2_{\mathrm{A}}), \alpha, \beta\right)$$

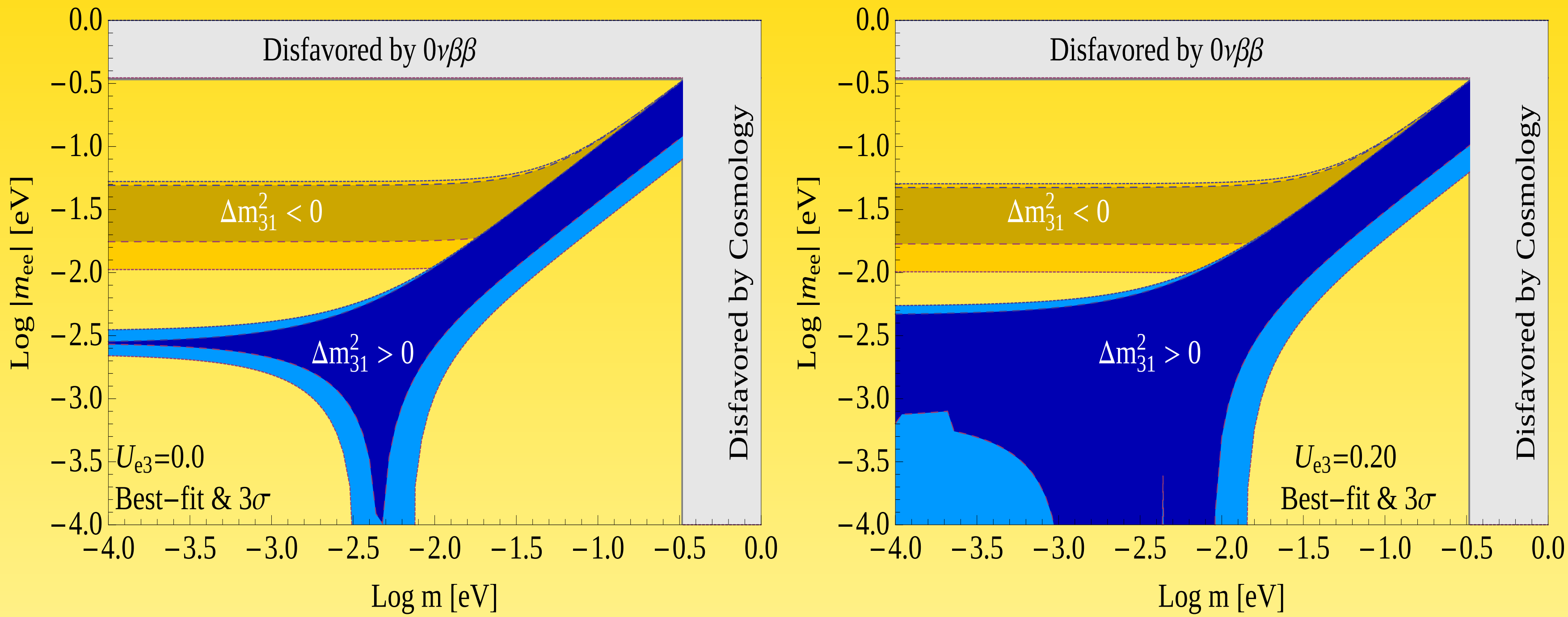
Kim, 1996; Minakata & Yasuda, 1996; Hirsch & Klapdor-Kleingrothaus, 1997; Bilenky, Giunti & Monteno, 1997; Fukuyama, Matsuda & Nishiura, 1997; Bilenky, Giunti, Kim & Monteno, 1998; Fukuyama, Matsuda & Nishiura, 1998; Vissani, 1999; Giunti, 1999; Bilenky, Giunti, Grimus, Kayser & Petcov, 1999; Ma, 1999; Wodecki & Kaminsky, 2000; Kalliomaki & Maalampi, 2000; Rodejohann, 2000; Matsuda, Takeda, Fukuyama & Nishiura, 2000; Klapdor-Kleingrothaus, Päs & Smirnov, 2001; Falcone & Tramontano, 2001; Bilenky, Pascoli & Petcov, 2001; Xing, 2001; Osland & Vigdel, 2001; Pascoli & Petcov, 2001; Barger, Glashow, Marfatia & Whisnant, 2002; Hambye, 2002; Minakata & Sugiyama, 2002; Klapdor-Kleingrothaus & Sarkar, 2002; Xing, 2002; Haba & Suzuki, 2002; Pakvasa & Roy, 2002; Rodejohann, 2002; Haba, Nakamura & Suzuki, 2002; Päs & Weiler, 2002; Barger, Glashow , Langacker, Marfatia, 2002; Civitarese & Suhonen, 2002; Pascoli, Petcov & Rodejohann, 2002; Sugiyama, 2002; Avignone & King, 2002; Minakata & Sugiyama, 2002; Cheung, 2003; Abada & Bhattacharyya, 2003; Giunti, 2003; Pascoli & Petcov, 2003; Elliott, 2003; Stoica, 2004; Brahmachari, 2004; Bilenky, Fäßler & Simkovic, 2004; Pascoli & Petcov; 2004; Deppisch, Päs & Suhonen, 2004; Joniec & Zralek, 2004; Pascoli & Petcov, 2005; Pascoli, Petcov & Schwetz, 2005; Goswami & Rodejohann, 2005; Choubey & Rodejohann, 2005; Bilenky, Fäßler, Gutsche, & Simkovic, 2005; Lindner, Merle & Rodejohann, 2005;

parameter	best-fit$^{+1\sigma}_{-1\sigma}$	2σ	3σ
Δm^2_{21} [10^{-5} eV^2]	$7.59^{+0.23}_{-0.18}$	$7.22 - 8.03$	$7.03 - 8.27$
$\|\Delta m^2_{31}\|$ [10^{-3} eV^2]	$2.40^{+0.12}_{-0.11}$	$2.18 - 2.64$	$2.07 - 2.75$
$\sin^2\theta_{12}$	$0.318^{+0.019}_{-0.016}$	$0.29 - 0.36$	$0.27 - 0.38$
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	$0.39 - 0.63$	$0.36 - 0.67$
$\sin^2\theta_{13}$	$0.013^{+0.013}_{-0.009}$	≤ 0.039	≤ 0.053

Schwetz, Tortola, Valle, 0808.2016v3 (Feb 2010)



Our plots (`A. Merle`) are blue and yellow. . .



Note: importance of U_{e3}

Testing Inverted Ordering

Nature gives us a scale:

$$|m_{ee}|^{\rm IH}_{\rm min} = \left(1 - |U_{e3}|^2\right) \sqrt{|\Delta m^2_{\rm A}|} \left(1 - 2 \sin^2\theta_{12}\right) = \left\{ \begin{array}{cc} (0.015 \ldots 0.020) \text{ eV} & 1\sigma \\ (0.010 \ldots 0.024) \text{ eV} & 3\sigma \end{array} \right.$$

Desiderata:

- small $|U_{e3}|$
- large $|\Delta m^2_{\rm A}|$
- small $\sin^2\theta_{12}$

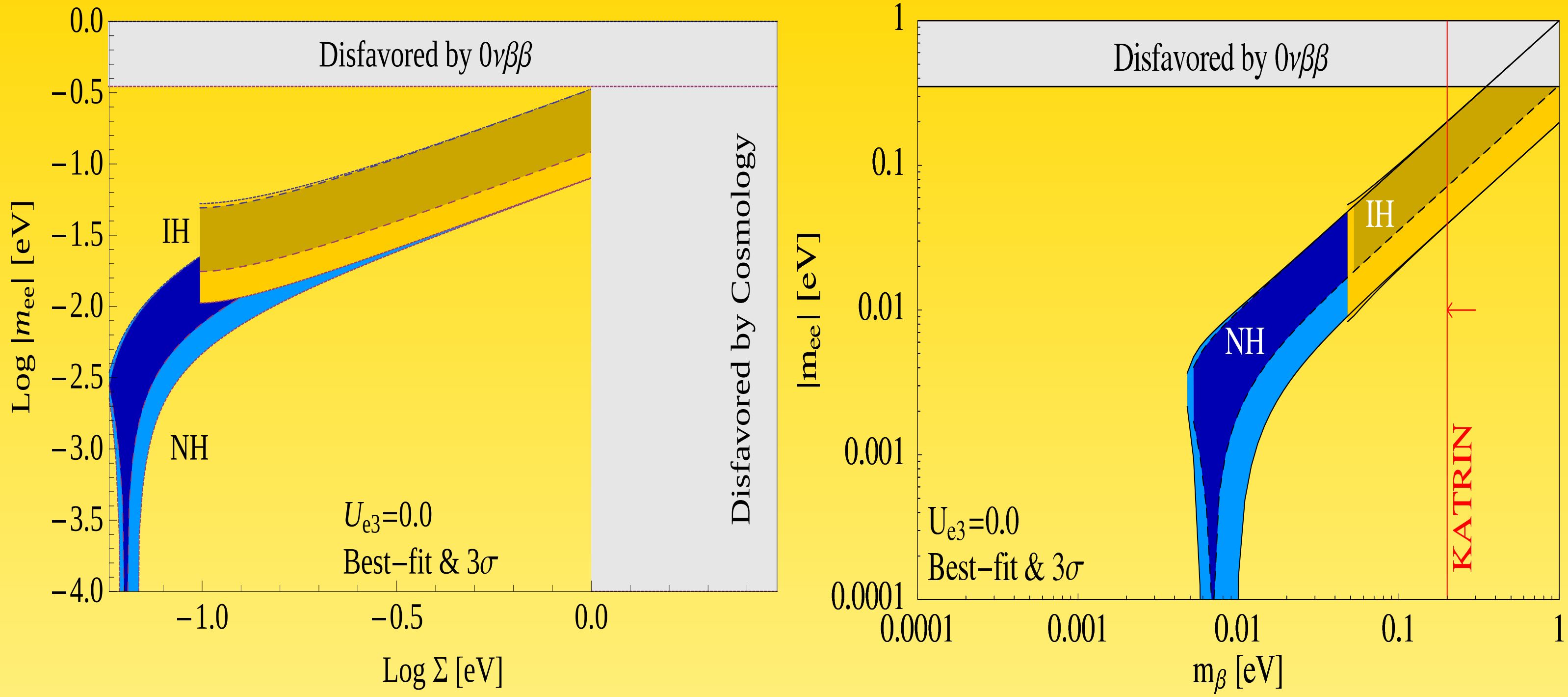
Testing Inverted Ordering

Nature gives us another scale:

$$|m_{ee}|^{\rm IH}_{\rm max} = \left(1 - |U_{e3}|^2\right) \sqrt{|\Delta m^2_{\rm A}|} = \left\{ \begin{array}{cc} (0.047 \ldots 0.050) \text{ eV} & 1\sigma \\ (0.043 \ldots 0.052) \text{ eV} & 3\sigma \end{array} \right.$$

Desiderata:

- small $|U_{e3}|$
- large $|\Delta m^2_{\rm A}|$



if experiments point to something outside the blue or yellow areas: interesting scenarios arise

(example: Klapdor's claim in conflict with cosmological neutrino mass bounds)

Ideal Case: Statistical Analysis of QD

Scenario	m_3 [eV]	$\|m_{ee}\|$ [eV]	m_β [eV]	Σ [eV]
$\mathcal{QD}$	0.3	$0.11 - 0.30$	0.30	0.91

- effective mass

$$\text{“experimental error”: } \sigma(|m_{ee}|_{\rm exp}) = \frac{|m_{ee}|_{\rm exp}}{2} \, \frac{\sigma(\Gamma_{\rm obs})}{\Gamma_{\rm obs}}$$

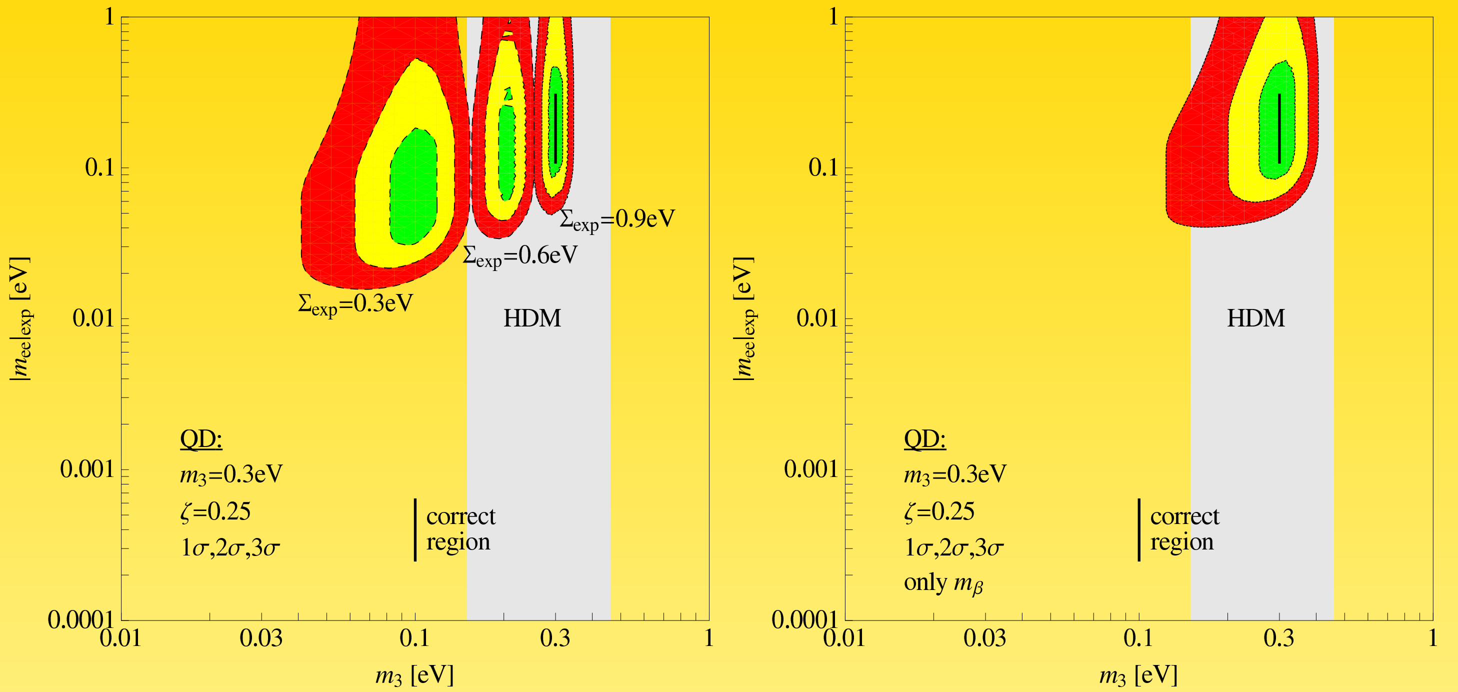
GERDA: $\sigma(\Gamma_{\rm obs})/\Gamma_{\rm obs} \simeq 23.3\%$ (Phase I: 6 ± 1.4 events if Klapdor is right)

$$\text{“theoretical error”: } \sigma(|m_{ee}|) = (1+\zeta) \left(|m_{ee}| + \sigma(|m_{ee}|_{\rm exp}) \right) - |m_{ee}|$$

- $\sigma(m_\beta^2) = 0.025$ eV^2 and $\sigma(\Sigma) = 0.05$ eV

Maneschg, Merle, W.R., EPL **85**, 51002 (2009)

(see also Pascoli, Petcov, Schwetz, NPB **734**, 24 (2006); Hannestad, 0710.1952; Lisi, talk at Erice 09)

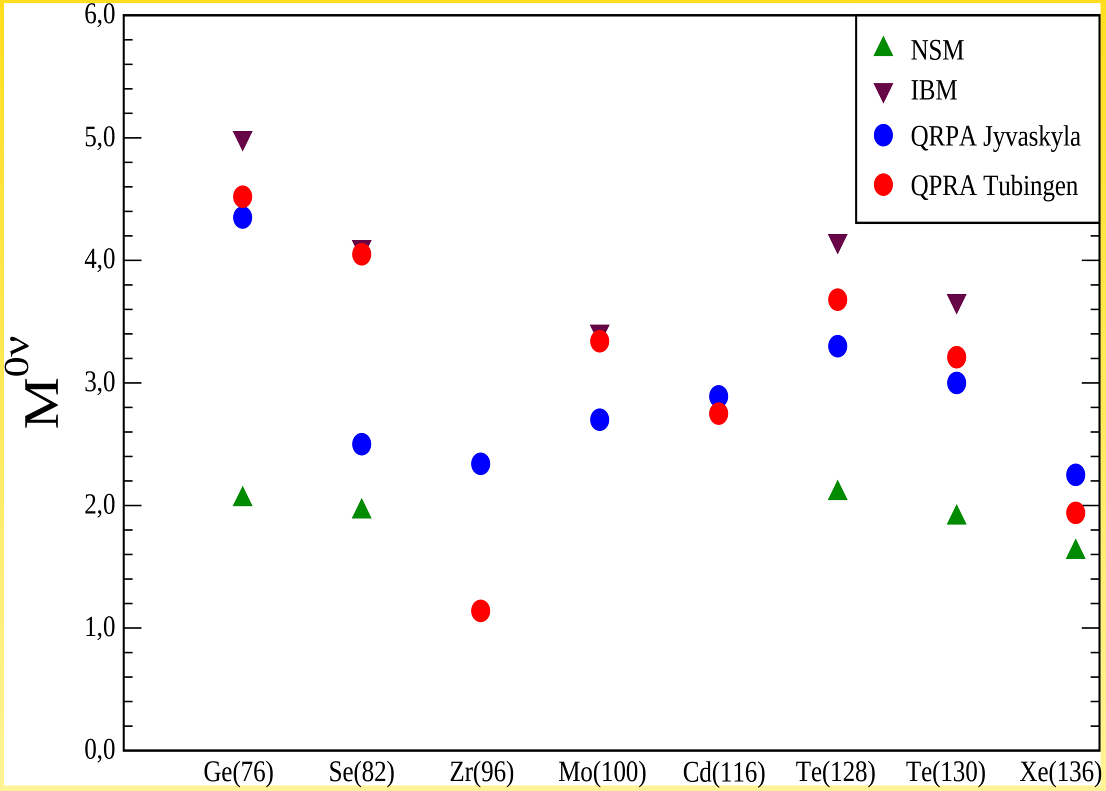


$\mathcal{QD}$ with $|m_{ee}|_{\rm exp} = 0.20$ eV

- if $\zeta(\text{NME}) = 0$: $\sigma(m_3) \simeq 15\%$ at 3σ
- if $\zeta(\text{NME}) = 0.25$: $\sigma(m_3) \simeq 25\%$

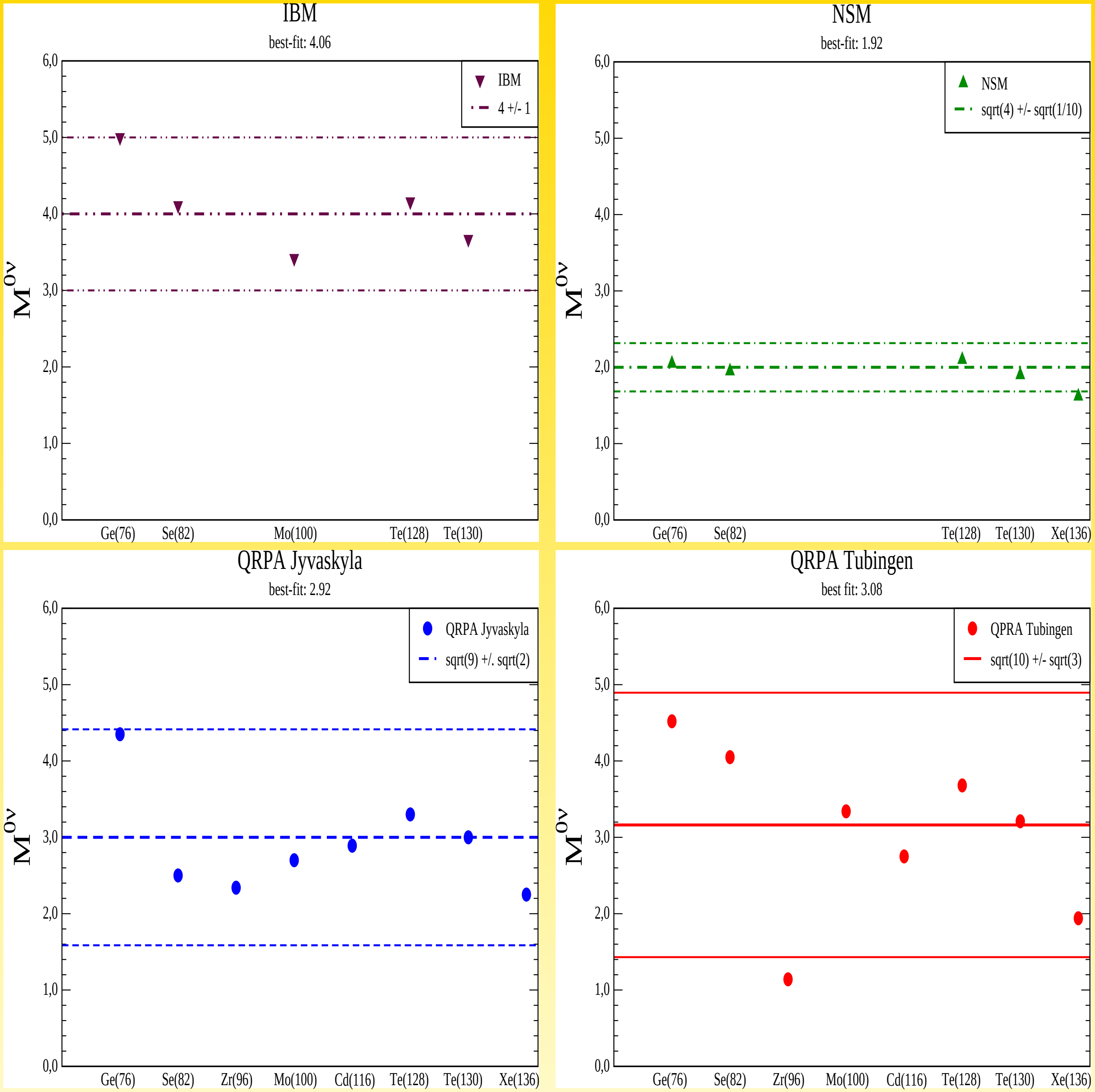
Maneschg, Merle, W.R., EPL **85**, 51002 (2009)

Nuclear Matrix Elements



Rodin, 0910.5866

Talk by Fedor Simkovic on Friday


IBM
best-fit: 4.06
IBM
4 +/- 1
NSM
best-fit: 1.92
NSM
sqrt(4) +/- sqrt(1/10)
QRPA Jyvaskyla
best-fit: 2.92
QRPA Jyvaskyla
sqrt(9) +/. sqrt(2)
QRPA Tubingen
best fit: 3.08
QPRA Tubingen
sqrt(10) +/- sqrt(3)
M^0ν
0,0
1,0
2,0
3,0
4,0
5,0
6,0
Ge(76)
Se(82)
Zr(96)
Mo(100)
Cd(116)
Te(128)
Te(130)
Xe(136)

Neutrino mass models and $0\nu\beta\beta$

Example: 58 models based on A_4 leading to tri-bimaximal mixing:

Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	-	[1–11] [12]$^{\#}$
A2				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[13, 14]
A3				$\underline{1}, \underline{3}$	[15]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[4, 16–21]$^{\#}$ [22, 23]* [24–35]
B2				$\underline{1}, \underline{3}$	[36]$^{\#}$
C1	$\underline{3}$	$\underline{3}$	-	-	[2]
C2				$\underline{1}$	[37, 38] [39]$^{\#}$
C3				$\underline{1}, \underline{3}$	[40]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[41]
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	-	[42, 43]* [44, 45]
D2				$\underline{1}$	[46] [47]*
D3				$\underline{1}'$	[48]*
D4				$\underline{1}', \underline{3}$	[49]*
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[50, 51]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[52]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[53]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[54]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[55]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[56, 57]
K	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	$\underline{1}$	[58]*
L	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}$	-	[59]*

Barry, W.R., PRD **81**, 093002 (2010)

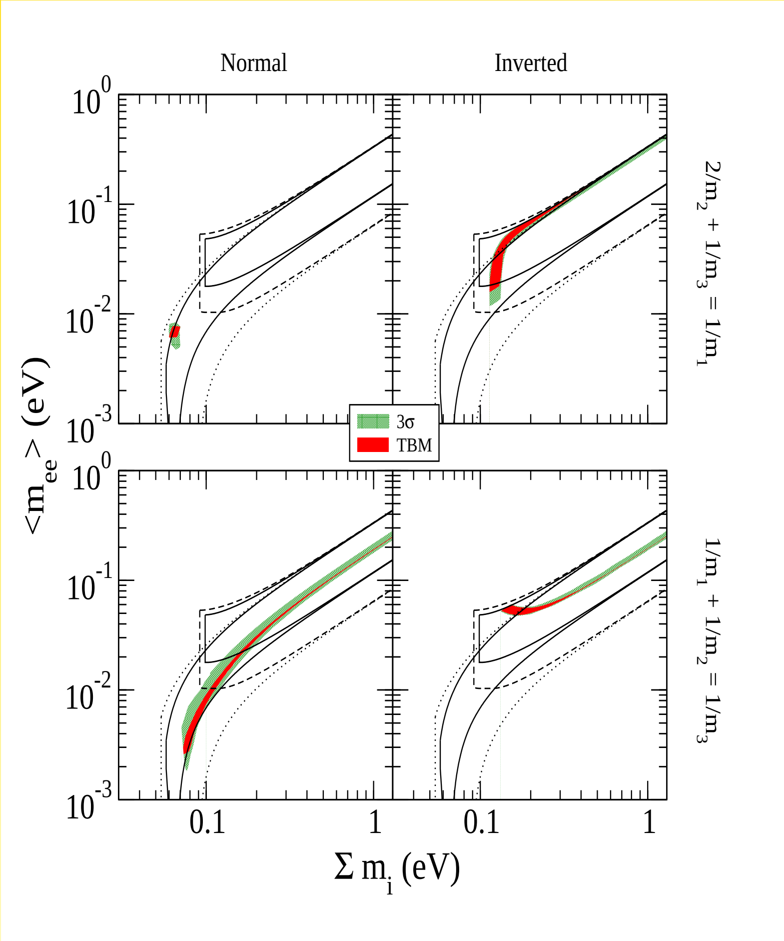
Sum-rules in Models and $0\nu\beta\beta$

$$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$$

Altarelli-Feruglio

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$$

AF + 1 singlet



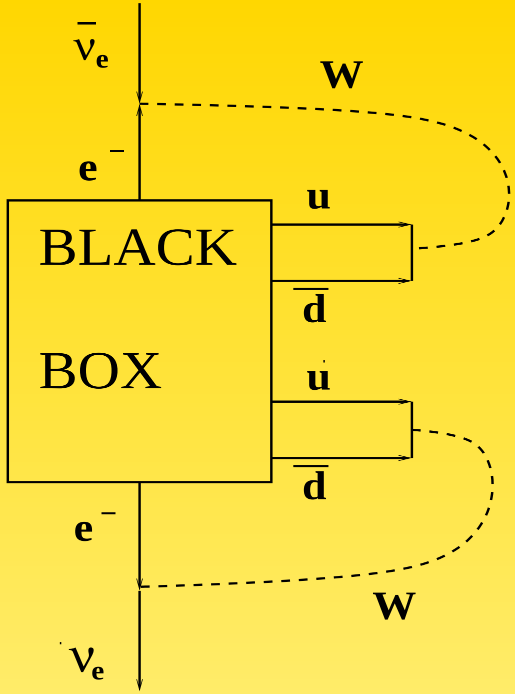
Barry, W.R., in preparation

Non-Standard Interpretations

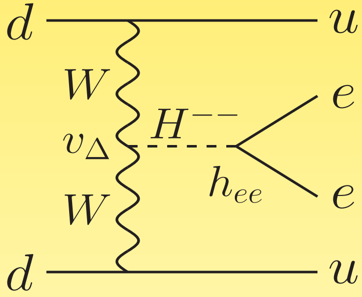
There is at least one other mechanism leading to Neutrinoless Double Beta Decay and its contribution is at least of the same order as the light neutrino exchange mechanism

Clear experimental signature:

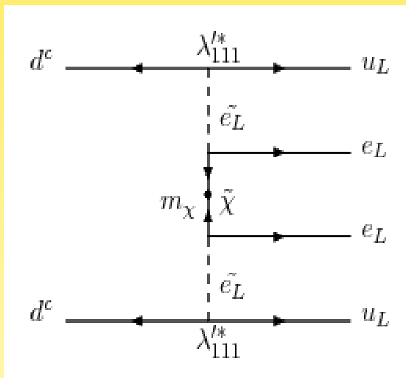
KATRIN (and cosmology) sees nothing but "$|m_{ee}| > 0.5$ eV"



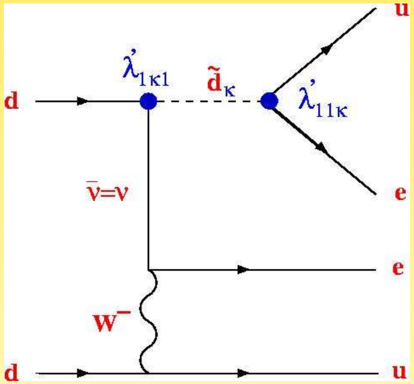
ν̄e
W
e −
u
BLACK
BOX
d̄
u
d̄
e −
W
νe



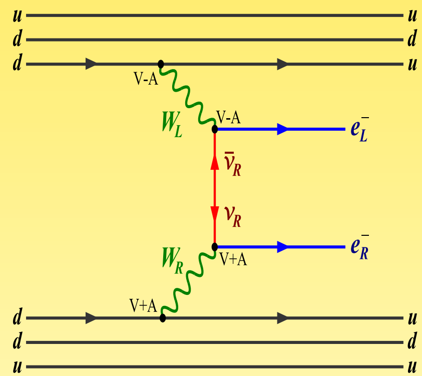
d
u
W
vΔ
H−−
hee
e
e
W
d
u



dc
λ′*111
uL
ẽL
eL
mχ
χ̃
eL
ẽL
dc
λ′*111
uL



λ′1κ1
d̃κ
λ′11κ
u
d
e
ν̄=ν
e
W−
d
u

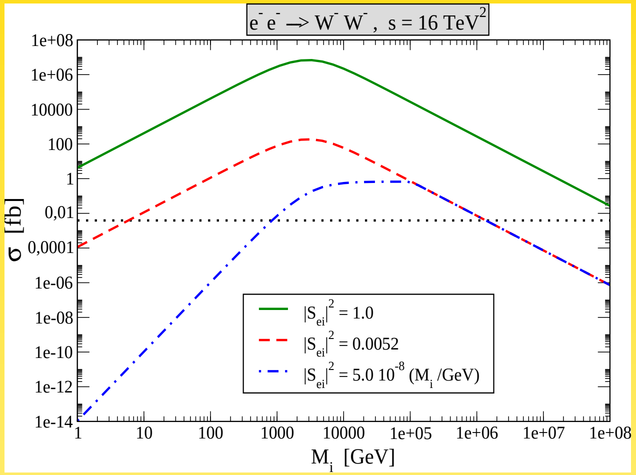


u
d
d
u
d
u
V-A
WL
V-A
e−L
ν̄R
νR
WR
V+A
e−R
V+A
d
d
u
u
d
u

Examples

Fermions[a] and no RHC:

$$\mathcal{A} \propto \frac{m_F}{q^2 - m_F^2} \rightarrow \begin{cases} m_F & \text{for } q^2 \gg m_F^2 \\ \frac{1}{m_F} & \text{for } q^2 \ll m_F^2 \end{cases}$$



$$e^- e^- \rightarrow W^- W^-$$

heavy scalar:

$$\mathcal{A} \propto \frac{1}{q^2 - m_S^2} \rightarrow \frac{1}{m_S^2}$$

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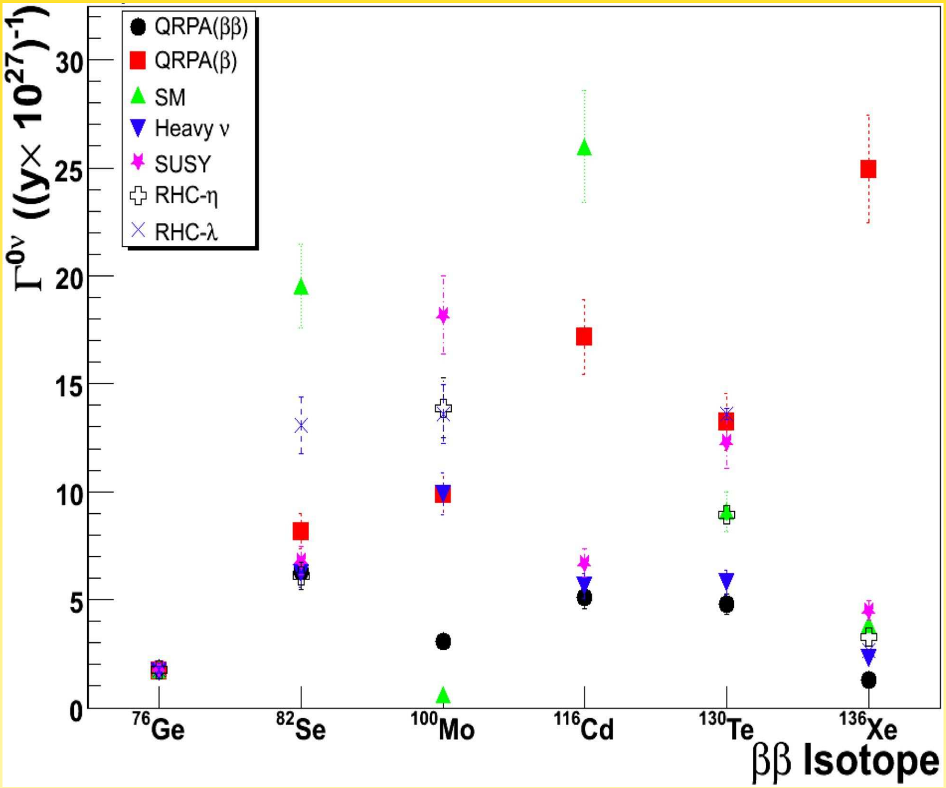
[a] Note: maximum $\mathcal{A}$ corresponds to $\langle E \rangle \simeq m_F$: interesting limits on $\mathcal{O}(m_K)$ Majorana neutrinos from e.g. $K^+ \rightarrow \pi^- \mu^+ e^+$ (Atre *et al.*, JHEP **0905**, 030 (2009); Helo, Kovalenko, Schmidt, 1005.1607)

mechanism	amplitude	limit	literature
light ν	$G_F^2 \frac{\lvert m_{ee}\rvert}{q^2}$	0.5 eV	0.5 eV
heavy ν	$G_F^2 \frac{S_{ei}^2}{M_i}$	$5 \cdot 10^{-8}$ GeV^{-1}	$5 \cdot 10^{-8}$ GeV^{-1}
Higgs triplet	$G_F^2 \frac{h_{ee}\, v_\Delta}{m_\Delta^2}$	$5 \cdot 10^{-8}$ GeV^{-1}	??
$\not{R}_P$ SUSY I	$g_i^2 \frac{\lambda'^2_{111}}{\Lambda^5_{\rm SUSY}}$	$7 \cdot 10^{-17}$ GeV^{-5}	$3 \cdot 10^{-17}$ GeV^{-5}
$\not{R}_P$ SUSY II	$G_F\, m_{d_k} \frac{\lambda'_{1k1}\, \lambda'_{11k}}{\Lambda^3_{\rm SUSY}}$	$7 \cdot 10^{-12}$ GeV^{-3} $3 \cdot 10^{-13}$ GeV^{-3} $1 \cdot 10^{-14}$ GeV^{-3}	$7.7 \cdot 10^{-12}$ GeV^{-3} $4.0 \cdot 10^{-13}$ GeV^{-3} $1.7 \cdot 10^{-14}$ GeV^{-3}
RHC$_1$ ("$\langle\lambda\rangle$")	$G_F^2 \frac{1}{q} \frac{m_W^2}{m_{W_R}^2} U_{ei}\, V_{ei}$	$5 \cdot 10^{-9}$	$1 \cdot 10^{-6}$
RHC$_2$ ("$\langle\eta\rangle$")	$G_F^2 \frac{1}{q} \tan\zeta\, U_{ei}\, V_{ei}$	$5 \cdot 10^{-9}$	$6 \cdot 10^{-9}$
Majoron $n = 1\,(3)$	$\Gamma \propto (G_F^2\, \langle g_\chi\rangle)^2 \frac{Q^{5+n}}{q^{n/2-2}}$	$2 \cdot 10^{-4}\,(1)$	$0.4 \cdot 10^{-4}\,(1.5)$

Non-Neutrino Physics and $0\nu\beta\beta$

Difference of standard to non-standard mechanism (RH currents, SUSY, etc.) is nuclei-dependent:

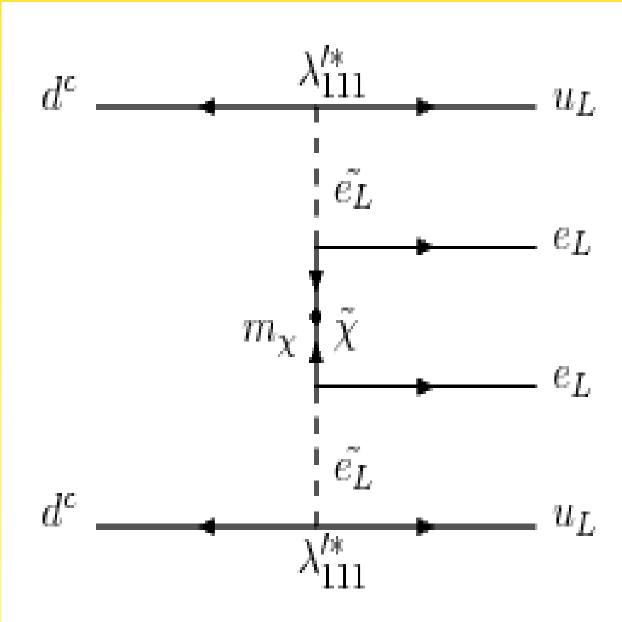


Gehman, Elliott, JPG **34**, 667 (2007); see also Deppisch, Päs, PRL **98**, 232501 (2007); Simkovic, Vergados, Faessler, 1006.0571

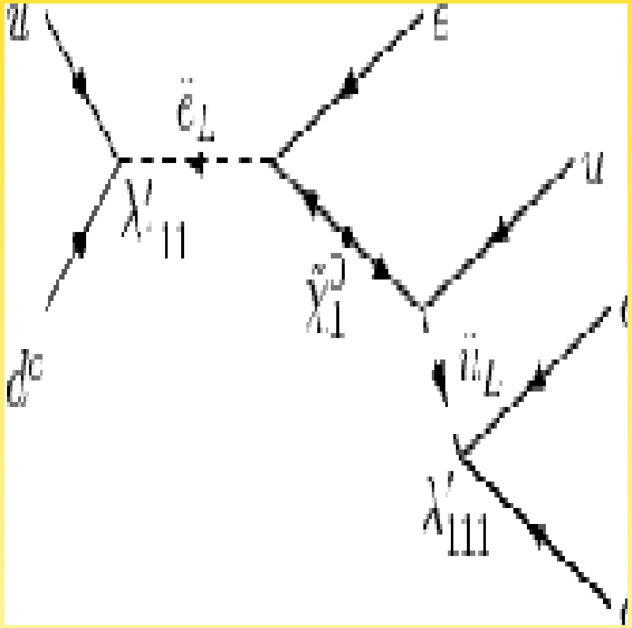
LHC and $0\nu\beta\beta$

$R\!\!\!/_P$ SUSY

Allanach, Kom, Päs, PRL **103**, 091801 (2009)



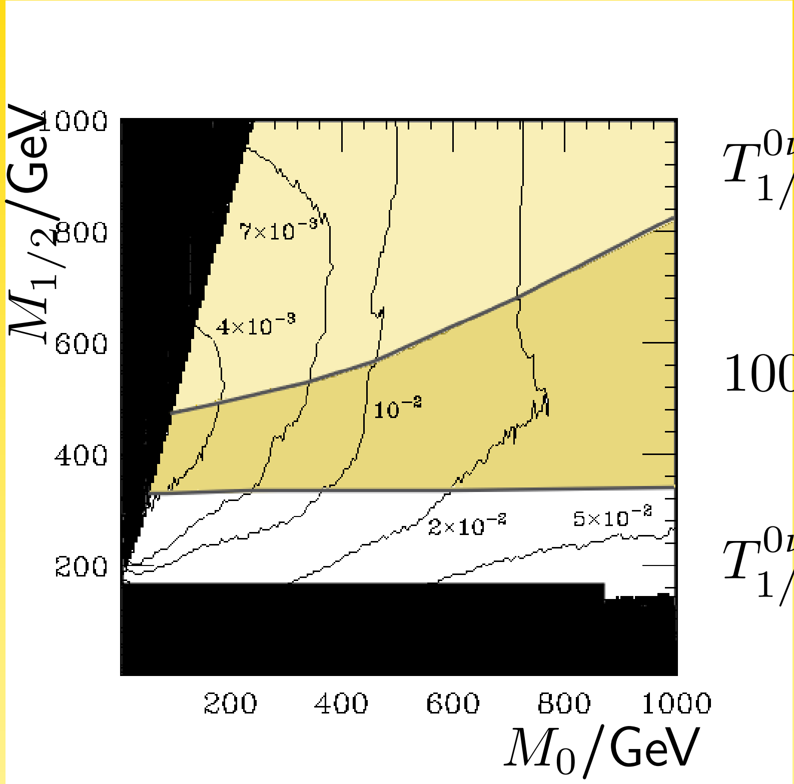
$0\nu\beta\beta$



resonant selectron production

$u\,\overline{d} \to e^+ e^+\, \overline{u}\, d$

$\tan\beta = 10,\ A_0 = 0,\ 10\ \mathrm{fb}^{-1}$



$T_{1/2}^{0\nu\beta\beta}(\mathrm{Ge}) > 1 \times 10^{27}$ yrs

$100 > T_{1/2}^{0\nu\beta\beta}(\mathrm{Ge})/10^{25}$ yrs > 1.9

$T_{1/2}^{0\nu\beta\beta}(\mathrm{Ge}) < 1.9 \cdot 10^{25}$ yrs

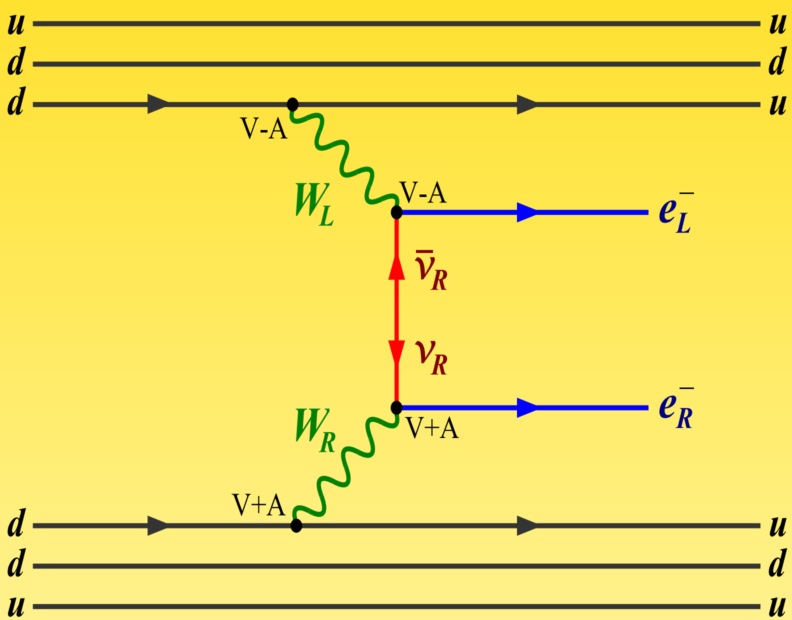
$\longrightarrow$ observation in white region in conflict with $0\nu\beta\beta$

$\longrightarrow$ if $0\nu\beta\beta$ observed: dark yellow region tests $R\!\!\!/_P$ SUSY mechanism

$\longrightarrow$ light yellow region: no significant $R\!\!\!/_P$ contribution to $0\nu\beta\beta$

SuperNEMO

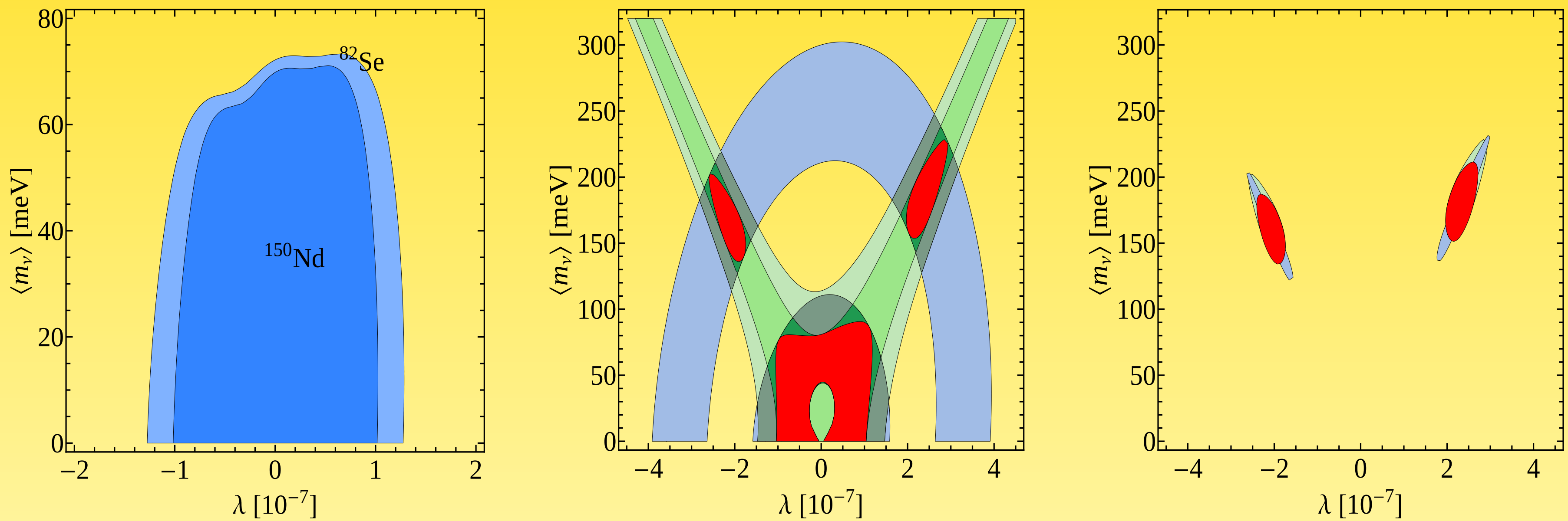
unique tracking calorimeter: energy of individual electrons, angular distribution of electrons, multi-isotopes (?)



$$\Gamma^{0\nu\beta\beta} = A\,|m_{ee}|^2 + B\,\lambda^2 + C\,|m_{ee}|\,\lambda \quad \text{with } \lambda = \left(\frac{M_W}{M_{W_R}}\right)^2 U_{ei}\,V_{ei}$$

talk in $\simeq 100$ min by Ruben Saakyan

Asymmetry	^{82}Se		^{150}Nd	
$\mathcal{A}_\theta$	0.44	-0.40	0.45	-0.40
$\mathcal{A}_\mathrm{E}$	0.33	-0.54	0.32	-0.55



sensitivity

$0.3\, m_\nu + 0.7$ RHC

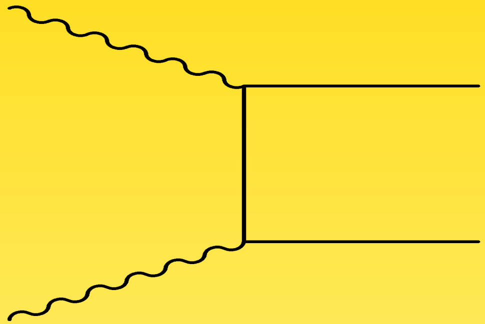
only ^{82}Se

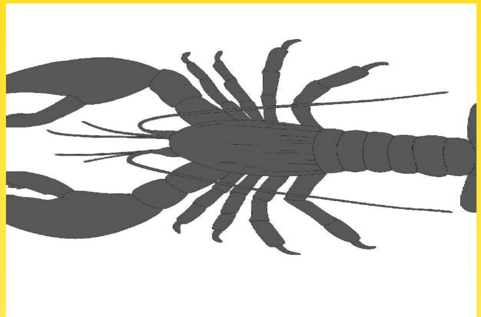
^{82}Se + ^{150}Nd

Arnold *et al.*, 1005.1241

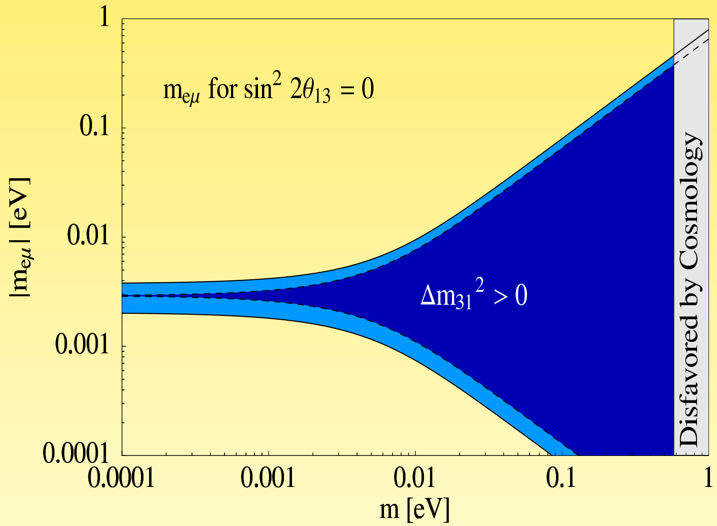
Analogous Processes

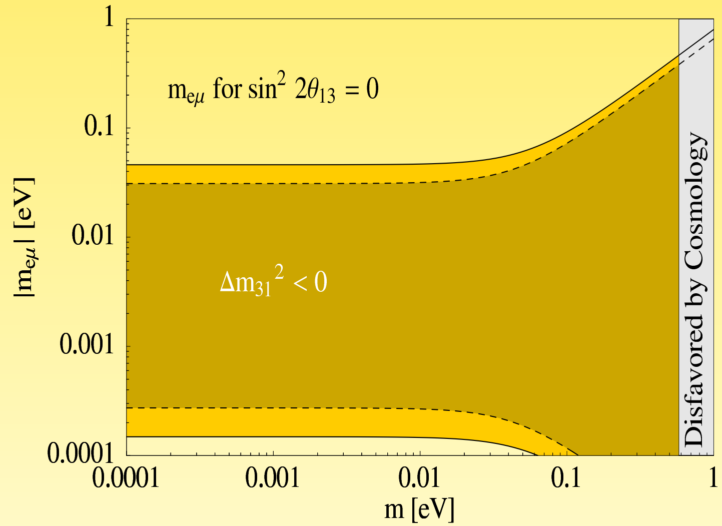
"The lobster"





$$\mathrm{BR}(K^+ \to \pi^- e^+ \mu^+) \propto |m_{e\mu}|^2 = \left| \sum U_{ei}\, U_{\mu i}\, m_i \right|^2 \sim 10^{-30} \left(\frac{|m_{e\mu}|}{\mathrm{eV}} \right)^2$$





Inverse Neutrinoless Double Beta Decay

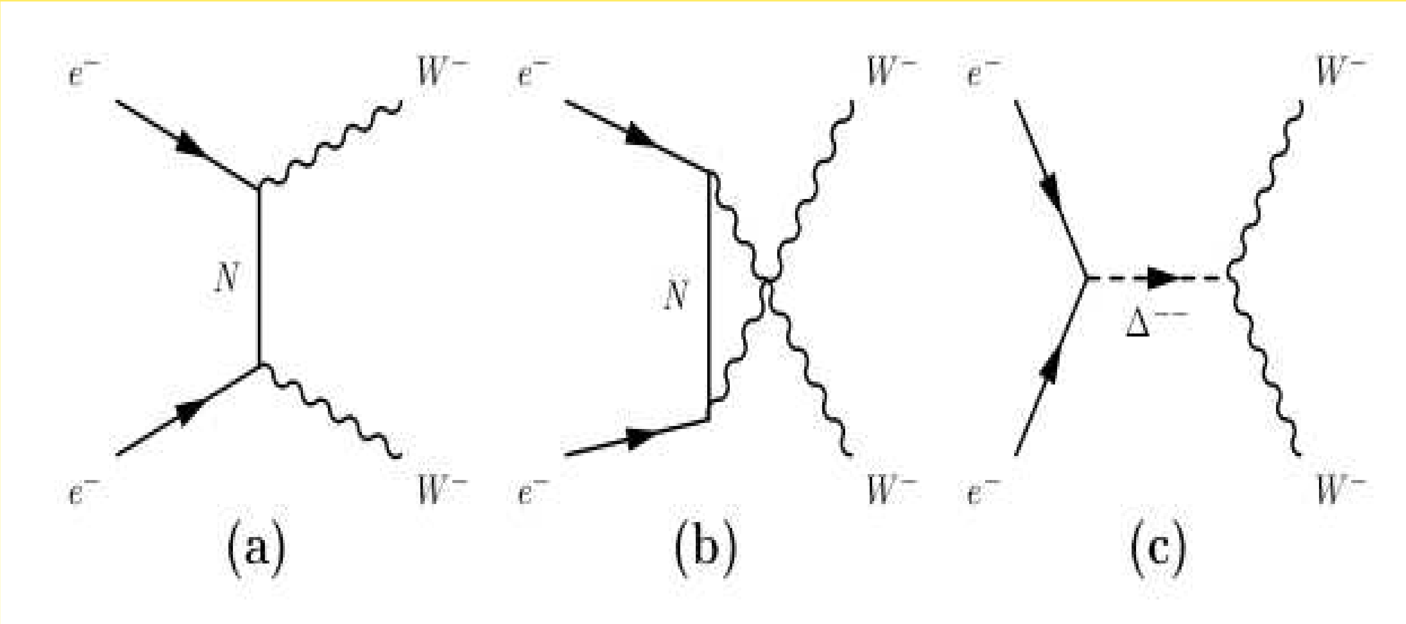
this is not

$$^{76}\mathrm{Se}^{++} + e^- + e^- \to \ ^{76}\mathrm{Ge}$$

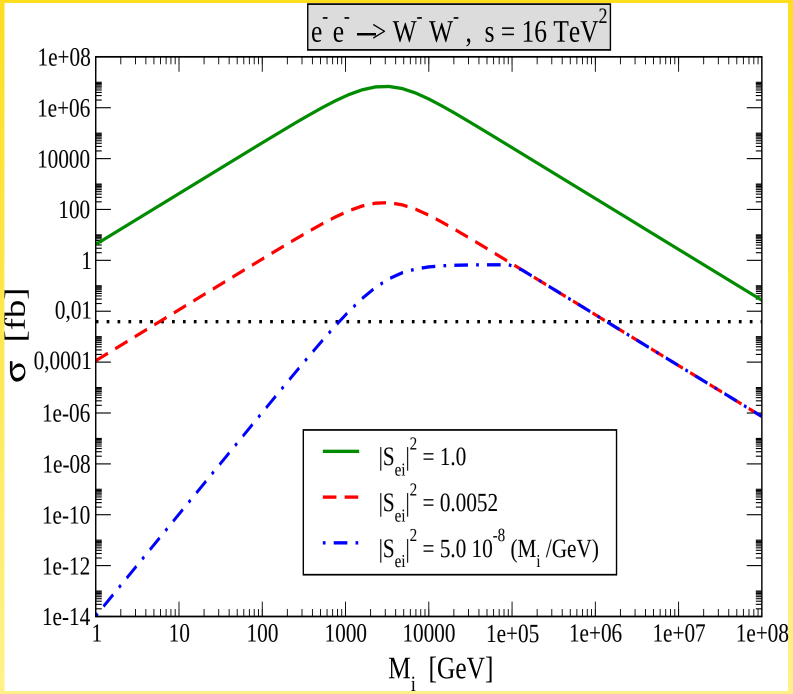
but rather

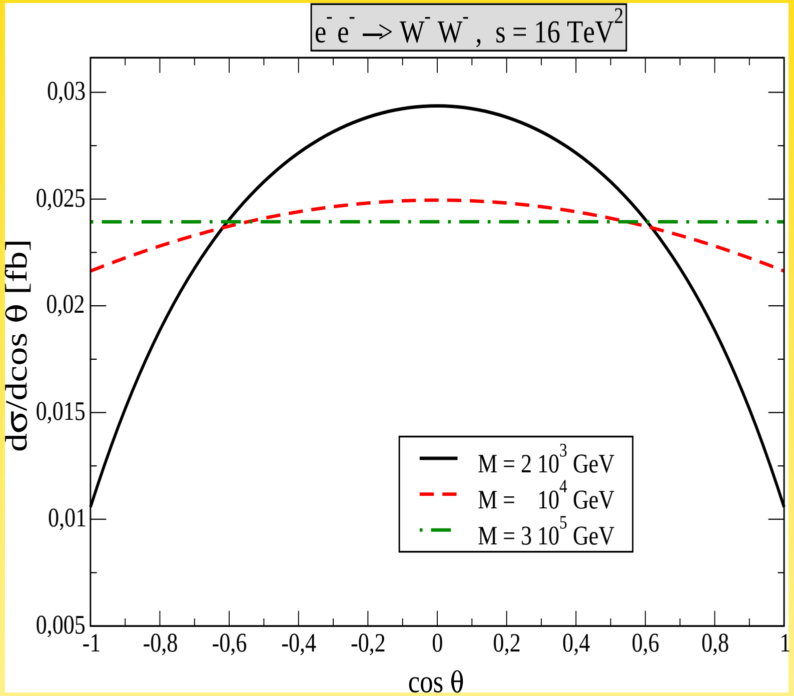
$$e^- + e^- \to W^- + W^-$$

Rizzo; Heusch, Minkowski; Gluza, Zralek; Cuypers, Raidal;...



Inverse Neutrinoless Double Beta Decay





W.R., PRD **81**, 114001 (2010)

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\,\pi}\left\{\sum (m_\nu)_i\,\mathcal{U}_{ei}^2\left(\frac{t}{t-(m_\nu)_i}+\frac{u}{u-(m_\nu)_i}\right)\right\}^2$$

Inverse Neutrinoless Double Beta Decay

Extreme limits:

- light neutrinos:

$$\sigma(e^- e^- \to W^- W^-) = \frac{G_F^2}{4\,\pi}\,|m_{ee}|^2 \leq 4.2 \cdot 10^{-18} \left(\frac{|m_{ee}|}{1\,\mathrm{eV}} \right)^2 \mathrm{fb}$$

$\Rightarrow$ way too small

- heavy neutrinos:

$$\sigma(e^- e^- \to W^- W^-) = 2.6 \cdot 10^{-3} \left(\frac{\sqrt{s}}{\mathrm{TeV}} \right)^4 \left(\frac{S_{ei}^2/M_i}{5 \cdot 10^{-8}\,\mathrm{GeV}^{-1}} \right)^2 \mathrm{fb}$$

$\Rightarrow$ too small

- $\sqrt{s} \to \infty$:

$$\sigma(e^- e^- \to W^- W^-) = \frac{G_F^2}{4\pi} \left(\sum \mathcal{U}_{ei}^2\,(m_\nu)_i \right)^2$$

$\Rightarrow$ amplitude grows with $\sqrt{s}$? Unitarity??

Unitarity

high energy limit $\sqrt{s} \to \infty$:

$$\sigma(e^- e^- \to W^- W^-) = \frac{G_F^2}{4\pi} \left(\sum \mathcal{U}_{ei}^2 \, (m_\nu)_i \right)^2$$

$\leftrightarrow$ amplitude grows with $\sqrt{s}$?

Answer: exact see-saw relation $\mathcal{U}_{ei}^2 \, (m_\nu)_i = 0$

$$\mathcal{M} = \left(\begin{array}{cc} 0 & m_D \\ m_D^T & M_R \end{array} \right) = \mathcal{U} \left(\begin{array}{cc} m_\nu^{\rm diag} & 0 \\ 0 & M_R^{\rm diag} \end{array} \right) \mathcal{U}^T$$

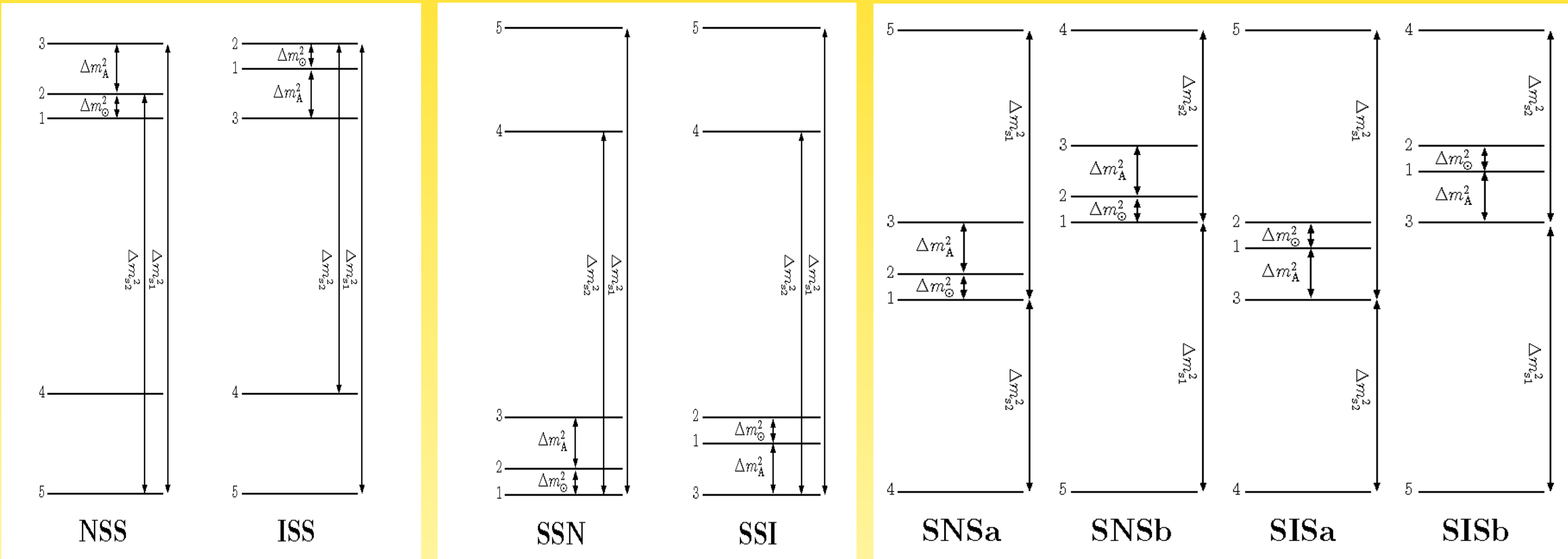
if Higgs triplet is present: unitarity also conserved

$$\sigma(e^- e^- \to W^- W^-) = \frac{G_F^2}{4\pi} \left((\mathcal{U}_{ei}^2 \, (m_\nu)_i - (m_L)_{ee} \right)^2 = 0$$

Something in between Standard and Non-Standard: Light Sterile Neutrinos

$\leftrightarrow$ LSND/MiniBooNE requires (at least) 2 additional ν

Karagiorgi, talk on Monday



8 schemes with interesting $|m_{ee}|$, m_β, Σ phenomenology

Goswami, W.R., JHEP **0710**, 073 (2007); see also Giunti, Laveder, 1005.4599

Summary

Chi l'ha visto?

Ettore Majorana, ordinario di fisica teorica all'Università di Napoli, è misteriosamente scomparso dagli ultimi di marzo. Di anni 31, alto metri 1,70, snello, con capelli neri, occhi scuri, una lunga cicatrice sul dorso di una mano. Chi ne sapesse qualcosa è pregato di scrivere al R. P. E. Marianecci, Viale Regina Margherita 66 - Roma.

An exact See-Saw Relation

Full mass matrix:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} = \mathcal{U} \begin{pmatrix} m_\nu^{\rm diag} & 0 \\ 0 & M_R^{\rm diag} \end{pmatrix} \mathcal{U}^T \text{ with } \mathcal{U} = \begin{pmatrix} N & S \\ T & V \end{pmatrix}$$

- N is the PMNS matrix
- S describes mixing of heavy neutrinos with SM leptons

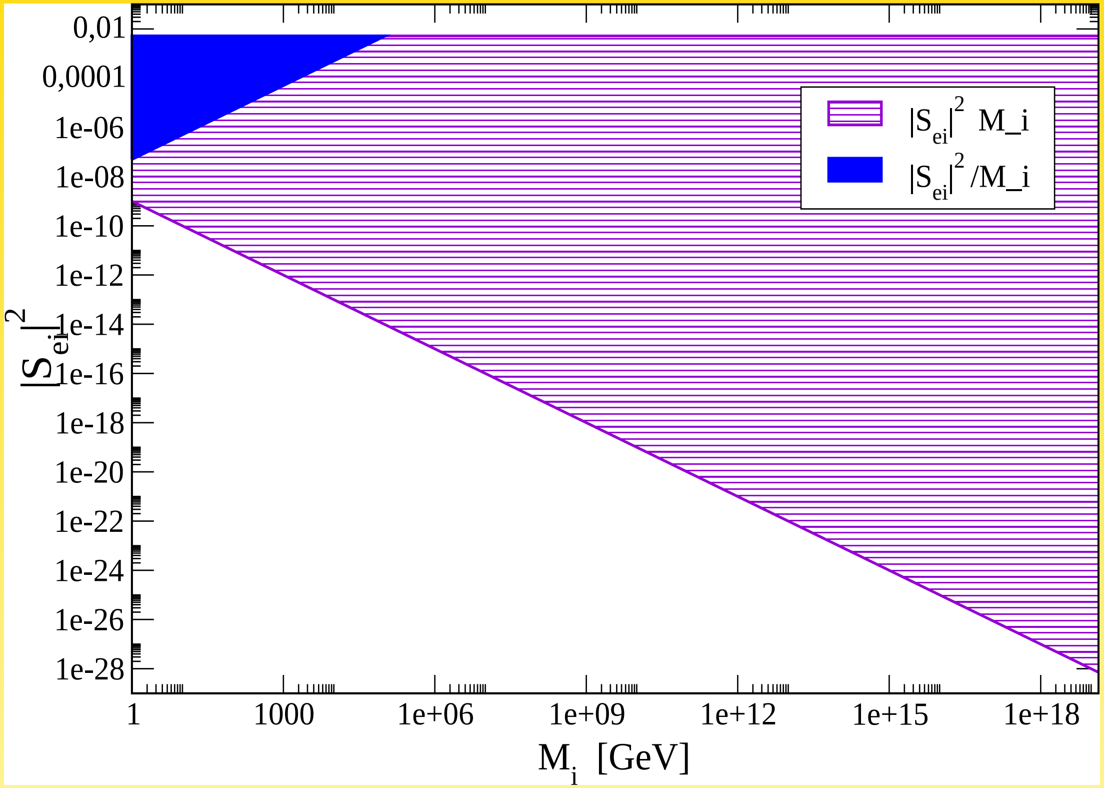
The upper left 0 in $\mathcal{M}$ gives exact see-saw relation $\mathcal{U}_{\alpha i} \, (m_\nu)_i \, \mathcal{U}_{i\beta} = 0$, or:

$$|N_{ei}^2 \, m_i| = |S_{ei}^2 \, M_i|$$

Xing, PLB **679**, 255 (2009); W.R., PLB **684**, 40 (2010)

compare with $\dfrac{S_{ei}^2}{M_i} < 5 \cdot 10^{-8} \, {\rm GeV}^{-1}$

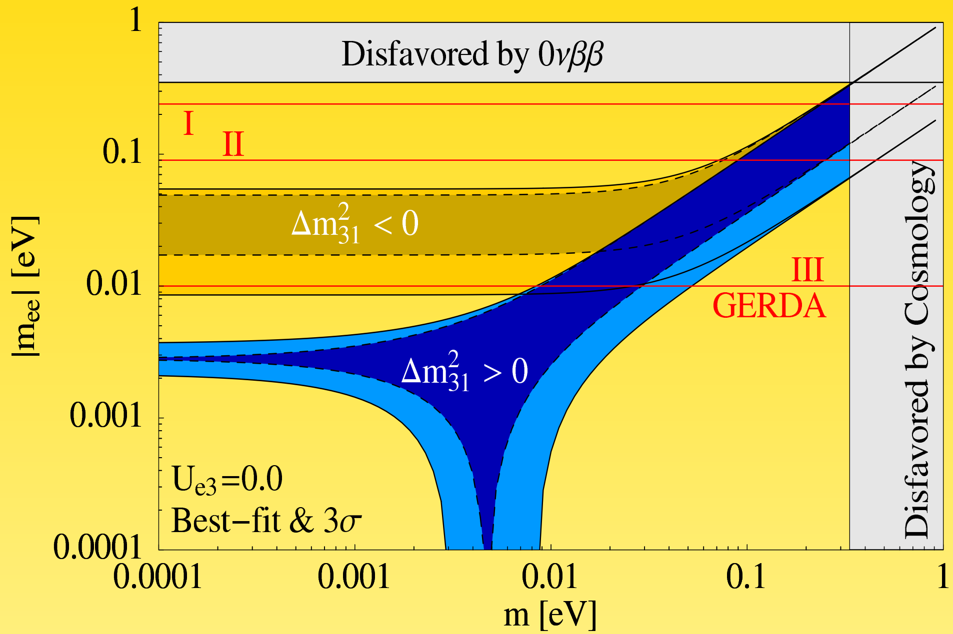
exact see-saw relation gives stronger constraints!



Xing, PLB **679**, 255 (2009); W.R., PLB **684**, 40 (2010)

also saves unitarity in $e^- e^- \to W^- W^-$ (Belanger *et al.*, PRD **53**, 6292 (1996); W.R., PRD **81**, 114001 (2010))

If $|m_{ee}| = 0$, does it stay zero?



The "chimney"

- RG effects!
- actually:

$$\mathcal{M} \propto \frac{U_{ei}^2 \, m_i}{q^2 - m_i^2} \simeq \frac{|m_{ee}|}{q^2} + \mathcal{O}(m_i^3/q^4)$$

	Σ	m_β	$\lvert m_{ee}\rvert$
NH	$\sqrt{\Delta m_{\rm A}^2}$	$\sqrt{\Delta m_\odot^2 + \lvert U_{e3}\rvert^2 \Delta m_{\rm A}^2}$	$\left\lvert \sqrt{\Delta m_\odot^2} + \lvert U_{e3}\rvert^2 \sqrt{\Delta m_{\rm A}^2}\, e^{i(\alpha-\beta)} \right\rvert$
IH	$2\sqrt{\Delta m_{\rm A}^2}$	$\sqrt{\Delta m_{\rm A}^2}$	$\sqrt{\Delta m_{\rm A}^2}\sqrt{1 - \sin^2 2\theta_\odot \sin^2 \alpha}$
QD	$3\, m_0$	m_0	$m_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2 \alpha}$

corrections due to splitting of masses very small;

corrections due to non-unitary PMNS matrix can be larger. . .

W.R., PLB **684**, 40 (2010)