

η Photoproduction in the Jülich-Bonn Model

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The 12th International Workshop on the Physics of Excited Nucleons

Supported by



DOE DE-AC05-06OR23177
& DE-SC0016582



HPC support by JSC grant *jikp07*

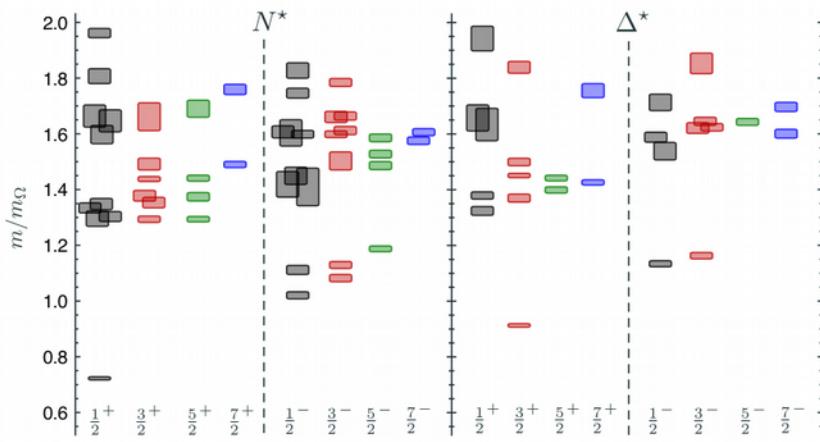
Degrees of freedom: Quarks or hadrons?

The Missing Resonance Problem

- above 1.8 GeV much more states are predicted than observed,

“Missing resonance problem”

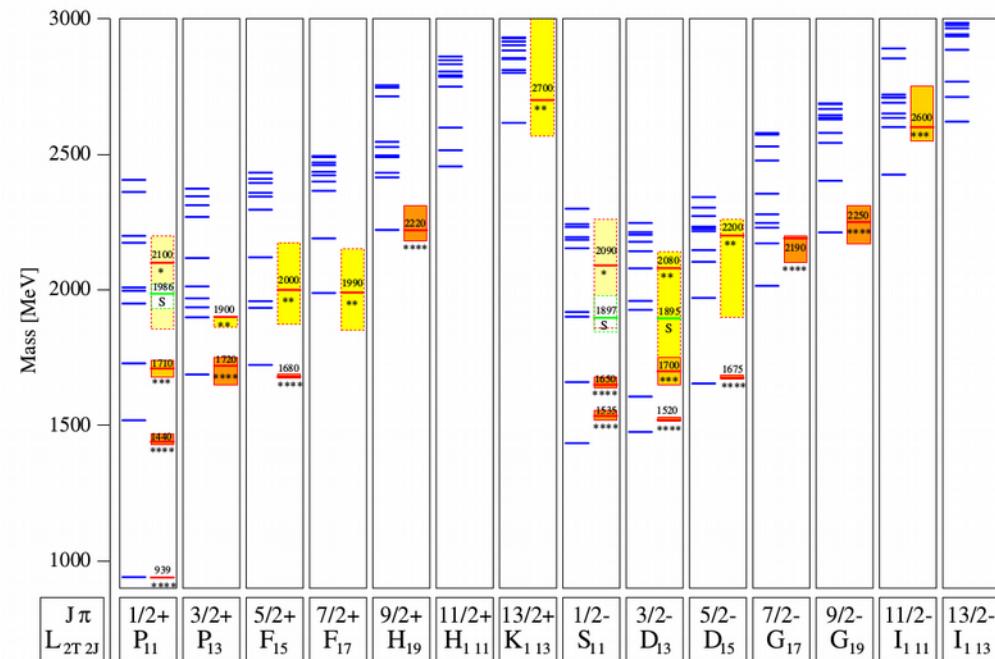
Lattice calculation (single hadron approximation):



[Edwards *et al.*, Phys.Rev. D84 (2011)]

- only 15 established N^* states (PDG 2015)
- ~ 48% of the states have **** or *** status (PDG 1982: 58% with **** or ***)

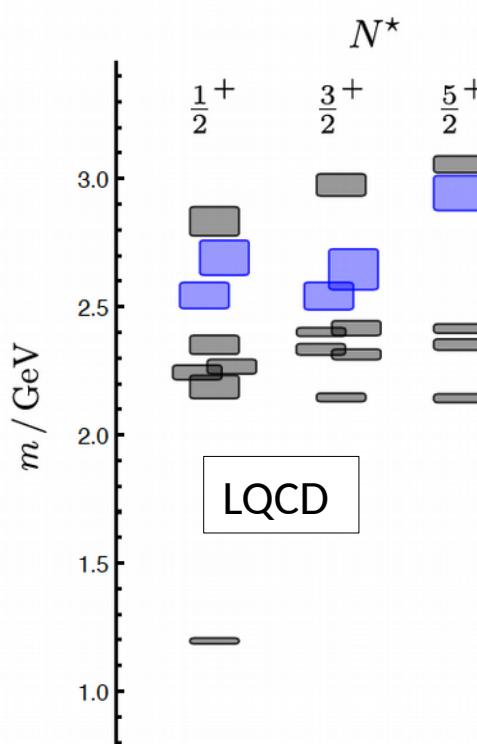
N^* spectrum in a relativistic quark model:



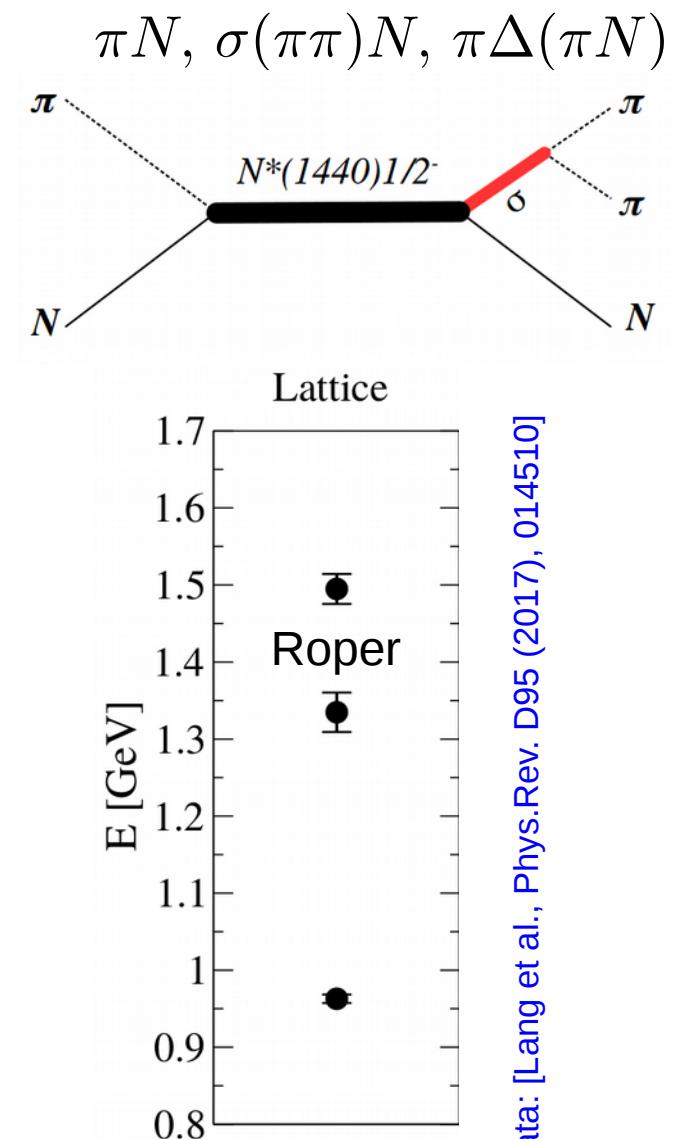
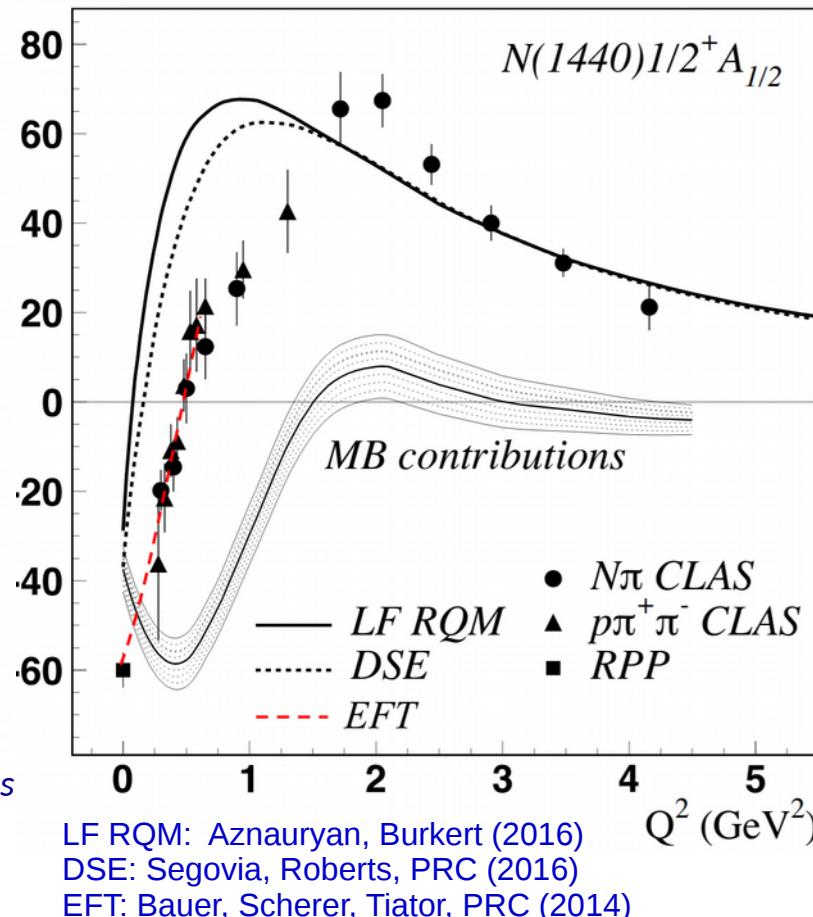
Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Overviews: Crede, Roberts, Rep. Prog. Phys. 76 (2013)
Aznauryan *et al.*, Int. J. Mod. Phys. E 22 (2013)

Hybrid Baryons



J.J. Dudek and R.G. Edwards
PRD85 (2012)



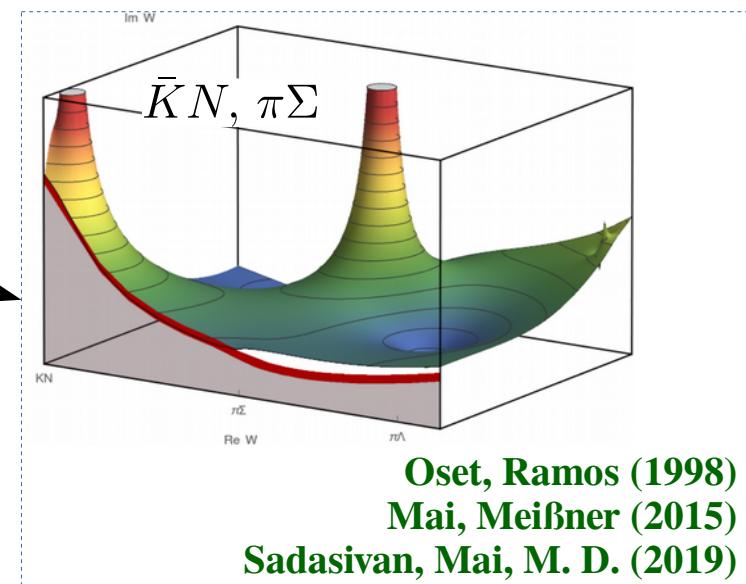
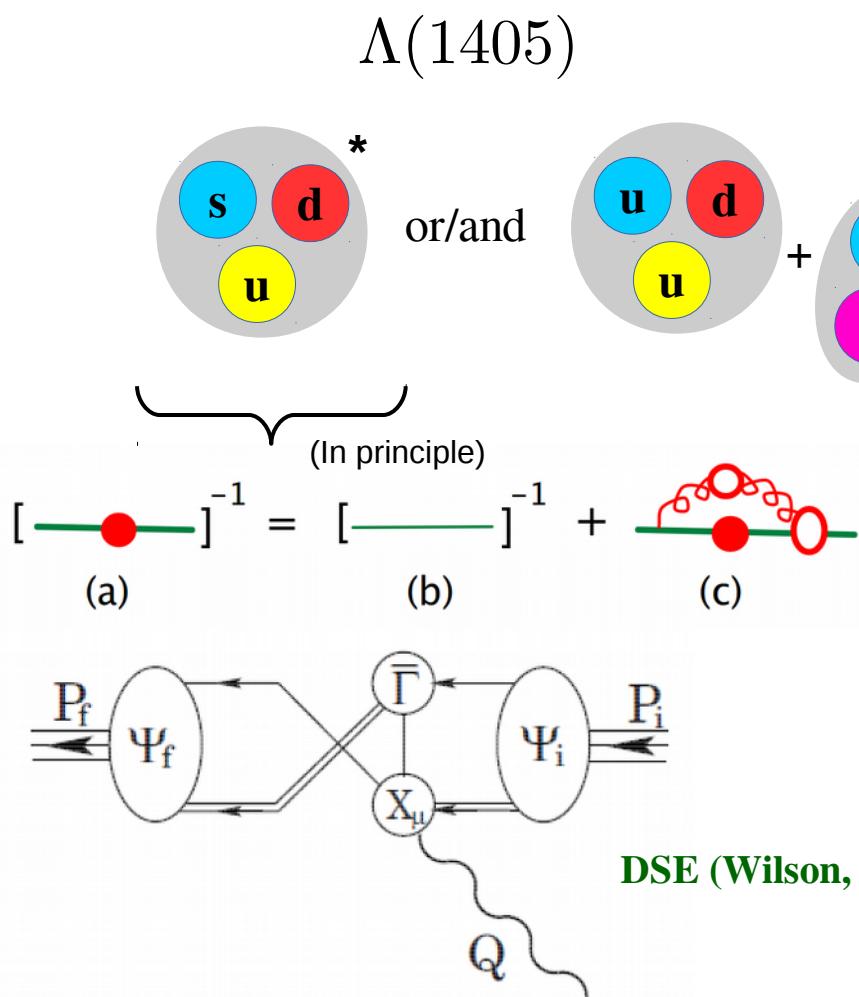
Hybrid states: same J^P values as q^3 baryons.
Identification? Measure Q^2 dependence of
electro-couplings (**CLAS 12**)

[parts of slide courtesy of V. Burkert]

- QCD at low energies
 - Non-perturbative dynamics
 - Q1: how many are there?
 - Q2: what are they?

- mass generation & confinement
- rich spectrum of excited states
(missing resonance problem)

(2-quark/3-quark, hadron molecules,
exotics,...)



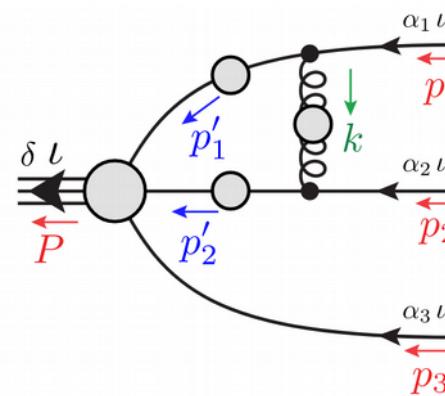
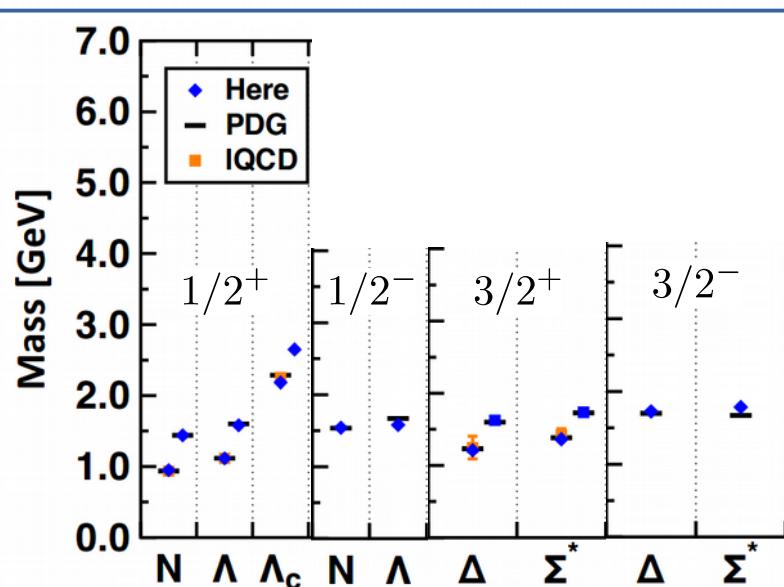
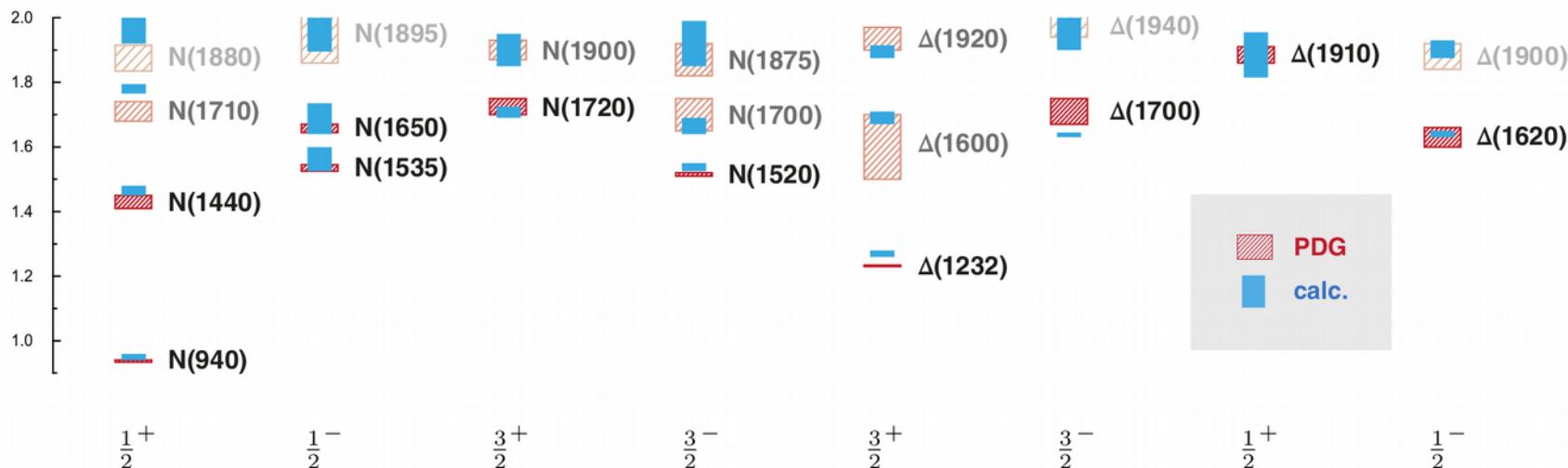
Talks on Thursday by Maxim Mai and Daniel Sadasivan

New results in dynamical quark picture

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

M [GeV]

[parts of slide courtesy of G. Eichmann, Few Body 2018]

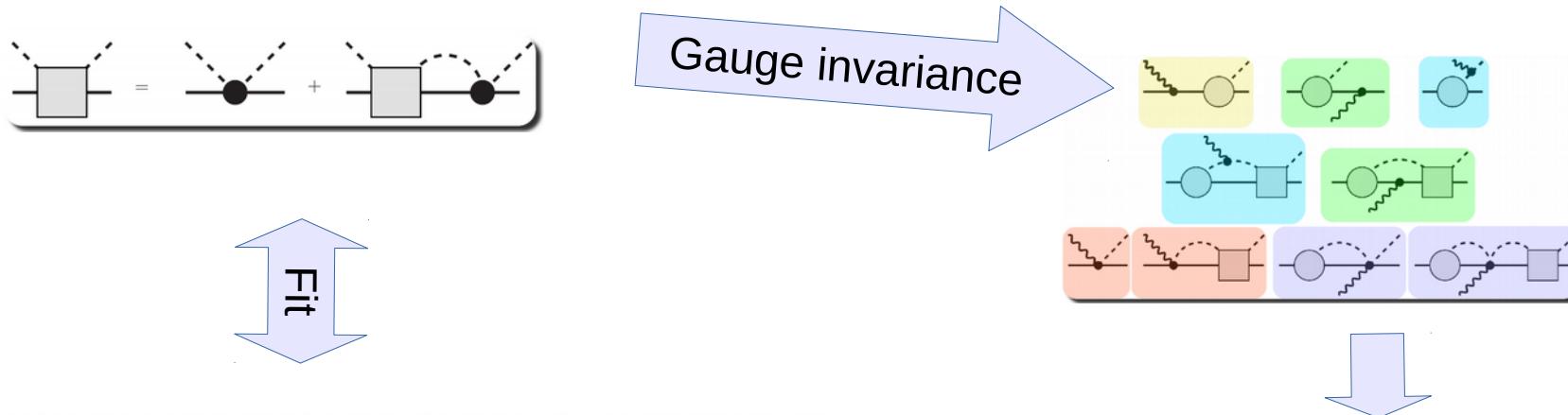


Poincaré-covariant analysis of heavy-quark baryons, Qin, Roberts, Schmidt, PRD (2018)
Spectrum of light- and heavy-baryons, Qin, Roberts, Schmidt, Few Body Syst. 60 (2019)

Using ONLY meson-baryon degrees of freedom (no explicit quark dynamics):

Manifestly gauge invariant approach based on full BSE solution

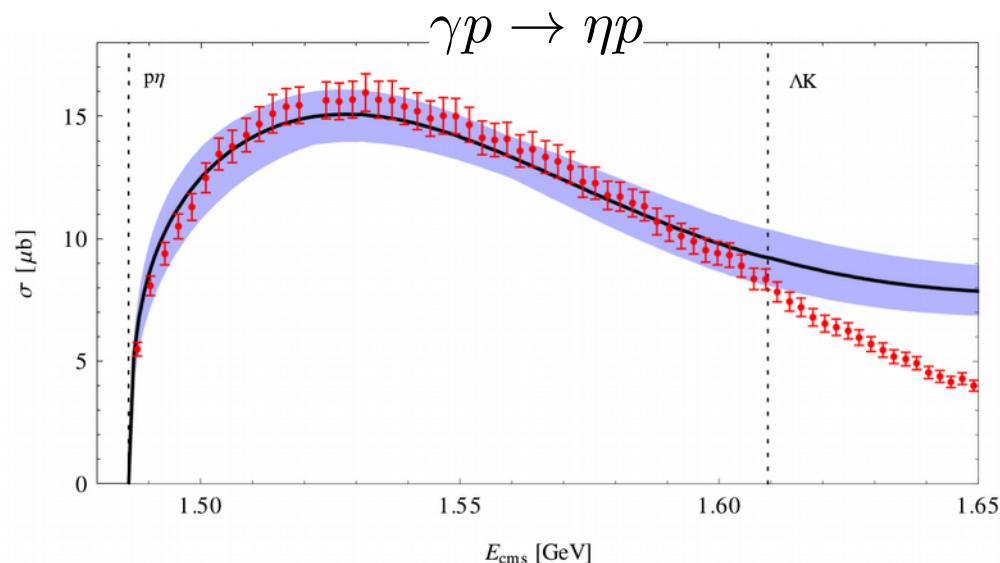
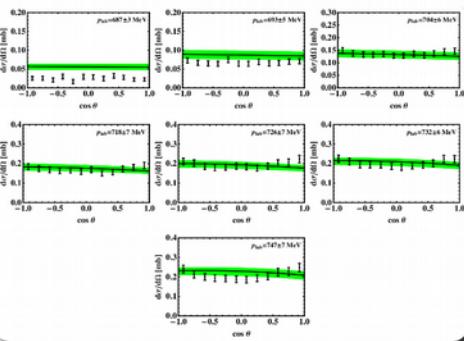
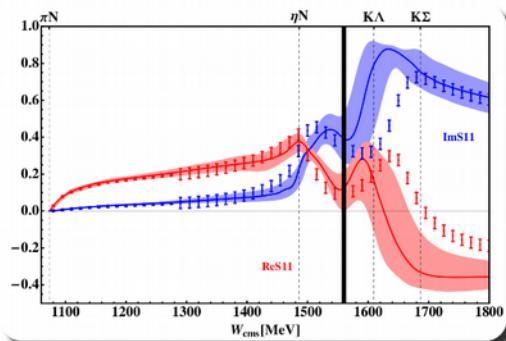
[Ruic, M. Mai, U.-G. Meissner PLB 704 (2011)]



- Exact unitary meson-baryon scattering amplitude T with parameters, fixed to reproduce:
 - πN -partial wave S_{11} and S_{31} for $\sqrt{s} < 1560$ MeV
 - $\pi^- p \rightarrow \eta n$ differential cross sections

Arndt et al. (2012)

Prakhov et al. (2005)

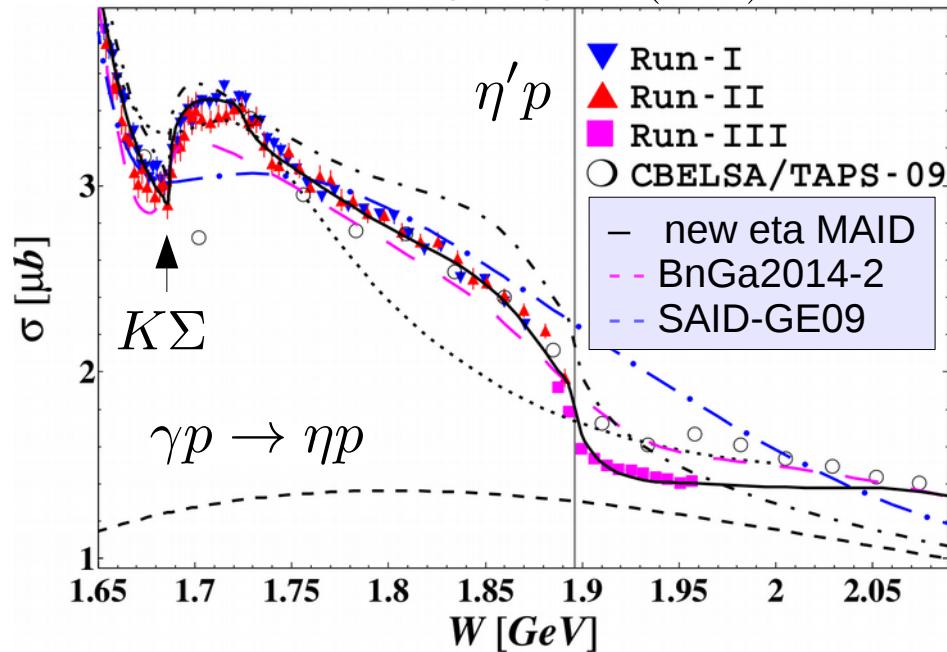


→ Making the “Missing resonance problem” worse ?!

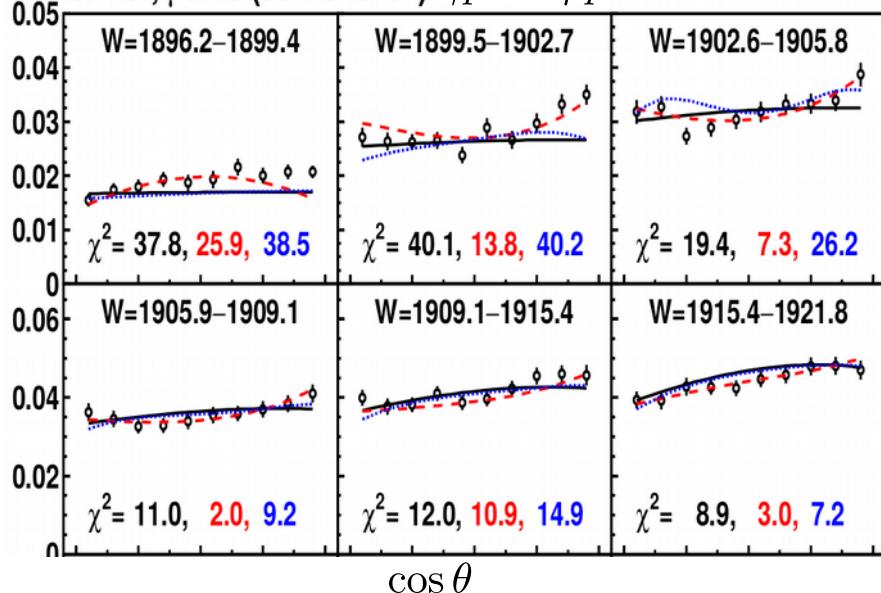
Phenomenology

Resonances or not?

A2 MAMI, PRL 118 (2017) $S_{11}(1895)$



$d\sigma/d\Omega, \mu\text{b}/\text{sr}$ (MAMI 2017) $\gamma p \rightarrow \eta' p$



BnGa
PLB785 (2018):

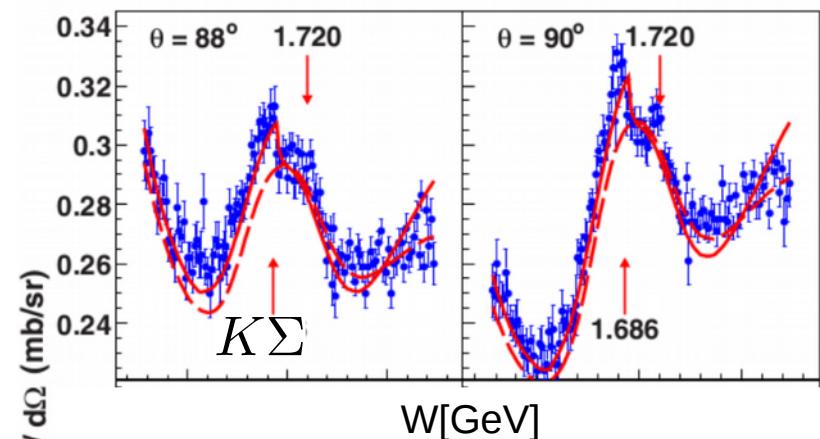
No narrow resonance

3/2⁻ narrow Resonance

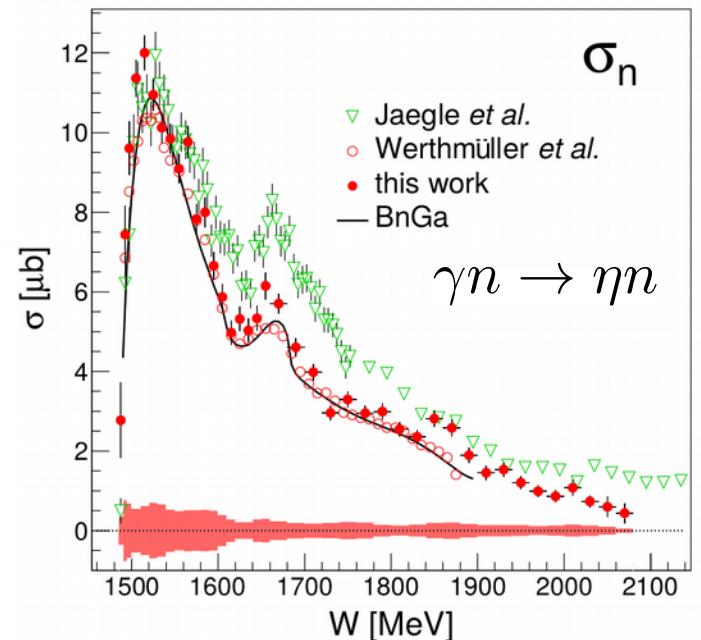
5/2⁻ narrow Resonance

Data: A2.Mami
PRL 118 (2017)

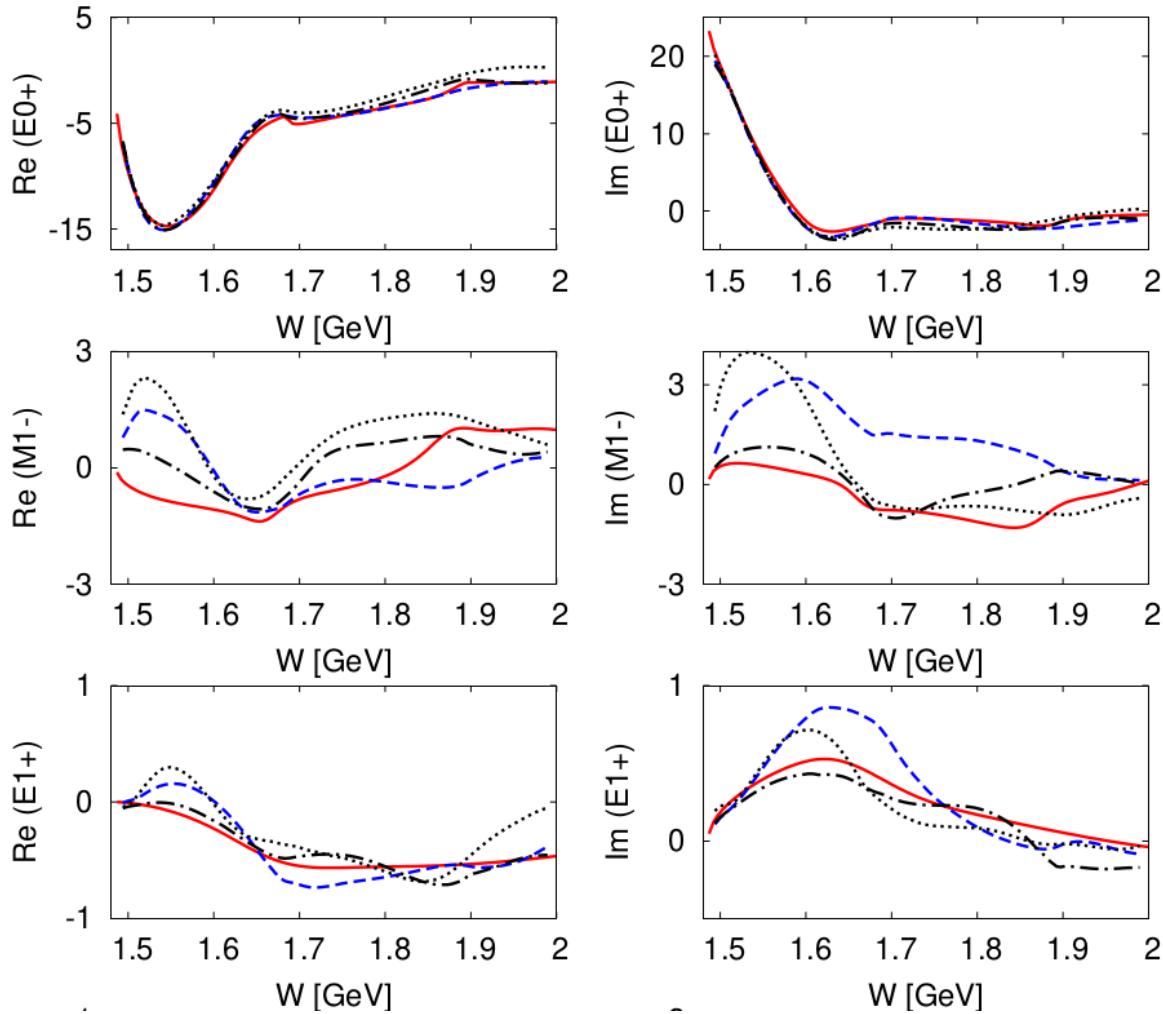
$\pi N \rightarrow \pi N$
EPECUR/SAID PRC 93 (2016)



[CBELSA/TAPS EPJA 53 (2017)]



Current state in η photoproduction: Multipoles from different groups



From: **EtaMAID2018**
[Tiator et al., EPJA54 (2018)]
Analyzes:

$\gamma p \rightarrow \eta p$

$\gamma p \rightarrow \eta' p$

$\gamma n \rightarrow \eta n$

$\gamma n \rightarrow \eta' n$

EtaMAID2018

BnGa [PLB 772 (2017)]

JuBo (dotted) [EPJA 54 (2018)]

KSU [1804.06031]

Review: Krusche, Wilkins,
[Prog.Part.Nucl.Phys. 80 (2014)]

Observable	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z
$p\pi^0$	✓	✓	✓			✓	✓	✓	✓							
$n\pi^+$	✓	✓	✓			✓	✓	✓								
$p\eta$	✓	✓				✓			✓	✓	✓					
$p\eta'$	✓	✓	✓			✓			✓	✓	✓	✓				
$K^+\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^+\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$p\omega/\phi$	✓	✓	✓			✓	✓	✓	✓						✓ SDME	
$K^{+*}\Lambda$	✓				✓										SDME	
$K^{0*}\Sigma^+$	✓	✓										✓	✓			SDME
$p\pi^-$	✓	✓				✓	✓	✓								
$p\rho^-$	✓	✓				✓	✓	✓								
$K^-\Sigma^+$	✓	✓				✓	✓	✓								
$K^0\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^0\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^{0*}\Sigma^0$	✓	✓									✓	✓				

Phys.Lett. B771 (2017)
Phys.Lett. B755 (2016)



$\gamma p \rightarrow X$

$\gamma n \rightarrow X$

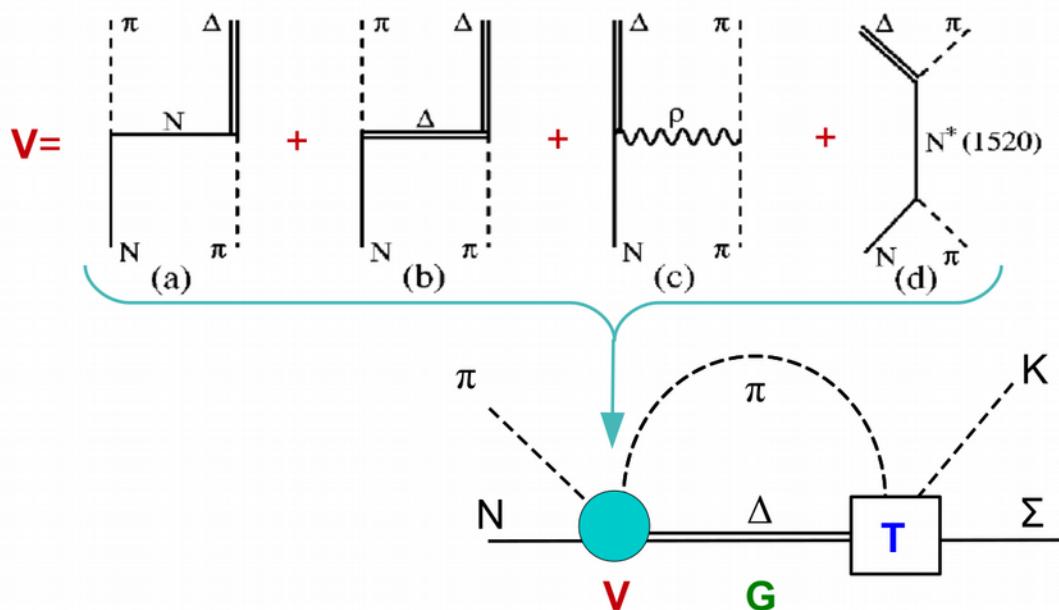
The Julich-Bonn Dynamical Coupled-Channel Approach

e.g. EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial-wave basis

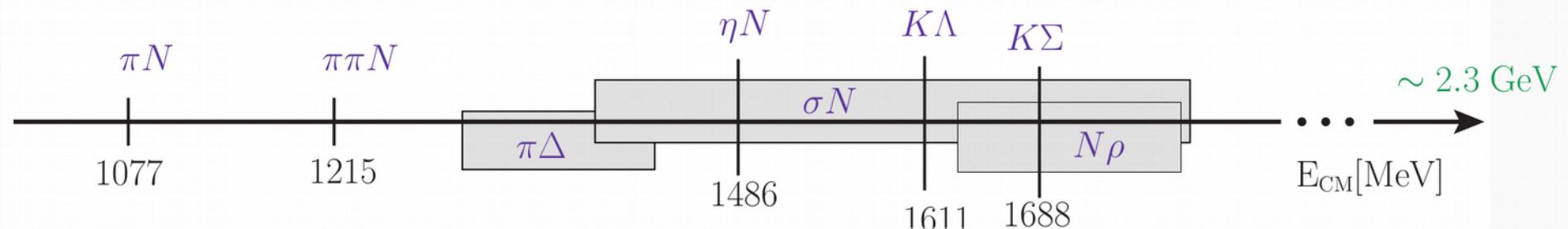
$$\langle L'S'p' | \textcolor{blue}{T}_{\mu\nu}^{IJ} | LS p \rangle = \langle L'S'p' | \textcolor{red}{V}_{\mu\nu}^{IJ} | LS p \rangle + \sum_{\gamma, L''S''} \int_0^\infty dq \quad q^2 \quad \langle L'S'p' | \textcolor{red}{V}_{\mu\gamma}^{IJ} | L''S''q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q | \textcolor{blue}{T}_{\gamma\nu}^{IJ} | LS p \rangle$$



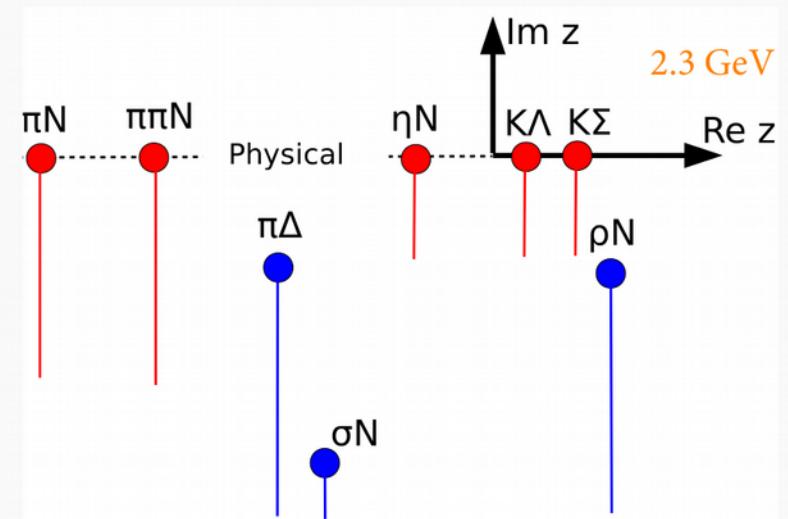
- potentials $\textcolor{red}{V}$ constructed from effective \mathcal{L}
- s -channel diagrams: T^P
genuine resonance states
- t - and u -channel: T^{NP}
dynamical generation of poles
partial waves strongly correlated

JuBo: Channels and Analytic Structure

Channels included:



- (2-body) unitarity and analyticity respected
- 3-body $\pi\pi N$ channel:
 - parameterized effectively as $\pi\Delta$, σN , ρN
 - $\pi N/\pi\pi$ subsystems fit the respective phase shifts
- ↳ branch points move into complex plane



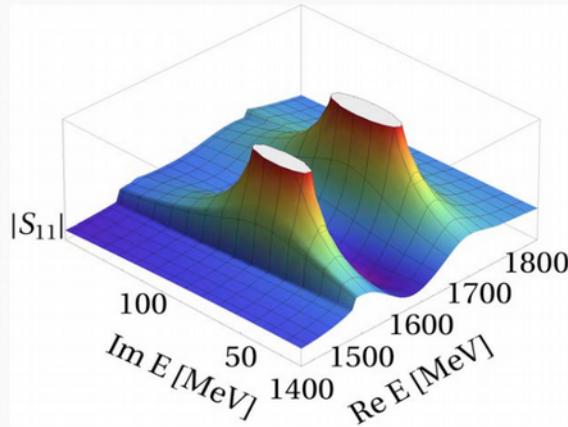
JuBo: Data base

[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]

Reaction	Observables (# data points)	p./channel
$\pi N \rightarrow \pi N$	PWA GW-SAID WI08 (ED solution)	3,760
$\pi^- p \rightarrow \eta n$	$d\sigma/d\Omega$ (676), P (79)	755
$\pi^- p \rightarrow K^0 \Lambda$	$d\sigma/d\Omega$ (814), P (472), β (72)	1,358
$\pi^- p \rightarrow K^0 \Sigma^0$	$d\sigma/d\Omega$ (470), P (120)	590
$\pi^- p \rightarrow K^+ \Sigma^-$	$d\sigma/d\Omega$ (150)	150
$\pi^+ p \rightarrow K^+ \Sigma^+$	$d\sigma/d\Omega$ (1124), P (551) , β (7)	1,682
$\gamma p \rightarrow \pi^0 p$	$d\sigma/d\Omega$ (10743), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (467), F (397), $C_{x_L'}$ (74), $C_{z_L'}$ (26)	17,564
$\gamma p \rightarrow \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,748
$\gamma p \rightarrow \eta p$	$d\sigma/d\Omega$ (5680), Σ (403), P (7), T (144), F (144), E (129)	6,507
$\gamma p \rightarrow K^+ \Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \rightarrow K \Sigma$	Plenary talk D. Rönchen tomorrow	
		in total 48,050

Resonance Couplings

Resonance states: Poles in the T -matrix on the 2nd Riemann sheet



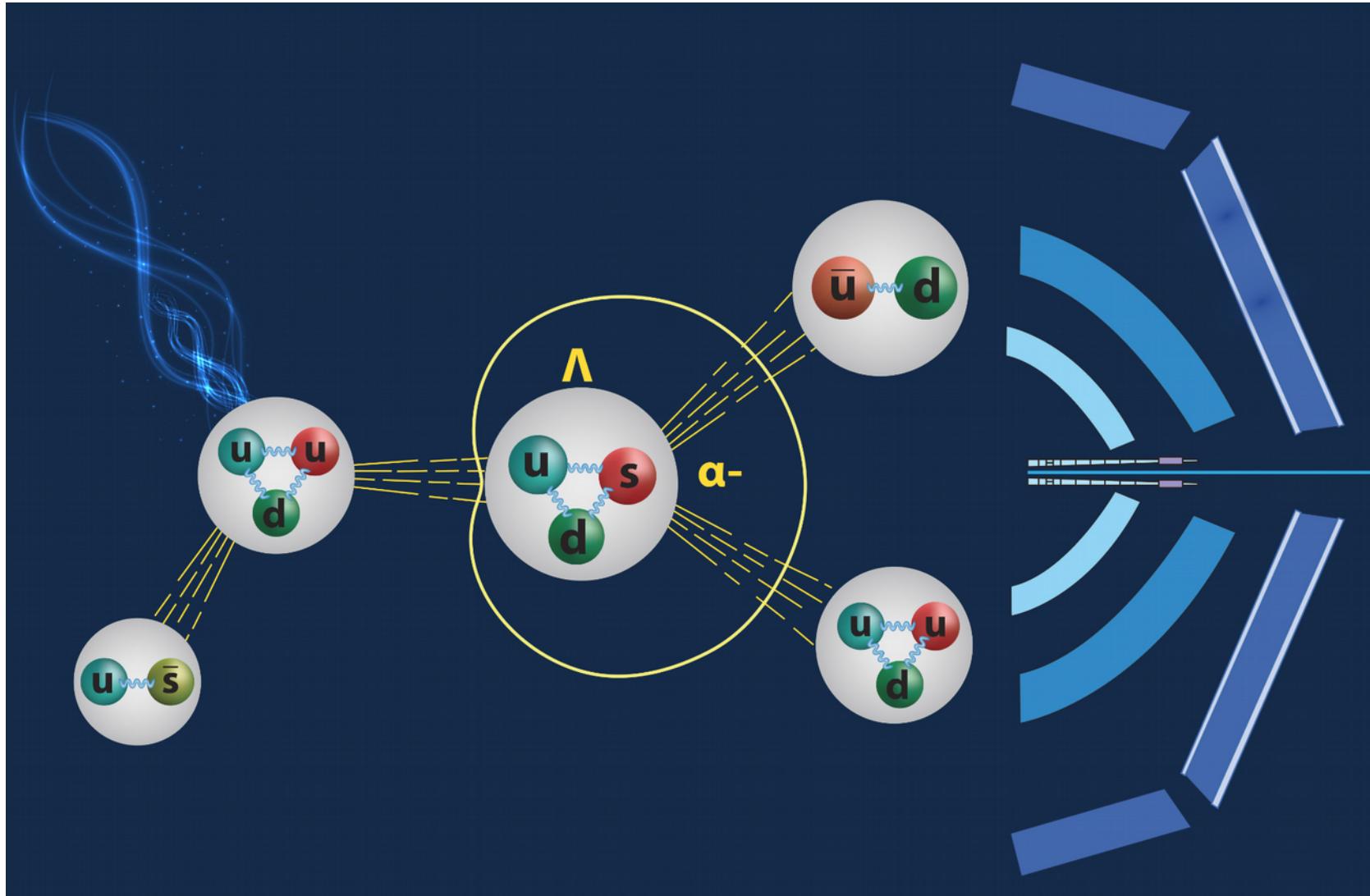
- $\text{Re}(E_0)$ = "mass", $-2\text{Im}(E_0)$ = "width"
- elastic πN residue ($|r_{\pi N}|, \theta_{\pi N \rightarrow \pi N}$), normalized residues for inelastic channels ($\sqrt{\Gamma_{\pi N} \Gamma_\mu} / \Gamma_{\text{tot}}, \theta_{\pi N \rightarrow \mu}$)
- photocouplings at the pole: $\tilde{A}_{\text{pole}}^h = A_{\text{pole}}^h e^{i\vartheta^h}$, $h = 1/2, 3/2$

Inclusion of $\gamma p \rightarrow K^+ \Lambda$ in JüBo ("JuBo2017-1"): 3 additional states

	z_0 [MeV]	$\frac{\Gamma_{\pi N}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{\eta N}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{K\Lambda}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{K\Sigma}}{\Gamma_{\text{tot}}}$
N(1900)3/2 ⁺	$1923 - i 108.4$	1.5 %	0.78 %	2.99 %	69.5 %
N(2060)5/2 ⁻	$1924 - i 100.4$	0.35 %	0.15 %	13.47 %	27.02 %
$\Delta(2190)1/2^+$	$2191 - i 103.0$	33.12 %			3.78 %

- N(1900)3/2⁺: s-channel resonances, seen in many other analyses of kaon photoproduction (BnGa), 3 stars in PDG
- N(2060)5/2⁻: dynamically generated, 2 stars in PDG, seen e.g. by BnGa
- $\Delta(2190)3/2^+$: dyn. gen., no equivalent PDG state

How do results change when the $\Lambda \rightarrow \pi^- p$ decay parameter changes?

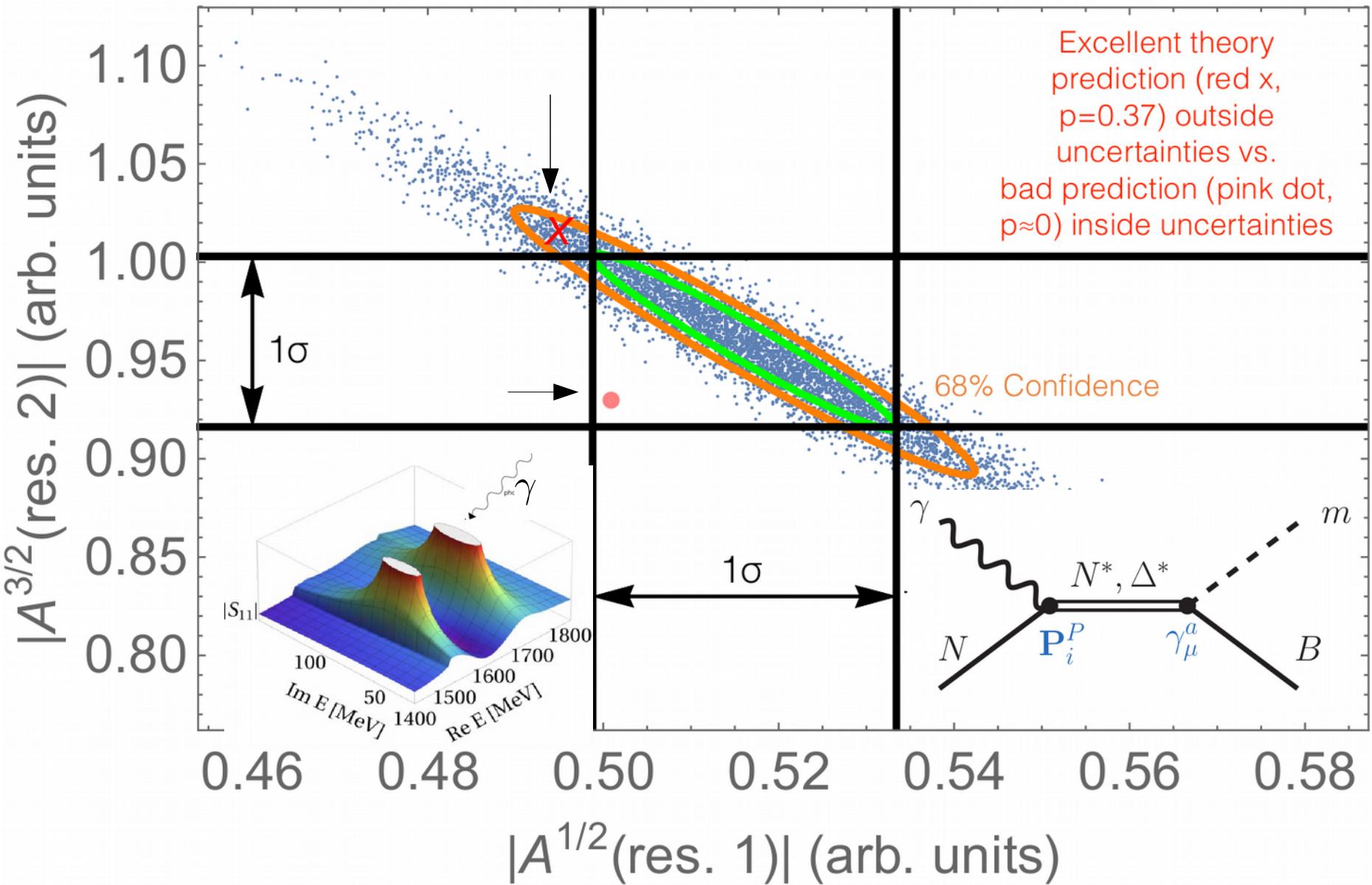


- Determination α_- : Talk by D. Irland on Thursday (arXiv:1904.07616 [nucl-ex])
- Changes in resonance spectrum: Talk by D. Rönchen tomorrow (in progress)

How to quantify the impact of new measurements?

Consider correlations of helicity couplings extracted from experiment

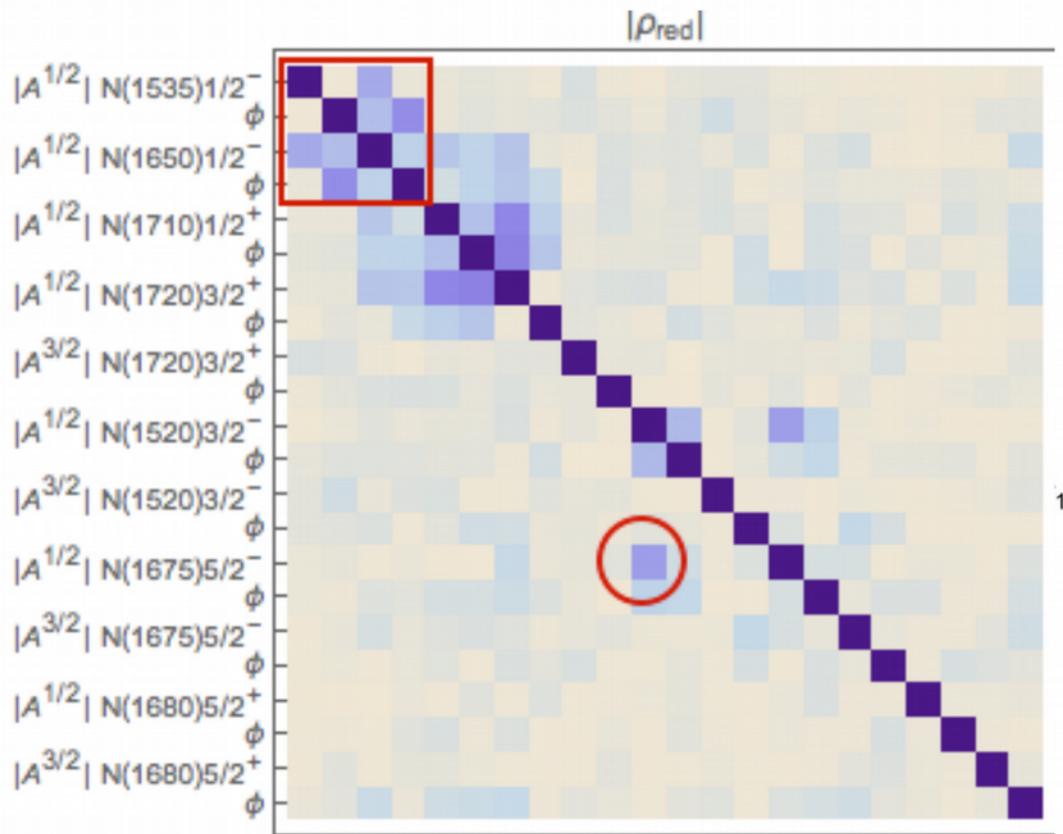
[D. Sadasivan, M.D., M. Mai, in preparation]



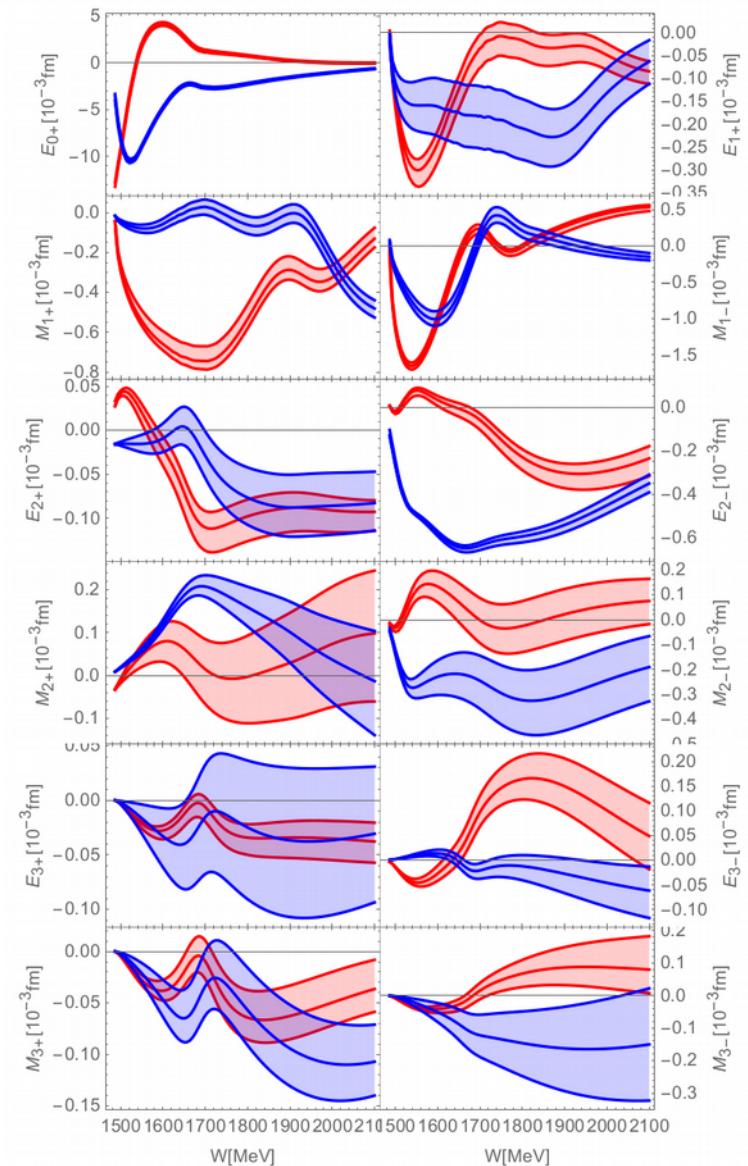
Results from analysis of world data of η photoproduction

[D. Sadasivan,M.D., M. Mai, in preparation]

Here $A = |A|e^{i\phi}$ defined at the resonance pole.



Correlation matrix



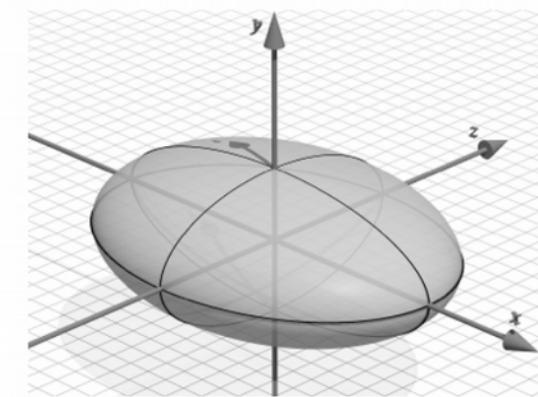
Bulk properties of uncertainties from different data sets

Helicity Coupling	All	No E	No F	No T	No Σ
Number of Data Points	6425	6369	6281	6281	6022
Generalized Variance	<u>0.0494</u>	0.0521	0.1288	0.1239	<u>6.664</u>
$\sqrt{\text{Tr } C}$	10.4965	10.51	12.00	11.423	19.85
Multicollinearity	8.173	8.203	9.280	9.5323	10.371
Condition number	133.61	132.10	173.664	164.1	322.66

C=Covariance Matrix

Generalized Variance
 $= \text{Det}[C] \sim \text{Volume of}$
 the Error Ellipsoid

Helicity Coupling	No artificial data	Cx	Cz	Cx and Cz
Number of Data Points	6425	6569	6569	6713
Generalized Variance	0.0494	0.03758	0.0362	<u>0.0132</u>
$\sqrt{\text{Tr } C}$	10.4965	10.72	10.487	10.102
Multicollinearity	8.173	7.599	6.770	6.157
Condition number	133.61	112.47	109.69	107.683

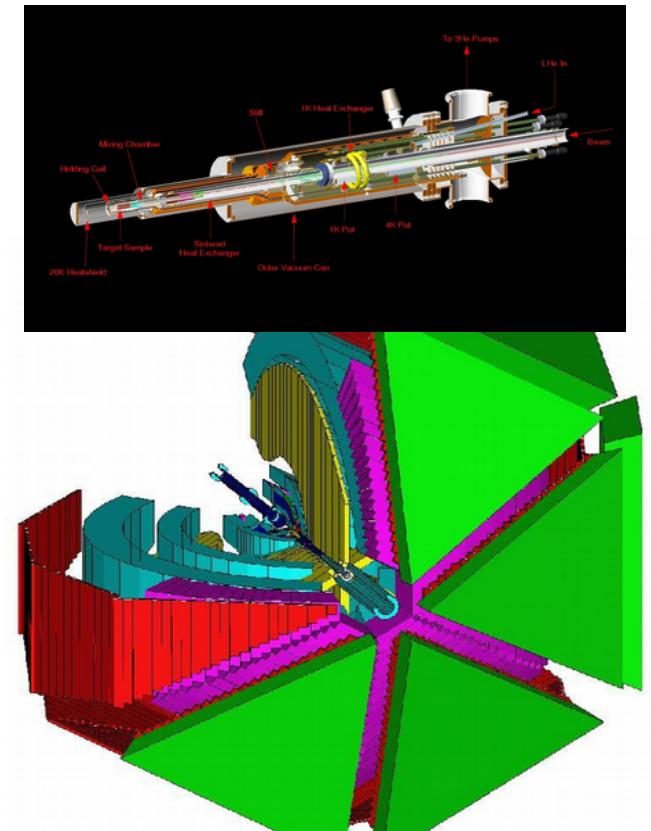
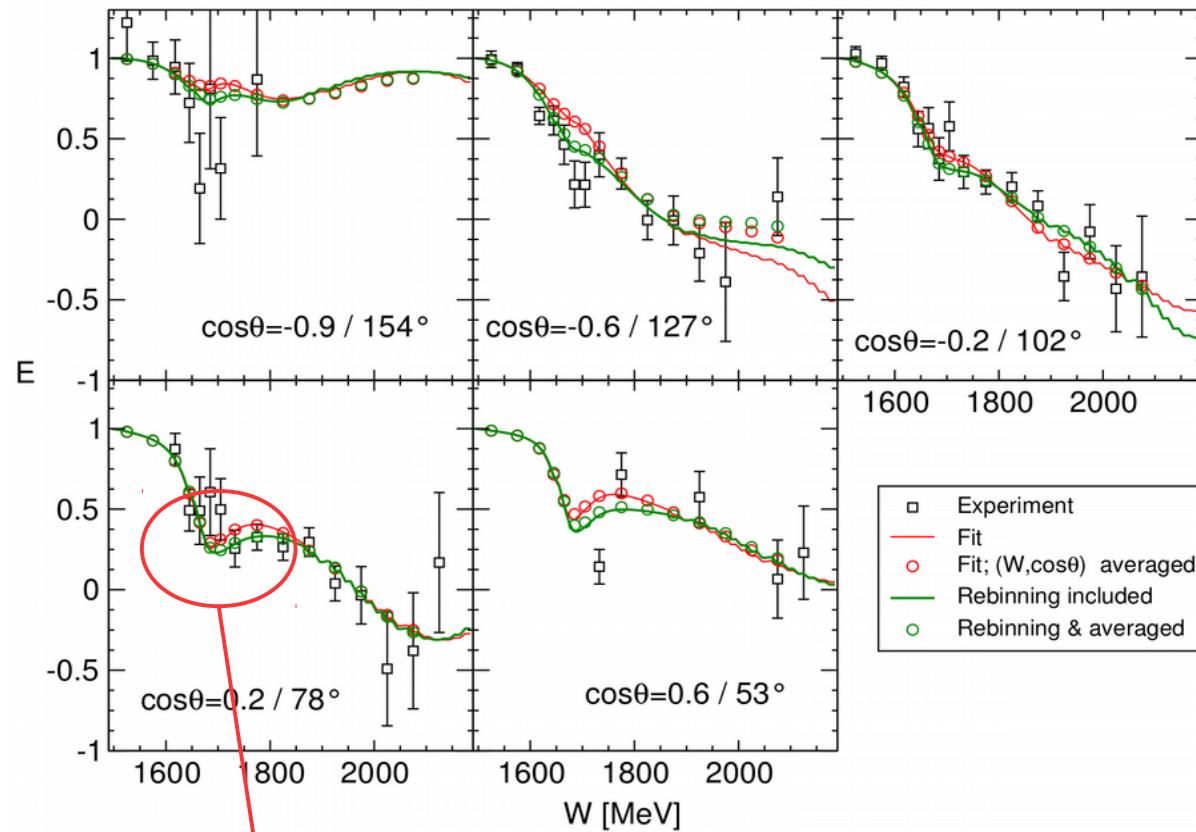


- Allows to trace quantitatively the impact of data sets and observables
- Helpful in design of new measurements
- Correlations allow to assess quality of theory predictions

Resonances and other structures

CLAS/JuBo (M. D., D. Rönchen), Phys.Lett. B755 (2016)

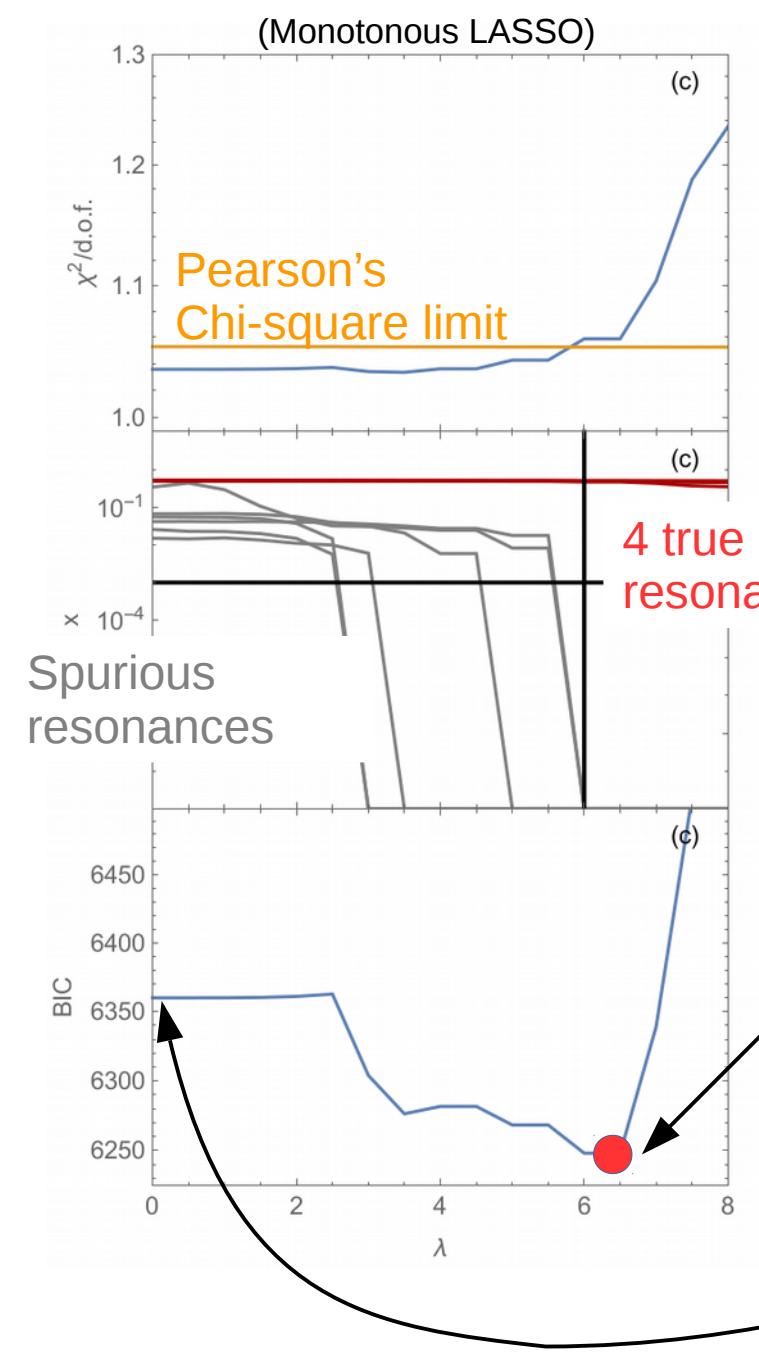
- First-ever measurement of observable E in η photo-production, enabled through the CLAS FROST target



Is this a new narrow baryonic resonance?
→ Conventional explanation in terms of interference effects.

Resonance selection

$(K^- p \rightarrow K\Xi)$

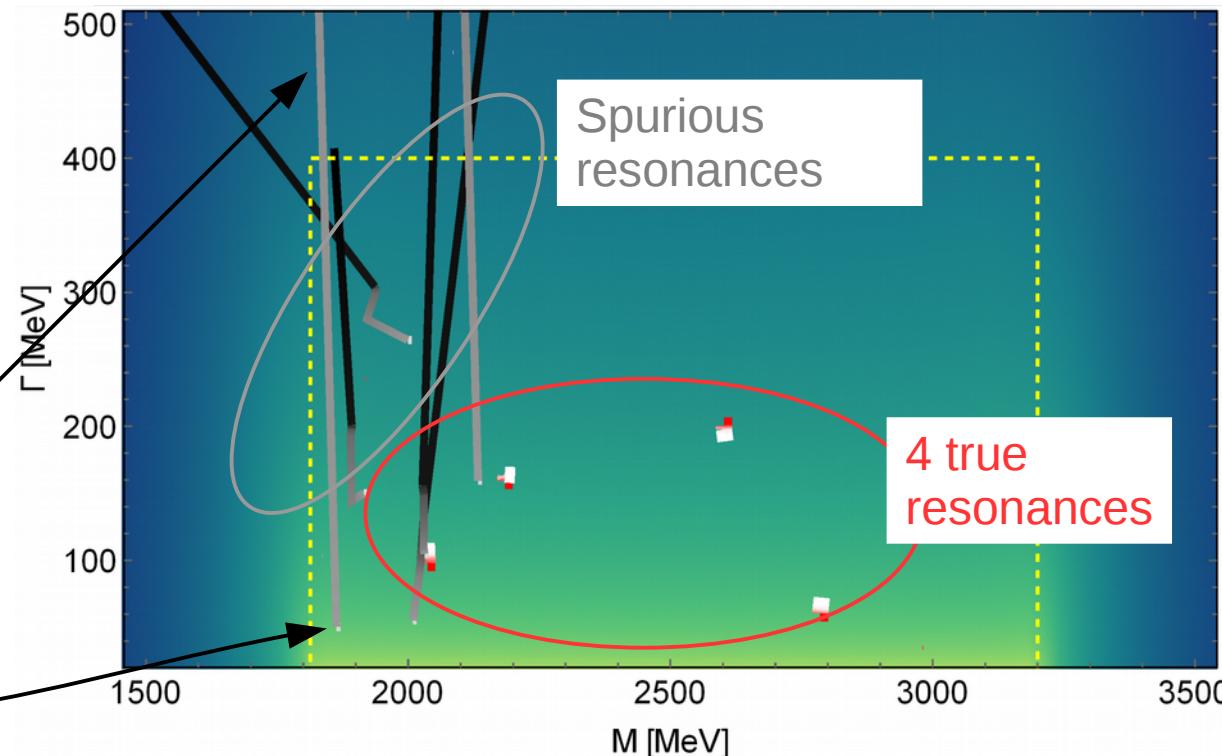


[M.D., J. Landay, H. Haberzettl, M. Mai, K. Nakayama, PRC 2019]

- Ten partial waves; 10 resonances in Ansatz
- Penalty: $\chi^2 = \chi_{\text{stat.}}^2 + P$

$$P(\lambda) = \lambda^5 \sum_{i=0}^9 \int_{m_K+m_\Xi}^{3200} \frac{\partial^2}{\partial W^2} \left(| -x_i e^{i\Phi_i} \frac{\Gamma_i}{2(W - M_i + i\frac{\Gamma_i}{2})} |^2 \right) dW$$

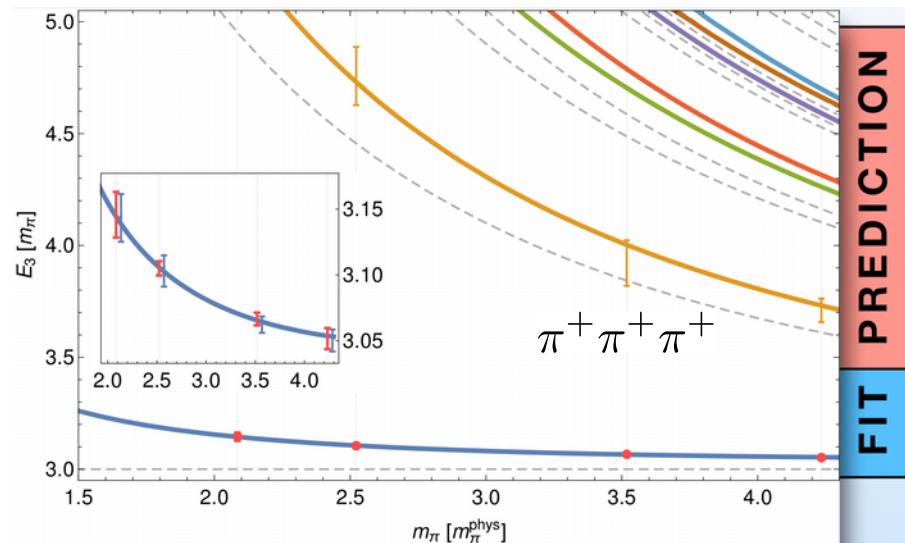
- LASSO picks the 4 correct ones



Summary

- Complicated phenomenology of excited baryons through coupled-channel and three-body effects

→ Conceptual progress
needed to connect to
lattice QCD calculations.

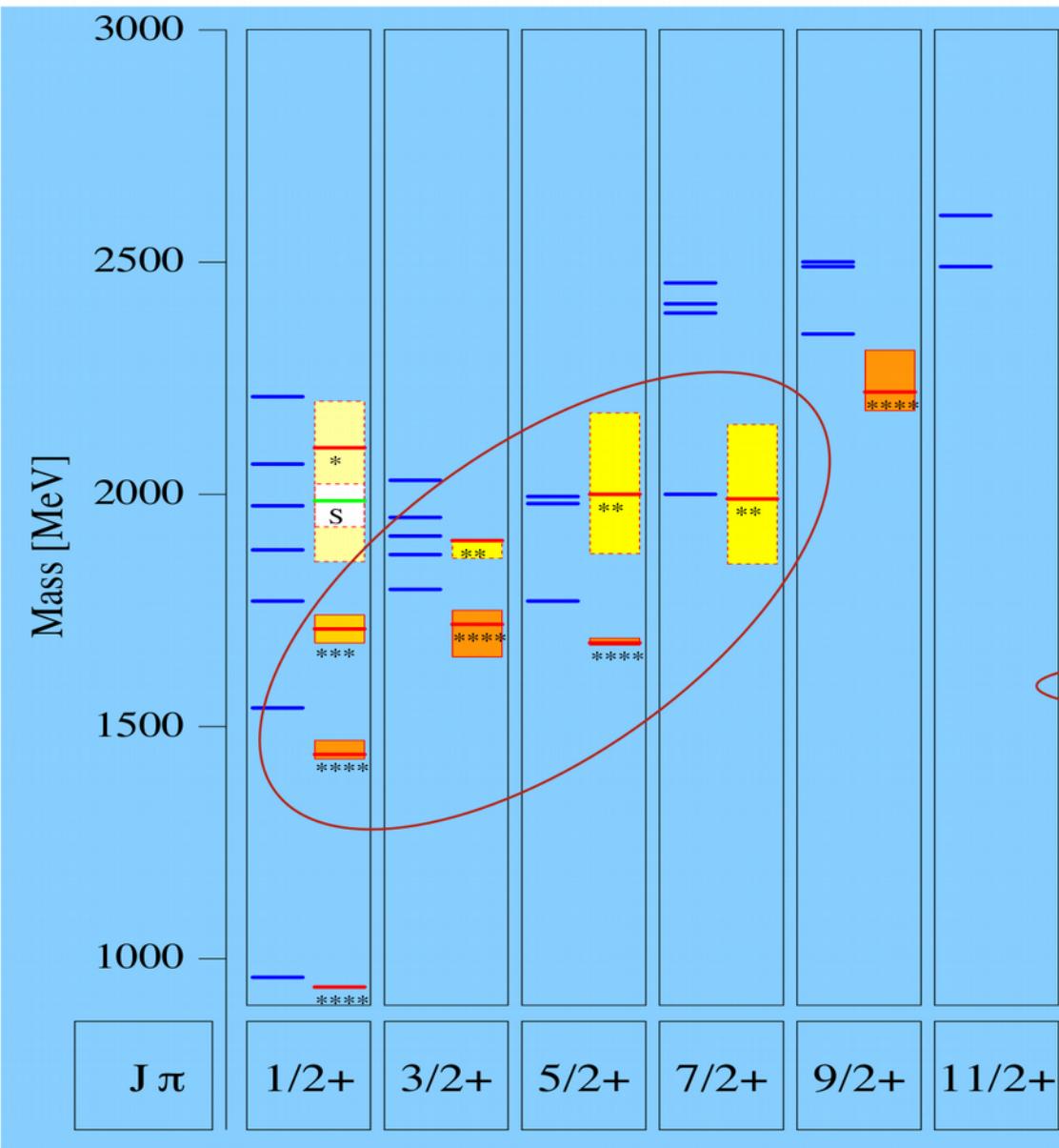


- η photoproduction ideally suitable to study excited baryons
 - Isospin filter, good channel for missing resonances
- Global analyses of pion and photon-induced reactions
 - Jülich-Bonn analysis confirms new states in analysis of photoproduction
- Model selection techniques to extract minimal spectrum of excited baryons

[M. Mai, MD, PRL (2019)]

Spare slides

Spectrum of N^* resonances

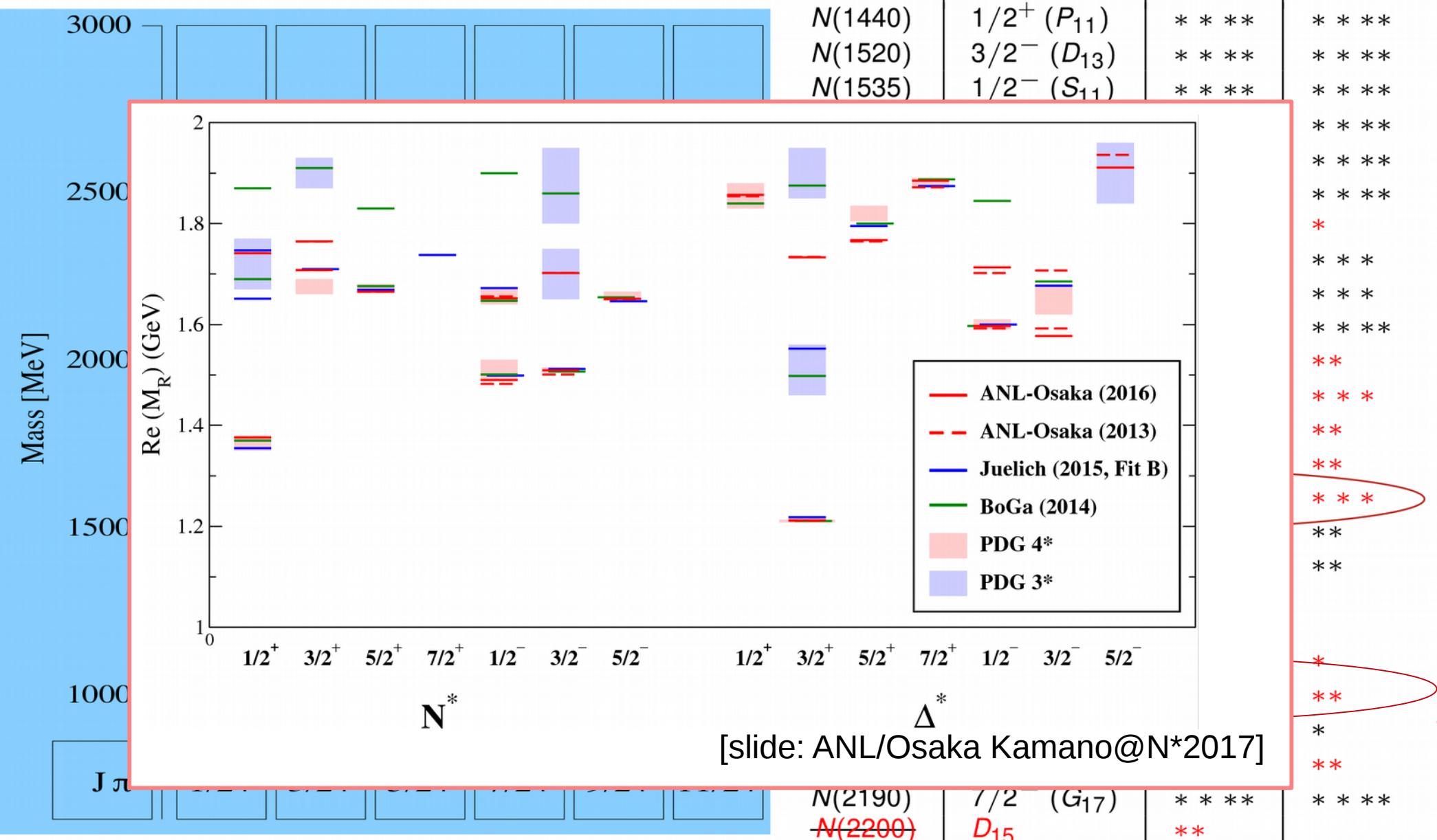


N^*	$J^P (L_{2I}, 2J)$	2010	2014
$N(1440)$	$1/2^+ (P_{11})$	****	****
$N(1520)$	$3/2^- (D_{13})$	****	****
$N(1535)$	$1/2^- (S_{11})$	****	****
$N(1650)$	$1/2^- (S_{11})$	****	****
$N(1675)$	$5/2^- (D_{15})$	****	****
$N(1680)$	$5/2^+ (F_{15})$	****	****
$N(1685)$			*
$N(1700)$	$3/2^- (D_{13})$	***	***
$N(1710)$	$1/2^+ (P_{11})$	***	***
$N(1720)$	$3/2^+ (P_{13})$	****	****
$N(1860)$	$5/2^+$		**
$N(1875)$	$3/2^-$		***
$N(1880)$	$1/2^+$		**
$N(1895)$	$1/2^-$		**
$N(1900)$	$3/2^+ (P_{13})$	**	***
$N(1990)$	$7/2^+ (F_{17})$	**	**
$N(2000)$	$5/2^+ (F_{15})$	**	**
$N(2080)$	D_{13}		**
$N(2090)$	S_{11}		*
$N(2040)$	$3/2^+$		*
$N(2060)$	$5/2^-$		**
$N(2100)$	$1/2^+ (P_{11})$	*	*
$N(2120)$	$3/2^-$		**
$N(2190)$	$7/2^- (G_{17})$	****	****
$N(2200)$	D_{15}	**	

- Most new resonances by Bonn-Gatchina group;
- Many from kaon photoproduction

[Slide: V. Crede/Nstar 2017, slight modifications]
[See also: Crede, Roberts, Rep. Prog. Phys. 76 (2013)]

Spectrum of N^* resonances



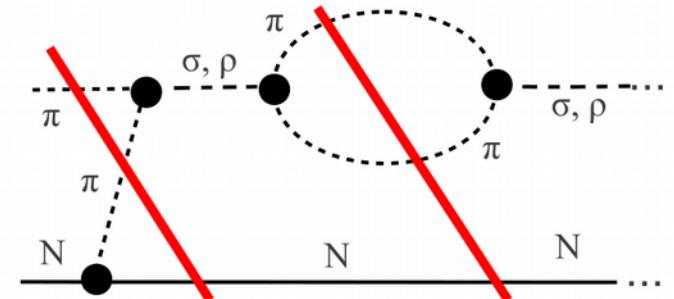
- Most new resonances by Bonn-Gatchina group;
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[Slide: V. Crede/Nstar 2017, slight modifications]
[See also: Crede, Roberts, Rep. Prog. Phys. 76 (2013)]

$$S = \mathbb{1} + iT$$

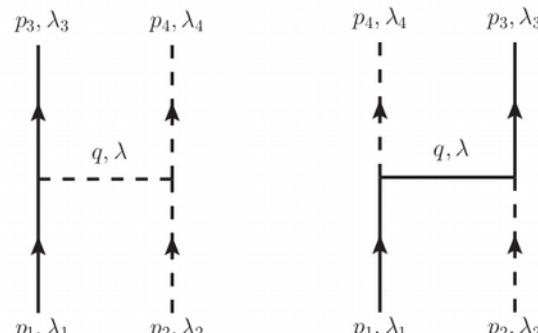
Unitarity: $SS^\dagger = 1 \Leftrightarrow -i(T - T^\dagger) = T T^\dagger$

- 3-body unitarity:
discontinuities from t -channel exchanges
→ Meson exchange from requirements of the S -matrix



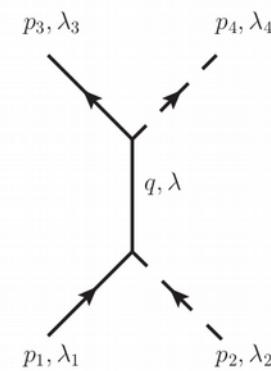
Other cuts

- to approximate left-hand cut → Baryon u -channel exchange
- σ, ρ exchanges from crossing plus analytic continuation.

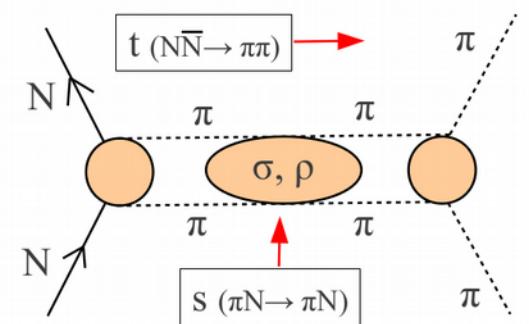


$$\vec{q} = \vec{p}_1 - \vec{p}_3$$

$$\vec{q} = \vec{q}_1 - \vec{p}_4$$

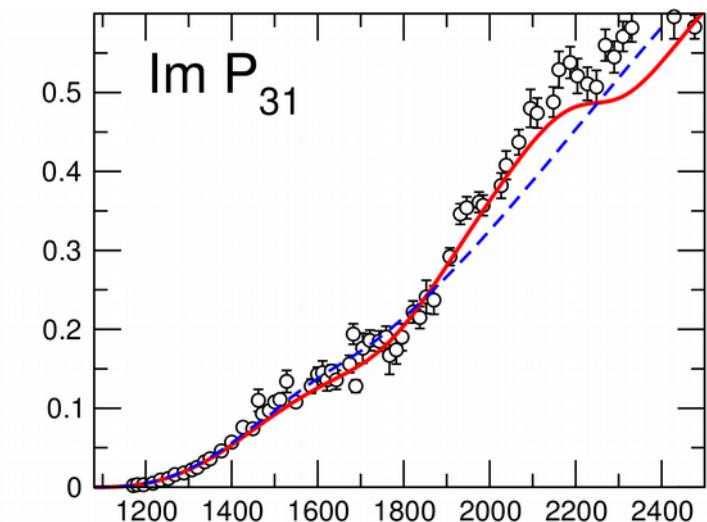
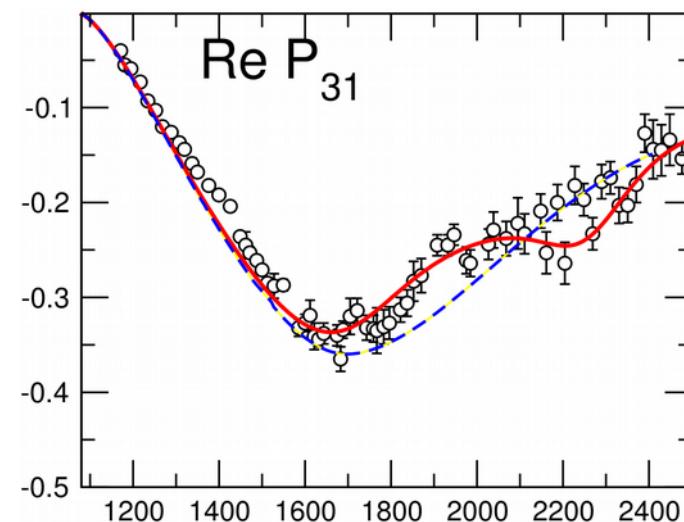


$$\vec{q} = \vec{p}_1 + \vec{p}_2 = 0$$

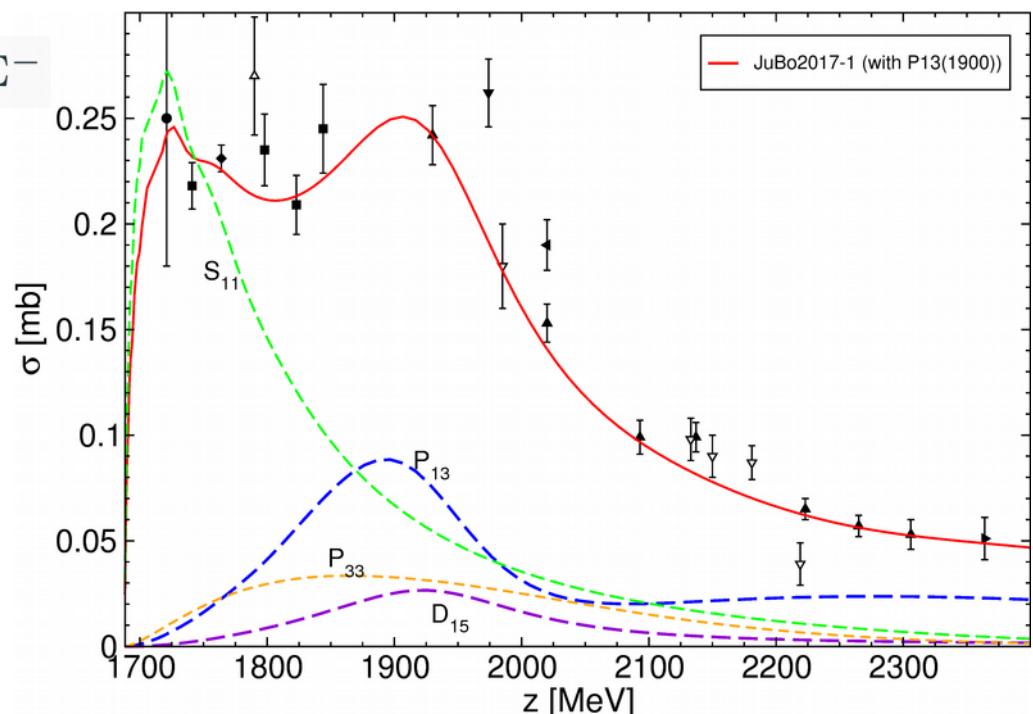


Visible influence of new states

$\Delta(2190)3/2^+$ in πN PW



$N(1900)3/2^+, N(2060)5/2^-$ in σ_{tot} in $\pi^- p \rightarrow K^+ \Sigma^-$



Amplitude parametrization

Chiral constraints from Lagrangian formalism

Isobar models (Jlab; JM15) and others

- Disp. rel. (Aznauryan, Burkert,...)
- KT equations, t-channel analyticity; Restoration of crossing symmetry via dispersion relations (Aitchison, Kubis, Szczepaniak, Tiator)

Two-body unitarity

K-matrix parametrization
Coupled channels
Analytic continuation of phase space factors below threshold?

Real part of 2-particle propagation from dispersion relation (“N/D”, Chew-Mandelstam)

Non-factorizing Integral-equation implementation of amplitude

No Yes

Giessen

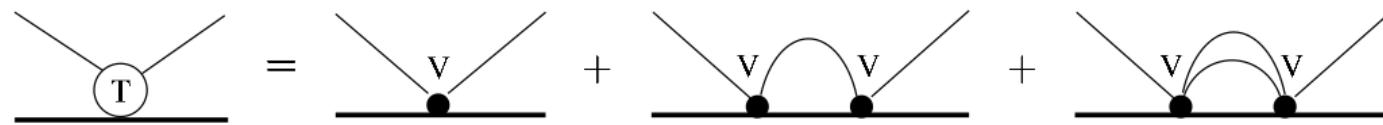
Bonn-Gatchina I
Bonn-Gatchina II

SAID

MAID
DMT

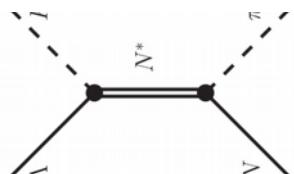
Zagreb

Julich-Bonn
ANL-Osaka



$$T = V + VGT,$$

Genuine Resonance:

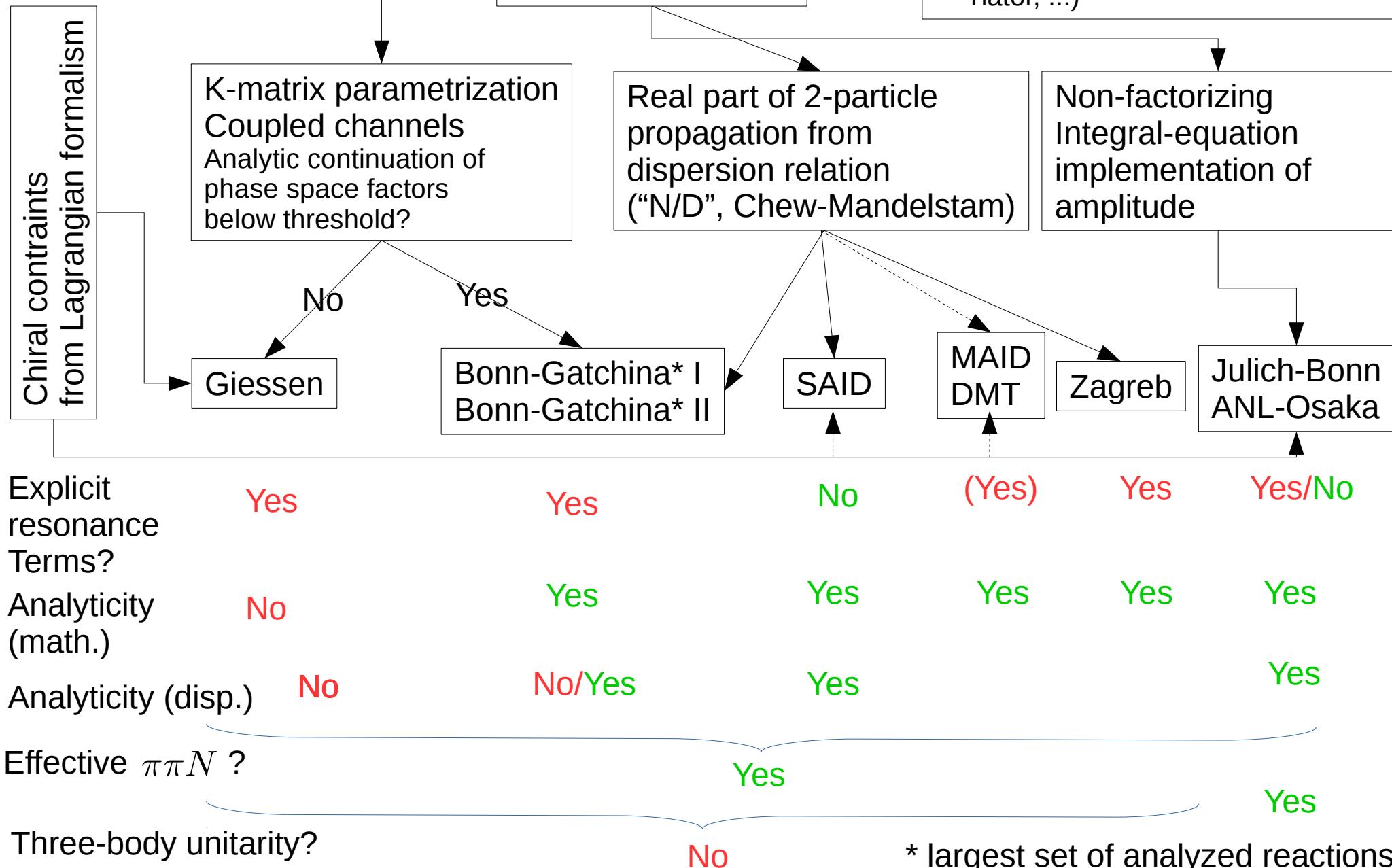


Unitarity loop G:

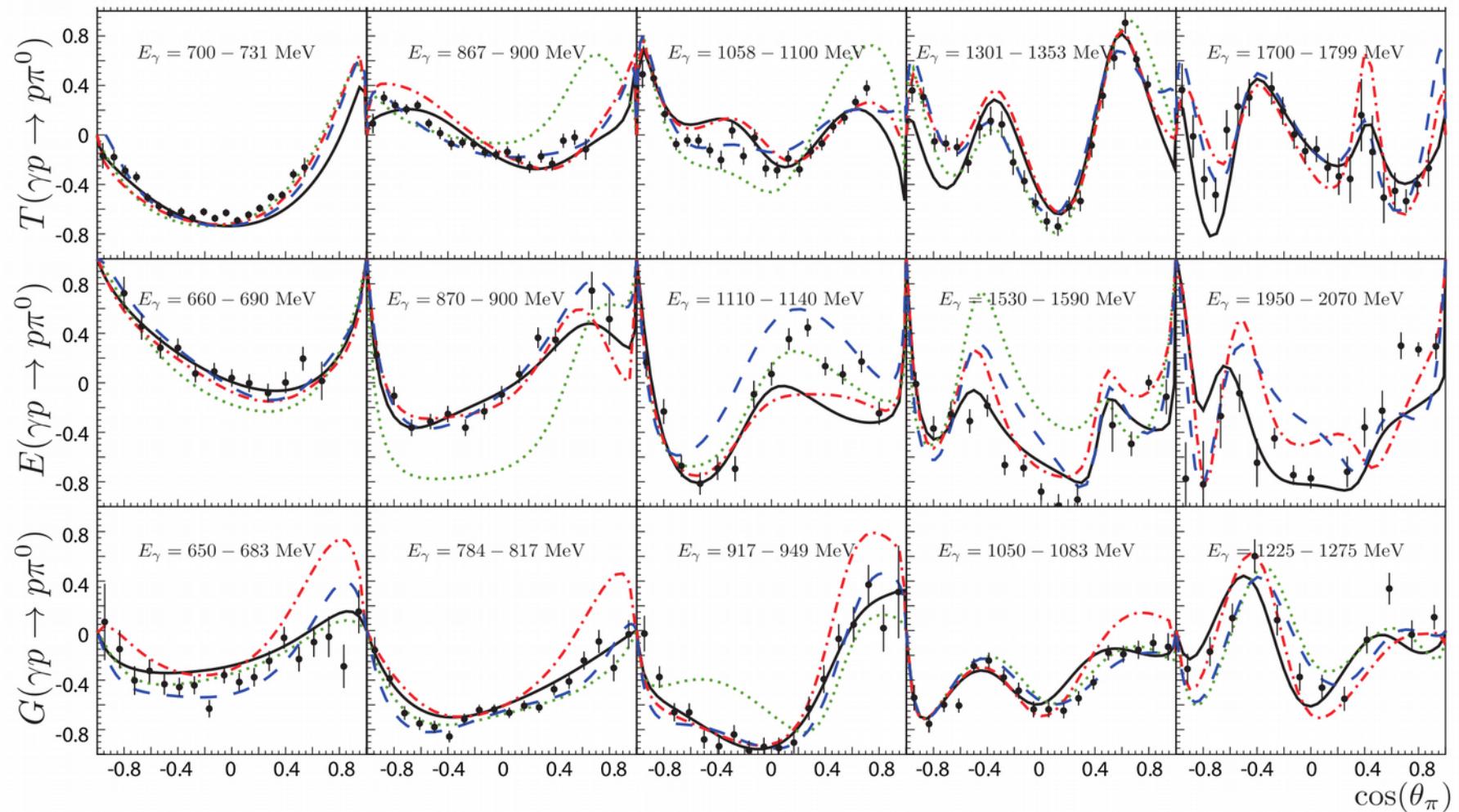
- $\text{Re } G \rightarrow 0$: K-matrix
- V point-like: SAID

Integral equation: Julich-Bonn, ANL-Osaka

Amplitude parametrization



Impact of data

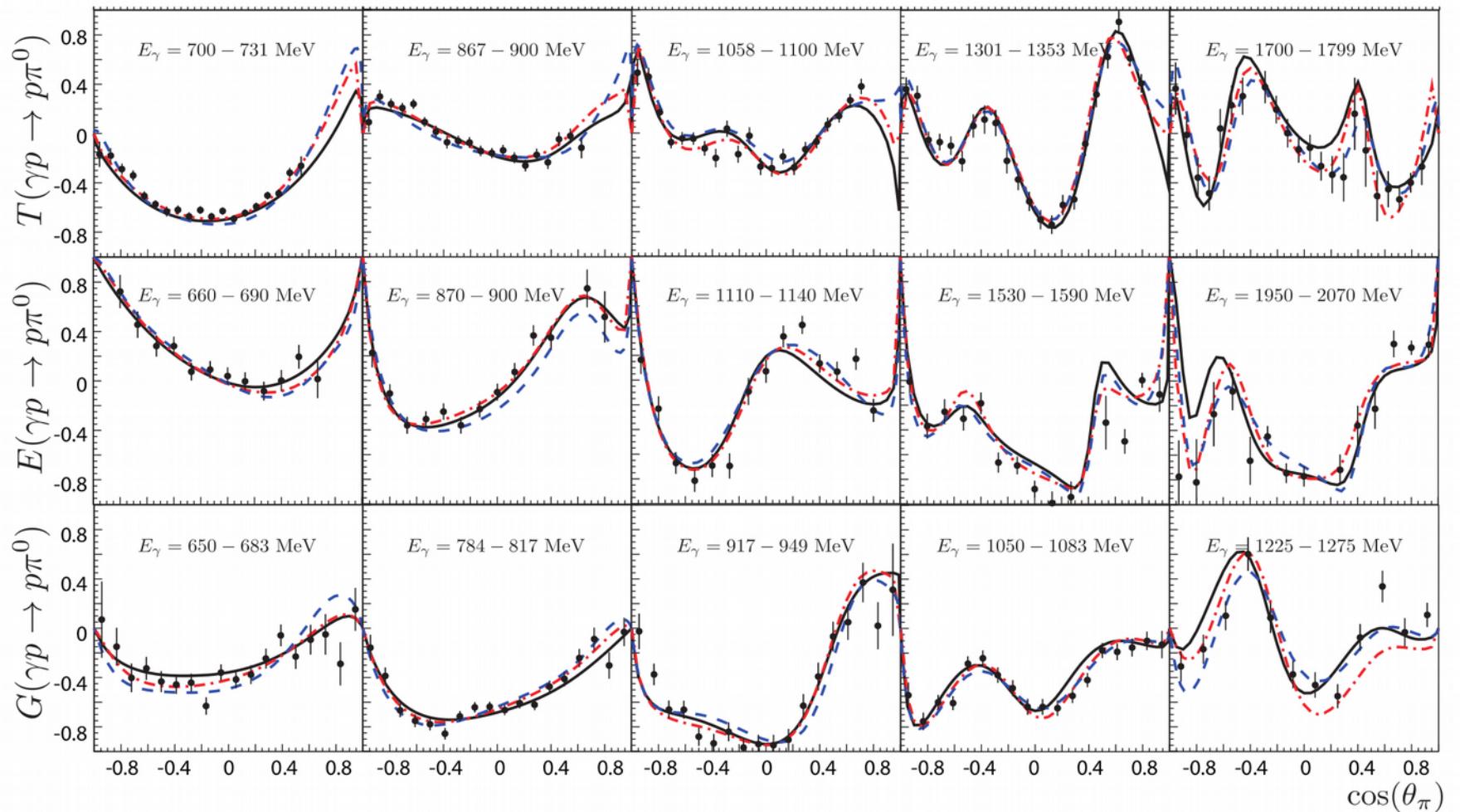


Data: CBELSA/TAPS Collaboration (T : Hartmann et al. PLB 748, 212 (2015) , E : Gottschall et al. PRL 112, 012003 (2014), G : Thiel et al. PRL 109, 102001 (2012), Thiel et al. arXiv:1604.02922)

Predictions: black solid lines: BnGa, red dash-dotted: SAID, blue dashed: JüBo, green dotted: MAID

Impact of new data

EPJA 52, 284 (2016)



Data: CBELSA/TAPS Collaboration (T : Hartmann et al. PLB 748, 212 (2015) , E : Gottschall et al. PRL 112, 012003 (2014), G : Thiel et al. PRL 109, 102001 (2012), Thiel et al. arXiv:1604.02922)

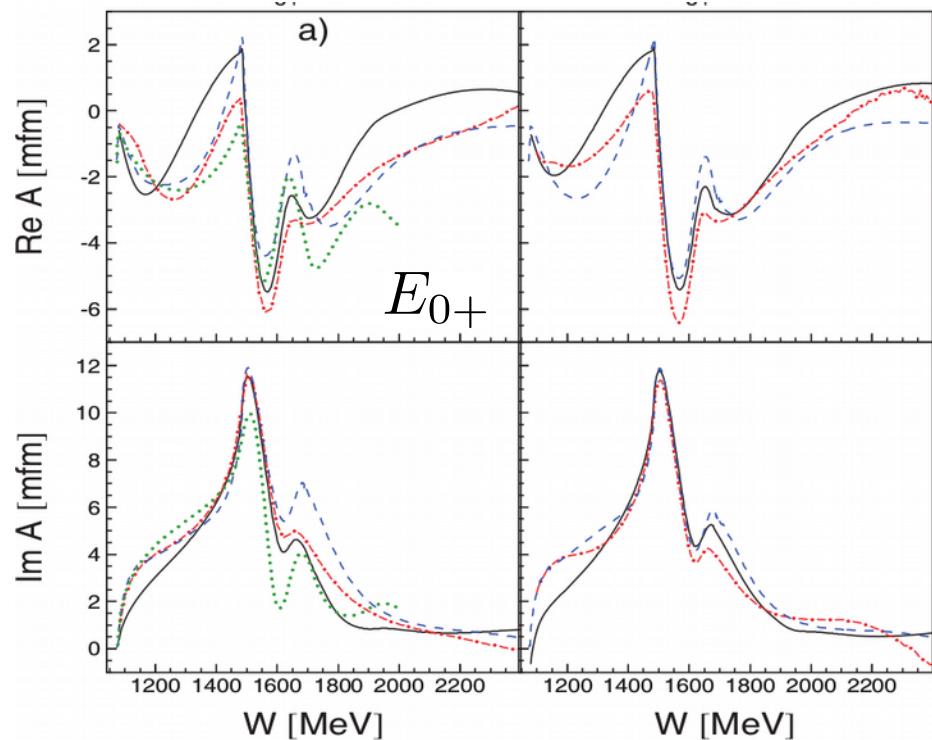
Fits: black solid lines: BnGa, red dash-dotted: SAID, blue dashed: JüBo

Impact of new data

EPJA 52, 284 (2016)

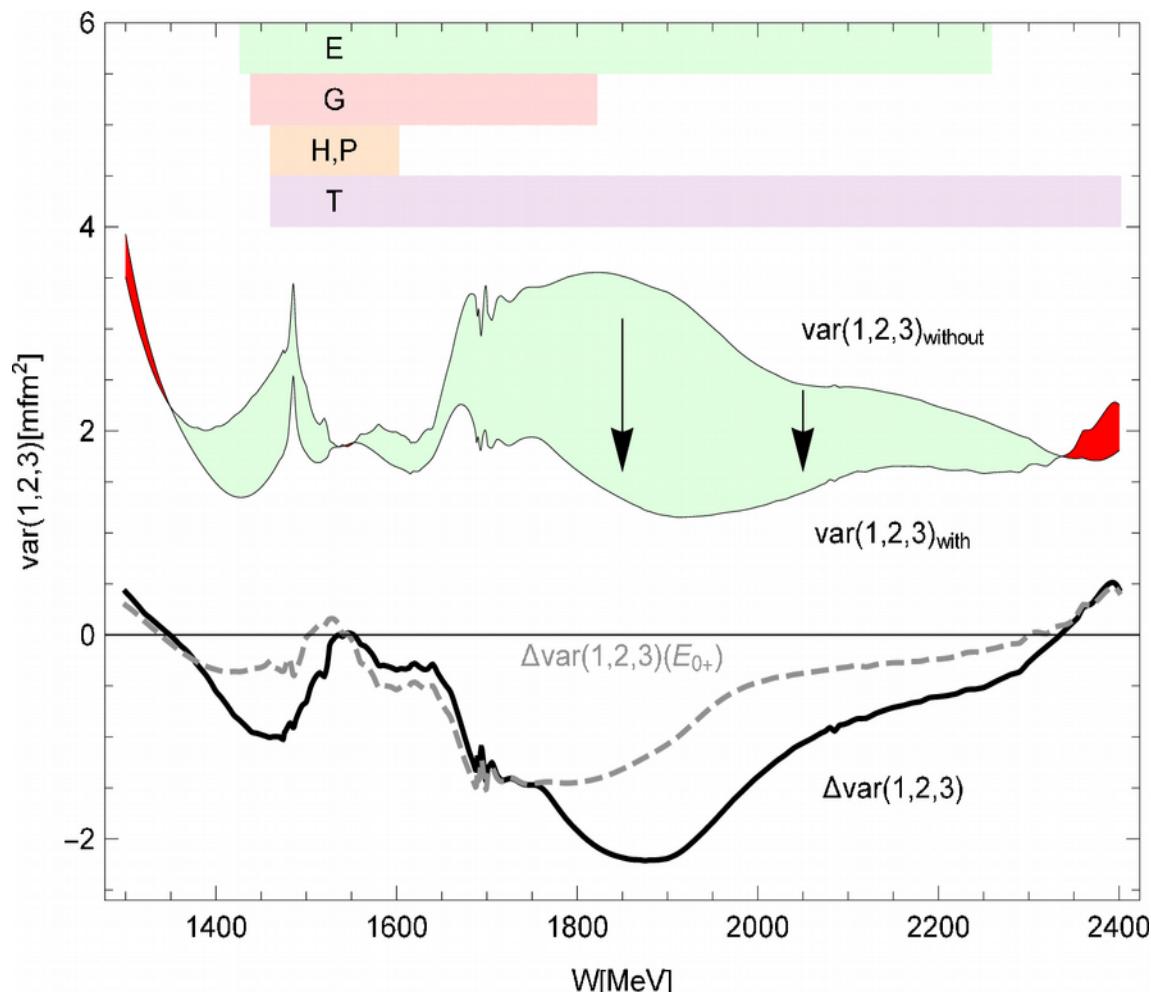
Before

After



Julich-Bonn, BnGa, SAID

$$\text{var}(1, 2) = \frac{1}{2} \sum_{i=1}^{16} (\mathcal{M}_1(i) - \mathcal{M}_2(i))(\mathcal{M}_1^*(i) - \mathcal{M}_2^*(i)). \quad (31)$$

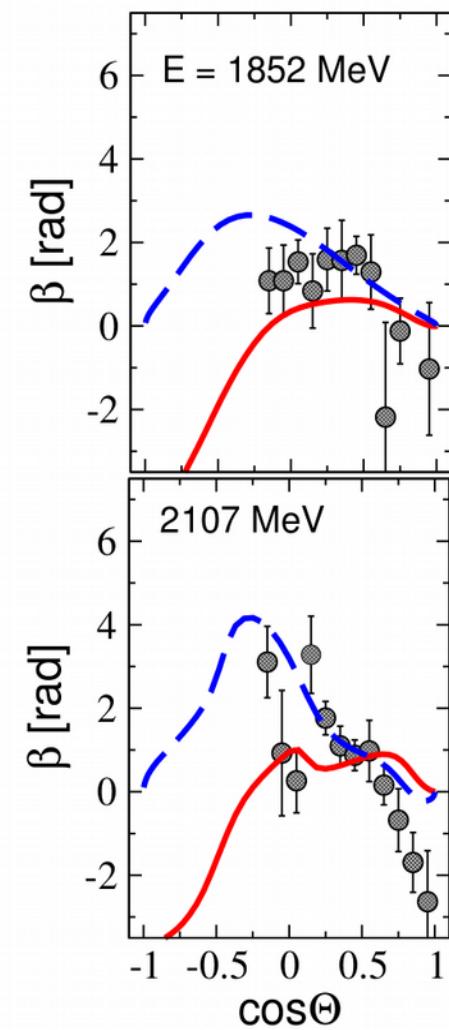
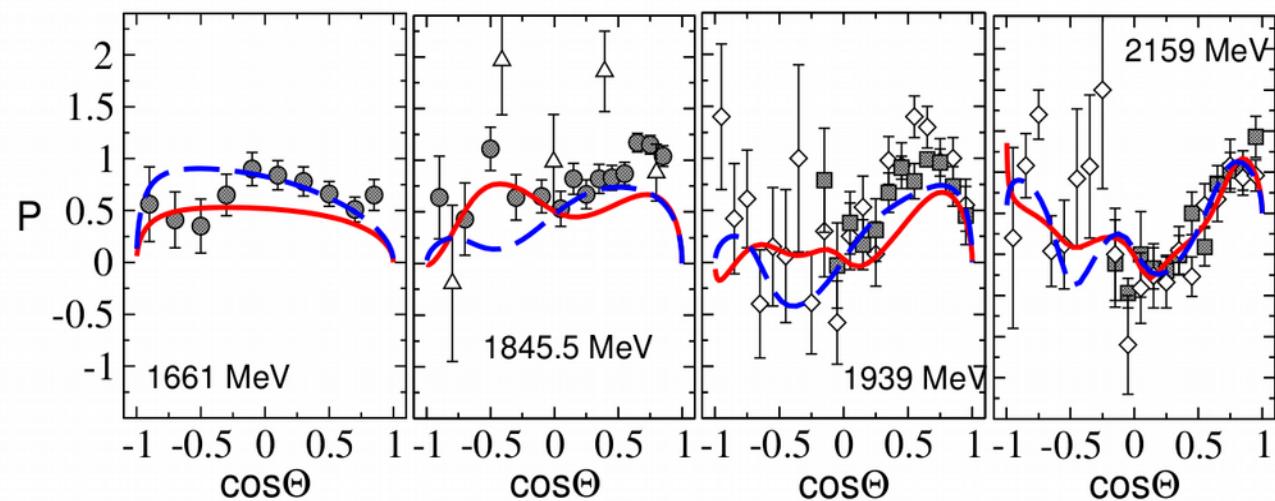
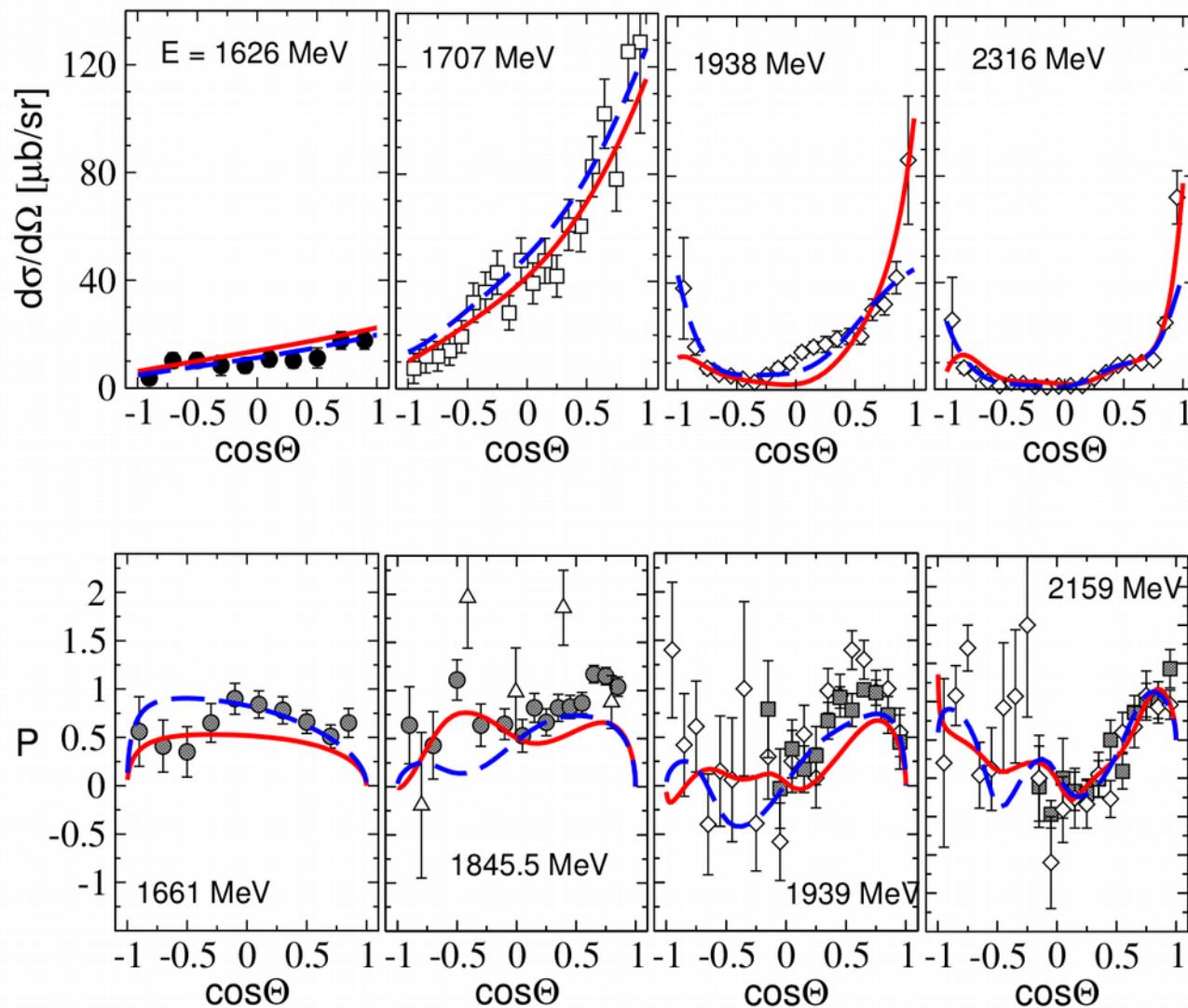


- Multipole solutions approach each other
- Remaining discrepancies

Fit to world data on $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ ($\sim 10^5$ exp. points)

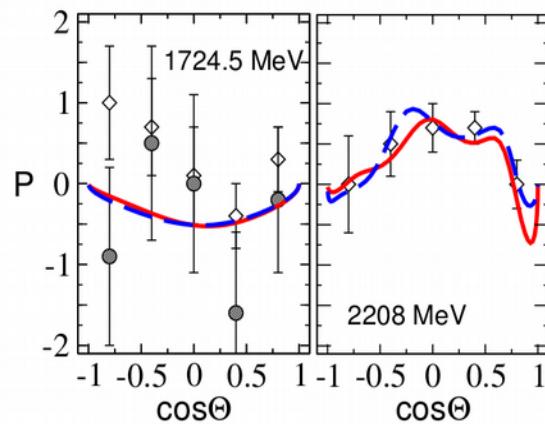
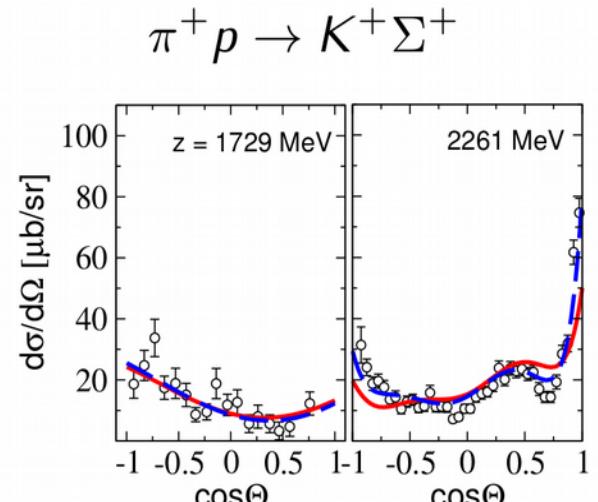
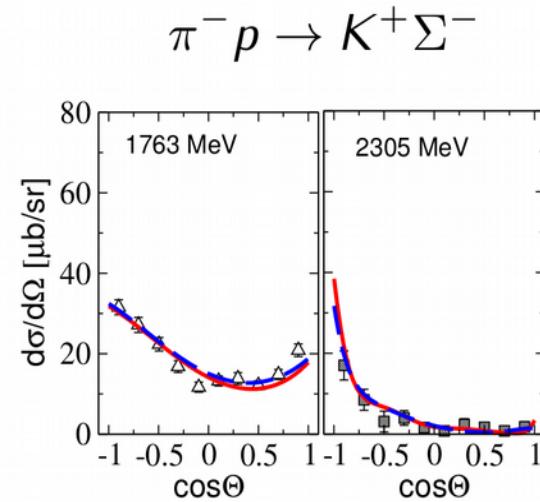
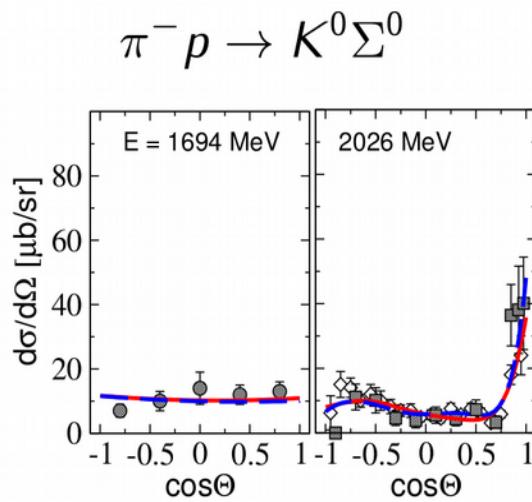
[Rönchen, M.D. et al., EPJA 49 (2013)]

Selected results for $\pi^- p \rightarrow K^0 \Lambda$ [almost complete experiment]

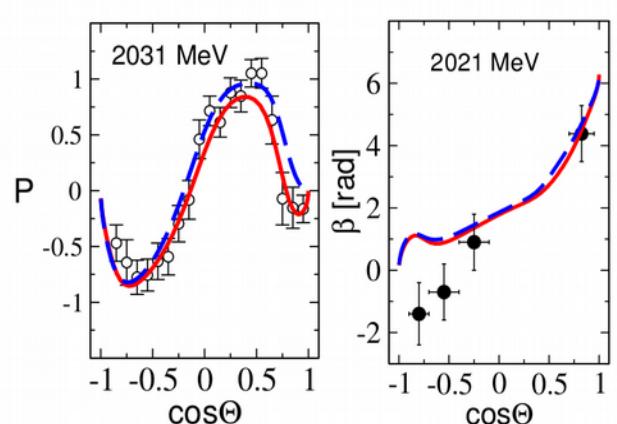


Re-measuring hadron-induced reactions

Fits: D. Rönchen, M.D., et al., EPJ A49 (2013)



No polarization data!

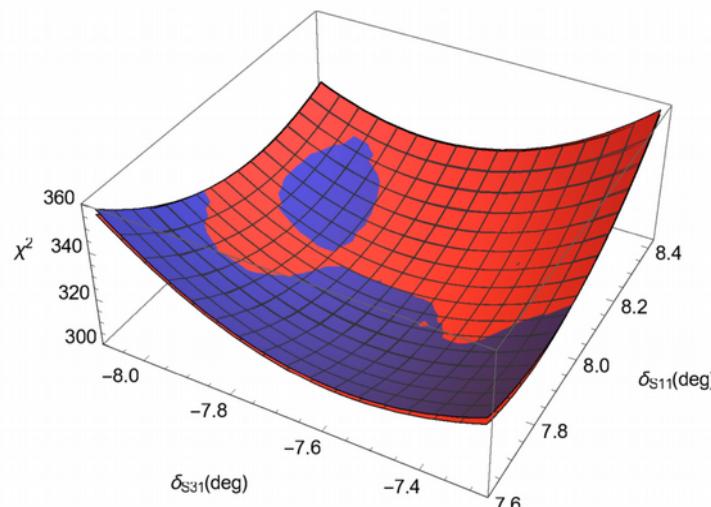


→ *Physics Opportunities with meson beams*,
Briscoe, M.D., Haberzettl, Manley, Naruki, Strakovsky, Swanson, EPJ A51 (2015)

Toward Data-driven Analyses

[M.D., Revier, Rönchen, Workman, arXiv:1603.07265, PRC 2016]

- Multi-channel analyses to detect faint resonance signals
- All groups use GW/SAID partial waves for $\pi N \rightarrow \pi N$
 - The chi-square obtained in fits to single-energy solutions is not related to chi-square of a fit to data → **Statistical interpretation of resonance signals difficult.**
- Provide online covariance matrices etc. to allow other groups to perform *correlated chi-square* fits.



Slight adaptation of their code allows other groups to obtain a χ^2 (almost) as if they fitted to $\pi N \rightarrow \pi N$ directly.

$$\begin{aligned}\chi^2(\mathbf{A}) &= \chi^2(\hat{\mathbf{A}}) + (\mathbf{A} - \hat{\mathbf{A}})^T \hat{\Sigma}^{-1} (\mathbf{A} - \hat{\mathbf{A}}) \\ &\quad + \mathcal{O}(\mathbf{A} - \hat{\mathbf{A}})^3\end{aligned}$$

Covariance matrices etc. can be downloaded on the SAID and JPAC web pages.

Amplitude reconstruction from complete experiments and truncated partial-wave expansions

[Workman, Tiator, Wunderlich, M.D.,
H. Haberzettl, PRC (2017)]

How do complete experiment and truncated partial wave complete experiment compare.
Depending on which partial-wave content is admitted in the amplitude?

Set	Included Partial Waves	CEA	TPWA	Complete Sets for TPWA
1	$L = 0 (E_{0+})$	1(1)	1(1)1	$I[1]$
2	$J = 1/2 (E_{0+}, M_{1-})$	4(4)	4(4)1	$I[1], \check{P}[1], \check{C}_x[1], \check{C}_z[1]$
			4(3)2	$I[2], \check{P}[1], \check{C}_x[1]$
3	$L = 0, 1 (E_{0+}, M_{1-}, E_{1+})$	6(6)	6(6)1	$I[1], \check{\Sigma}[1], \check{T}[1], \check{P}[1], \check{F}[1], \check{G}[1]$
			6(4)2	$I[2], \check{\Sigma}[1], \check{T}[2], \check{P}[1]$
			6(3)3	$I[3], \check{\Sigma}[1], \check{T}[2]$
4	$L = 0, 1 (E_{0+}, M_{1-}, E_{1+}, M_{1+})$ full set of 4 S, P wave multipoles	†		TPWA at 1 angle not possible
			8(5)2	$I[2], \check{\Sigma}[1], \check{T}[2], \check{P}[2], \check{F}[1]$
			8(4)3	$I[3], \check{\Sigma}[1], \check{F}[2], \check{H}[2]$
5	$L = 0, 1, 2 (E_{0+}, M_{1-}, E_{1+}, E_{2-})$	8(8)	8(8)1	$I[1], \check{\Sigma}[1], \check{T}[1], \check{P}[1], \check{F}[1], \check{G}[1], \check{C}_x[1], \check{O}_x[1]$
			8(4)2	$I[2], \check{\Sigma}[2], \check{T}[2], \check{P}[2]$
			8(3)3	$I[3], \check{\Sigma}[2], \check{T}[3]$
6	$J \leq 3/2 (E_{0+}, M_{1-}, E_{1+}, M_{1+}, E_{2-}, M_{2-})$	†		TPWA at 1 or 2 angles not possible
			12(5)3	$I[3], \check{\Sigma}[2], \check{T}[3], \check{P}[2], \check{F}[2]$
			12(4)4	$I[4], \check{\Sigma}[2], \check{F}[3], \check{H}[3]$
7	$L = 0, 1, 2 (E_{0+}, \dots, M_{2+})$ full set of 8 S, P, D wave multipoles	†		TPWA at 1 or 2 angles not possible
			16(6)3	$I[3], \check{\Sigma}[3], \check{T}[3], \check{P}[3], \check{F}[3], \check{G}[1]$
			16(5)4	$I[4], \check{\Sigma}[3], \check{T}[3], \check{P}[3], \check{F}[3]$
			16(4)5	$I[5], \check{\Sigma}[3], \check{F}[4], \check{H}[4]$

Four are enough!

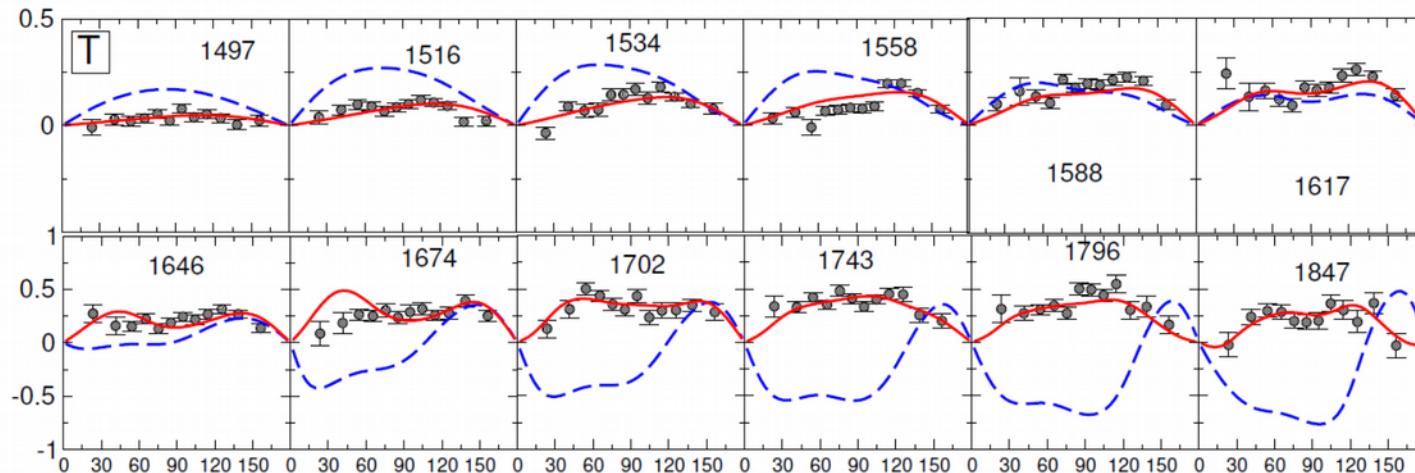
Order:

of different measurements,

of different observables

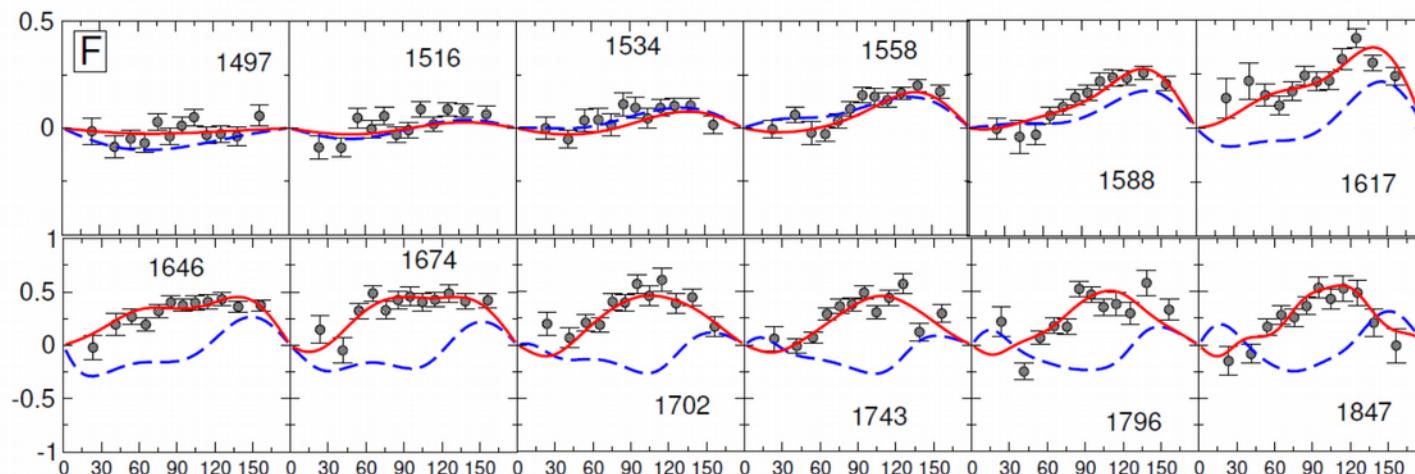
of different angles

Data: Akondi *et al.* (A2 at MAMI) PRL 113, 102001 (2014)



--- prediction
— fit

Beam	Target	Recoil
0	+y	0
0	-y	0



Beam	Target	Recoil
+1	+x	0
-1	+x	0

