Undressing the Nucleon with Photons

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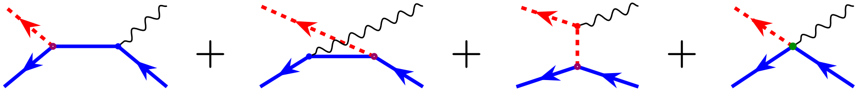
Defining the Problem

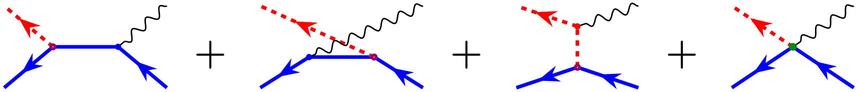
Consider meson photoproduction: (photon may be real or virtual)



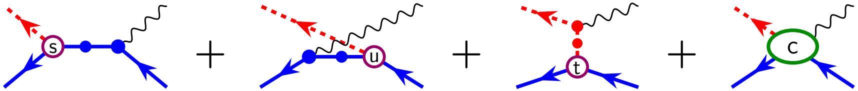
This structure is only based on topology resulting from attaching a photon to external legs and interior of the hadronic vertex.

Defining the Problem

Consider meson photoproduction: (photon may be real or virtual)



This structure is only based on topology — reality is much more complicated:

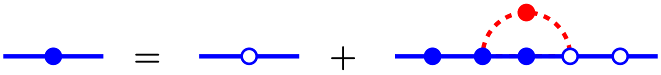


M_s^μ M_u^μ M_t^μ M_{int}^μ

All propagators and vertices need to be fully dressed

Defining the Problem: Dressing Effects

Propagator:



Dyson-Schwinger equations

Defining the Problem: Dressing Effects

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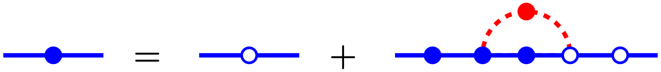

Dyson-Schwinger equations

Hadronic πNN Vertex:

U: 1-particle irreducible interaction

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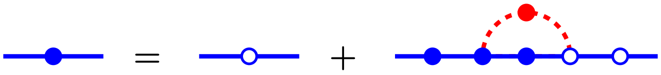

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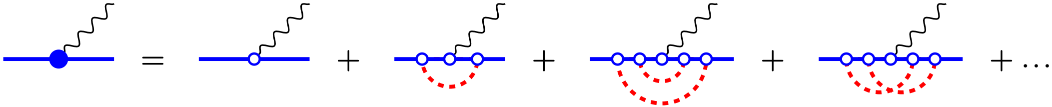

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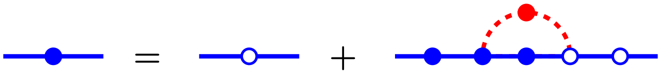

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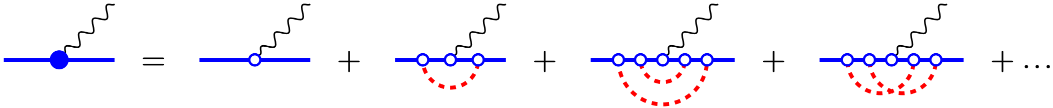
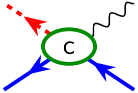


Interaction Current:

$M^{\mu}_{\text{int}} =$ 

Contains full final-state interaction

Details, see PRC**56**,2041(1997), PRC**62**,034605(2000), PRC**74**,045202(2006), PRC**83**,065502(2011)

Outline

- Defining the problem — *You just heard it!*
- Tool Needed: How to determine a current
- Spin 0: Pion
- Spin $\frac{1}{2}$: Nucleon
- Consequences for calculating photoprocesses
- If there is time: Spin 1. — Just a teaser anyway...
- Summary

How to Determine a Current

Minimal substitution for a particle of mass m, charge Q, and four-momentum p: $\boldsymbol{p^\mu \to p^\mu - QA^\mu}$

applied to connected Green's function

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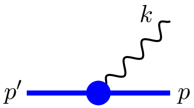
Electromagnetic Current for Single Hadron:



$$\boldsymbol{J^\mu(p',p) = \{P^{-1}(p)\}^\mu + T^\mu(p',p) \equiv \{P^{-1}(p)\}^\mu_{\text{ext}}}$$

simplified notation

$P(p)$: particle propagator

$\{\cdots\}^\mu$: gauge derivative [PRC**56**,2041(1997)]

$T^\mu(p',p)$: transverse contribution (contains all e.m. form factors)

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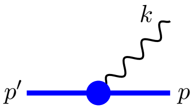

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Ward-Takahashi Identity:

Current is locally gauge invariant

$$k_\mu J^\mu(p',p) = e\left[P^{-1}(p') - P^{-1}(p)\right]$$

Important: WTI does not depend on form factors.

Spin 0: Pion Current for $\gamma(k) + \pi(q) \to \pi(q')$

Bare current:

bare propagator: $\Delta_\pi(q) = \dfrac{1}{q^2 - \mu^2}$ $\Rightarrow$ $J^\mu_\pi(q', q) = \{q^2 - \mu^2\}^\mu = Q_\pi (q' + q)^\mu$ bare current

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Fully dressed current:

$$\Delta_\pi(q) = \frac{1}{(q^2-\mu^2)\Pi(q^2)} \qquad \Rightarrow \qquad J^\mu_\pi(q',q) = \{\Delta_\pi^{-1}(q^2)\}^\mu_S = \Big[Q_\pi(q'+q)^\mu + T^\mu_\pi(q',q)\Big]\frac{\Delta_\pi^{-1}(q'^2)-\Delta_\pi^{-1}(q^2)}{q'^2-q^2}$$

Exact!

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Exact!

Transverse current contribution:

$$T^\mu_\pi(q',q) = Q_\pi \underbrace{\left[(q'+q)^\mu - k^\mu \frac{q'^2-q^2}{k^2}\right]}_{\text{transverse coupling}} \underbrace{\left[F_\pi(k^2) - 1 + \frac{k^2}{\mu^2} \times \textit{(off shell)}\right]}_{\text{e.m. form factor dependence}}$$

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Check WTI:

$$k_\mu J_\pi^\mu(q', q) = Q_\pi \left[\Delta_\pi^{-1}(q'^2) - \Delta_\pi^{-1}(q^2)\right]$$

Okay!

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Exact!

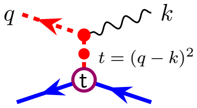
Important technical detail:

scalar coupling: $\{\Pi(q^2)\}_S^\mu = Q_\pi(q' + q)^\mu \dfrac{\Pi(q'^2) - \Pi(q^2)}{q'^2 - q^2}$

Spin 0: Half-on shell case

Consider fully dressed propagator and current in half-on-shell configuration:

t-channel kinematics



$$J_\pi^\mu(\underline{q}, q-k)\Delta_\pi(t) = \left[Q_\pi(2q-k)^\mu + T_\pi^\mu(\underline{q}, q-k)\right]\frac{1}{t-\mu^2}$$

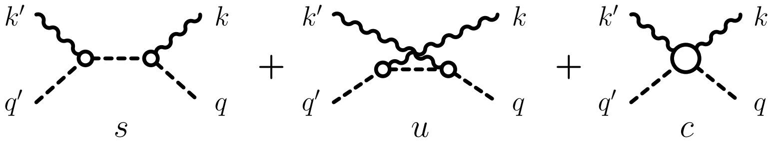
Hadronic dressing completely cancels!

Rewriting:

$$J_\pi^\mu(\underline{q}, q-k)\Delta_\pi(t) = Q_\pi\frac{(2q-k)^\mu}{t-\mu^2}F_\pi(k^2) + \underbrace{Q_\pi\frac{(2q-k)^\mu}{\mu^2}\frac{k^2}{\mu^2}D_H(t;k^2)}_{\text{e.m. off-shell contact term}}$$

This structure is exact — no approximation!

Spin 0: Real Compton scattering on the pion



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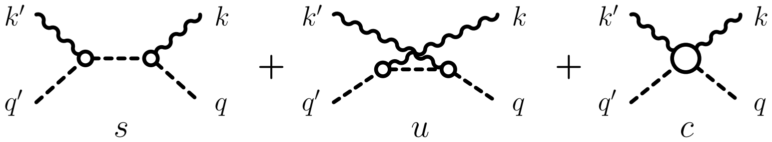

Use **fully dressed** **propagators and currents:**

$$C_\pi^{\nu\mu}(q',q) = e^2 \frac{(2q'+k')^\nu (2q+k)^\mu}{s-\mu^2} + e^2 \frac{(2q'-k)^\nu (2q-k')^\mu}{u-\mu^2} - 2e^2 g^{\mu\nu} \qquad \text{(on shell)}$$

Hadronic dressing effects completely cancel!

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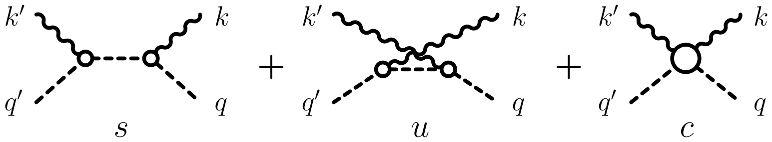

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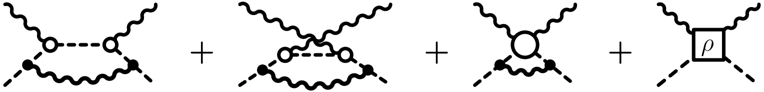
Higher-order effects: vector-meson loops



⇒ polarization effects

Spin 1/2: Nucleon Current for $\gamma(k) + N(p) \to N(p')$

Bare current:

bare propagator: $S(p) = \dfrac{1}{\not{p} - m}$ $\quad\Rightarrow\quad$ $J_N^\mu(p', p) = \{\not{p} - m\}^\mu = Q_N \gamma^\mu$ bare current

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Fully dressed propagator:

$$S(p) = \frac{1}{\not{p} A(p^2) - m B(p^2)} \,, \qquad \text{where} \qquad \begin{cases} A(m^2) = B(m^2) & \text{pole condition} \\ A(m^2) + 2m^2 \dfrac{d\left[A(p^2) - B(p^2)\right]}{dp^2}\Bigg|_{p^2=m^2} = 1 & \text{residue condition} \end{cases}$$

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Another important technical detail:

Recall scalar coupling for spin 0: $\quad \{\Pi(q^2)\}_S^\mu = Q_\pi (q' + q)^\mu \dfrac{\Pi(q'^2) - \Pi(q^2)}{q'^2 - q^2}$

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Now, e.m. coupling must ensure equivalence of $\dfrac{1}{\not p - m} \equiv \dfrac{\not p + m}{p^2 - m^2}$ to make KG equation true $\Rightarrow$ $\left\{\dfrac{1}{\not p - m}\right\}^\mu \stackrel{!}{=} \left\{\dfrac{\not p + m}{p^2 - m^2}\right\}^\mu$

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One finds:

$$\{f(p^2)\}_D^\mu = Q_N \left[(p' + p)^\mu + i\sigma^{\mu\nu}k_\nu\right] \frac{f(p'^2) - f(p^2)}{p'^2 - p^2}$$

Dirac version of scalar coupling

$f(p^2)$: any scalar function

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Fully dressed current:

$$D_f(p'^2, p^2) = 2m^2 \frac{f(p'^2) - f(p^2)}{p'^2 - p^2} \quad \text{for } f = A, B$$

$$J_N^\mu(p', p) = \left\{ \frac{\not{p}A(p^2) + A(p^2)\not{p}}{2} - mB(p^2) \right\}_D^\mu$$

$$= \Gamma_N^\mu(p', p) \frac{A(p'^2) + A(p^2)}{2} + \frac{(p'^2 + p^2)\Gamma_N^\mu(p', p) + 2\not{p}'\Gamma_N^\mu(p', p)\not{p}}{4m^2} D_A(p'^2, p^2) - \frac{\not{p}'\Gamma_N^\mu(p', p) + \Gamma_N^\mu(p', p)\not{p}}{2m} D_B(p'^2, p^2)$$

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$\Gamma_N^\mu(p',p) = Q_N \gamma^\mu + T_N^\mu(p',p)$, where transverse current, $T_N^\mu(p',p) = Q_N \underbrace{\left[\left(\gamma^\mu - k^\mu \frac{\not{p}' - \not{p}}{k^2} \right) f_1^N(p',p) + \frac{i\sigma^{\mu\nu}k_\nu}{2m} f_2^N(p',p) \right]}_{\text{contains e.m. form factors}}$

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WTI:

$$k_\mu J_N^\mu(p',p) = Q_N\left[S^{-1}(p') - S^{-1}(p)\right]$$

On-shell limit:

$$J_N^\mu(p',p) = \Gamma_N^\mu(p',p) = e\gamma^\mu \underbrace{F_1^N(k^2)}_{\text{Dirac}} + \kappa_N \frac{i\sigma^{\mu\nu}k_\nu}{2m} \underbrace{F_2^N(k^2)}_{\text{Pauli}}$$

Spin 1/2: Relationship to Ball-Chiu Current [PRD**22**,2542(1980)]

Fully dressed propagator:

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Ball-Chiu current for proton: nonsingular expression proposed to reproduce WTI for fully dressed propagator

$$J^\mu_{\mathrm{BC}}(p',p) = Q_p \gamma^\mu \frac{A(p'^2)+A(p^2)}{2} + Q_p \frac{(p'+p)^\mu}{2m} \left[\frac{\not{p}'+\not{p}}{2m} D_A(p'^2,p^2) - D_A(p'^2,p^2) \right]$$

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Relationship: (proton)

put $T^\mu_N = 0$: $$J_N(p',p) \quad \to \quad J_0(p',p) = J^\mu_{\rm BC}(p',p) + Q_p \frac{\not{p}' i\sigma^{\mu\nu}k_\nu + i\sigma^{\mu\nu}k_\nu \not{p}}{4m^2} D_A(p'^2,p^2) - Q_p \frac{i\sigma^{\mu\nu}k_\nu}{2m} D_B(p'^2,p^2)$$

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One finds:

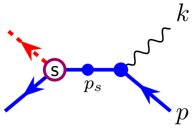
$$J^\mu_{\mathrm{BC}}(p',p) = \{S^{-1}(p)\}^\mu_S \;, \qquad \text{whereas} \qquad J^\mu_0(p',p) = \{S^{-1}(p)\}^\mu_D$$

wrong on-shell limit — correct on-shell limit $\Rightarrow$ homework problem

Spin 1/2: Half-on shell case

Consider fully dressed propagator and current in half-on-shell configuration:

s-channel kinematics



$$S(p_s)J_N^\mu(p_s,p) = \frac{1}{\not p_s - m}\Gamma_N^\mu(p_s,p)$$

Hadronic dressing completely cancels!

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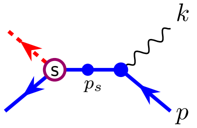

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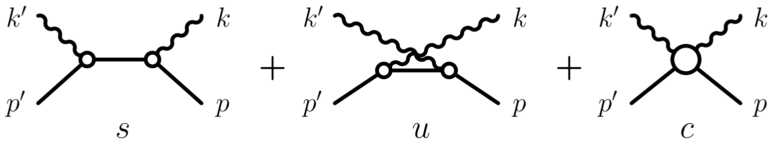
Rewriting:

$$S(p_s)J_N^\mu(p_s,p) = \frac{Q_N}{\not{p}_s - m}\left[\gamma^\mu F_1^N(k^2) + \kappa_N \frac{i\sigma^{\mu\nu}k_\nu}{2m}F_2^N(k^2)\right] + \underbrace{\frac{k^2}{m^2}Q_N C^\mu(p_s;k)}_{\text{e.m. off-shell contact term}}$$

This structure is exact — no approximation!

Spin 1/2: Real Compton scattering on the proton



π^0 t-channel exchange omitted

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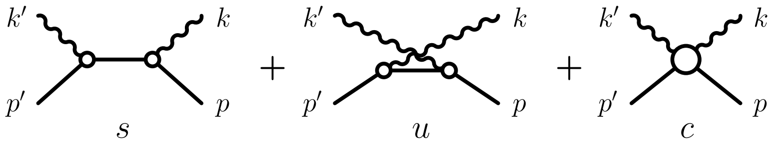

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Use fully dressed propagators and currents:

$$C_p^{\nu\mu}(q',q) = \Gamma_p^\nu \frac{1}{\not{p}_s - m} \Gamma_p^\mu + \Gamma_p^\mu \frac{1}{\not{p}_u - m} \Gamma_p^\nu$$ (on shell)

$$\Gamma_p^\mu = e\gamma^\mu + \kappa_p \frac{i\sigma^{\mu\tau} k_\tau}{2m}$$

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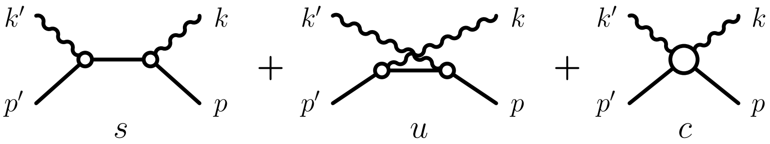

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(on shell)

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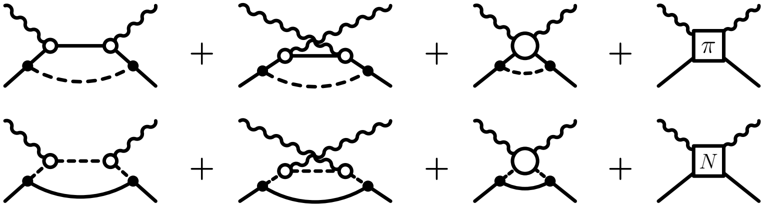
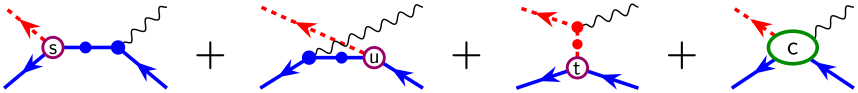
Higher-order effects: meson loops and nucleon loops



$\Rightarrow$ polarization effects

Consequences for Calculating Photoprocesses

Previously: (photon may be real or virtual)



$$M_s^\mu \quad + \quad M_u^\mu \quad + \quad M_t^\mu \quad + \quad M_{\rm int}^\mu$$

All propagators and vertices need to be fully dressed

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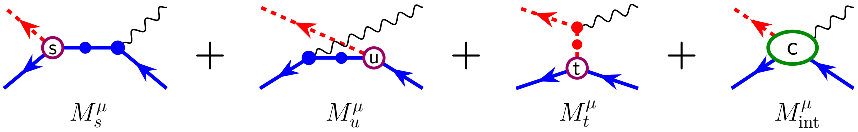

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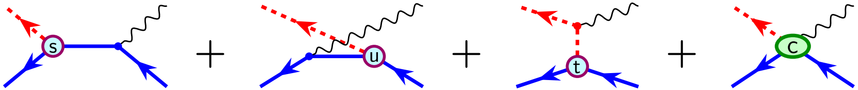
Now: (external hadrons on shell)



Only hadronic πNN vertices need to be fully dressed

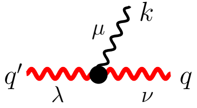
The only remnants of intermediate hadron dressing effects are physical masses

Spin 1: Just a teaser. . .

arXiv:1905.06299 [nucl-th] (2019)

Dressed propagator:

$$P^{\lambda\nu}(q) = \frac{-g^{\lambda\nu} - \frac{q^\lambda q^\nu}{m^2} N_{\mathsf{gauge}}(q^2)}{q^2 - m^2 - \Sigma(q^2)}$$



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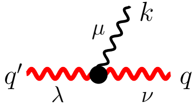

Current without hadronic dressing:

$$J_1^{\lambda\mu\nu}(q',q) = -e(q'+q)^\mu g^{\lambda\nu} - eG_2(k^\lambda g^{\nu\mu} - k^\nu g^{\mu\lambda}) - e\left(\frac{G_1 - 1}{k^2} g^{\lambda\nu} - \frac{G_3}{2k^2}\frac{k^\lambda k^\nu}{m^2}\right)\left[(q'+q)^\mu k^2 - k^\mu(q'^2 - q^2)\right]$$

three form factors related to charge e, magnetic moment μ, and quadrupole moment Q

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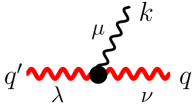

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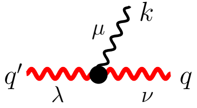

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Model-independent correlation:

$$G_3(0) = 0 \quad \Rightarrow \quad \boldsymbol{2m\mu + m^2 Q = e}$$

Known to be true at the tree-level, but actually true at all orders

In general, spin 1 is a nontrivial problem because of the gauge dependence. Plays no role for all-hadronic initial and final states because of the transverse hadronic vertex couplings, but in Compton scattering, for example, gauge-dependent contact terms are required to make the problem well defined.

Summary

- It was shown here that consistently constructed fully dressed propagators and currents exhibit a large degree of cancellations of dressing effects when combined in the form (propagator)×(current).
- For spin-0 and spin-1/2, cancellations are complete for stable hadrons and real photons.
- The only remaining effects of dressing are physical mass parameters.
- Currents consistent with hadronic propagation are locally gauge-invariant as a matter of course, i.e., Ward-Takahashi identities are satisfied.
- For unstable states, remaining effect of dressing for current is a factor depending on dressing function:

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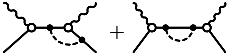
- Hadronic loop effects, like  $+ \cdots$ for Compton scattering, etc., are taken into account implicitly to all orders.
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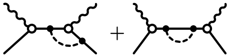
- Hadronic loop effects, like  $+\cdots$ for Compton scattering, etc., are taken into account implicitly to all orders.
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