$\Lambda(1405)$ and $\Sigma^*(1430)$ production in $\Lambda_c \to \pi\pi\pi\Sigma$ decay

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$\Lambda_c \to \pi^+ \bar{K}^* N$
Triangle singularities
The dynamical origin of the $\Lambda(1405)$, and an extra $\Sigma(1430)$
The $\Lambda_c \to \pi^+ \pi^0 \pi^0 \Sigma^0$ and isospin forbidden $\Lambda(1405)$ production
The $\Lambda_c \to \pi^+ \pi^- \pi^0 \Sigma^+$ and the new $\Sigma(1430)$ production

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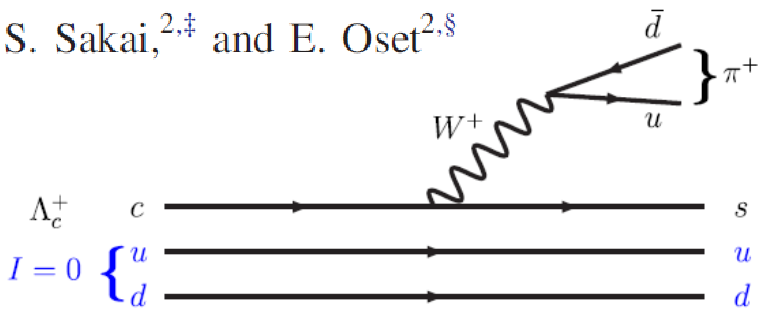
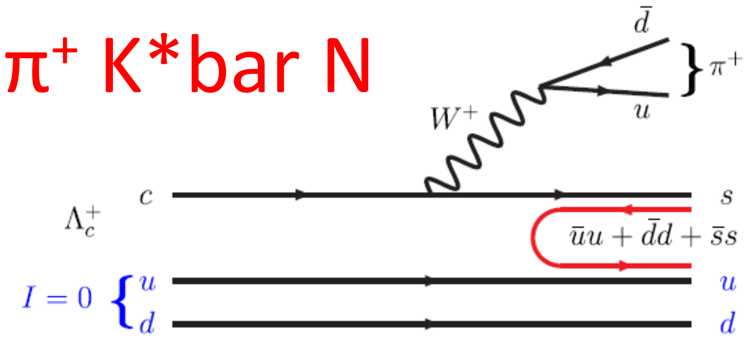
PRD 97 (2018)

$\Lambda_c \to \pi^+ K^*\text{bar } N$

u c t
d s b



(a)



(b)

$$H = \sum_{i=1}^{3} s\bar{q}_i q_i \frac{1}{\sqrt{2}}(ud - du) = \sum_{i=1}^{3} M_{3i} q_i \frac{1}{\sqrt{2}}(ud - du)$$

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),$$
$$\rho^+ = u\bar{d}, \quad \rho^- = d\bar{u}, \quad K^{*0} = d\bar{s},$$
$$K^{*-} = s\bar{u}, \quad K^{*+} = u\bar{s}, \quad \bar{K}^{*0} = s\bar{d},$$
$$\phi = s\bar{s}.$$

$$M \to V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

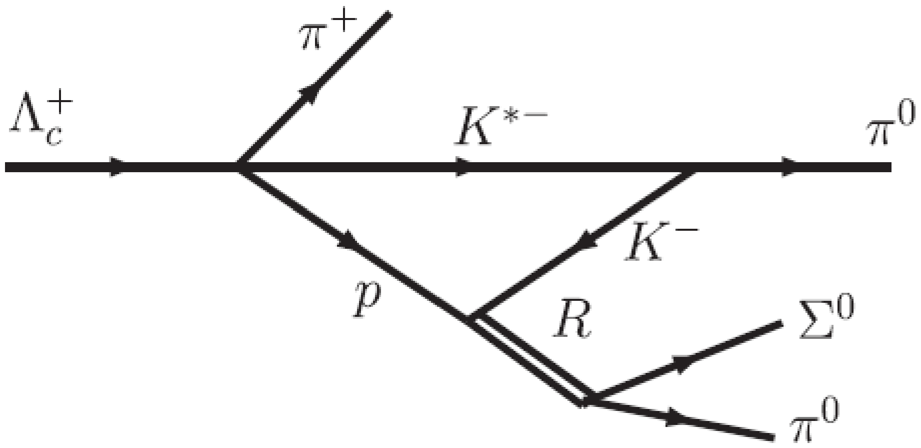
$$H = K^{*-} u \frac{1}{\sqrt{2}}(ud - du) + \bar{K}^{*0} d \frac{1}{\sqrt{2}}(ud - du) + \phi s \frac{1}{\sqrt{2}}(ud - du).$$

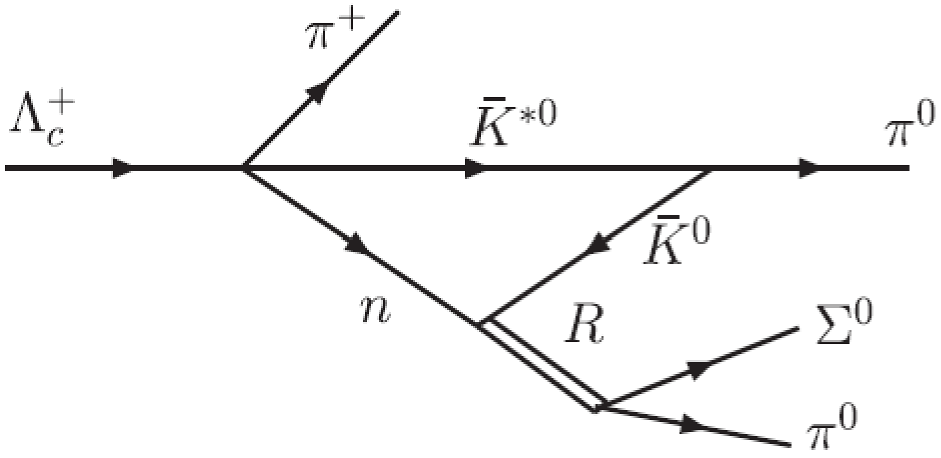
$$p = \frac{1}{\sqrt{2}} u(ud - du), \quad \Lambda = \frac{1}{2\sqrt{3}}[u(ds - sd) + d(su - us) - 2s(ud - du)]$$

$$n = \frac{1}{\sqrt{2}} d(ud - du), \quad \Sigma^0 = \frac{1}{2}[u(ds - sd) - d(su - us)].$$

$$H = K^{*-} p + \bar{K}^{*0} n - \sqrt{\frac{2}{3}} \phi \Lambda$$

Triangle mechanism with singularity





Isospin forbidden. Cancellation of diagrams if equal masses. The different masses of the Kaons make the cancellation partial and we can see the Λ(1405)

Triangle singularity: if three intermediate particles are on shell and K* and π0 are parallel -> the mechanisms generates a singularity in the amplitude for zero width of the K*, or a peak if the width is considered. L.D. Landau NP 13 (1959)

Kbar N interaction

$$K^- p,\ \bar{K}^0 n,\ \pi^0 \Lambda,\ \pi^0 \Sigma^0,\ \pi^+ \Sigma^-,\ \pi^- \Sigma^+,\ \eta \Lambda,\ \eta \Sigma^0,\ K^0 \Xi^0 \text{ and } K^+ \Xi^-$$

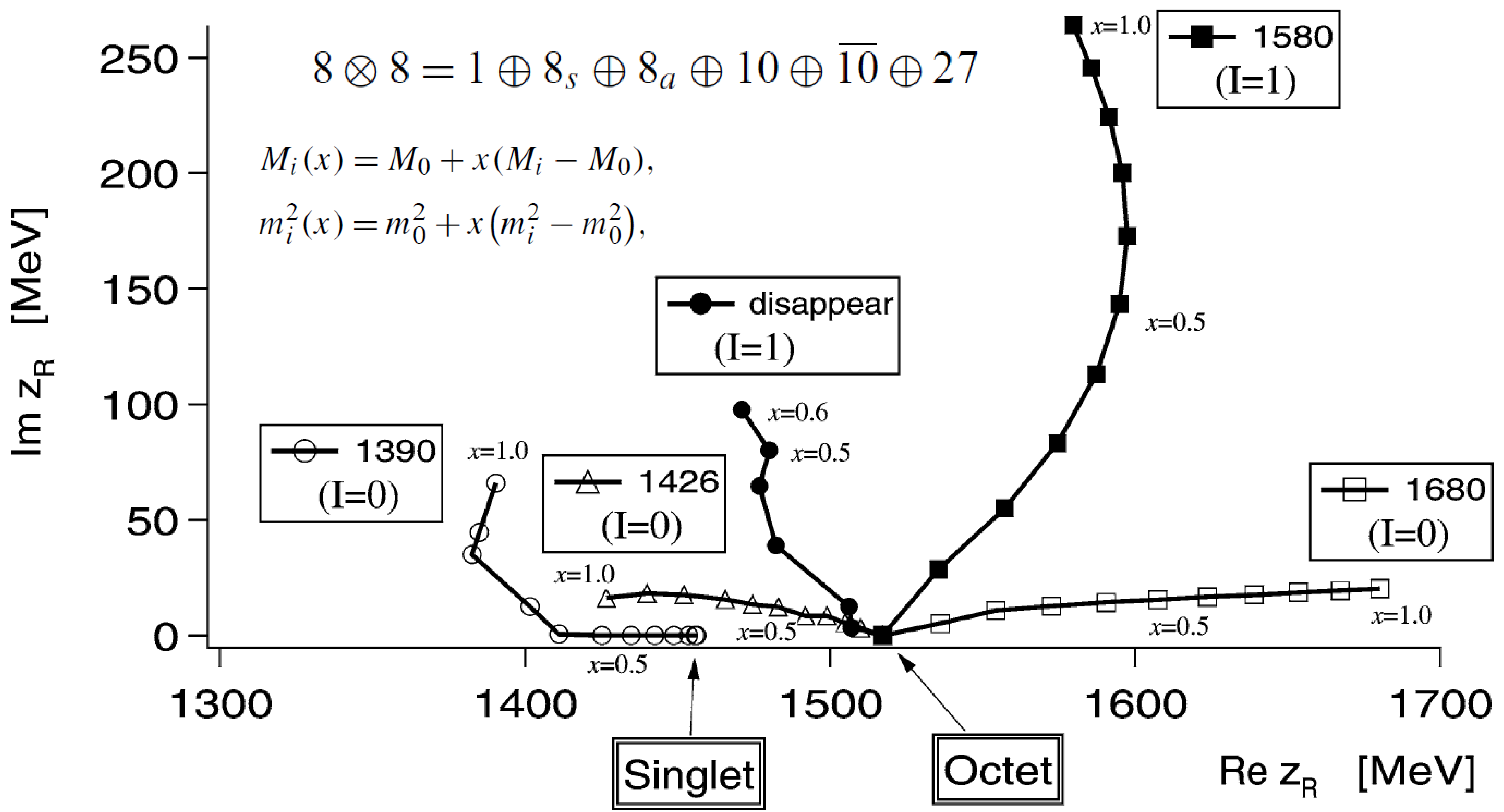
$$T = [1 - VG]^{-1} V$$

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (2\sqrt{s} - M_i - M_j) \left(\frac{M_i + E}{2M_i} \right)^{1/2} \left(\frac{M_j + E'}{2M_j} \right)^{1/2}$$

The V_{ij} are obtained from the chiral Lagrangians. G are the meson baryon loop functions

Very good reproduction is obtained of scattering data and threshold parameters

Two Λ(1405) are generated from this interaction



$8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \overline{10} \oplus 27$
$M_i(x) = M_0 + x(M_i - M_0),$
$m_i^2(x) = m_0^2 + x\left(m_i^2 - m_0^2\right),$
1580 (I=1)
disappear (I=1)
1390 (I=0)
1426 (I=0)
1680 (I=0)
x=1.0
x=0.5
x=0.6
Singlet
Octet
Im z_R [MeV]
Re z_R [MeV]
0
50
100
150
200
250
1300
1400
1500
1600
1700

$$t_{\Lambda_c^+ \to \pi^+ K^{*-} p} = A \vec{\sigma} \cdot \vec{\epsilon},$$

$$\frac{d\Gamma_{\Lambda_c^+ \to \pi^+ K^{*-} p}}{dM_{\rm inv}(K^{*-} p)} = \frac{1}{(2\pi)^3} \frac{2M_{\Lambda_c^+} 2M_p}{4M_{\Lambda_c^+}^2} p_{\pi^+} \tilde{p}_{K^{*-}} \times \overline{\sum} \sum |t_{\Lambda_c^+ \to \pi^+ K^{*-} p}|^2,$$

By calculating the width of this decay, using the experimental branching ratio of this decay $\mathrm{Br}(\Lambda_c^+ \to \pi^+ K^{*-} p) = (1.5 \pm 0.5) \times 10^{-2}$ [19], we can determine the value of the constant $|A|$.

$$t_{\Lambda_c^+ \to \pi^+ \pi^0 \pi^0 \Sigma^0} = -A \frac{1}{\sqrt{2}} g \vec{\sigma} \cdot \vec{k} \, t_{K^- p \to \pi^0 \Sigma^0} \, t_T$$

For the first diagram

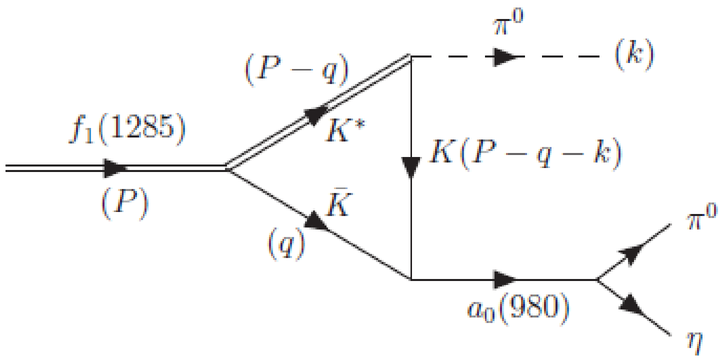
$$t_T = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_p}{q^2 - M_p^2 + i\epsilon} \frac{1}{(P-q)^2 - m_{K^{*-}}^2 + i\epsilon} \times \frac{1}{(P-q-k)^2 - m_{K^-}^2 + i\epsilon} \left(2 + \frac{\vec{q} \cdot \vec{k}}{\vec{k}^2} \right),$$

$$\frac{1}{\Gamma_{\Lambda_c^+}} \frac{d^2 \Gamma}{dM_{\rm inv}(\pi^0 \Lambda(1405)) dM_{\rm inv}(\pi^0 \Sigma^0)} = \frac{1}{(2\pi)^5} \frac{M_{\Sigma^0}}{M_{\Lambda_c^+}} \tilde{p}_{\pi^+} \tilde{q}_{\Sigma^0} \frac{1}{2} g^2 \frac{A^2}{\Gamma_{\Lambda_c^+}} |\vec{k}|^3 \times | t_T(m_{K^{*-}}, M_p, m_{K^-}) t_{K^- p \to \pi^0 \Sigma^0} - t_T \times (m_{\bar{K}^{*0}}, M_n, m_{\bar{K}^0}) t_{\bar{K}^0 n \to \pi^0 \Sigma^0} |^2.$$

Includes the two diagrams

Triangle singularities

THE $\pi a_0(980)$ DECAY MODE OF THE $f_1(1285)$



$$t_T = i\int \frac{d^4q}{(2\pi)^4}\,\vec{\epsilon}_{f_1}\cdot\vec{\epsilon}_{K^*}\;\vec{\epsilon}_{K^*}\cdot(2\vec{k}+\vec{q})\,\frac{1}{q^2-m_K^2+i\epsilon}\,\frac{1}{(P-q)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}}\,\frac{1}{(P-q-k)^2-m_K^2+i\epsilon}$$

$$\tilde{t}_T = \int \frac{d^3q}{(2\pi)^3}\left(2+\frac{\vec{k}\cdot\vec{q}}{\vec{k}^2}\right)\frac{1}{8\,\omega(q)\omega'(q)\omega^*(q)}\,\frac{1}{k^0-\omega'(q)-\omega^*(q)+i\epsilon}\,\frac{1}{P^0-\omega^*(q)-\omega(q)+i\epsilon}$$
$$\times\,\frac{2P^0\omega(q)+2k^0\omega'(q)-2(\omega(q)+\omega'(q))(\omega(q)+\omega'(q)+\omega^*(q))}{(P^0-\omega(q)-\omega'(q)-k^0+i\epsilon)(P^0+\omega(q)+\omega'(q)-k^0-i\epsilon)}\,,$$

$$\omega(q) = \sqrt{\vec{q}^{\,2}+m_K^2},\;\; \omega'(q) = \sqrt{(\vec{q}+\vec{k})^2+m_K^2},\;\; \omega^*(q) = \sqrt{\vec{q}^{\,2}+m_{K^*}^2}$$

Poles in the integration

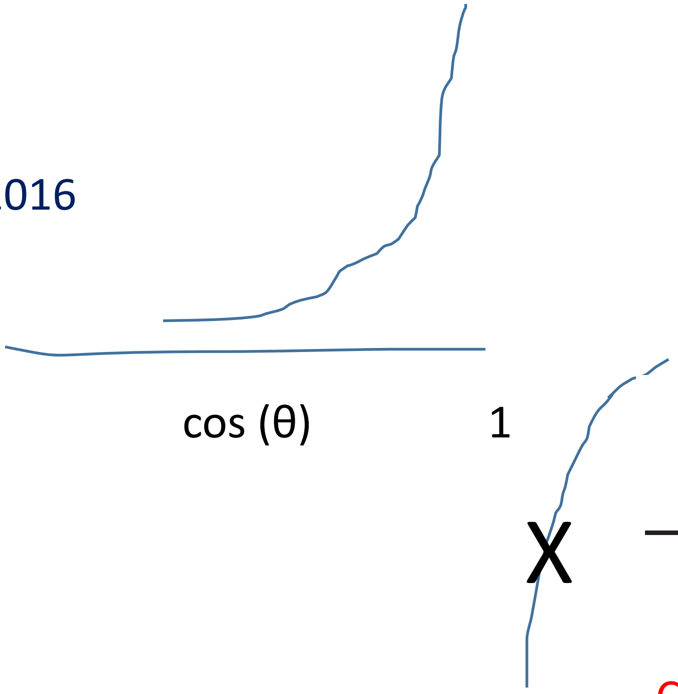
$$P^0-\omega^*(q)-\omega(q)+i\epsilon = 0\,, \qquad q_{\rm on+} = q_{\rm on} + i\,\epsilon \quad \text{with} \quad q_{\rm on} = \frac{1}{2M}\sqrt{\lambda(M^2,m_1^2,m_2^2)}$$

$$P^0-\omega(q)-\omega'(q)-k^0+i\epsilon = 0$$

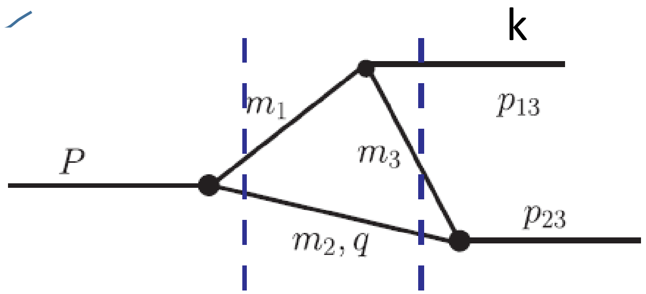
$$P^0 - \omega(q) - \omega'(q) - k^0 + i\epsilon \quad = \quad 0 \qquad\qquad \omega'(q) \;=\; \sqrt{(\vec{q}+\vec{k})^2 + m_K^2}$$

If we fix cos (θ)= ± 1 and we make this expression zero, then in the integral of cos(θ) one cannot cancel the divergence with the principal value, and the divergence remains. Θ is the angle between $\vec{k}$ and $\vec{q}$.

Bayar, Aceti, Guo, E. O, PRD 2016



For cos(θ)=-1



Calculate solution when p_{23} is at rest (p_2*) and make a boost

$$q_{a+} = \gamma\,(v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(v\,E_2^* - p_2^*) - i\,\epsilon$$

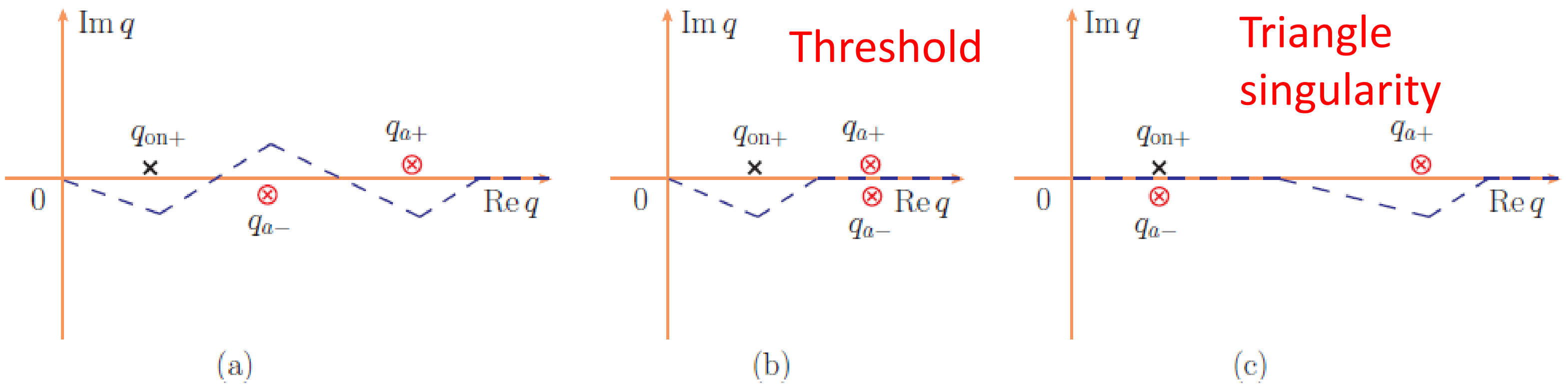
$$v = \frac{k}{E_{23}}\,, \qquad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}}\,,$$

$$E_2^* = \frac{1}{2m_{23}}\left(m_{23}^2 + m_2^2 - m_3^2\right), \qquad p_2^* = \frac{1}{2m_{23}}\sqrt{\lambda(m_{23}^2, m_2^2, m_3^2)}$$

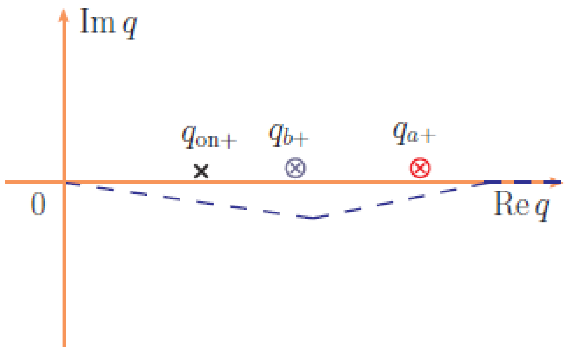
For cos(θ)=1

$$q_{b+} = \gamma\,(-v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{b-} = -\gamma\,(v\,E_2^* + p_2^*) - i\,\epsilon$$

For cos(θ)=-1

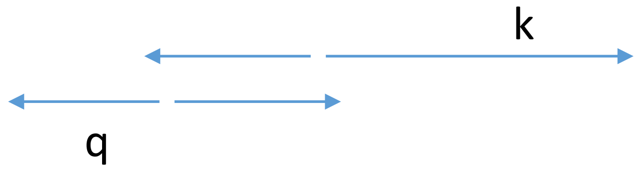


For cos(θ)=1

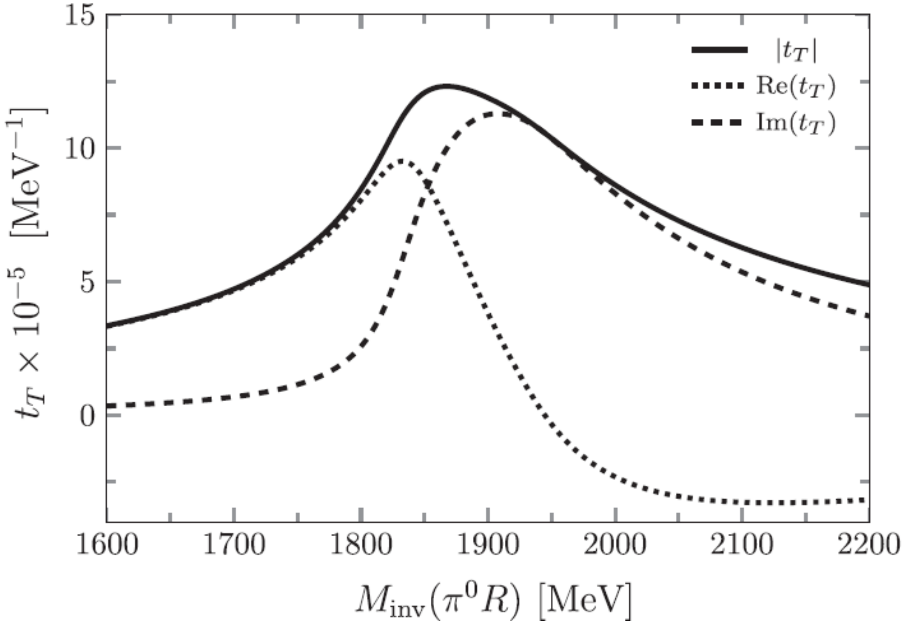


Triangle singularity

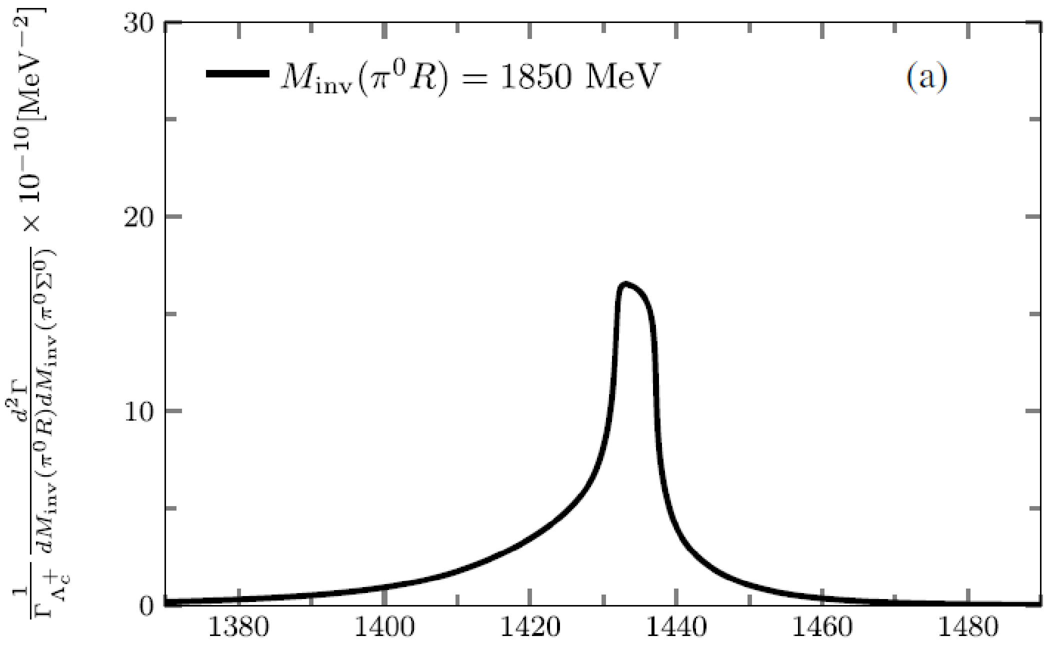
$$\lim_{\epsilon\to 0}(q_{on+} - q_{a-}) = 0$$

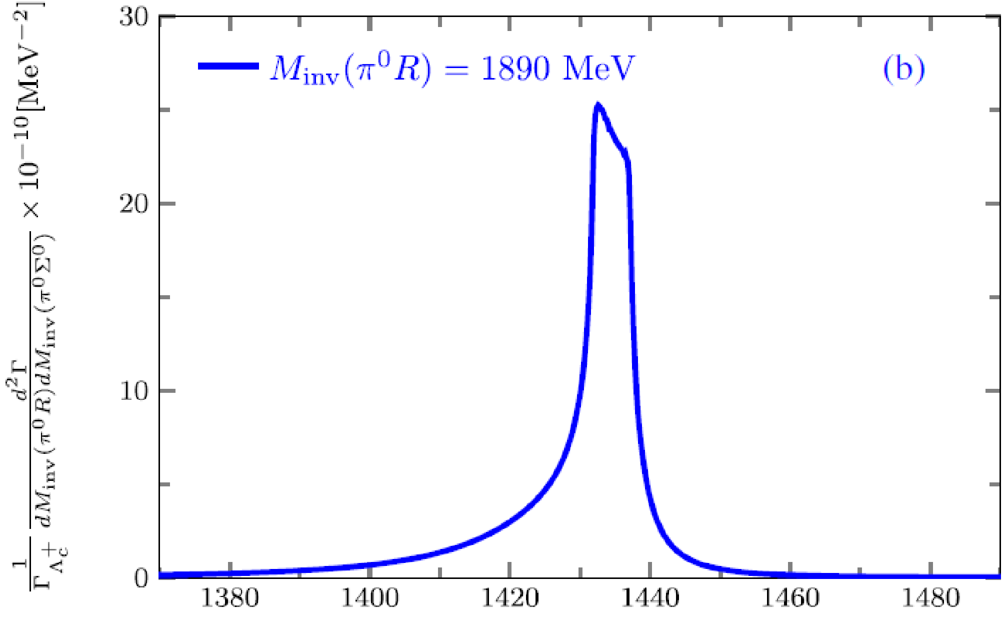


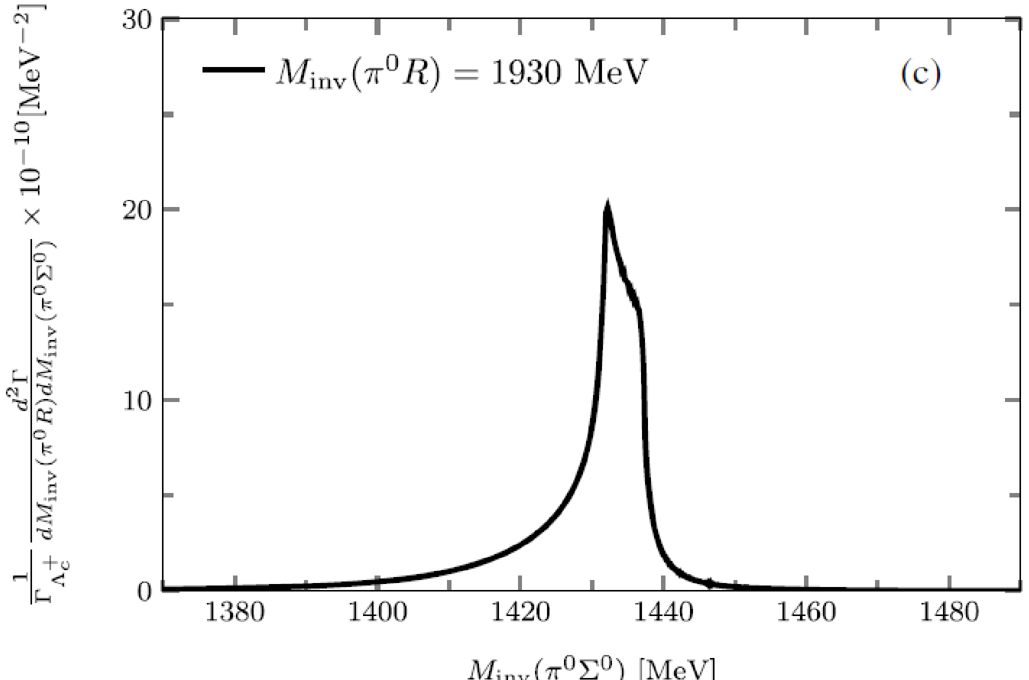
Very simple expression to see where the TS appears , and to explain the Coleman-Norton theorem, Nuovo Cim. 1965, (TS appears when the decays in the loop can occur at the classical level).



$\Lambda_c \to \pi^+ \pi^0 \pi^0 \Sigma^0$







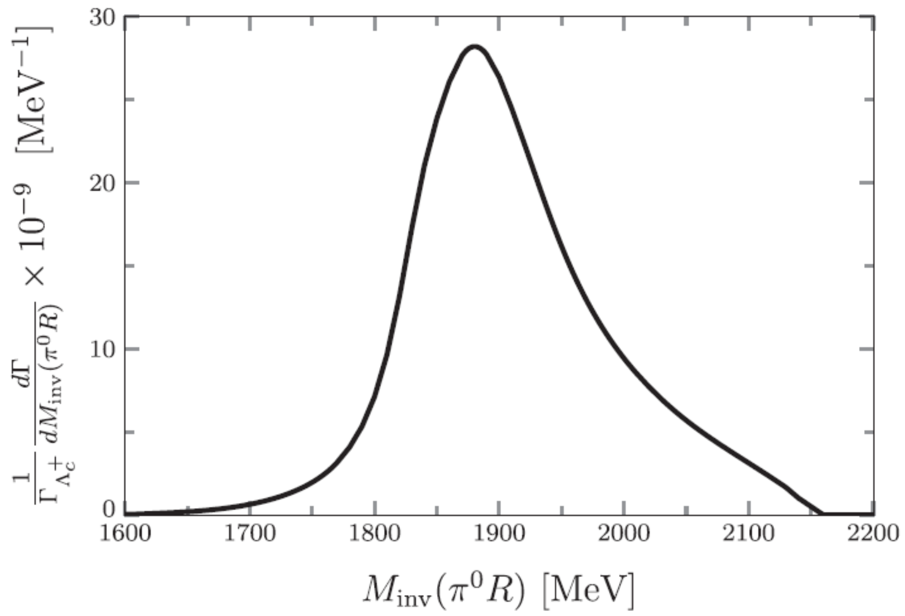
Note the unusual narrow witdh of the Λ(2405)

7 MeV !!

The apperance of a narrow resonance in the isospin forbidden reactions due to different Kaon masses also appears in the f0(980) or a0(980) isospín forbidden production in

J/ψ -> phi pi0 eta production , Hanhart, Kubis, Pelaez PRD (2007), Roca PRD (2013)

eta(1405) -> pi0 f0(980) , Aceti, Liang, Oset, Wu, Zou PRD (2012)

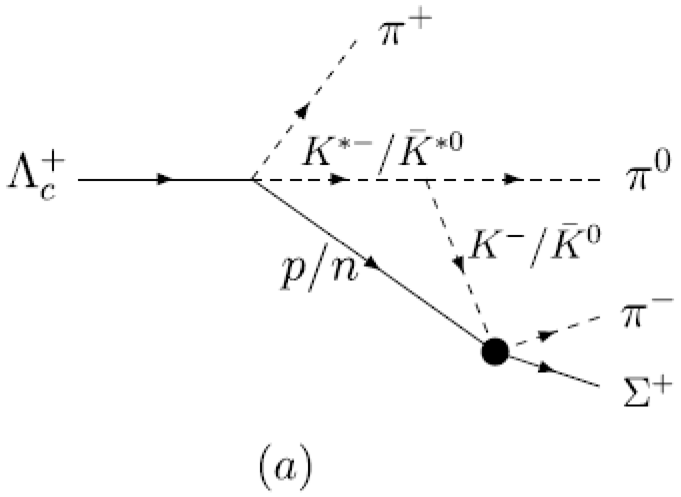


This peak is due to the triangle singularity

$$\mathrm{Br}(\Lambda_c^+ \to \pi^+\pi^0\Lambda(1405); \Lambda(1405) \to \pi^0\Sigma^0)$$
$$= (4.17 \pm 1.39) \times 10^{-6}.$$

Search for the Σ^* state in $\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+$ decay by triangle singularity

Ju Jun Xie, E.O.   PLB 792 (2019)



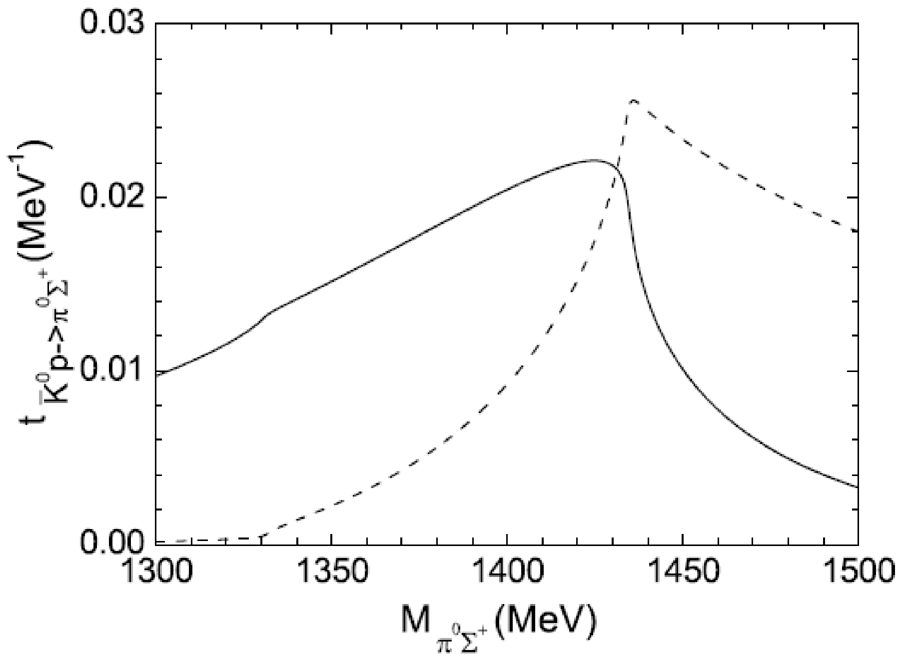
$$t_{\text{total}} = -\frac{Ag}{\sqrt{2}}\left(\vec{\sigma}\cdot\vec{k}_a t_T^a \mathcal{M}^a + \sqrt{2}\vec{\sigma}\cdot\vec{k}_b t_T^b \mathcal{M}^b\right)$$

$$\mathcal{M}^a = t_{K^-p\to\pi^-\Sigma^+} - t_{\bar{K}^0 n\to\pi^-\Sigma^+},$$
$$\mathcal{M}^b = t_{\bar{K}^0 p\to\pi^0\Sigma^+},$$

$$\frac{d^3\Gamma}{dM_{\pi^0\pi^-\Sigma^+}dM_{\pi^-\Sigma^+}dM_{\pi^0\Sigma^+}} = \frac{g^2|A|^2}{128\pi^5}\frac{m_{\Sigma^+}}{M_{\Lambda_c^+}}\widetilde{p}_{\pi^+}$$

$$\times\frac{M_{\pi^-\Sigma^+}M_{\pi^0\Sigma^+}}{M_{\pi^0\pi^-\Sigma^+}}\left(|\vec{k}_a|^2|t_T^a\mathcal{M}_a|^2 + 2|\vec{k}_b|^2|t_T^b\mathcal{M}_b|^2\right.$$

$$\left.+2\sqrt{2}\mathrm{Re}[t_T^a\mathcal{M}_a(t_T^b\mathcal{M}_b)^*]\vec{k}_a\cdot\vec{k}_b\right),$$



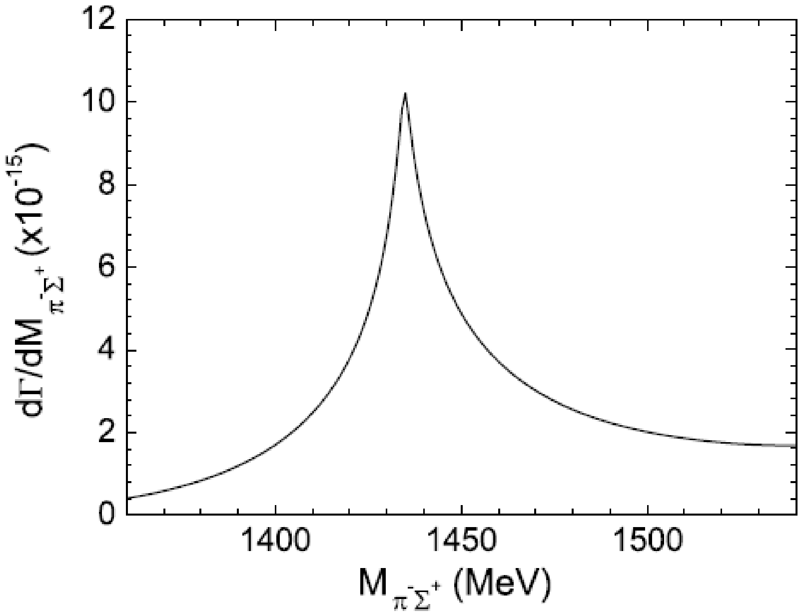


Fig. 6. Invariant $\pi^-\Sigma^+$ mass distribution of $\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+$ decay.

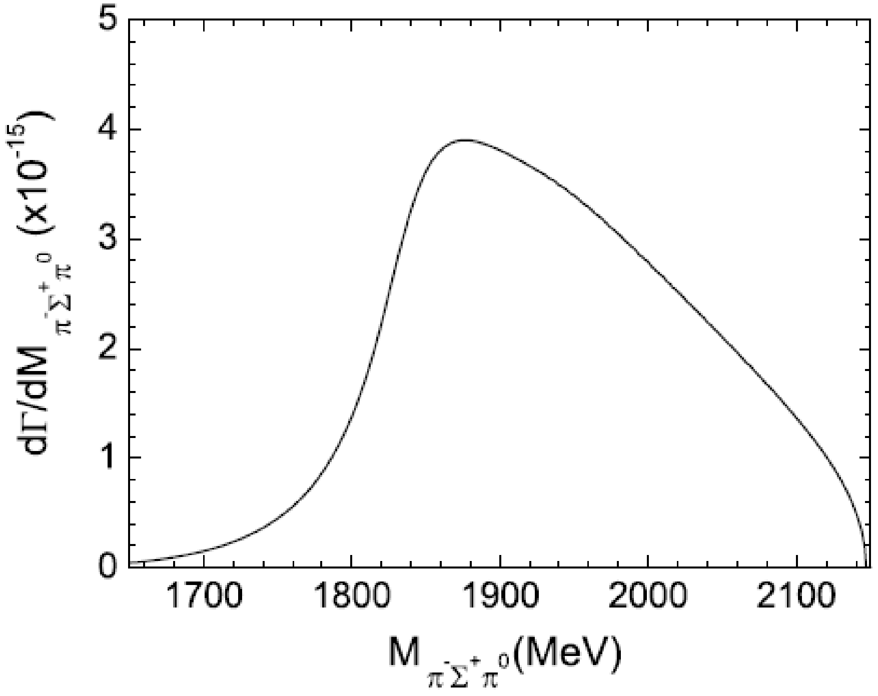


Fig. 5. Invariant $\pi^0\pi^-\Sigma^+$ mass distribution of $\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+$ decay.

$Br(\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+)$ is about $(3 \pm 1) \times 10^{-4}$

Conclusions:

Triangle singularities show a great potential to enhance suppressed processes

In the present case we showed how the Λ(1405) could be produced in an isospín forbidden mode

Resulting from cancellation of diagrams involving the Kbar N -> πΣ amplitudes, it stresses the nature of this resonance as dynamically generated from the meson baryon interaction.

One signal of this is the narrow shape of the resonance, which would not be justified if the resonance was a genuine state

The triangle singularity also enhances the production of resonances that appear around the singular point. We took advange of this to produce a Σ*(1430) state predicted by the chiral unitary approach, filtering the spin channel and enhancing the production due to the triangle singularity.