Role of the triangle singularity in Λ(1405) production in $\pi^- p \rightarrow K^0 \pi \Sigma$ and $pp \rightarrow pK^+ \pi \Sigma$ reactions

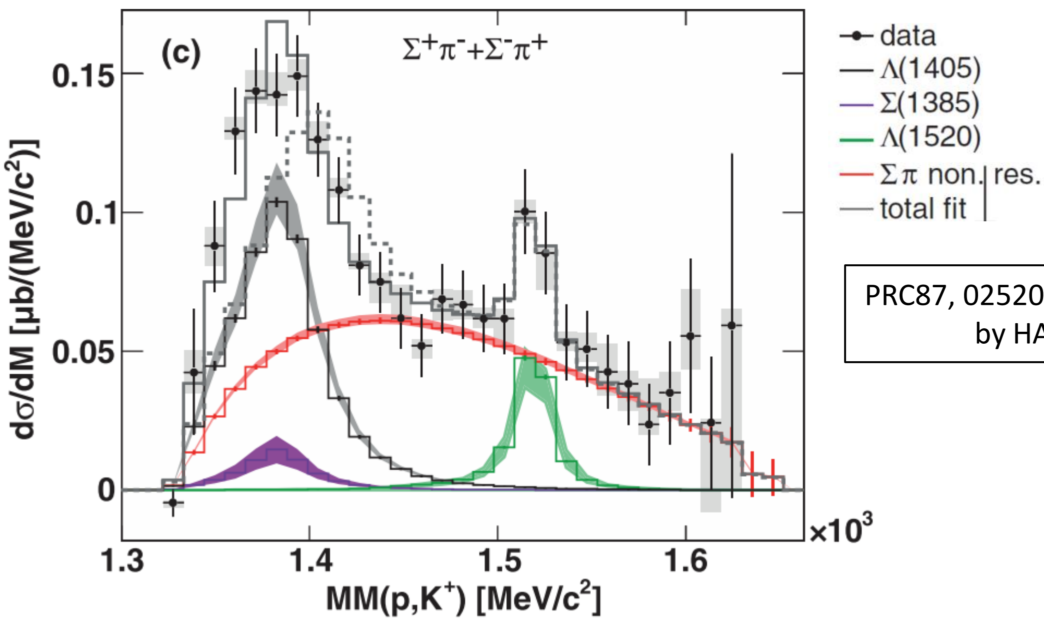
M. Bayar, R. Pavao, **S. Sakai**, and E. Oset

Phys. Rev. C 97, 035203 (2018)

Shuntaro Sakai [Institute of Theoretical Physics, CAS (China)]

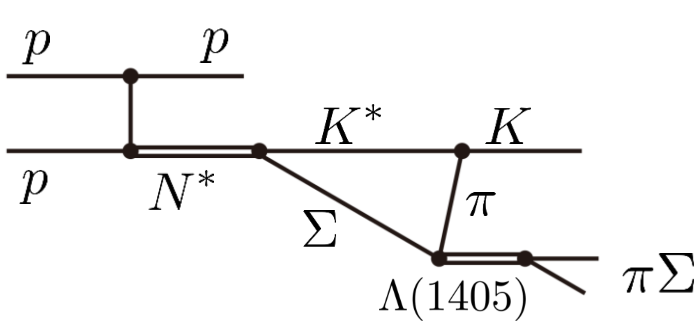
HADES Experiment

$pp \to \Sigma^{\pm} + \pi^{\mp} + K^{+} + p$
with 3.5 GeV kinetic energy beam



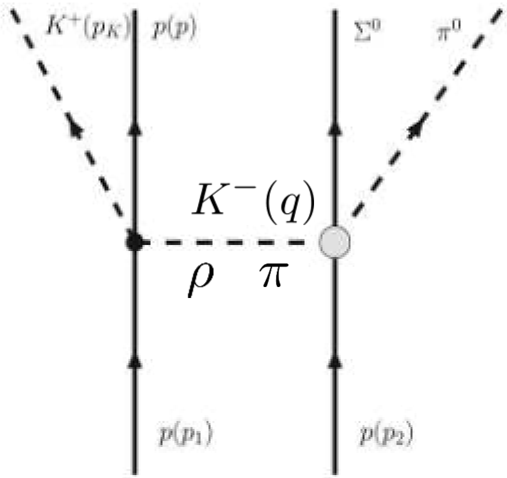
PRC87, 025201 (2013)
by HADES Collab.

Possible description with triangle mechanism

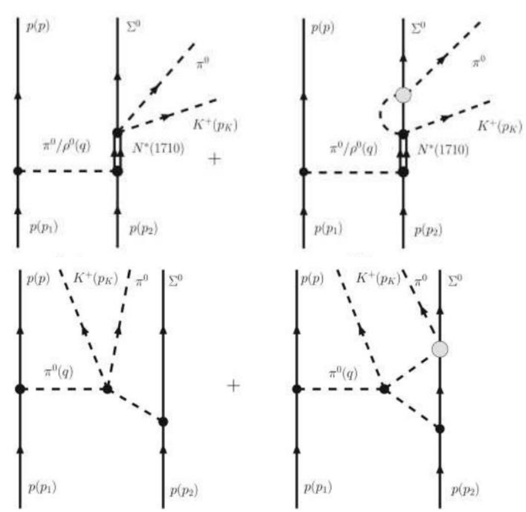


Theoretical study of $pp \to pK^+\pi\Sigma$

Geng-Oset (2007)



■ π and ρ exchange



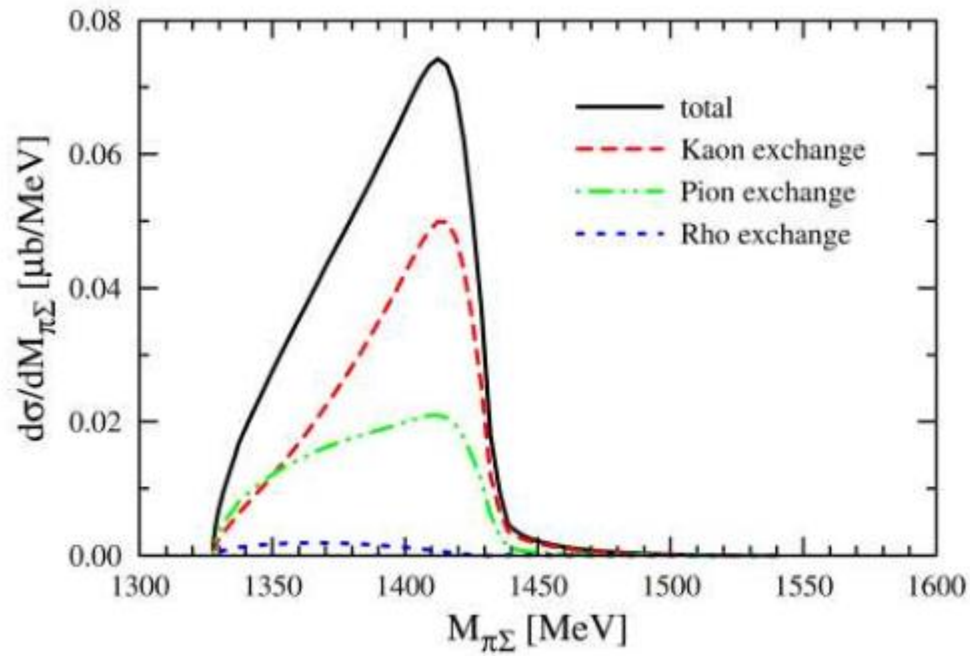
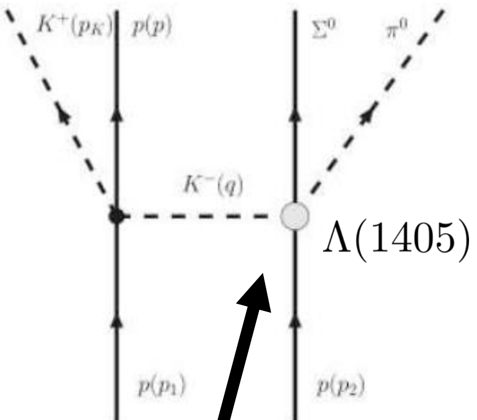


Fig. 6. The contribution of the three different mechanisms with $G_{\rho N N^*} = -0.62$.

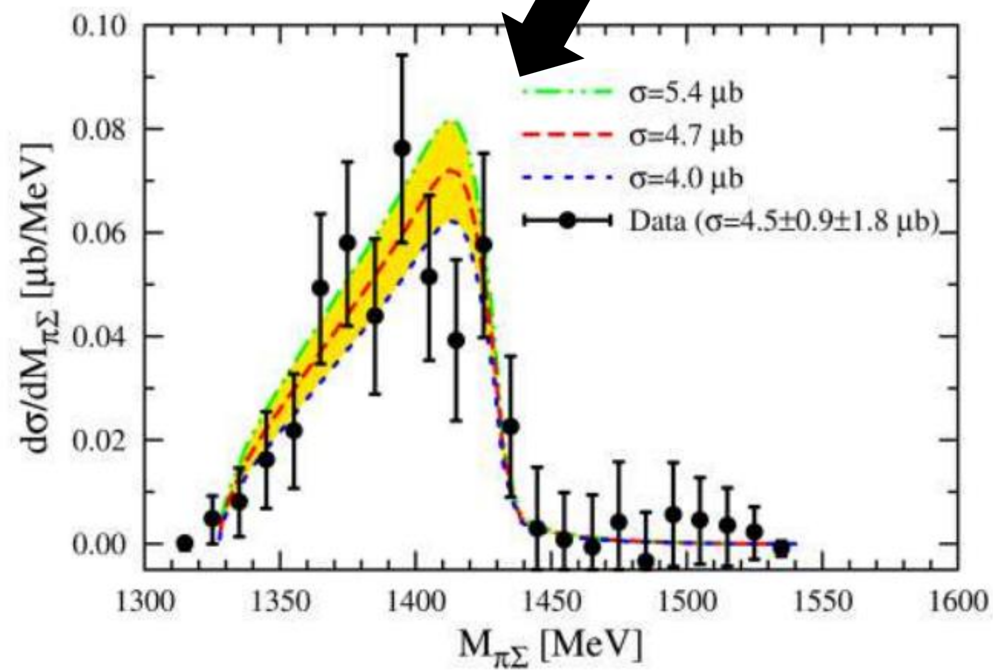
Theoretical study of $pp \to pK^+\pi\Sigma$

Geng-Oset (2007)



$\Lambda(1405)$
in $K^- p \to \pi^0 \Sigma^0$

Theor. Peak @1420MeV

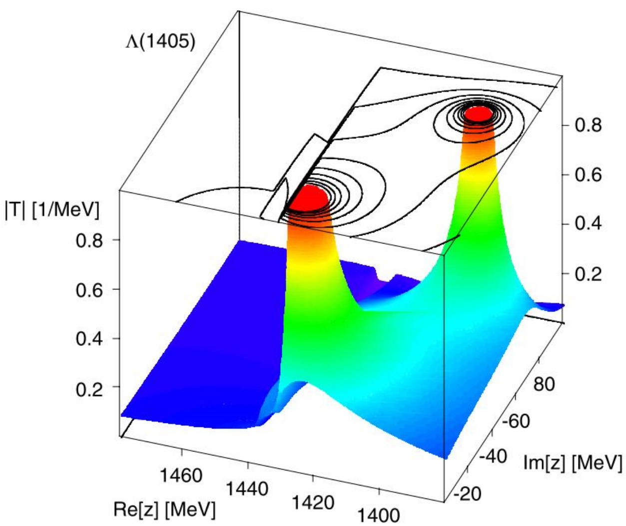


data: COSY-Julich [PLB660, 167 (2008)]

✓ Pole structure of Λ(1405)

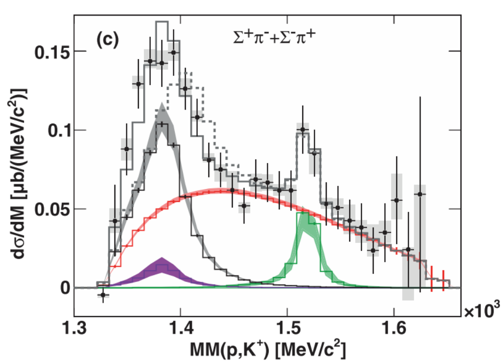
[based on chiral SU(3)]

Oset-Ramos (1998), Oller-Meissner (2001),…

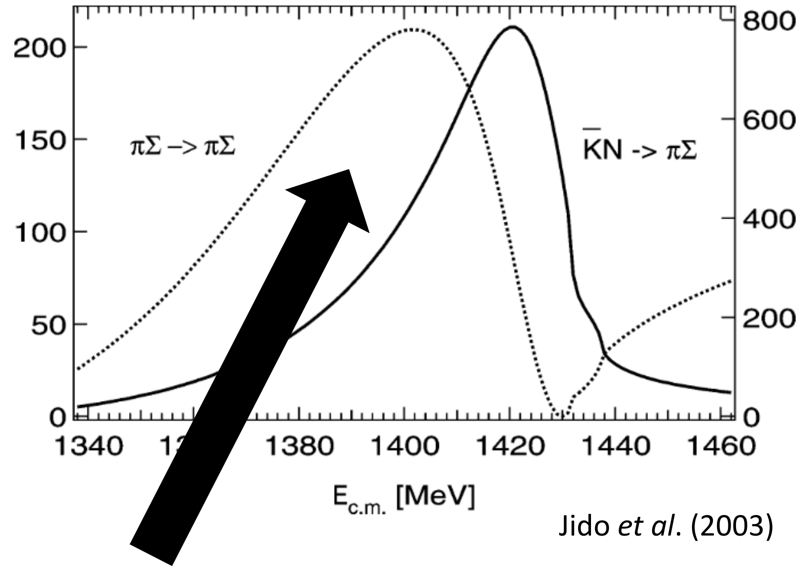


Hyodo-Jido (2012)

$\Lambda(1405)$	
$1390 - 66i$	$1426 - 16i$
$\to \pi\Sigma$	$\to \bar{K}N$

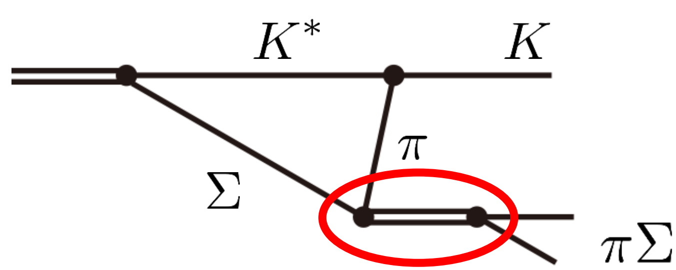


✓ Mass distribution of Λ(1405)➔πΣ



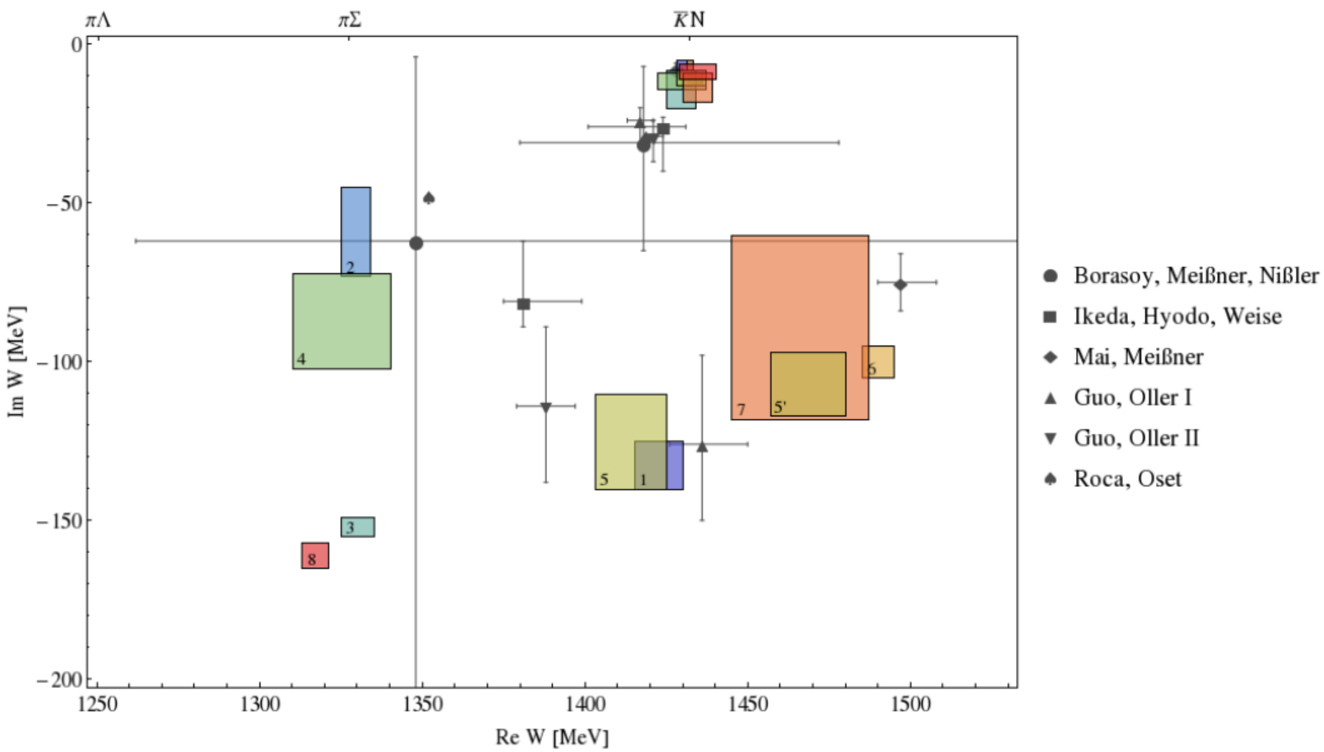
Jido *et al*. (2003)

Large contribution of lower pole for πΣ➔ πΣ peak



$$\pi\Sigma \to \Lambda(1405) \to \pi\Sigma$$

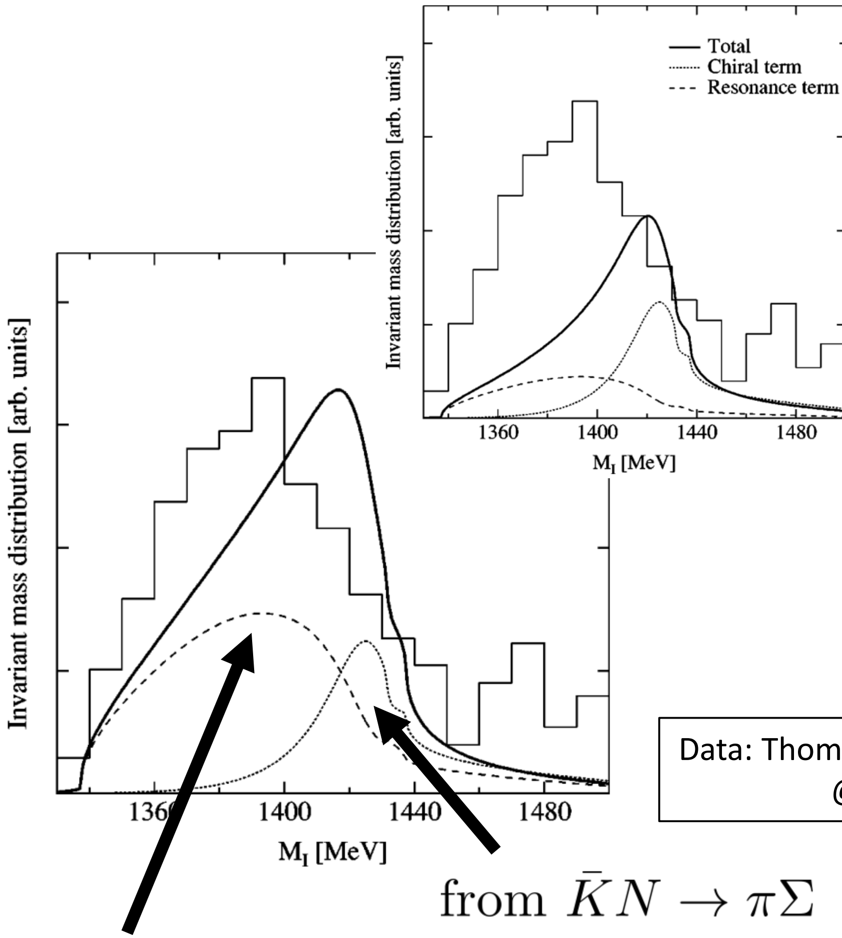
Analyses

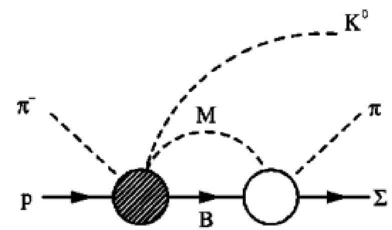


Mai-Meissner (2015)

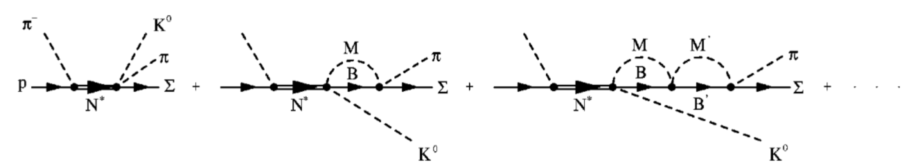
Theoretical study of $\pi^- p \to K^0 \pi\Sigma$

Hyodo *et al.* (2003)





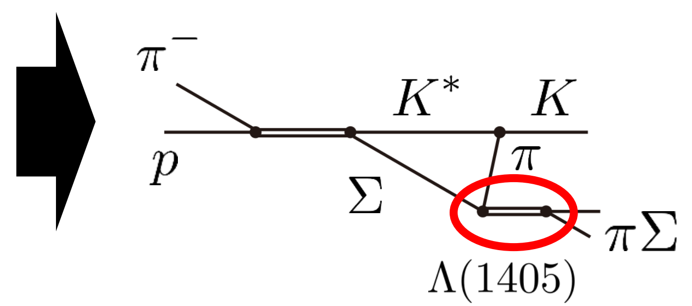
with



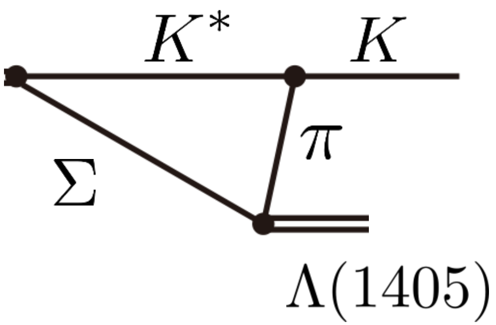
Data: Thomas *et al.*(1973)
@**W~2020 MeV**

from $\bar{K}N \to \pi\Sigma$

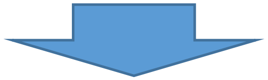
from intermediate $N^*(1710)$
[induce $\pi\Sigma \to \pi\Sigma$]



Triangle mechanism from $K^*\Sigma\pi$ loop



: singularity@2140 MeV in $M_{K\Lambda(1405)}$

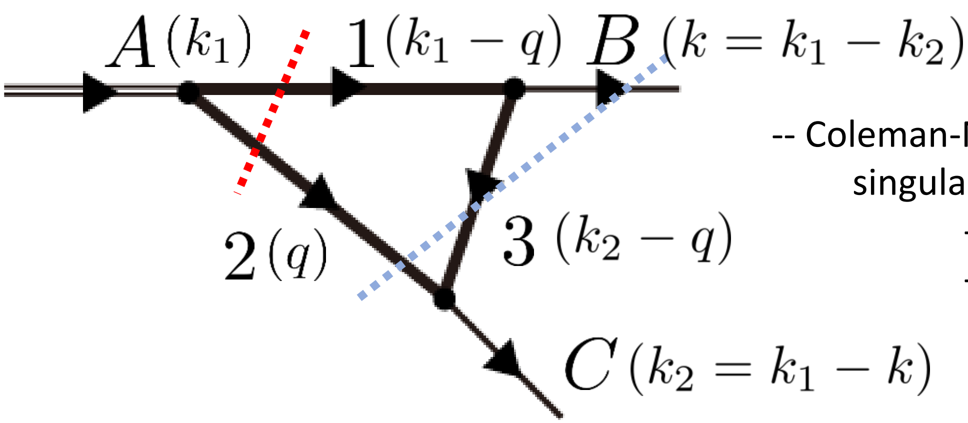


Large contribution in the process

@$M_{K\Lambda(1405)}$=2140MeV

Triangle singularity

Landau (1959),….



-- Coleman-Norton theorem (1965):
singularity~ process occurs classically
- internal particle: onshell
- energy-mom. of each vertex: conserved

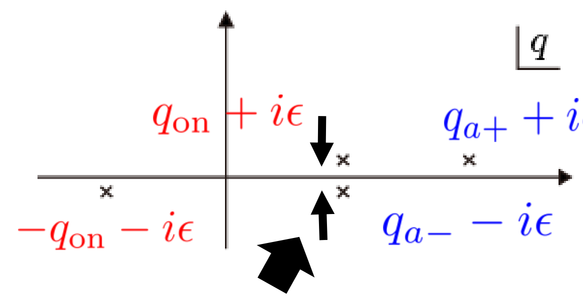
$$i\int\frac{d^4q}{(2\pi)^4}\frac{1}{[q^2-m_2^2+i\epsilon][(k_1+q)^2-m_1^2+i\epsilon][(k_2+q)^2-m_3^2+i\epsilon]}$$

$$\sim\int dq\,d\cos\theta\,\frac{1}{[E_A-\omega_1(q)-\omega_2(q)+i\epsilon][E_{23}-\omega_2(q)-\omega_3(\vec{q}+\vec{k})+i\epsilon]}$$

$q=\pm q_{\rm on}\pm i\epsilon$

$q=q_{a\pm}\pm i\epsilon$ with $\cos\theta=-1$

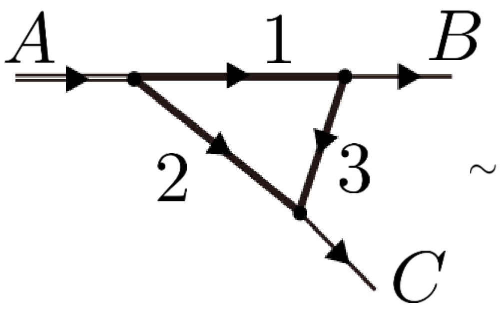
$[\omega_i(\vec{q})\ \sqrt{m_i^2+|\vec{q}|^2}]$
$E_{23}=E_A-E_B$



Pinching
$@q_{\rm on}=q_{a-}$

Singularity

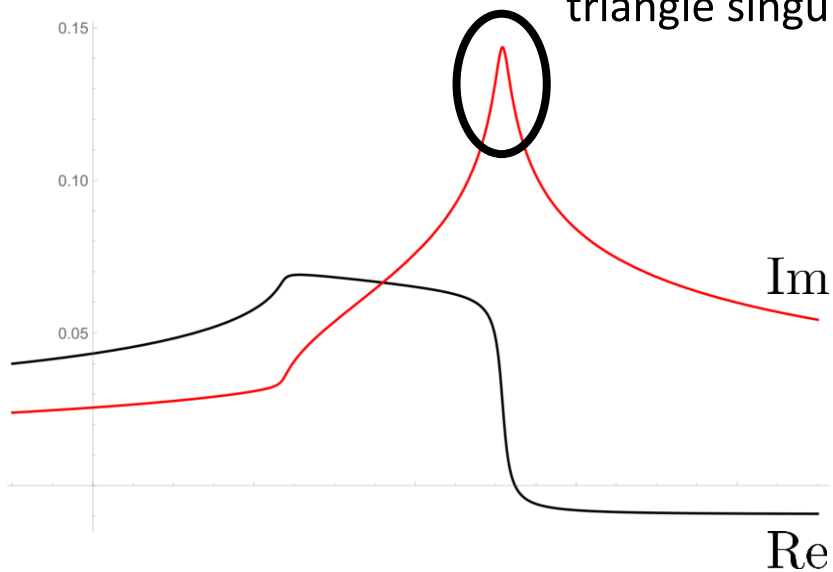
[Bayar *et al*. (2016)]



$$\sim i\int\frac{d^4q}{(2\pi)^4}\frac{1}{[q^2-m_1^2+i\epsilon][(k_1+q)^2-m_2^2+i\epsilon][(k_2+q)^2-m_3^2+i\epsilon]}$$

triangle singularity ⇐ $q_{\rm on}=q_{a-}$

-- determined by masses of particles



-- finite width of particles: smear the singularity

-- momentum or angle dependence of vertices: modify the structure

⇒ **<u>Necessity of phenomenological study of triangle mechanism</u>**

Role of triangle mechanism

-- Enhancement of Isospin-violating $\pi^0 f_0(980)$ production in $\eta(1405/1475)$ decay

>> Wu *et al*. (2012,2013), Aceti *et al*. (2012),...

-- "a1(1420)" peak in p-wave $\pi^- f_0(980)$

>> Mikhasenko-Ketzer-Sarantsev (2015), Aceti-Dai-Oset (2016),...

-- $\gamma p \rightarrow K\Lambda(1405)$ process

>> Wang *et al*. (2017)

-- Near-threshold $\pi^0 N(1535)$ photoproduction

>> Debastiani-Sakai-Oset (2017)

-- P_c peak in $J/\psi N$ distribution by LHCb

>> Mikhasenko (2015), Guo *et al*. (2015),...

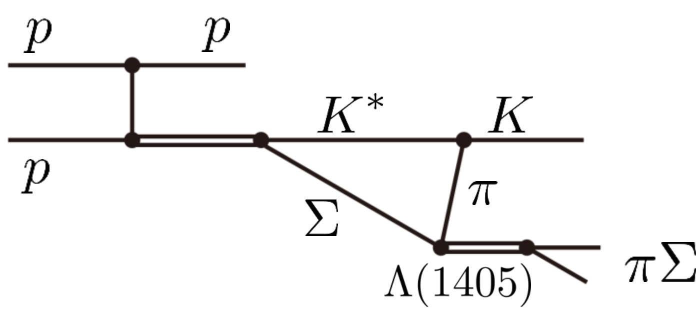
-- B_c or B decays

>> Liu-Meissner (2017), Nakamura (2019),...

-- and so on…

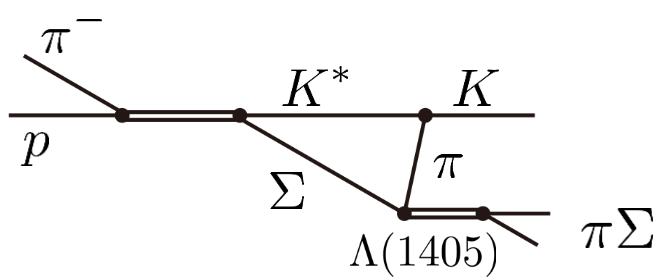
This work

- $pp \to pK^+\pi\Sigma$



No exp. data ~2.14 GeV (πN ene.)

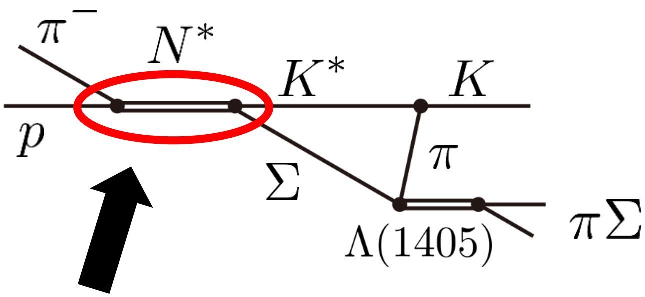
- $\pi^- p \to K^0\pi\Sigma$

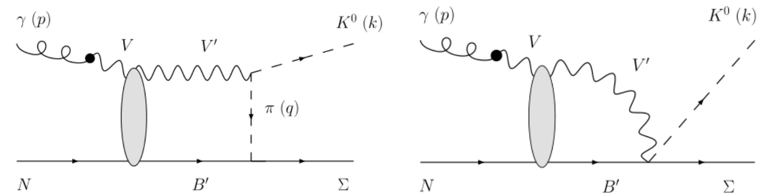


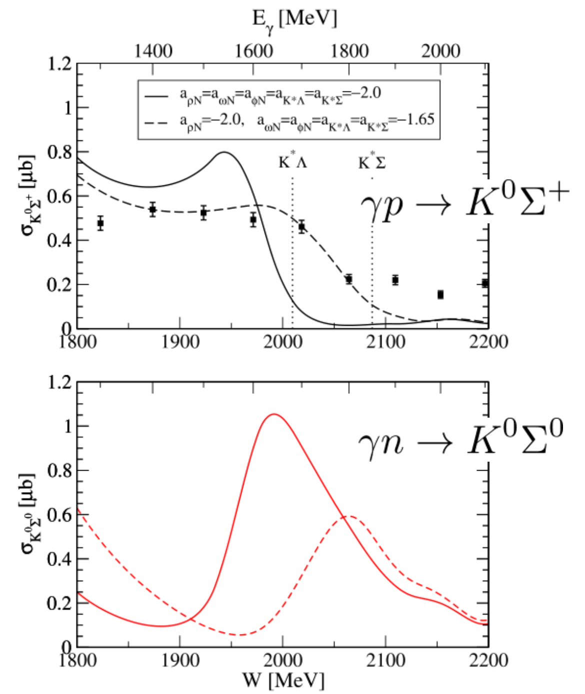
Formalism

- $\pi^- p \to K^0 \pi \Sigma$

✓ $\pi N \to K^* \Sigma$







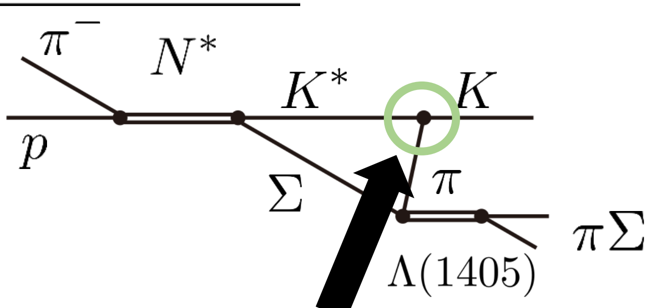
K*Σ: dominantly produced by N*(2030) in s wave

-- large coupling: Oset-Ramos(2010,2013)←from γN→KΣ

-- used in the γp→KΛ(1405) study (Wang *et al*. (2017))

※ No information of πN-N* coupling:
estimation with typical value of N*→πN width (~70MeV)

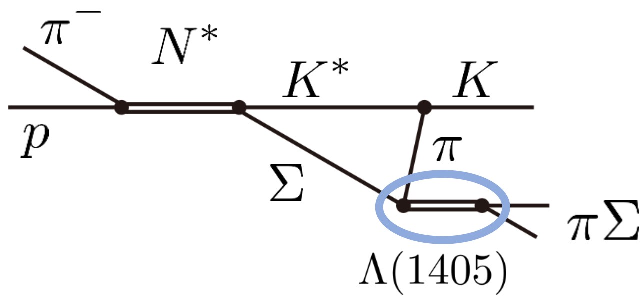
✓ $K^* \to \pi K$



K*πK: effective Lagrangian

$$\mathcal{L} = -ig\,\mathrm{tr}\left([P, \partial^\mu P] V_\mu\right)$$

✓ $\pi\Sigma \to \Lambda(1405) \to \pi\Sigma$



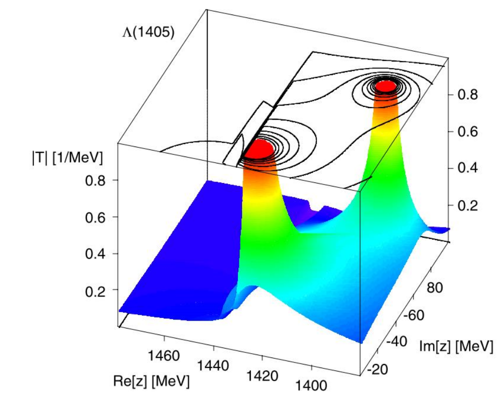
-- Chiral unitary approach

$$t_{ij} = ([1 - vg]^{-1} v)_{ij}$$ ← elastic unitarity

$(i, j : \pi\Sigma, \bar{K}N, \eta\Lambda, \pi\Lambda, \eta\Sigma, K\Xi)$

$v :$ interaction kernel

$\leftarrow$ from LO SU(3) chiral Lagrangian

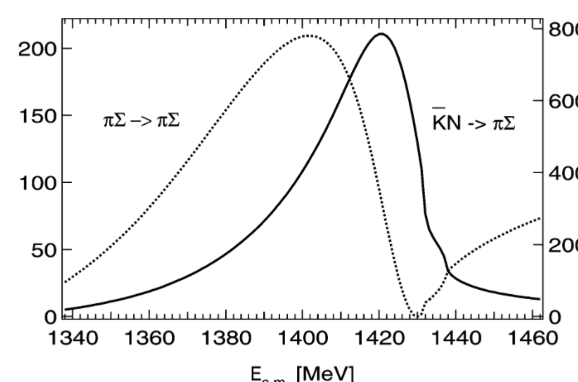


Hyodo-Jido (2012)

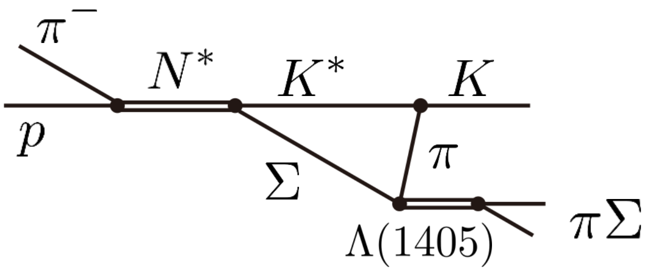
Two poles:

Higher pole(~1420MeV):
large coupling to $\bar{K}N$

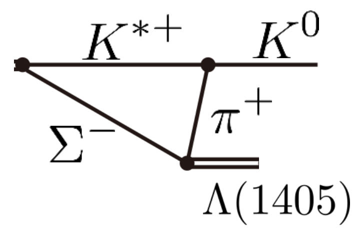
Lower pole(~1390MeV):
large coupling to $\pi\Sigma$

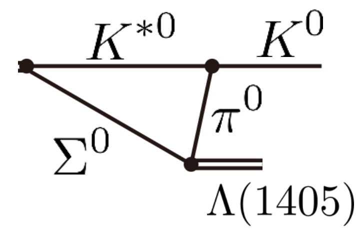


Jido *et al*. (2003)



$$t = -\frac{2M_\Sigma}{3\sqrt{3}} \frac{g g^{I=1/2}_{\pi N, N^*} g^{I=1/2}_{N^*, K^*\Sigma}}{\sqrt{s} - M_{N^*} + i\Gamma_{N^*}/2} \vec{\sigma} \cdot \vec{k}\, t_T \left(t_{\Sigma^-\pi^+, \Sigma\pi} + \frac{1}{2} t_{\Sigma^0\pi^0, \Sigma\pi} \right)$$





t_T : triangle loop

$$t_T = i \int \frac{d^4 q}{(2\pi)^4} \left(2 + \frac{\vec{k} \cdot \vec{q}}{k^2} \right) \frac{1}{[q^2 - m_\Sigma^2 + i\epsilon][(P-q)^2 - m_{K^*}^2 + i\epsilon][(P-q-k)^2 - m_\pi^2 + i\epsilon]}$$

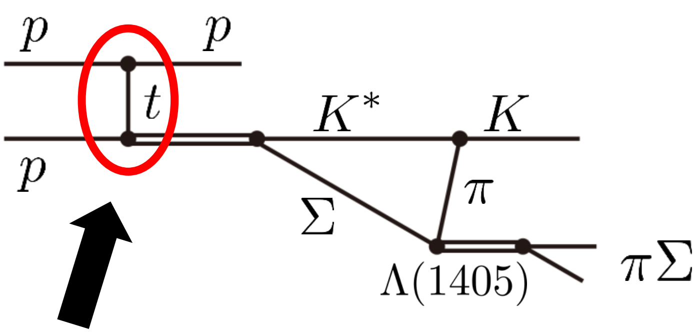
$$\sim \int \frac{d^3 q}{(2\pi)^3} \frac{\left(2 + \frac{\vec{k}\cdot\vec{q}}{k^2}\right)}{8\omega\omega'\omega^*} \frac{2P^0\omega + 2k^0\omega' - 2[\omega + \omega'][\omega + \omega' + \omega^*]}{[k^0 - \omega' - \omega^* + i\epsilon][P^0 + \omega + \omega' - k^0][P^0 - \omega - \omega' - k^0 + i\epsilon][P^0 - \omega^* - \omega + i\epsilon]}$$

$$\frac{d\sigma_{K^0\pi\Sigma}}{dm_{\pi\Sigma}} = \frac{M_p M_\Sigma |\vec{k}||\vec{\tilde{p}}_\pi|}{2(2\pi)^3 s |\vec{p}_\pi|} \overline{\sum} \sum |t_{\pi^- p \to K^0 \pi\Sigma}|^2$$

Formalism

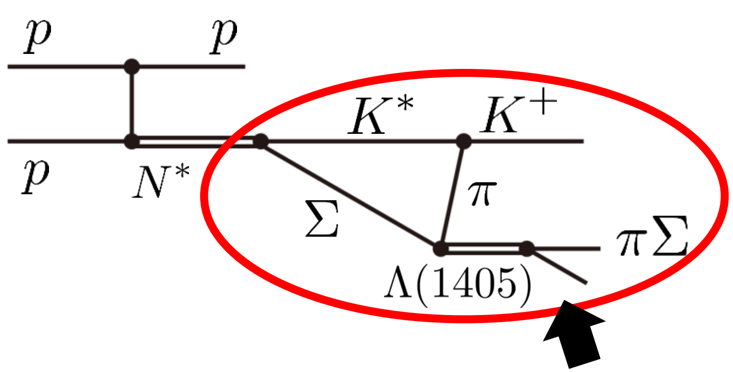
- $pp \to pK^+\pi\Sigma$

✓ $pp \to pN^*$



C : t $(\cos\theta)$ dependence

← averaged

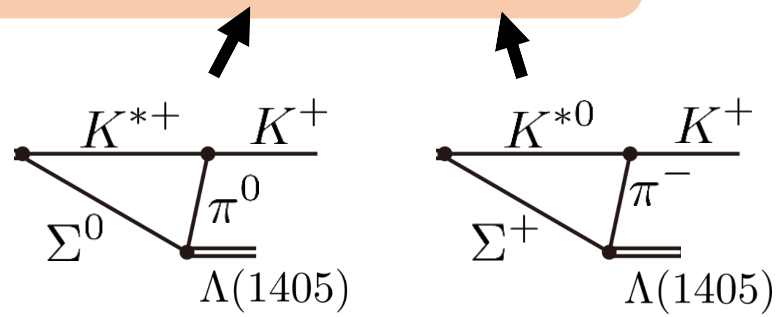


Same as πN→KΛ(1405)

K*Σ: large coupling to N*: Oset-Ramos(2010)

$pp \to K\Lambda(1405)$ amplitude

$$t = -\frac{2M_\Sigma}{3\sqrt{3}} \frac{C g g^{I=1/2}_{N^*,K^*\Sigma}}{M_{\rm inv} - M_{N^*} + i\Gamma_{N^*}/2} \vec{\sigma}\cdot\vec{k}\, t_T \left(t_{\Sigma^-\pi^+,\Sigma\pi} + \frac{1}{2} t_{\Sigma^0\pi^0,\Sigma\pi} \right)$$



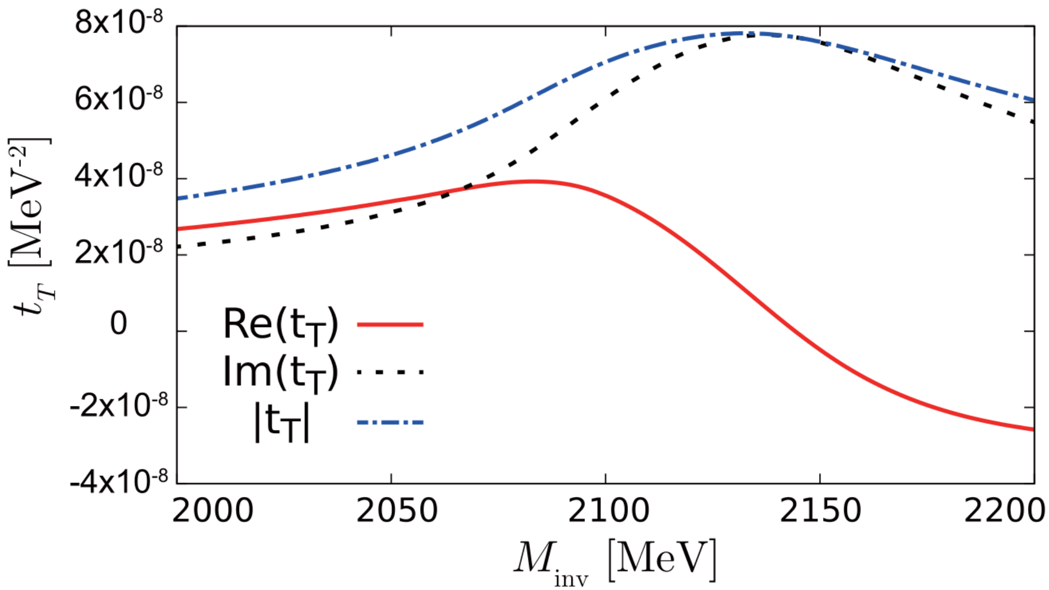
$$\frac{d^2\sigma}{dM_{K\Lambda^*}\, dm_{\pi\Sigma}} = \frac{C p_1 p_a}{|M_{\rm inv} - M_{N^*} + i\Gamma_{N^*}/2|^2} \times |\tilde{p}_\pi| k^3 |t_T|^2 |t_{\Sigma^+\pi^-,\Sigma\pi} + \frac{1}{2} t_{\Sigma^0\pi^0,\Sigma\pi}|^2$$

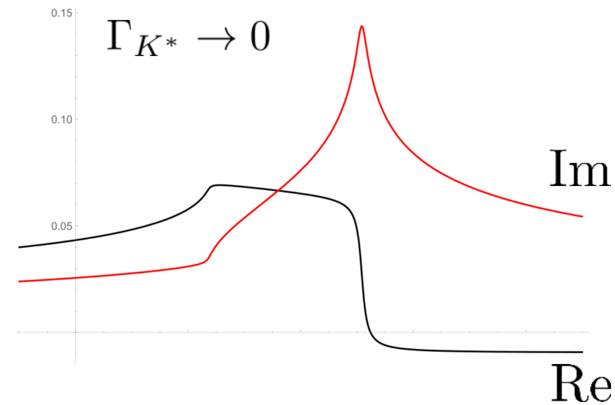
○ pp energy: coincide with experiment

Results

Triangle amplitude

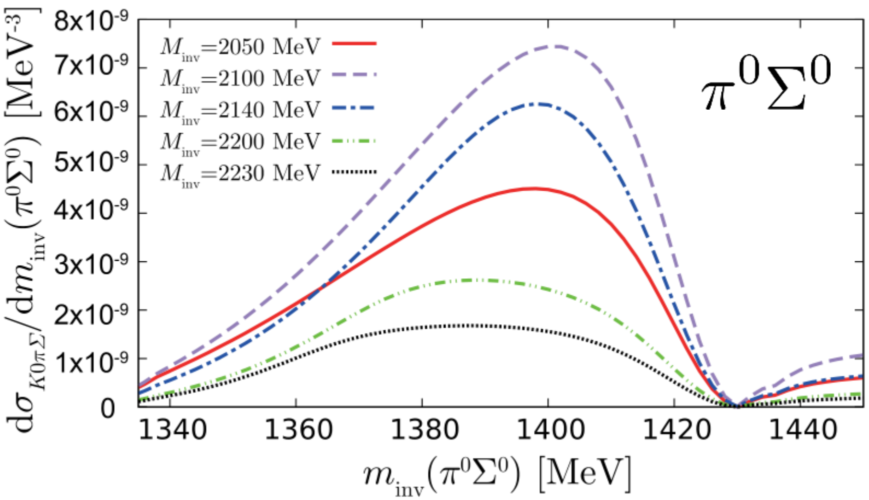
$$t_T = \int \frac{d^3 q}{(2\pi)^3} \frac{\left(2 + \frac{\vec{k}\cdot\vec{q}}{k^2}\right)}{8\omega\omega'\omega^*} \frac{2P^0\omega + 2k^0\omega' - 2[\omega+\omega'][\omega+\omega'+\omega^*]}{[k^0-\omega'-\omega^*+i\epsilon][P^0+\omega+\omega'-k^0][P^0-\omega-\omega'-k^0+i\epsilon][P^0-\omega^*-\omega+i\epsilon]}$$

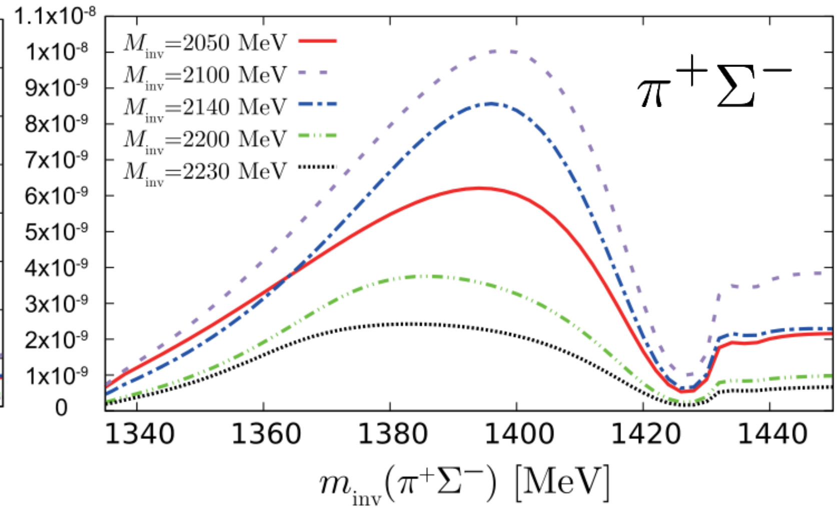


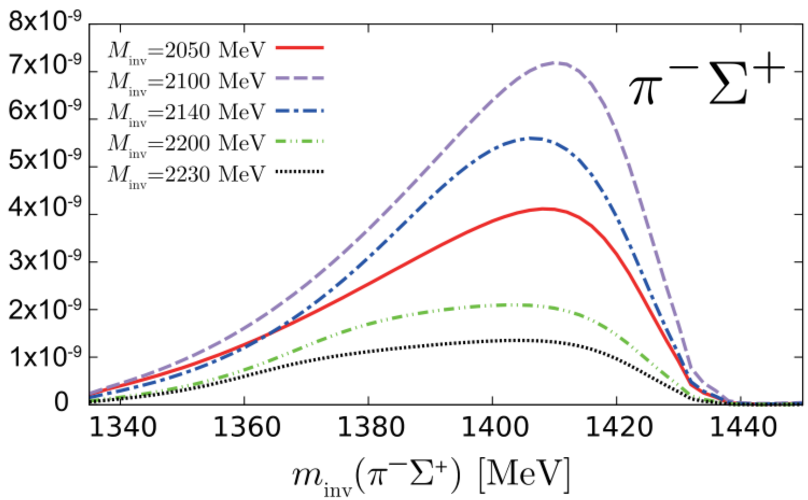


Peak @2140MeV
from triangle singularity

$$\frac{d\sigma_{\pi^- p \to K^0 \pi \Sigma}}{dm_{\pi\Sigma}}$$







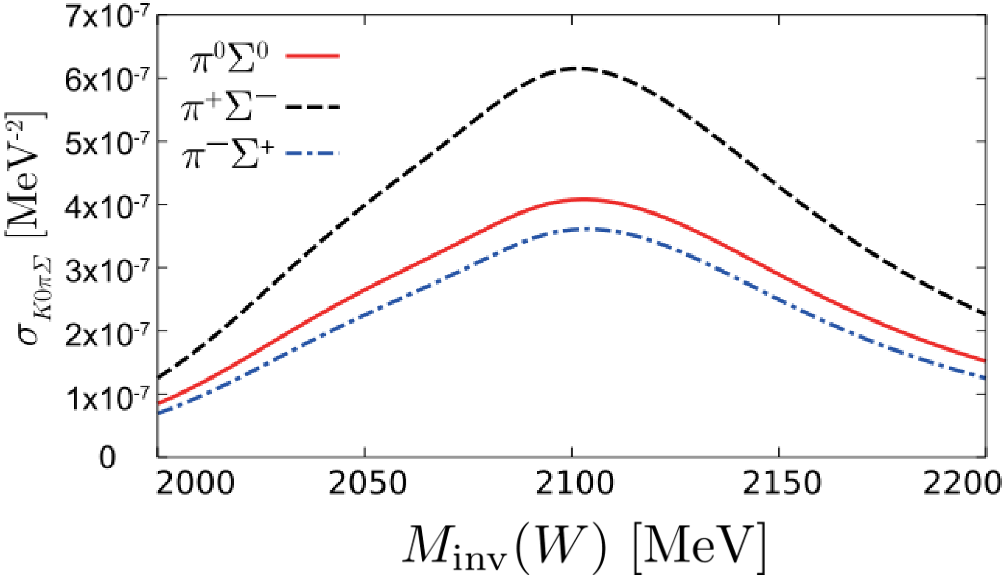
Peaks around @m_{inv}~1380-1410MeV

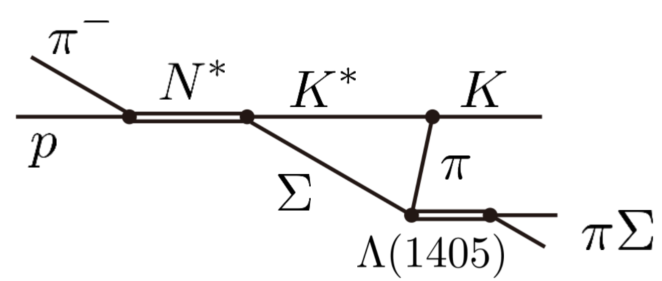
-- Role of lower pole of Λ(1405)

-- Peak position: lower than 1420MeV

Maximum @W~2100MeV

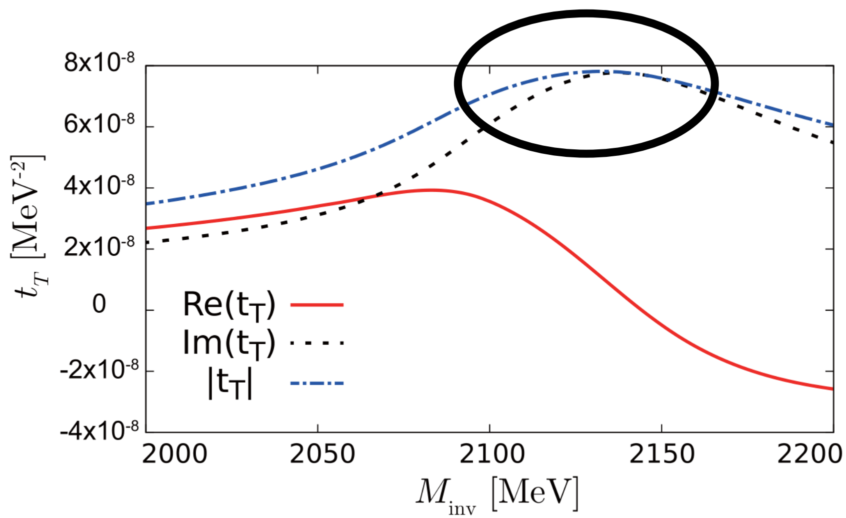
$$\sigma_{\pi^- p \to K^0 \pi \Sigma}(W)$$



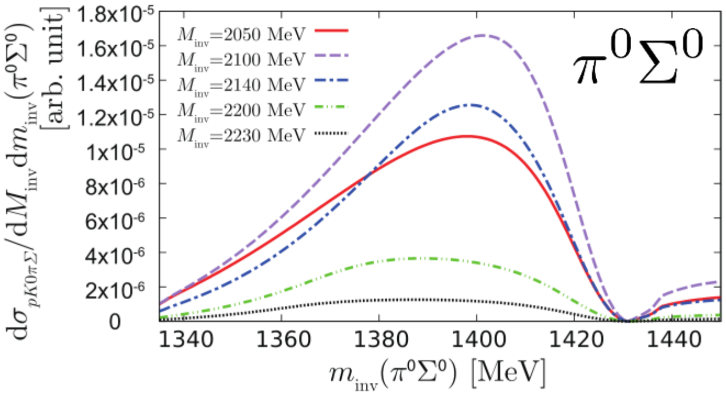


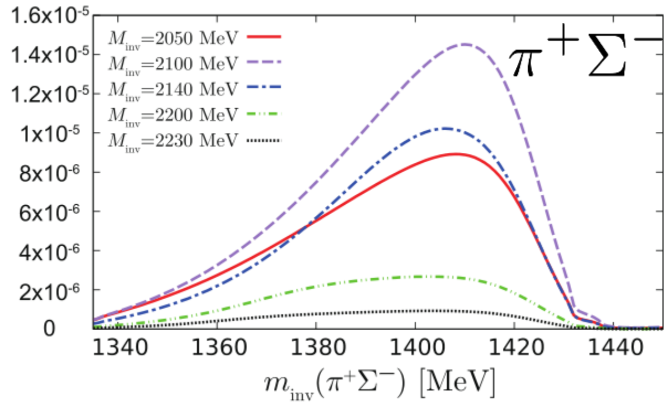
■ Peak @2100MeV
from triangle mechanism

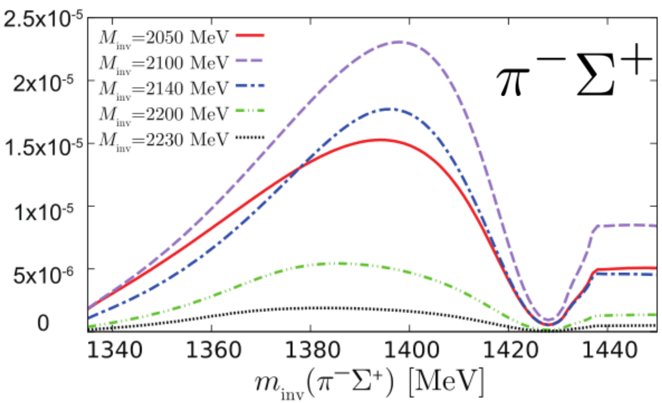
[lowered by the resonance amplitude
@2030MeV]

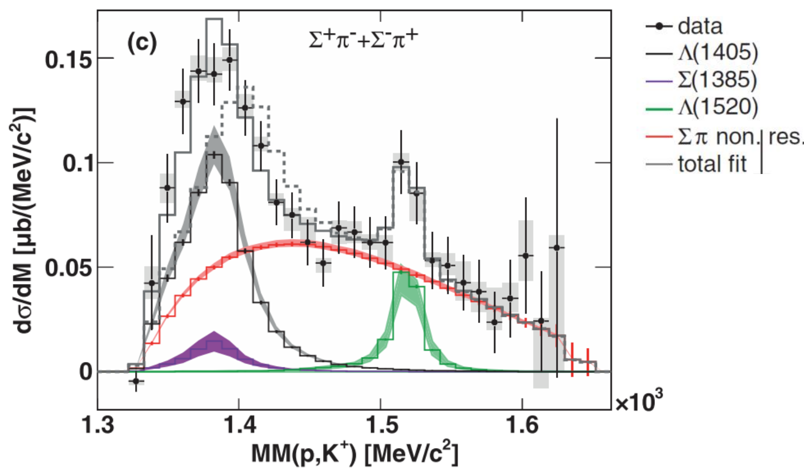


$$\frac{d^2\sigma_{pp\to pK^+\pi\Sigma}}{dM_{K\Lambda^*}\,dm_{\pi\Sigma}}$$









Peaks around @m_{inv}~1380-1410MeV

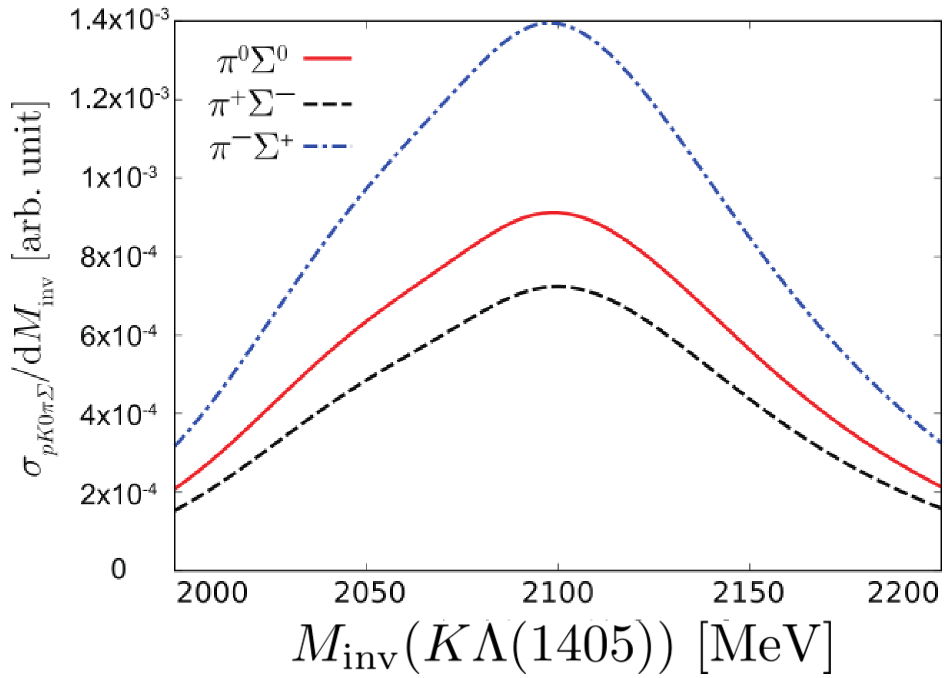
-- Role of lower pole of Λ(1405)

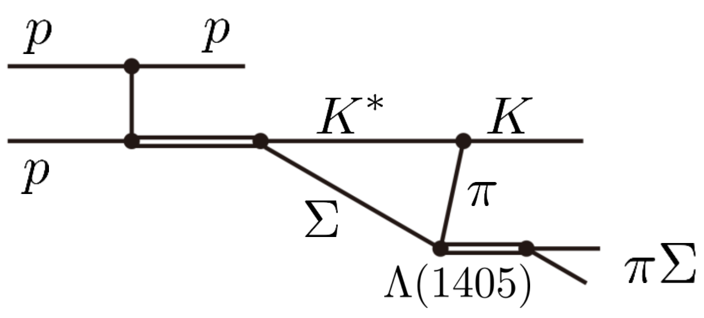
-- Peak position: lower than 1420MeV

Maximum @$M_{K\Lambda^*}$~2100MeV

-- Role of triangle mechanism and N* resonance

$$\frac{d\sigma_{pp\to pK^+\pi\Sigma}}{dM_{\rm inv}(K\Lambda(1405))}$$





- Peak @2100MeV
  from triangle mechanism

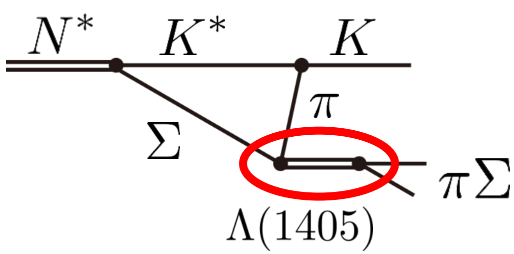
[lowered by the resonance amplitude @2030MeV]

Summary

$$\begin{cases} \text{-- } \pi^- p \to K^0 \pi\Sigma \\ \text{-- } pp \to pK^+ \pi\Sigma \end{cases}$$

with triangle mechanism

○ $K^*\Sigma\pi$ loop: peak @2140MeV



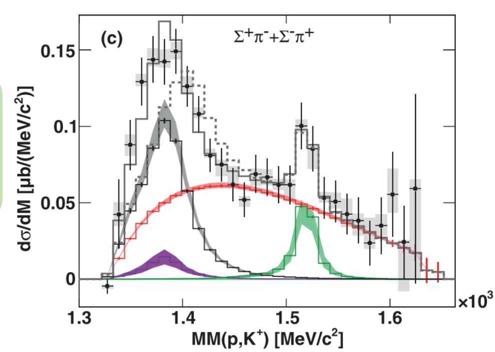
⇦ $\pi\Sigma \to \pi\Sigma$ amplitude

: <u>larger weight of **lower Λ(1405) pole**</u>

⬇

$$\frac{d\sigma_{\pi^- p \to K^0 \pi\Sigma}}{dm_{\pi\Sigma}} \text{ and } \frac{d^2\sigma_{pp \to pK^+\pi\Sigma}}{dM_{K\Lambda^*}\, dm_{\pi\Sigma}}$$: peak @1380-1410MeV

-- explain experimental peak position



<u>No exp. data @W=2140MeV</u> ↗

$$\sigma_{\pi^- p \to K^0 \pi\Sigma} \text{ and } \frac{d\sigma_{pp \to pK^+\pi\Sigma}}{dM_{K\Lambda^*}}$$: peak @2100MeV

🡐 from triangle mechanism and resonance amplitude